Modified Noor iterative procedure for uniformly continuous mappings in Banach spaces.

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Abstract. In this paper, a strong convergence theorem is obtained for three uniformly continuous mappings in real Banach spaces. Our results extend, improve and generalize the recent results of Chang et al.(2009) among others.

Resumen. En este trabajo, se obtiene un teorema de la convergencia fuerte para tres funciones uniformemente continuas en espacios de Banach reales. Nuestro resultado amplia, mejora y generaliza los resultados recientes de Chang et al. (2009), entre otros.

1   Introduction

Let E be an arbitrary real Banach Space and let $J : E \to 2^{E^*}$ be the normalized duality mapping defined by

$$J(x) = \{f \in E^* :< x, f >= \|x\|^2 = \|f\|^2\}, \forall x \in E$$

where E^* denotes the dual space of E and $< .,. >$ denotes the generalized duality pairing between E and E^*. The single-valued normalized duality mapping is denoted by j. Let $y \in E$ and $j(y) \in J(y)$; note that $< ., j(y) >$ is a Lipschitzian map.
Let K be a nonempty closed convex subset of E and $T : K \to K$ be a map.
The mapping T is said to be uniformly L- Lipschitzian if there exists a constant $L > 0$ such that

$$\|T^n x - T^n y\| \le L\|x - y\|$$

for any $x, y \in K$ and $\forall n \ge 1$.
The mapping T is said to be asymptotically pseudocontractive if there exists a

sequence $(k_n) \subset [1, \infty)$ with $lim_{n\to\infty} k_n = 1$ and for any $x, y \in K$ there exists $j(x - y) \in J(x - y)$ such that

$$< T^n x - T^n y, j(x - y) > \leq k_n \|x - y\|^2, \forall n \geq 1.$$

The concept of asymptotically pseudocontractive mappings was introduced by Schu[14].
Recently, Chang et al.[4] pointed out some gaps in the proofs of result in [12] and then proved a strong convergence theorem for a pair of L-Lipschitzian mappings instead of a single map used in [12]. In fact,they proved the following theorem :

Theorem 1.1 ([4]). Let E be a real Banach space, K be a nonempty closed convex subset of E, $T_i : K \to K$,$(i = 1, 2)$ be two uniformly L_i-Lipschitzian mappings with $F(T_1) \cap F(T_2) \neq \phi$, where $F(T_i)$ is the set of fixed points of T_i in K and ρ be a point in $F(T_1) \cap F(T_2)$. Let $k_n \subset [1, \infty)$ be a sequence with $k_n \to 1$. Let $\{x_n\}$ and $\{\beta_n\}$ be two sequences in $[0, 1]$ satisfying the following conditions:
(i)$\sum_{n=1}^{\infty} \alpha_n = \infty$ (ii)$\sum_{n=1}^{\infty} \alpha_n^2 < \infty$ (iii)$\sum_{n=1}^{\infty} \beta_n < \infty$
(iv) $\sum_{n=1}^{\infty} \alpha_n(k_n - 1) < \infty$.
For any $x_0 \in K$, let $\{x_n\}$ be the iterative sequence defined by

$$x_{n+1} = (1 - \alpha_n)x_n + \alpha_n T_1^n y_n$$

$$y_n = (1 - \beta_n)x_n + \beta_n T_2^n x_n.$$

If there exists a strictly increasing function $\varphi : [0, \infty) \to [0, \infty)$ with $\varphi(0) = 0$ such that

$$< T_1^n x_n - \rho, j(x_n - \rho) > \leq k_n \|x_n - \rho\|^2 - \varphi(\|x_n - \rho\|)$$

for all $j(x - \rho) \in J(x - \rho)$ and $x \in K$, (i=1,2), then $\{x_n\}$ converges strongly to ρ.
The result above extends and improves the corresponding results of [12] from one uniformly Lipschitzian asymptotically pseudocontractive mapping to two uniformly Lipschitzian asymptotically pseudocontractive mappings. In fact, if the iteration parameter $\{\beta_n\}$ in Theorem 1.1 above is equal to zero for all n and $T_1 = T_2 = T$ then we have the main result of Ofoedu [12].
Within the past 5years or so, considerable research efforts have been devoted to developing iterative methods for approximating the common fixed points (assuming existence) for families of two or more maps for several classes of non-linear mappings (see,[4] and [13]).
Rafiq [13], introduced a new type of iteration- the modified three-step iteration process, to approximate the common fixed point of three strongly pseudocontractive mappings in a real Banach space. It is defined as follows:

