Quasifibrations

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Abstract

Quasifibrations are defined and shown to be spreads or proper maximal partial spreads. Examples are given of quasifibrations producing proper maximal partial flocks of quadratic cones

1 Introduction.

Recently, there has been interest in developing the theory of finite flocks of quadric sets to the infinite or general case(see e.g. [2], [3], [4]). Corresponding to flocks are certain translation planes with spreads in PG(3,K) ,where K is a field, such that the spreads are covered by reguli with various line intersection properties. Furthermore, certain partial flocks of quadric sets also correspond to translation planes. For example, translation planes admitting certain Baer collineation groups correspond to partial flocks of deficiency one (i.e. what would be a flock minus a conic) (see e.g. Johnson [5]).

Recently, De Clerck and Van Maldeghem [2] consider flocks of infinite quadratic cones and provide some examples of what might be called generalized Thas-Fisher-Walker flocks and generalized Kantor flocks.

In [3], the authors note that analogous to the generalized Thas-Fisher-Walker flocks, there are some fascinating maximal partial spreads and maximal partial flocks of a type which do not arise in the finite case. These are called generalized Betten partial spreads due to the fact that Betten [1] considered spreads defined over the real number field which we generalize to produce maximal partial spreads.

In this article, we notice that there is a theory of such maximal partial spreads which is similar to the theory of spreads and which we develop herein.

Received by the editors May 1995.
Communicated by J. Thas.
1991 *Mathematics Subject Classification* : 51E23, 51A40.

We call the constructed partial spreads *quasifibrations* and show that such structures may be coordinatized by systems which are almost quasifields. We call the corresponding coordinate systems *quasi quasifields*.

More generally, we consider what we call class-covered nets which are nets that contain an antiflag (P, L) such that the points of L are covered by intersections with the lines of the net incident with the point P.

In section 2, we provide the basic theory of class-covered nets and quasifibrations. Here we show that a quasifibration is either a spread or a proper maximal partial spread.

The maximal partial spreads that we found in [3] are quasifibrations. In the present article, we show in section 3 that net replacement procedures that normally are considered within translation planes are also valid in quasifibrations. This allows construction of an enormous number of such structures.

In section 4, we note that derivation of a quasifibration produces another quasifibration.

Moreover, in section 5, we also show that we may construct quasifibrations from the general outlines set forth in De Clerck and Van Maldeghem for generalized Kantor flocks which then in turn may be derived to other quasifibrations.

2 Class-covered nets.

Definition 2.1 *Let N be an arbitrary net. Let P be an affine point and L_P the set of lines which are incident with P. Let α be any parallel class and let M be a line of α which is not incident with P. The net N shall be called class-covered with respect to M and P if and only if the points of M are contained in the intersections of M with L_P.*

Theorem 2.2 *A class-covered net with respect to line M and point P is either an affine plane or is a proper maximal net which contains no transversals containing P.*

Proof: Assume that net N is not an affine plane. Let T be a transversal to N (a set of points which intersects each line of each parallel class uniquely and lies in the union) and assume that P is incident with T. Since T intersects M uniquely in the point Q, there is a line Z incident with Q and P of the net by definition. Hence, there can be no transversals containing P.

Note that it is obvious that finite class-covered nets are affine planes. Also, at this point, it is not clear that proper class-covered nets actually exist. We shall see that such nets exist and to formulate this, we consider "translation" class-covered nets.

Definition 2.3 *A translation class-covered net is a translation net which is also class-covered. That is, there is an automorphism group G that acts regularly on the affine points and fixes each parallel class of the net.*

The net is said to be Abelian, elementary Abelian, or a vector space class-covered net provided there is a translation group which is Abelian, elementary Abelian or forms a vector space over some skewfield.

It was noted in Johnson and Ostrom [7] that Sprague's basic results for finite nets(see reference [34] in [7])are valid for infinite nets as well. In particular, for a given point P, we may define the subtranslation groups, called "components", which act transitively on points on lines incident with P. If two of these subgroups are normal then all subgroups are isomorphic and the translation group is the direct product of the two normal subgroups. Furthermore, if three of the subgroups are normal then the translation group is Abelian and is the direct product of any two of the subgroups.

