Invariant points under strict contractive conditions.

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Abstract. This paper is intended to consider a new approach for obtaining common fixed points under strict contractive conditions in metric spaces without assuming continuity or completeness (or closedness) of the range of any one of the involved maps. The results proved by us can be extended to the nonexpansive or Lipschitz type mapping pairs. Our results substantially improve the results of Pant [Discontinuity and fixed points, J. Math. Anal. Appl. 240, (1999), 284-289], Imdad et al. [Coincidence fixed points in symmetric spaces under strict contractions, J. Math. Anal. Appl. 320, (2006), 352-360], Jin-xuan and Yang [Common fixed point theorems under strict contractive conditions in Menger spaces, Nonlinear Analysis 70, (2009), 184–193] and Pant and Pant [Common fixed points under strict contractive conditions, J. Math. Anal. Appl. 248,(2000), 327-332].

Resumen. Este artículo tiene la intención de considerar un nuevo enfoque para la obtención de puntos fijos comunes bajo estrictas condiciones contractivas en espacios métricos sin asumir la continuidad o integridad (o cercanía) de la gama de cualquiera de los mapas implicados. Los resultados obtenidos por nosotros se puede extender a los mapas de tipo no expansivos o Lipschitz. Nuestros resultados mejoran sustancialmente los resultados de Pant [Discontinuity and fixed points, J. Math. Anal. Appl. 240, (1999), 284-289], Imdad et al. [Coincidence fixed points in symmetric spaces under strict contractions, J. Math. Anal. Appl. 320, (2006), 352-360], Jin-xuan and Yang [Common fixed point theorems under strict contractive conditions in Menger spaces, Nonlinear Analysis 70, (2009), 184–193] y Pant y Pant [Common fixed points under strict contractive conditions, J. Math. Anal. Appl. 248,(2000), 327-332].

1. Introduction:

In metric fixed point theory, strict contractive conditions constitute a very important class of mappings and include contraction mappings as their subclass. It may be observed that strict contractive conditions do not ensure the existence of common fixed points unless some strong condition is assumed either on the space or on the mappings. In such cases either the space is taken to be compact or some sequence of iterates is assumed to be Cauchy sequence. The study of common fixed points of strict contractive conditions using noncompatibility was initated by Pant [9]. Motivated by Pant [9] researchers of this domain obtained common fixed point results for strict contractive conditions under generalized metric spaces [1, 3, 7, 10, 11, 16]. The significance of this paper lies in the fact that we can obtain fixed point theorems for conditionally reciprocally continuous mappings under generalized strict contractive conditions without assuming any strong conditions on the space or on the mappings.

The question of continuity of contractive maps in general and of continuity at fixed points in particular, emerged with the publication of two research papers by R. Kannan [5, 6] in 1968 and 1969 respectively. These papers generated unprecedented interest in the fixed point theory of contractive maps which, in turn, resulted in vigorous research on the existence of fixed points of contractive maps and the question of continuity of contractive maps at their fixed points turned into an open question.

This problem was settled in the affirmative by Pant [8] in 1998 when he introduced the notion of reciprocal continuity and as an application of this concept obtained the first result that established a situation in which a collection of mappings has a fixed point which is a point of discontinuity for all the mappings.

Definition 1.1. [8]. Two selfmappings f and g of a metric space (X, d) are called reciprocally continuous iff $\lim_n fgx_n = ft$ and $\lim_n gfx_n = gt$, whenever $\{x_n\}$ is a sequence such that $\lim_n fx_n = \lim_n gx_n = t$ for some t in X.

In the setting of common fixed point theorems for compatible maps satisfying contractive conditions, continuity of one of the mappings f and g implies their reciprocal continuity but not conversely [8].

More recently, Pant and Bisht [10] further generalized reciprocal continuity by introducing the new concept of conditional reciprocal continuity, which turns out to be the necessary condition for the existence of common fixed points. This notion is applicable to compatible as well as noncompatible mappings.