Let $T_1, T_2, T_3 : K \to K$ be three mappings. For any given $x_0 \in K$, the modified Noor iteration $\{x_n\}_{n=0}^{\infty} \subset K$ is defined by

$$x_{n+1} = (1 - \alpha_n)x_n + \alpha_n T_1 y_n$$

$$y_n = (1 - \beta_n)x_n + \beta_n T_2 z_n$$

$$z_n = (1 - \gamma_n)x_n + \gamma_n T_3 x_n, \quad \geq 0 \qquad (1.1)$$

where $\{\alpha_n\}_{n=0}^{\infty}, \{\beta_n\}_{n=0}^{\infty}$ and $\{\gamma_n\}_{n=0}^{\infty}$ are three real sequences satisfying (i) $a_n, b_n \to 0$ as $n \to \infty$ and (ii)$\sum_{n=0}^{\infty} a_n = \infty$. It is clear that the iteration scheme (1.1) includes iterations defined in the theorems of Ofoedu[12] as special cases.

In fact, he proved the following theorem:

Theorem 1.2 ([13]). Let X be a real Banach space and K be a nonempty closed convex subset of X. Let T_1, T_2, T_3 be strongly pseudocontractive self maps of K with $T_1(K)$ bounded and T_1, T_3 be uniformly continuous. Let $\{x_n\}_{n=0}^{\infty}$ be the sequence defined by (1.1), where $\{a_n\}_{n=0}^{\infty}, \{b_n\}_{n=0}^{\infty}$ and $\{c_n\}_{n=0}^{\infty}$ are the three real sequences in [0,1] satisfying the conditions,

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} b_n = \lim_{n\to\infty} c_n = 0, \sum_{n=0}^{\infty} a_n = \infty.$$

If $F(T_1) \cap F(T_2) \cap F(T_3) \neq \phi$, then the sequence $\{x_n\}_{n=0}^{\infty}$ converges strongly to the common fixed point of T_1, T_2, T_3.

The purpose of this paper is to extend and improve the recent results of Chang et al.[4] which in turn is a correction, improvement and generalization of results in Ofoedu[12]. We remove the conditions $\sum_{n=1}^{\infty} \alpha_n^2 < \infty$ and $\sum_{n=1}^{\infty} \alpha_n(k_n - 1) < \infty$ from Theorem 1.1 and, replace them with a weaker condition $\alpha_n, \beta_n \to 0$ as $n \to \infty$. We equally extend their pair of maps to three maps. Furthermore, we use a more general iteration procedure. Also, the L- Lipschitzian assumption imposed on T_i in Theorem 1.1 is replaced by more general uniformly continuous mappings. Our method is different from [4].

In order to obtain the main results, the following lemmas are needed.

Lemma 1.1[2].Let E be real Banach Space and $J : E \to 2^{E^*}$ be the normalized duality mapping.Then,for any $x, y \in E$

$$\|x + y\|^2 \leq \|x\|^2 + 2 < y, j(x + y) >, \forall j(x + y) \in J(x + y)$$

Lemma 1.2[8]. Let $\Phi : [0, \infty) \to [0, \infty)$ be an increasing function with $\Phi(x) = 0 \Leftrightarrow x = 0$ and let $\{b_n\}_{n=0}^{\infty}$ be a positive real sequence satisfying

$$\sum_{n=0}^{\infty} b_n = +\infty \quad \text{and} \quad \lim_{n\to\infty} b_n = 0.$$

Suppose that $\{a_n\}_{n=0}^{\infty}$ is a nonnegative real sequence. If there exists an integer $N_0 > 0$ satisfying

$$a_{n+1}^2 < a_n^2 + o(b_n) - b_n \Phi(a_{n+1}), \quad \forall n \geq N_0$$

where $\lim_{n\to\infty} \frac{o(b_n)}{b_n} = 0$, then $\lim_{n\to\infty} a_n = 0$.

2.Main results

Theorem 2.1. Let X be a real Banach space , K a nonempty closed and convex subset of X and $T_1, T_2, T_3 : K \to K$ be uniformly continuous mappings such that $T_1(K)$ is bounded and let suppose that $T_1(K), T_2(K)$ and $T_3(K)$ have only one common fixed point. Let $k_n \subset [1, \infty)$ be a sequence with $k_n \to 1$ and $\{x_n\}$ be a sequence defined by (1.1) where $\{\alpha_n\}_{n=0}^{\infty}, \{\beta_n\}_{n=0}^{\infty}$ and $\{\gamma_n\}_{n=0}^{\infty}$ are three sequences in [0,1] satisfying

(i) $\lim_{n\to\infty} \alpha_n = \lim_{n\to\infty} \beta_n = \lim_{n\to\infty} \gamma_n = 0$
     and

(ii) $\sum_{n=1}^{\infty} \alpha_n = \infty$.