In general, we call such nets i-normal if there are i normal components.

In [7], there is a coordinate method established for such 2-normal translation nets.

Proposition 2.4 *(Johnson and Ostrom [7] (3.5)).*

Let N be a net (finite or infinite) which is an i-normal translation net for i ≥ 2. Then a coordinate system $(Q, +, *)$ may be chosen so that

(i) $(Q, +)$ is a group,

(ii) the ternary function is linear T(x,m,b) = x∗m + b for all b in Q and for all parallel classes (m),

(iii) (c+a)∗m = c∗m + a∗m for all a,c in Q and for all parallel classes (m).

(iv) When the net is an Abelian translation net then $(Q, +)$ is an Abelian group.

We now adapt the above proposition to i-normal translation class-covered nets.

Proposition 2.5 *Let N be an i-normal translation class-covered net for i $\geq$ 2. Then a coordinate system $(Q, +, *)$ may be chosen so that*

(i) $(Q, +)$ is a group,

(ii) for each m in Q, there is an associated parallel class (m) so that the ternary function is linear and defined for all m,b in Q : T(x,m,b) = x∗m + b. Moreover, for a $\neq$ b, and c, there is a unique solution to the equation x∗a = x∗b + c.

(iii) (c+a)∗m = c∗m + a∗m for all c,a,m of Q.

(iv) Conversely, a net with coordinate system satisfying (ii) is a class-covered net.

Proof: We choose the point P as (0,0) and the line M as x = 1 in the coordinate system where the net is class-covered with respect to the line M and point P. The lines incident with P have the general form y = x∗m. Since M = {(1,a) for all a in Q} and the net is class-covered, for each a in Q, there exists a line y = x∗m such that the line contains (1,a). Moreover, since a line is now represented generally in the form y = x∗m + b, then for a≠b,c, there exists a unique intersection between the lines y = x∗b + c and y = x∗a.

Note that a net with coordinate system satisfying (ii) is class covered with special line x = 1 and special point (0,0).

An interesting feature of such class-covered nets is that they "look" like affine planes even when they are not. In particular, we show that in the case when (Q,+) is an Abelian group, there is an associated vector space.

Proposition 2.6 *Let N be an Abelian translation class-covered net. Choose coordinates $(Q, +, *)$ as in proposition (2.5). Then there is a skew field K such that $(Q, +)$ is a $K-$vector space.*

Proof: Since N is Abelian then there is a coordinate system $(Q, +, *)$ such that $(Q, +)$ is an Abelian group. Consider the set S of mappings $\langle$ x→x∗m for all m in $Q\rangle$. Notice that an element x →x∗m is an endomorphism and is 1-1 and onto by (2.5)(ii). Since 1∗m = m in our coordinate system, it follows that the automorphism group $\langle S\rangle$ generated by S acts transitively on the nonzero elements of Q. Consider $(Q, +)$ as a $Z-$module so that $\langle S\ \rangle$ acts irreducibly as a group of $(Q, +)$ automorphisms. Hence, the centralizer K of S in $\mathrm{Hom}_Z(Q, +)$ is a skewfield by Schur's lemma.

Note that although $\langle S\rangle$ acts transitively on the nonzero elements of Q , we are not trying to say that S acts transitively on Q.

Definition 2.7 *We call the centralizer K of S in (2.6), the outer kernel of the net N(following the notation of Lüneburg [8] p. 23 for the outer kernel of a quasifield).*

We note that we may consider an action of K on the net as follows:

When we have an Abelian translation group G, and all "components" are isomorphic, we may identify any two of these components with the corresponding group $(Q, +)$. Then the net has the representation of points (x,y) for all x,y in Q. We then may consider an action of g in K on the net by considering (x,y) →(xg,yg). Since g commutes with the mappings x →x∗m for all m in Q, it follows that y = x∗ m is mapped onto y = x∗m by g for all m in Q. We call the associated skewfield of mappings the *kernel* of the net.

Note that we have circumvented the necessity of having a partition of an Abelian group. In fact, this is the only ingredient omitted from having a quasifield /translation plane setting.