Definition 1.2. [10]. Two selfmappings f and g of a metric space (X, d) are called conditionally reciprocally continuous (CRC) iff whenever the set of sequences $\{x_n\}$ satisfying $\lim_n fx_n = \lim_n gx_n$ is nonempty, there exists a sequence $\{y_n\}$ satisfying $\lim_n fy_n = \lim_n gy_n = t(say)$ for some t in X such that $\lim_n fgy_n = ft$ and $\lim_n gfy_n = gt$.

If f and g are reciprocally continuous then they are obviously conditionally reciprocally continuous but, as shown in Example 2,1 below, the converse is not true.

In 1986, Jungck [4] generalized the notion of weakly commuting maps [15] by introducing the concept of compatible maps.

Definition 1.3. [4]. Two selfmaps f and g of a metric space (X, d) are called compatible iff $\lim_n d(fgx_n, gfx_n) = 0$, whenever $\{x_n\}$ is a sequence in X such that $\lim_n fx_n = \lim_n gx_n = t$ for some t in X.

The definition of compatibility implies that the mappings f and g will be noncompatible if there exists a sequence $\{x_n\}$ in X such that for some t in X but $\lim_n d(fgx_n, gfx_n)$ is either non zero or nonexistent.

In a recent work, Aamri and Moutawakil [1] introduced the idea of (E.A.) property, which is more general than noncompatible mappings.

Definition 1.4. [1]. A pair (f, g) of selfmappings of a metric space (X, d) is said to satisfy the property (E.A.) if there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n \to \infty} fx_n = \lim_{n \to \infty} gx_n = t, \quad \text{for some } t \in X.$$

In 1997, Pathak et al. [13] further generalized the notion of weakly commuting maps [15] by introducing two new independent concepts of $R-$ weakly commuting of type (A_f) and (A_g).

Definition 1.5. [13] Two selfmappings f and g of a metric space (X, d) are called R-weakly commuting of type (A_f) if there exists some positive real number R such that $d(fgx, ggx) \le Rd(fx, gx)$ for all x in X.

Definition 1.6. [13] Two selfmappings f and g of a metric space (X, d) are called R-weakly commuting of type (A_g) if there exists some positive real number R such that $d(ffx, gfx) \le Rd(fx, gx)$ for all x in X.

On the other hand in the same year 1997, Pathak and Khan [12] further introduced some interesting generalized noncommuting conditions analogous

to the notion of compatibility by defining the notions of $f-$compatibility and $g-$compatibility.

Definition 1.7. [12]. Two selfmaps f and g of a metric space (X, d) are called $f-$compatible iff $\lim_n d(fgx_n, ggx_n) = 0$, whenever $\{x_n\}$ is a sequence in X such that $\lim_n fx_n = \lim_n gx_n = t$ for some t in X.

Definition 1.8. [12]. Two selfmaps f and g of a metric space (X, d) are called $g-$compatible iff $\lim_n d(ffx_n, gfx_n) = 0$, whenever $\{x_n\}$ is a sequence in X such that $\lim_n fx_n = \lim_n gx_n = t$ for some t in X.

It may be observed that $f-$compatibility or $g-$compaibility implies $R-$ weak commutativity of type (A_f) or (A_g) respectively, but the converse is not true in general. It may also be noted that both compatible and noncompatible mappings can be R-weakly commuting of type (A_g) or (A_f) [9].

The question whether there exists a contractive definition which is strong enough to generate a fixed point but which does not force the map to be continuous at the fixed point was reiterated by Rhoades in [14] as an existing open problem. Pant [8, 9], Pant and Pant [11], Pant and Bisht [10], Imdad et al.[3] and Singh et al.[16] have provided some solutions to this problem. It is of worth to note that in all the results proved by us, none of the mappings under consideration has been assumed continuous. In fact, the mappings become discontinuous at the fixed point. We, thus, also provide one more answer to the open problem of Rhoades [14].