If there exists a strictly increasing function $\varphi : [0, \infty) \to [0, \infty)$ with $\varphi(0) = 0$ such that

$$< T_i^n x_n - \rho, j(x_n - \rho) > \leq k_n \|x_n - \rho\|^2 - \varphi(\|x_n - \rho\|)$$

for all $j(x - \rho) \in J(x - \rho)$ and $x \in K$, (i=1,2,3), then $\{x_n\}$ converges strongly to ρ the unique common fixed point of T_1, T_2, T_3.

Proof. By assumption, we have $F(T_1) \cap F(T_2) \cap F(T_3) = \rho$, say. Let $D_1 = \|x_0 - \rho\| + \sup_{n\geq 0} \|T_1^n y_n - \rho\|$. We prove by induction that $\|x_n - \rho\| \leq D_1$ for all n. It is clear that, $\|x_0 - \rho\| \leq D_1$. Assume that $\|x_n - \rho\| \leq D_1$ holds. We will prove that $\|x_{n+1} - \rho\| \leq D_1$. Indeed, from (1.1), we obtain

$$\begin{aligned}
\|x_{n+1} - \rho\| &\leq \|(1 - \alpha_n)(x_n - \rho) + \alpha_n(T_1^n y_n - \rho)\| \\
&\leq (1 - \alpha_n)\|x_n - \rho\| + \alpha_n\|T_1^n y_n - \rho\| \\
&\leq (1 - \alpha_n)D_1 + \alpha_n D_1 = D_1.
\end{aligned}$$

Hence the sequence $\{x_n\}$ is bounded.
Using the uniformly continuity of T_3, we have $\{T_3^n x_n\}$ is bounded. Denote $D_2 = \max\{D_1, \sup\{\|T_3^n x_n - \rho\|\}\}$, then

$$\begin{aligned}
\|z_n - \rho\| &\leq (1 - \gamma_n)\|x_n - \rho\| + \gamma_n\|T_3^n x_n - \rho\| \\
&\leq (1 - \gamma_n)D_1 + \gamma_n D_2 \\
&\leq (1 - \gamma_n)D_2 + \gamma_n D_2 = D_2
\end{aligned}$$

By the virtue of the uniform continuity of T_2, we get that $\{T_2^n z_n\}$ is bounded. Set $D = \sup_{n\geq 0}\|T_2^n z_n - \rho\| + D_2$. From equation (1.1) we have, in view of Lemma 1.1, that

$$
\begin{aligned}
\|x_{n+1} - \rho\|^2 &= \langle x_{n+1} - \rho, j(x_{n+1} - \rho)\rangle \\
&\leq (1-\alpha_n)\|x_n - \rho\|\|x_{n+1} - \rho\| + \alpha_n\langle T_1^n y_n - \rho, j(x_{n+1} - \rho)\rangle \\
&= (1-\alpha_n)\|x_n - \rho\|\|x_{n+1} - \rho\| \\
&\quad + \alpha_n\langle T_1^n x_{n+1} - \rho, j(x_{n+1} - \rho)\rangle \\
&\quad + \alpha_n\langle T_1^n y_n - T_1^n x_{n+1}, j(x_{n+1} - p)\rangle \\
&\leq (1-\alpha_n)\|x_n - \rho\|\|x_{n+1} - \rho\| + \alpha_n\sigma_n\|x_{n+1} - \rho\| \\
&\quad + \alpha_n(k_n\|x_{n+1} - \rho\|^2 - \Phi(\|x_{n+1} - \rho\|))
\end{aligned}
$$
(2.1)

where $\sigma_n = \|T_1^n y_n - T_1^n x_{n+1}\|$. Observe that

$$
\begin{aligned}
\|x_{n+1} - y_n\| &= \beta_n\|x_n - T_2^n z_n\| + \alpha_n\|x_n - T_1^n y_n\| \\
&\leq \beta_n(\|x_n - \rho\| + \|T_2^n z_n - \rho\|) + \alpha_n(\|x_n - \rho\| + \|T_1^n y_n - \rho\|) \\
&\leq \beta_n(D_1 + D) + \alpha_n(D_1 + D)
\end{aligned}
$$