Hence, we obtain that the outer kernel K is isomorphic to the kernel of the net and we shall not distinguish notations from one to the other. It also works exactly as in the quasifield situation that there is a set of elements k(Q) of Q which associate and distribute on "the other side" with elements of Q and which forms a skewfield anti-isomorphic to K.

Since all components are isomorphic as Abelian groups, the direct product of any two is the full group, and the components of the group correspond to the lines thru (0,0), it follows that the lines y = x∗m are isomorphic as K -subspaces.

Proposition 2.8 *If N is an Abelian translation class-covered net then the net is a vector space V over a skewfield K such that the lines thru the zero vector are isomorphic K-subspaces such that any two such subspaces direct sum to the space V.*

We formulate this in terms of partial spreads.

Definition 2.9 *Let V be a vector space of the form $W\oplus W$ where W is a K- space for a skewfield K. A partial spread is a set of subspaces K-isomorphic to W such that the direct sum of any two distinct subspaces is V.*

Let B be a basis for W and choose a fixed vector e from $W\oplus 0$.A quasifibration Q is a set of mutually disjoint subspaces K isomorphic to W which contains $W\oplus 0$ and $0\oplus W$ and which has the property that for each vector of the form e + w for w in $0\oplus W$, there is a subspace of Q which contains e+w. Furthermore, K is called the kernel of the quasifibration if and only if K is maximal among skewfields L such that the subspaces indicated are L spaces.

Remark. Assume that Q is a finite quasifibration. Let W be a K - space of dimension n where K is isomorphic to GF(q). Then Q is a spread in PG(2n-1,q).

Proof: There are q^n elements of W so there are $1 + q^n$ mutually disjoint n-spaces so we must have a cover. Hence, there is an associated spread in PG(2n-1,q).

Remark. We shall say that the dimension of a quasifibration is the K - dimension of W where the underlying vector space is $W \oplus W$.

Theorem 2.10 *A Quasifibration is either a spread or a proper maximal partial spread.*

Proof: We may represent the quasifibration in the form x = 0, y = 0 , y = xM_w where (e, w) is a vector of y = xM_w and M_w is a K -automorphism of W. Any additional K -space isomorphic to W and disjoint from x = 0, y = 0 has the general form y = $x\sigma$ where σ is also a K - automorphism of W.

However, if $(e, e\sigma) = (e, v)$ for v in W, there is a unique M_v such that y = xM_v is an element of the quasifibration. However, then $\sigma -$ M_v is singular. Hence, a quasifibration is either a spread or a proper maximal partial spread.

Actually, our analysis of class-covered nets shows that a quasifibration may be produced via an Abelian translation class-covered net.

Theorem 2.11 *Abelian translation class-covered nets are equivalent to quasifibrations.*

Proof: Let N be an Abelian translation class-covered net. The previous propositions show that there is an associated coordinate system $(Q, +, *)$ and a skewfield K so that N is a K-vector space and the components are isomorphic K-spaces. The components may be given the form x = 0, y = x$*$m for all m in Q. Since 1 is an element of Q, we identify Q with W above and note that the translate 1+Q (that is, x = 1) is covered by the set of intersections (1,1$*$m = m) for all m in Q.

Conversely, suppose we have a quasifibration F. We define a net in the standard way by taking the points as vectors and lines as translates of the set defining the quasifibration. Hence, $\{(e, w)$ such that w is in $W\}$ corresponds to a line M of a given parallel class. Since the translate is covered by intersections with the given subspaces, we clearly have a class-covered net which admits an Abelian translation group.

Definition 2.12 *Let $(Q, +, *)$ be a triple so that*

(i) $(Q, +)$ is an Abelian group,

*(ii) $(Q - \{0\}, *)$ is a binary operation so that a$*$0 = 0$*$a = 0 and there is an element 1 in Q so that 1$*$m = m for all a,m in Q.*

(iii) (a+b)$*$m = a$*$m + b$*$m for all a,b,m in Q,

(iv) for a,$\neq$b, c in Q, there is a unique solution to the equation x$*$a = x$*$b + c.

Then we shall call $(Q, +, *)$ a quasi quasifield.