2. Main Results:

Theorem 2.1.

Let f and g be conditionally reciprocally continuous noncompatible selfmappings of a metric space (X, d) satisfying

(I). $fX \subseteq gX$

(II).

$$d(fx, fy) < max\left\{d(gx, gy), [d(fx, gx) + d(fy, gy)]/2, [d(fx, gy) + d(fy, gx)]/2\right\}$$

whenever the right hand side is positive. If f and g are either $g-$compatible or $f-$compatible then f and g have a unique common fixed point.

Proof:

Since f and g are noncompatible maps, there exists a sequence $\{x_n\}$ in X such that $fx_n \to t$ and $gx_n \to t$ for some t in X but either $\lim_n d(fgx_n, gfx_n) \neq 0$ or the limit does not exist. Since f and g are conditionally reciprocally continuous and $fx_n \to t, gx_n \to t$ there exists a sequence $\{y_n\}$ satisfying $\lim_n fy_n = \lim_n gy_n = u$ such that $\lim_n fgy_n = fu$ and $\lim_n gfy_n = gu$. Since $fX \subseteq gX$, for each y_n there exists z_n in X such that $fy_n = gz_n$. Thus $fy_n \to u, gy_n \to u$ and $gz_n \to u$ as $n \to \infty$. By virtue of this and using (ii) we obtain $fz_n \to u$. Therefore, we have

$$fy_n = gz_n \to u, gy_n \to u, fz_n \to u. \tag{1}$$

Now, suppose that f and g are $g-$compatible. Then $\lim_n d(ffy_n, gfy_n) = 0$, i.e., $ffy_n \to gu$. We assert that $fu = gu$. If not, using (ii) we get $d(ffy_n, fu) < max\{d(gfy_n, gu), [d(ffy_n, gfy_n) + d(fu, gu)]/2, [d(ffy_n, gu) + d(fu, gfy_n)/2]\}$. On letting $n \to \infty$ this yields $d(gu, fu) \leq \frac{1}{2}d(fu, gu)$, a contradiction unless $fu = gu$. Since $g-$compatibility implies commutativity at coincidence points, i.e., $fgu = gfu$ and, hence $ffu = fgu = gfu = ggu$. If $fu \neq ffu$ then by using (ii), we get $d(ffu, fu) < max\{d(gfu, gu), [d(ffu, gfu) + d(fu, gu)]/2, [d(ffu, gu) + d(fu, gfu)]/2\} = d(ffu, fu)$, a contradiction. Hence $fu = ffu = gfu$ and fu is a common fixed point of f and g.

Finally, suppose that f and g are $f-$compatible. Then $\lim_n d(fgz_n, ggz_n) = 0$. Using $ggz_n = gfy_n \to gu$, this yields $fgz_n \to gu$. If $fu \neq gu$, the inequality $d(fgz_n, fu) < max\{d(ggz_n, gu), [d(fgz_n, ggz_n) + d(fu, gu)]/2, [d(fgz_n, gu) + d(fu, ggz_n)]/2\}$, on letting $n \to \infty$ we get $d(gu, fu) \leq \frac{1}{2}d(fu, gu)$, a contradiction. This implies $fu = gu$. Again, $f-$compatibility of f and g implies that $fgu = gfu$ and, hence, $ffu = fgu = gfu = ggu$. If $fu \neq ffu$ then by using (ii), we get $d(ffu, fu) < max\{d(gfu, gu), [d(ffu, gfu) + d(fu, gu)]/2, [d(ffu, gu) + d(fu, gfu)]/2\} = d(ffu, fu)\}$, a contradiction. Hence $fu = ffu = gfu$ and fu is a common fixed point of f and g. This completes the proof of the theorem.