This implies that $\lim_{n\to\infty}\|x_{n+1} - y_n\| = 0$, since $\lim_{n\to\infty}\alpha_n = 0$, $\lim_{n\to\infty}\beta_n = 0$. Since T_1 is uniformly continuous, we have

$$
\sigma_n = \|T_1^n x_{n+1} - T_1^n y_n\| \to 0, \quad (n \to \infty)
$$
(2.2)

In view of the fact that $(a - 1)^2 \geq 0$, if $a = \|x_{n+1} - \rho\|$ then

$$
\|x_{n+1} - \rho\| \leq \frac{1}{2}(1 + \|x_{n+1} - \rho\|^2).
$$
(2.3)

Substituting (2.3) into (2.1), we obtain

$$
\begin{aligned}
\|x_{n+1} - \rho\|^2 &\leq \tfrac{1}{2}((1-\alpha_n)^2\|x_n - \rho\|^2 + \|x_{n+1} - \rho\|^2) + k_n\alpha_n\|x_{n+1} - \rho\|^2 \\
&\quad - \alpha_n\Phi(\|x_{n+1} - \rho\|) + \alpha_n\sigma_n \cdot \tfrac{1}{2}(1 + \|x_{n+1} - \rho\|^2)
\end{aligned}
$$

$$
(1 - 2k_n\alpha_n - \alpha_n\sigma_n)\|x_{n+1} - \rho\|^2 \leq (1-\alpha_n)^2\|x_n - \rho\|^2 - 2\alpha_n\Phi(\|x_{n+1} - \rho\|) + \alpha_n\sigma_n.
$$
(2.4)

Since $\lim_{n\to\infty} k_n\alpha_n = \lim_{n\to\infty}\alpha_n\sigma_n = 0$, there exists a natural number N_0 such that

$$
\frac{1}{2} < 1 - 2k_n\alpha_n - \alpha_n\sigma_n < 1
$$

for all $n > N_0$. Then, (2.4) implies that

$$
\begin{aligned}
\|x_{n+1} - \rho\|^2 &\leq \frac{(1-\alpha_n)^2}{1-2k_n\alpha_n-\alpha_n\sigma_n}\|x_n - \rho\|^2 - \frac{2\alpha_n}{1-2k_n\alpha_n-\alpha_n\sigma_n}\Phi(\|x_{n+1} - \rho\|) \\
&\quad + \frac{\alpha_n\sigma_n}{1-2k_n\alpha_n-\alpha_n\sigma_n} \\
&\leq \|x_n - \rho\|^2 + \alpha_n\frac{(\alpha_n+\sigma_n-2(1-k_n))}{1-2k_n\alpha_n-\alpha_n\sigma_n}\|x_n - \rho\|^2 \\
&\quad - \frac{2\alpha_n}{1-2k_n\alpha_n-\alpha_n\sigma_n}\Phi(\|x_{n+1} - \rho\|) + \frac{\alpha_n\sigma_n}{1-2k_n\alpha_n-\alpha_n\sigma_n}
\end{aligned}
\tag{2.5}
$$

Since $\|x_n - \rho\| \leq D_1$, it follows from (2.5) that $\forall\ n \geq N_0$,

$$
\begin{aligned}
\|x_{n+1} - \rho\|^2 &\leq \|x_n - \rho\|^2 + 2\alpha_n(\alpha_n + \sigma_n - 2(1 - k_n))D_1^2 \\
&\quad - 2\alpha_n\Phi(\|x_{n+1} - \rho\|) + 2\alpha_n\sigma_n \\
&= \|x_n - \rho\|^2 - 2\alpha_n\Phi(\|x_{n+1} - \rho\|) \\
&\quad + 2\alpha_n((\alpha_n + \sigma_n - 2(1 - k_n))D_1^2 + \sigma_n) \\
&\leq \|x_n - \rho\|^2 - 2\alpha_n\Phi(\|x_{n+1} - \rho\|) \\
&\quad + 2\alpha_n((\alpha_n + \sigma_n - 2(1 - k_n))D_1^2 + \sigma_n), \quad \forall n \geq N_0
\end{aligned}
\tag{2.6}
$$

Taking $b_n = 2\alpha_n$ and observing that

$$
\lim_{n\to\infty} \frac{2\alpha_n((\alpha_n + \sigma_n - 2(1 - k_n))D_1^2 + \sigma_n)}{2\alpha_n}
$$

$$
= \lim_{n\to\infty} ((\alpha_n + \sigma_n - 2(1 - k_n))D_1^2 + \sigma_n) = 0
$$

then (2.6) becomes

$$
a_{n+1}^2 \leq a_n^2 - b_n\Phi(a_{n+1}) + o(b_n), \quad \forall n \geq N_0
$$

This, with Lemma 1.2, showed that $a_n \to 0$ as $n \to \infty$, that is ,

$$
\lim_{n\to\infty} \|x_n - p\| = 0.
$$

This completes the proof.