The reader might note that a *quasifield* is defined in Lüneburg [8] p.22 as a system satisfying the above conditions and , in addition, satisfies:

(v) For a,c in Q, there is a unique solution to the equation a$*$x = c.

Corollary 2.13 *The following systems are equivalent:*
(1) Abelian translation class-covered nets,
(2) quasifibrations,
(3) quasi quasifields.

Proof: We need only show that a quasi quasifield produces an Abelian translation class-covered net. We take points as elements of $Q \times Q$ and lines as sets defined by the equations y = x∗m + b, x = c for m,b,c fixed in Q and parallel classes defined in the obvious way. We note that x = 1 is contained in the intersections of the other lines y = x∗m for all m in Q. It is easy to verify that we have defined a net which is class-covered and admits an Abelian translation group whose elements are given by the mappings (x,y) → (x+a,y+b) for a,b in Q.

Corollary 2.14 *A proper quasifibration in $PG(V, K)$, is a maximal partial spread which may not be embedded in any other partial spread within $PG(V, L)$ for any subfield L of K; a proper quasifibration is a maximal partial spread which is nonextendable to an affine plane.*

3 Construction techniques in class-covered nets.

One rather nice property of quasifibrations is that net replacement procedures preserve this property.

Theorem 3.1 *Let Q be a quasifibration. Let P be any partial spread of Q and let P^* denote a replacement partial spread for P.*

(1) Then $(Q - P) \cup P^*$ is a maximal partial spread. (2) The maximal partial spread is a quasifibration when the standard axes are not involved in the replacement.

(3) Moreover, the constructed quasifibration is a proper maximal partial spread if and only if the original quasifibration is a proper maximal partial spread.

Proof: Assume that the partial spread does not contain $0 \oplus W$ or $W \oplus 0$(the standard axes). There is a unique set P_W ={w in W such that (e,w) is in P}. For each point (e,w) in P ,there is a unique subspace P^*(w) of P^* containing (e,w) by the definition of replacement partial spread. Clearly, Q is a cover if and only if the constructed quasifibration is a cover.

In general, if the constructed partial spread is not maximal then since the set of covered points is the same in both the original and the constructed partial spreads, it follows that the quasifibration is not maximal.

In the previous result, we have noted that we may perform net replacement procedures on proper quasifibrations that produce other proper quasifibrations. We consider this more generally for derivable nets with class-covered nets.

We define a derivable net in the standard manner (see e.g. Johnson [6]).

Theorem 3.2 *Let N be a class-covered net and let D be a derivable subnet. Let D^* denote the associated derived net. Then the net whose points are the points of N and whose lines are the lines of N whose parallel classes are not in D together* *with the Baer subplanes of D form a maximal net N^*. N^* is also derivable with net D^*.*

Moreover, N^* is not an affine plane if and only if N is not an affine plane.

4 Quasifibrations of dimension 2.

Let Q be a quasifibration of $K-$dimension 2 with respect to e where K is a field. Choose a basis so that e = (0,1). Then the quasifibration can be represented by subspaces of the form x = 0, y = 0 and y = x$\begin{bmatrix} g(t,u) & f(t,u) \\ t & u \end{bmatrix}$ for all t,u in K where g and f are functions from $K \times K$ to K.

Theorem 4.1 *The derivation of a quasifibration of K-dimension 2 by a derivable net of the form*

$x = 0$, $y = $ x$\begin{bmatrix} u^\sigma & 0 \\ 0 & u \end{bmatrix}$ *for all u in K where σ is an automorphism of the field K , is a quasifibration.*

Proof: It follows easily that the components of the derived net have the basis form:

x = 0, y = x$^\sigma$ $\begin{bmatrix} u & 0 \\ 0 & u \end{bmatrix}$ for all u in K and where x$^\sigma$ = (x$_1^\sigma$, x$_2^\sigma$) where x$_i$ is in K for i = 1,2, and

y = x$\begin{bmatrix} -g(t,u)t^{-1} & f(t,u) - g(t,u)t^{-1}u \\ t^{-1} & t^{-1}u \end{bmatrix}$ for t nonzero.