Theorem 2.1 can be generalized further if we use the property (E.A.) instead of the notion of noncompatiblity. We do so in our next theorem.

Theorem 2.2.

Let f and g be conditionally reciprocally continuous selfmappings of a metric space (X, d) satisfying the property $(E.A.)$ and

(I). $fX \subseteq gX$

(II).
$d(fx, fy) < max\{d(gx, gy), [d(fx, gx) + d(fy, gy)]/2, [d(fx, gy) + d(fy, gx)]/2\},$

whenever the right hand side is positive. If f and g are either $g-$compatible or $f-$compatible then f and g have a unique common fixed point.

Proof:

Since f and g satisfy the property $(E.A.)$, there exists a sequence $\{x_n\}$ in X such that $fx_n \to t$ and $gx_n \to t$ for some t in X. Since f and g are conditionally reciprocally continuous and $fx_n \to t, gx_n \to t$ there exists a sequence $\{y_n\}$ satisfying $\lim_n fy_n = \lim_n gy_n = u$ such that $\lim_n fgy_n = fu$ and $\lim_n gfy_n = gu$. Rest of the proof follows on the same lines as in the corresponding part of the Theorem 2.1.

The next example illustrates the above theorem.

Example 2.1.

Let $X = [2, 20]$ and d be the usual metric on X. Define $f, g : X \to X$ as follows

$fx = 2$ if $x = 2$ or $x > 5$, $fx = 6$ if $2 < x \le 5$,

$g2 = 2$, $gx = 12$, if $2 < x \le 5$, $gx = \frac{(x+1)}{3}$ if $x > 5$.

Then f and g satisfy all the conditions of Theorem 2,2 and have a common fixed point at $x = 2$. It can be verified in this example that f and g satisfy the condition (ii). Furthermore, f and g are $g-$ compatible. It can also be noted that f and g are conditionally reciprocally continuous. To see this, let $\{x_n\}$ be the constant sequence given by $x_n = 2$. Then $fx_n \to 2$, $gx_n \to 2$. Also $fgx_n \to 2 = f2$ and $gfx_n \to 2 = g2$. Hence f and g are conditionally reciprocally continuous. It is also obvious that f and g are not reciprocally continuous but satisfy (E. A.) property. To see this, let $\{y_n\}$ be a sequence in X given by $y_n = 5 + \epsilon_n$ where $\epsilon_n \to 0$ as $n \to \infty$. Then $fy_n \to 2$, $gy_n = (2 + \frac{\epsilon_n}{3}) \to 2$, $\lim_n fgy_n = f(2 + \frac{\epsilon_n}{3}) = 6 \ne f2$ and $\lim_n gfy_n = g2 = 2$. Thus $\lim_n gfy_n = g2$ but $\lim_n fgy_n \ne f2$. Hence f and g are not reciprocally continuous mappings but satisfy (E. A.) property.

As a direct consequence of the above theorem we get the following corollary.

Corollary 2.1.

Let f and g be reciprocally continuous selfmappings of a metric space (X, d) satisfying

(i). $fX \subseteq gX$

(II).
$$d(fx, fy) < max\{d(gx, gy), [d(fx, gx) + d(fy, gy)]/2, [d(fx, gy) + d(fy, gx)]/2\},$$

whenever the right hand side is positive. Suppose f and g satisfy the property $(E.A.)$. If f and g are either g−compatible or f−compatible then f and g have a unique common fixed point.

The corollary follows from the main theorem since reciprocally continuous maps are conditionally reciprocally continuous.

The next theorem demonstrates the applicability of conditional reciprocal continuity and noncompatibility in diverse settings by establishing the existence of fixed point under the Lipschitz type analogue of a strict contractive condition.

Theorem 2.3.