Theorem 2.2. Let X be a real Banach space , K a nonempty closed and convex subset of X and $T : K \to K$ be uniformly continuous mappings such that $T(K)$ is bounded and let suppose that $F(T)$, the set of fixed points of T, has only one common fixed point. Let $k_n \subset [1, \infty)$ be a sequence with $k_n \to 1$ and let $\{x_n\}$ be a sequence defined by

$$
x_{n+1} = (1 - \alpha_n)x_n + \alpha_n T^n y_n
$$

$$y_n = (1 - \beta_n)x_n + \beta_n T^n z_n$$

$$z_n = (1 - \gamma_n)x_n + \gamma_n T^n x_n, \quad n \geq 0$$

where $\{\alpha_n\}_{n=0}^\infty, \{\beta_n\}_{n=0}^\infty$ and $\{\gamma_n\}_{n=0}^\infty$ are three sequences in [0,1] satisfying

(i) $\lim_{n\to\infty} \alpha_n = \lim_{n\to\infty} \beta_n = 0$

(ii) $\sum_{n=1}^\infty \alpha_n = \infty.$

If there exists a strictly increasing function $\varphi : [0, \infty) \to [0, \infty)$ with $\varphi(0) = 0$ such that

$$< T^n x_n - \rho, j(x_n - \rho) > \leq k_n \|x_n - \rho\|^2 - \varphi(\|x_n - \rho\|)$$

for all $j(x - \rho) \in J(x - \rho)$ and $x \in K$, then $\{x_n\}$ cconverges strongly to the fixed point of T.

Corollary 2.3. Let X be a real Banach space , K a nonempty closed and convex subset of X and $T_1, T_2 : K \to K$ be uniformly continuous mappings such that $T_1(K)$ is bounded and let suppose that $T_1(K)$ and $T_2(K)$ have only one common fixed point. Let $k_n \subset [1, \infty)$ be a sequence with $k_n \to 1$ and let $\{x_n\}$ be a sequence defined by

$$x_{n+1} = (1 - \alpha_n)x_n + \alpha_n T_1^n y_n$$

$$y_n = (1 - \beta_n)x_n + \beta_n T_2^n x_n$$

where $\{\alpha_n\}_{n=0}^\infty$ and $\{\beta_n\}_{n=0}^\infty$ are two sequences in [0,1] satisfying

(i) $\lim_{n\to\infty} \alpha_n = \lim_{n\to\infty} \beta_n = 0$

(ii) $\sum_{n=1}^\infty \alpha_n = \infty.$

If there exists a strictly increasing function $\varphi : [0, \infty) \to [0, \infty)$ with $\varphi(0) = 0$ such that

$$< T_i^n x_n - \rho, j(x_n - \rho) > \leq k_n \|x_n - \rho\|^2 - \varphi(\|x_n - \rho\|)$$

for all $j(x - \rho) \in J(x - \rho)$ and $x \in K$, (i=1,2), then $\{x_n\}$ converges strongly to the unique common fixed point of T_1, T_2.

Corollary 2.4. Let X be a real Banach space , K a nonempty closed and convex subset of X and $T : K \to K$ be uniformly continuous mappings such that $T(K)$ is bounded and let suppose that $F(T)$, the set of fixed points of T,

has only one common fixed point. Let $k_n \subset [1, \infty)$ be a sequence with $k_n \to 1$ and let $\{x_n\}$ be a sequence defined by

$$x_{n+1} = (1 - \alpha_n)x_n + \alpha_n T^n y_n$$

$$y_n = (1 - \beta_n)x_n + \beta_n T^n x_n$$

where $\{\alpha_n\}_{n=0}^{\infty}$ and $\{\beta_n\}_{n=0}^{\infty}$ are two sequences in [0,1] satisfying

(i) $\lim_{n \to \infty} \alpha_n = \lim_{n \to \infty} \beta_n = 0$

(ii) $\sum_{n=1}^{\infty} \alpha_n = \infty$.

If there exists a strictly increasing function $\varphi : [0, \infty) \to [0, \infty)$ with $\varphi(0) = 0$ such that

$$< T^n x_n - \rho, j(x_n - \rho) > \leq k_n \|x_n - \rho\|^2 - \varphi(\|x_n - \rho\|)$$

for all $j(x - \rho) \in J(x - \rho)$ and $x \in K$, then $\{x_n\}$ converges strongly to the fixed point of T.

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