(See e.g. Johnson [6] for the finite case.)The line x = (0,1) intersects the components of the net in (0,1,0,u), (0,1,t^{-1},t^{-1}u) for all u,t$\neq$ 0 of K so that the net is class-covered and hence we obtain a quasifibration.

Since all derivable nets when K is a field have the above form (see Johnson [6] for the finite case), we obtain:

Corollary 4.2 *The derivation of a quasifibration of K-dimension 2 by a derivable net containing the standard component whose parallel class contains the covering line of the quasifibration is a quasifibration.*

Recently, the authors have considered generally flocks of quadratic cones in PG(3,K) for arbitrary fields $K(see[3])$. Corresponding to a flock of a quadratic cone is a spread in PG(3, K) represented in the following way in the associated vector space: x =0, y = 0, y = x$\begin{bmatrix} u + g(t) & f(t) \\ t & u \end{bmatrix}$ for all u,t in K where f and g are functions on K and f is 1-1 and onto. If we relax the condition that f is 1-1 and onto to simply that f is 1-1, then we may obtain a quasifibration which produces a proper maximal partial spread and a corresponding maximal partial flock of a quadratic cone (see below)

Also, in [4], the second author has considered infinite flocks of hyperbolic quadrics. We may similarly potentially determine maximal partial flocks of hyperbolic quadrics using the idea of a quasifibration.

Theorem 4.3 *If there is a quasifibration Q of V_4 over field a K whose corresponding translation net admits an elation group such that any component orbit union the axis forms a regulus in PG(3,K) then there is an associated partial flock of a quadratic cone which is either a flock or a proper maximal partial flock.*

Theorem 4.4 *If a quasifibration admits a homology group such that any component orbit union the axis and coaxis forms a regulus in PG(3,K), then there is an associated partial flock of a hyperbolic quadric which is either a flock or a proper maximal partial flock.*

Proof: By [3], we may represent the quasi fibration in the form x =0 , y = 0 , y = x$\begin{bmatrix} u+g(t) & f(t) \\ t & u \end{bmatrix}$ for all u,t in K provided we can change bases as above choosing e = (0,1) and maintaining that $\langle \begin{bmatrix} 1 & 0 & u & 0 \\ 0 & 1 & 0 & u \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \mid u \text{ is in } K \rangle = E$ is the form of the elation group. However, we may choose a basis change of the general form $\begin{bmatrix} A & 0 \\ 0 & A \end{bmatrix}$ where A is a 2×2 $K-$matrix. Since E is left invariant under the basis change, we have the representation as maintained.

The proof of the second result is very similar, where the elation group is replaced by the homology group $\langle \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & u & 0 \\ 0 & 0 & 0 & u \end{bmatrix} \mid u \text{ is in } K - \{0\} \rangle$. The general form for the partial spread in this case is x = 0, y = x$\begin{bmatrix} v & 0 \\ 0 & v \end{bmatrix}$, $y = x \begin{bmatrix} f(u)t & g(t)u \\ u & tu \end{bmatrix}$ for all v,u,t in K and u nonzero. (See [5].)

For the partial conical flock, clearly, f(t) is 1-1 or otherwise there are differences which are singular. Consider, the vector (1,0,0,c). This vector is incident with one of the elements of the quasifibration if and only if f(t) = c. Hence, we obtain a cover precisely when f(t) is 1-1 and onto.

There are similar restrictions on f (and g) where partial hyperbolic flocks are considered.

5 Examples.

In [3], the authors discuss the quasifibrations which may be obtained via what we call generalized Betten partial spreads. Here, we repeat some of this discussion for clarity.

In [1], Betten discusses some topological projective planes. The planes are translation planes with spreads within PG(3,K) for certain fields K. In particular, Betten's construction is valid when K is the field of real numbers. Furthermore, there are finite analogues when K is GF(q) and q$\equiv -1 mod 3$.

Recently, the authors noted that the Betten planes also produce flocks of quadratic cones in PG(3,K)(see [3] and also note that this example appears in the work of De

Clerck and Van Maldeghem [2]). Furthermore, it was noted that there are fields K which produce partial flocks when the polynomial $x^2 - x + 1/3$ is irreducible and the partial flocks are flocks provided in addition that the function C defined by C(x) = x^3 is 1-1 and onto.