Let f and g be conditionally reciprocally continuous noncompatible selfmappings of a metric space (X, d) satisfying

(I). $fX \subseteq gX$

(II).
$$d(fx, fy) < max\{d(gx, gy), k[d(fx, gx) + d(fy, gy)]/2, [d(fx, gy) + d(fy, gx)]/2\},$$
$$1 \le k < 2,$$

whenever the right hand side is positive. If f and g are either g−compatible or f−compatible then f and g have a unique common fixed point.

Proof:

Since f and g are noncompatible maps, there exists a sequence $\{x_n\}$ in X such that $fx_n \to t$ and $gx_n \to t$ for some t in X but either $\lim_n d(fgx_n, gfx_n) \neq 0$ or the limit does not exist. Since f and g are conditionally reciprocally continuous and $fx_n = gx_n \to t$ there exists a sequence $\{y_n\}$ satisfying $\lim_n fy_n = \lim_n gy_n = u$ such that $\lim_n fgy_n = fu$ and $\lim_n gfy_n = gu$. Since $fX \subseteq gX$, for each y_n there exists z_n in X such that $fy_n = gz_n$. Thus $fy_n \to u$, $gy_n \to u$ and $gz_n \to u$ as $n \to \infty$. By virtue of this and using (ii) we obtain $fz_n \to u$. Therefore, we have

$$fy_n = gz_n \to u, gy_n \to u, fz_n \to u. \tag{2}$$

Now, suppose that f and g are g−compatible. Then $\lim_n d(ffy_n, gfy_n) = 0$, i.e., $ffy_n \to gu$. We assert that $fu = gu$. If not, using (ii) we get $d(ffy_n, fu) < max\{d(gfy_n, gu), k[d(ffy_n, gfy_n) + d(fu, gu)]/2, [d(ffy_n, gu) + d(fu, gfy_n)/2]\}$. On letting $n \to \infty$ this yields $d(gu, fu) \le \frac{k}{2}d(fu, gu)$, a contradiction unless

$fu = gu$. Since g−compatibility implies commutativity at coincidence points, i.e., $fgu = gfu$ and, hence $ffu = fgu = gfu = ggu$. If $fu \neq ffu$ then by using (ii), we get $d(ffu, fu) < max\,\{d(gfu, gu), k[d(ffu, gfu) + d(fu, gu)]/2,$ $[d(ffu, gu) + d(fu, gfu)]/2\} = d(ffu, fu)$, a contradiction. Hence $fu = ffu = gfu$ and fu is a common fixed point of f and g.

Finally, suppose that f and g are f−compatible. Then $\lim_n d(fgz_n, ggz_n) = 0$. Using $ggz_n = gfy_n \to gu$, this yields $fgz_n \to gu$. If $fu \neq gu$, the inequality $d(fgz_n, fu) < max\,\{d(ggz_n, gu), k[d(fgz_n, ggz_n) + d(fu, gu)]/2,$ $[d(fgz_n, gu) + d(fu, ggz_n)]/2\}$, on letting $n \to \infty$ we get $d(gu, fu) \leq \frac{k}{2}d(fu, gu)$, a contradiction. This implies $fu = gu$. Again, f−compatibility of f and g implies that $fgu = gfu$ and, hence, $ffu = fgu = gfu = ggu$. If $fu \neq ffu$ then by using (ii), we get $d(ffu, fu) < max\,\{d(gfu, gu), k[d(ffu, gfu) + d(fu, gu)]/2,$ $[d(ffu, gu) + d(fu, gfu)]/2\} = d(ffu, fu)$, a contradiction. Hence $fu = ffu = gfu$ and fu is a common fixed point of f and g. This completes the proof of the theorem.

Theorem 2.3 can be generalized further if we use the property (E.A.) instead of the notion of noncompatiblity as patterned in Theorem 2.2.

Remark 2.1.

If f and g fail to be reciprocally continuous then there exists a sequence $\{x_n\}$ in X such that $fx_n \to t$ and $gx_n \to t$ for some t in X but either $\lim_n fgx_n \neq ft$ or $\lim_n gfx_n \neq gt$ or one of fgx_n, gfx_n is not convergent. It is also pertinent to mention here that if f and g are not reciprocally continuous then they necessarily satisfy the (E. A.) property, however the mappings satisfying (E. A.) property may be reciprocally continuous (see Example 11 [10]).

Theorem 2.4.

Let f and g be conditionally reciprocally continuous selfmappings of a metric space (X, d) satisfying

(i). $fX \subseteq gX$

(ii).
$d(fx, fy) < max\,\{d(gx, gy), [d(fx, gx) + d(fy, gy)]/2, [d(fx, gy) + d(fy, gx)]/2\},$

whenever the right hand side is positive. Suppose f and g are not reciprocally continuous. If f and g are either g−compatible or f−compatible then f and g have a unique common fixed point.

Proof:

Since f and g are not reciprocally continuous, there exists a sequence $\{x_n\}$ in X such that $fx_n \to t$ and $gx_n \to t$ for some t in X but either $\lim_n fgx_n \neq ft$ or $\lim_n gfx_n \neq gt$ or one of fgx_n, gfx_n is not convergent. Rest of the proof can be completed on the similar lines as has been done in Theorem 2.1.

Example 2.1. also illustrates Theorem 2.4.

Theorem 2.5.

The conclusions of all the above Theorems 2.1, 2.2, 2,3 and 2.4 respectively remain true if the noncommuting condition, i.e., $f-$compatible or $g-$compatible is replaced by the strong noncommuting notion of $R-$ weakly commuting of type (A_f) or (A_g) respectively.

Proof:

Theorem 2.5 follows from the fact that $f-$compatible or $g-$compatible are $R-$ weakly commuting of type (A_f) or (A_g) respectively.

Remark 2.2.

In all the above results we have not asummed any of the strong condition, i.e., continuity or completeness (or closedness) of the range of any one of the involved mappings. Our results substantially improve the results of Pant [9], Imdad et. al. [3], Kubiaczyk and Sharma [7] and many others.

Remark 2.3.

In this paper we have proved all the results using strict contractive conditions. It may be observed that strict contractive conditions do not ensure the existence of common fixed points unless the space is assumed compact or the strict contractive condition is replaced by some strong conditions, e.g., a Banach type contractive condition or a $\phi-$ contractive condition or a Meir-Keeler type contractive condition.

Remark 2.4.

In all the results established in this paper, we have not assumed any mapping to be continuous. We now show that f and g are discontinuous at the common fixed point $u = fu = gu$. If possible, suppose f is continuous. Then taking the sequence $\{y_n\}$ of (1) (Theorem 2.1) we get $ffy_n \to fu = u$ and $fgy_n \to fu = u$. g-compatibility now implies that $\lim_n d(ffy_n, gfy_n) = 0$, i.e., $gfy_n \to fu = u$ This, in turn, yields $\lim_n d(fgy_n, gfy_n) = 0$. This contradicts the fact that $\lim_n d(fgy_n, gfy_n)$ is either nonzero or nonexistent. Hence f is discontinuous at the fixed point. Next, suppose that g is continuous. Then, for the sequence $\{y_n\}$, we get $gfy_n \to gu = u$ and $ggy_n \to gu = u$. In view of these limits, the inequality $d(fgy_n, fu) < max\{d(ggy_n, gu), [d(fgy_n, ggy_n) + d(fu, gu)]/2,$ $[d(fgy_n, gu) + d(fu, ggy_n)]/2\}$, yields a contradiction unless $fgy_n \to fu = u$. But $fgy_n \to u$ and $gfy_n \to u$ contradicts the fact that $\lim_n d(fgy_n, gfy_n)$ is either nonzero or nonexistent. Thus we provide more answers to the problem posed by Rhoades [14] regarding existence a contractive definition which is strong enough to generate a fixed point, but which does not force the map to be continuous at the fixed point.

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