We note that $(x^3 - (x-1)^3)/3 = x^2 - x + 1/3$ so that it follows that if C is 1-1 then $x^2 - x + 1/3$ is irreducible. Conversely, if C is not 1-1, let $u^3 = v^3$ where u is not equal to v.

Then writing v as u - w for w nonzero, we obtain $u^3 - v^3 = 3u^2w - 3uw^2 + w^3 = 0$. Dividing by $3w^3$ and letting $u/w = x$, we obtain $x^2 - x + 1/3 = 0$.

Hence, we have for the existence of the partial spread that C is 1-1 and for the existence of the spread that C is 1-1 and onto.

We need to show that we do not obtain a spread unless C is onto.

Theorem 5.1 *(see Jha-Johnson [3]) Let K be a field in which the function C defined by C(x) = x^3 is 1-1 but not onto. Let V_4 denote a 4-dimensional vector space over K. Then the following equations define a quasifibration which is a proper maximal partial spread:x = 0, y =* $x\begin{bmatrix} u - s^2 & -s^3/3 \\ s & u \end{bmatrix}$ *for all u,s in K and where x and y are 2-vectors over K.*

Proof: By (6.1) and (6.2) of [2] and the above note, the indicated 2-dimensional subspaces form a partial spread provided C is 1-1.

Consider the vector (1,0,0,t). In order that this vector is covered by a 2-dimensional subspace of the form listed in the statement of the theorem, it must be that $(-s^3/3) = t$ for some s in K. As t varies over K, it follows that this is possible if and only if the function C is onto.

We may now apply the main theorem on quasifibratons to see that the partial spread is actually maximal. To see a particular version of the argument, suppose not. Then there is a 2-dimensional K-space which we may represent in the form y = $x\begin{bmatrix} a & b \\ c & d \end{bmatrix}$. In order that we obtain a partial spread which includes the extra 2-dimensional K-space, the differences of the indicated matrix with all of the other matrices above in the statement of the theorem must be nonsingular. However, since a,b,c,d are in K, it follows that if we subtract $\begin{bmatrix} d - c^2 & -c^3/3 \\ c & d \end{bmatrix}$ from $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, this forces $b = -c^3/3$ and $a = d - c^2$. That is, the 2-dimensional K-space is not exterior to the set of 2-dimensional K-spaces listed above. Hence, the partial spread is maximal and proper.

Definition 5.2 *We shall call any partial spread as in the above theorem, a generalized Betten partial spread over the field K. There is a corresponding partial flock of a quadratic cone which we shall call a generalized Betten partial flock.*

Corollary 5.3 *(Jha-Johnson [3]) (1) If K is a field in which the function C defined by C(x) = x^3 is 1-1 but not onto then the corresponding generalized Betten partial spread is a proper maximal partial spread.*

(2) If K is a field in which the function C is 1-1 but not onto then the generalized Betten partial flock of a quadratic cone in PG(3,K) is proper and maximal.

Proof: Statement (1) is simply a restatement of theorem A. By [3], corresponding to a partial spread of the form indicated is a partial flock of a quadratic cone in PG(3,K). This partial flock is a flock if and only if the partial spread is a spread. Now assume that the partial flock can be properly extended to a partial flock. Then it is immediate that the corresponding partial spread can be properly extended to a partial spread. Hence the partial flock is also maximal.

Example 5.4 *(Jha-Johnson [3]). Let Q denote the field of rationals. Clearly, $x^2 - x + 1/3$ is irreducible over Q as the discriminant of the polynomial is (-1/3). Hence the function C defined by $C(x) = x^3$ is 1-1 but not onto. Hence, there is a corresponding maximal proper partial generalized Betten flock in PG(3,Q). More generally, we may take any subfield K of the real field which contains the rationals but does not contain all cube roots of the rationals to produce other maximal proper partial generalized Betten flocks in PG(3,K).*

In [2], there is an infinite flock of a quadratic cone which, in the finite case, corresponds to the Knuth semifield flocks. Furthermore, there are associated generalized quadrangles which correspond to some quadrangles of Kantor. We shall refer to the following possible type as generalized Kantor -Knuth flocks or partial flocks : The associated translation plane has spread x = 0, y = 0, y = x$\begin{bmatrix} u & mt^{\sigma} \\ t & u \end{bmatrix}$ for all u,t in a field K where σ is an automorphism of K and m is a fixed nonzero constant.

In terms of quasifibrations, we could consider the slightly more general potential partial spread :

x = 0, y = x$\begin{bmatrix} u^{\rho} & mt^{\sigma} \\ t & u \end{bmatrix}$ for all t,u in K .

Letting $K =$ k(x) the field of rational functions over k = the field of rationals and σ an automorphism of K which leaves the natural valuation invariant with m = x, it is shown in [2] that there is an associated flock of a quadratic cone (when $\rho = 1$ in the more general situation).

More generally, if we let σ and ρ denote monomorphisms of K which leaves the parity of the natural valuation invariant, then we obtain a quasifibration which produces proper maximal partial flocks.

For example, consider the mappings {x$\rightarrow$x^{2a+1} } and extend such mappings to monomorphisms σ_b ,ρ_c of k(x).

Theorem 5.5 *There are quasifibrations $Q_{(\sigma_b,\rho_c)}$ for any positive integers b,c which produce proper maximal proper spreads.*

When $\rho_c = 1 (c = 0)$,there are corresponding infinite generalized Kantor - Knuth semifield quasifibrations which produce maximal and proper partial flocks of a quadratic cone which are not flocks.

Proof: For example, consider σ_3. We need to show that u^2 - mt t$^{\sigma}$ can never be zero for all u,t in the field K. Choose m = x and let u = r(x)/s(x) with degrees r,s respectively, and t = q(t)/n(t) with degress q,n respectively. Taking the valuation on degrees, we see that we obtain the degree equation

2(r-s) - 1 - 3(q-n) - (q-n) = 0 which obviously cannot occur.

More generally, we need to show that u u^{ρ_c} − m t t^{σ_b} can never be zero. Thus, applying the same argument as above, we obtain (2c+2)(r-s) - 1 - (2b+2)(q-n) = 0 which is a contradiction.

Now all of the above examples may be derived in various ways.

Theorem 5.6 *Any generalized Kantor-Knuth or generalized Betten quasifibration may be derived using any regulus net of the spread. Any such derived net is also a class-covered net and defines a quasifibration.*

(1) If we choose to derive the net x = 0, y = $x\begin{bmatrix} u+g(t_o) & f(t_o) \\ t_o & u \end{bmatrix}$ *for all u in K and fixed* t_o*, we obtain the following derived quasifibration:*

$$x = 0,\ y = x\begin{bmatrix} v & 0 \\ 0 & v \end{bmatrix},$$

$$y = x\begin{bmatrix} -t^{-1}(u+g(t)-g(t_o)) & f(t)-f(t_o)-t^{-1}(u+g(t)-g(t_o))u \\ t^{-1} & t^{-1}u \end{bmatrix}$$

for all v,u,t$\neq$ 0 *of* K.

(2) For a derived generalized Kantor-Knuth quasifibration, the forms are:

$$x = 0,\ y = x\begin{bmatrix} v & 0 \\ 0 & v \end{bmatrix},\ y = x\begin{bmatrix} -t^{-1}u & (t-t_o)^{\sigma}-t^{-1}u^2 \\ t^{-1} & t^{-1}u \end{bmatrix}$$ *for all* v,u,t$\neq$ 0 *of* K, t_0 *fixed.*

(3) For a derived generalized Betten quasifibration, the forms are:

$$x = 0,\ y = x\begin{bmatrix} v & 0 \\ 0 & v \end{bmatrix},$$

$$y = x\begin{bmatrix} -t^{-1}(u-t^2-t_o^2) & -(t^3-t_o^3)-t^{-1}(u-t^2-t_o^2)u \\ t^{-1} & t^{-1}u \end{bmatrix}$$

for all v,u, t$\neq$ 0 *in* K *and* t_o *constant.*

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