Complete hypersurfaces with constant scalar curvature in a hyperbolic space

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Abstract. In this paper, we characterize the n-dimensional ($n \geq 3$) complete hypersurfaces M^n in a hyperbolic space H^{n+1} with constant scalar curvature and with two distinct principal curvatures. We show that if the multiplicities of such principal curvatures are greater than 1, then M^n is isometric to $S^k(r) \times H^{n-k}(-1/(r^2+1))$. On the other hand, let M^n be the complete hypersurfaces in H^{n+1} with constant scalar curvature $n(n-1)R$ and nonnegative sectional curvature, if $R+1 \geq 0$, then M^n is totally umbilical, or is isometric to $S^{n-1}(r) \times H^1(-1/(r^2+1))$.

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Introduction

Let $R^{n+1}(c)$ be an $(n+1)$-dimensional connected Riemannian manifold with constant sectional curvature c. According to $c > 0, c = 0$ and $c < 0$, it is called sphere space, Euclidean space or hyperbolic space, respectively, and it is denoted by $S^{n+1}(c), R^{n+1}$ or $H^{n+1}(c)$. As it is well known that there are many rigidity results for hypersurfaces with constant mean curvature or with constant scalar curvature in $S^{n+1}(c)$ or R^{n+1}, for example, see[1], [2], [3], [4], [6] and [10] etc., but less are obtained for hypersurfaces immersed into a hyperbolic space. S.Y.Cheng and Yau[2] proved that an n-dimensional ($n \geq 2$) complete hypersurface M^n with constant scalar curvature in R^{n+1} is isometric to a sphere, a hyperplane or a generalized cylinder $S^k(c) \times R^{n-k}, 1 \leq k \leq n-1$, if the sectional curvature of M^n is nonnegative. They also proved that an n-dimensional compact hypersurface M^n with constant scalar curvature $n(n-1)R$ satisfying $R \geq 1$ in the unit sphere $S^{n+1}(1)$ is isometric to a sphere, or a Riemannian product $S^k(c_1) \times S^{n-k}(c_2), 1 \leq k \leq n-1$, if the sectional curvature of M^n is nonnegative. In [6], Li extended the results due to S.Y.Cheng and Yau[2] in terms of the squared norm of the second fundamental form of M^n. Cheng [3] and [4] characterized the hypersurface

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$S^k(c) \times R^{n-k}$ in a Euclidean space R^{n+1} and the hypersurface $S^k(c_1) \times S^{n-k}(c_2)$ in a unit sphere $S^{n+1}(1)$, respectively.

On the other hand, Morvan-Wu[9], Wu[13] proved some rigidity theorems for complete hypersurfaces M^n in a hyperbolic space $H^{n+1}(c)$ under the assumption that the mean curvature is constant and the Ricci curvature is non-negative. To our best knowledge, there are almost no intrinsic rigidity results for the hypersurfaces with constant scalar curvature in a hyperbolic space until Liu and Su[8] obtained the following :

Theorem 1.1 ([8]) *Let M^n be an n-dimensional $(n > 2)$ complete hypersurface with constant scalar curvature $n(n-1)R$ in H^{n+1}. If $\overline{R} = R + 1 \geq 0$ and the norm square $|h|^2$ of the second fundamental form of M^n satisfies*

$$n\overline{R} \leq \sup |h|^2 \leq \frac{n}{(n-2)(n\overline{R}-2)}[n(n-1)\overline{R}^2 - 4(n-1)\overline{R} + n],$$

then either $\sup |h|^2 = n\overline{R}$ and M^n is a totally umbilical hypersurface; or $\sup |h|^2 = \frac{n}{(n-2)(n\overline{R}-2)}[n(n-1)\overline{R}^2 - 4(n-1)\overline{R}+n]$, and M^n is isometric to $S^{n-1}(r) \times H^1(-1/(r^2+1))$, for some $r > 0$.

In this paper, we shall firstly investigate the complete hypersurfaces of constant scalar curvature with two distinct principal curvatures whose multiplicities are greater than 1, and obtain a characteristic Theorem, see Theorem 3.1. Secondly, we study the complete hypersurfaces of constant scalar curvature with nonnegative sectional curvature and obtain another characteristic Theorem, see Theorem 3.2.

2 Preliminaries

We simply denote $H^{n+1}(-1)$ by H^{n+1}. Let M^n be an n-dimensional hypersurface in H^{n+1}. We choose a local orthonormal frame $e_1, \cdots, e_{n+1}$ in H^{n+1} such that $e_1, \cdots, e_n$ are tangent to M^n. Let $\omega_1, \cdots, \omega_{n+1}$ be the dual coframe. We use the following convention on the range of indices:

$$1 \leq A, B, C, \cdots \leq n+1; \quad 1 \leq i, j, k, \cdots \leq n.$$

The structure equations of H^{n+1} are given by

$$d\omega_A = \sum_B \omega_{AB} \wedge \omega_B, \quad \omega_{AB} + \omega_{BA} = 0, \tag{2.1}$$

$$d\omega_{AB} = \sum_C \omega_{AC} \wedge \omega_{CB} + \Omega_{AB}, \tag{2.2}$$

where

$$\Omega_{AB} = -\frac{1}{2} \sum_{C,D} K_{ABCD} \omega_C \wedge \omega_D, \tag{2.3}$$

$$K_{ABCD} = -(\delta_{AC}\delta_{BD} - \delta_{AD}\delta_{BC}). \tag{2.4}$$

Restricting to M^n,

$$\omega_{n+1} = 0. \tag{2.5}$$

$$\omega_{n+1i} = \sum_j h_{ij}\omega_j, \quad h_{ij} = h_{ji}. \tag{2.6}$$

The structure equations of M^n are

$$d\omega_i = \sum_j \omega_{ij} \wedge \omega_j, \qquad \omega_{ij} + \omega_{ji} = 0, \tag{2.7}$$

$$d\omega_{ij} = \sum_k \omega_{ik} \wedge \omega_{kj} - \frac{1}{2}\sum_{k,l} R_{ijkl}\omega_k \wedge \omega_l, \tag{2.8}$$

$$R_{ijkl} = -(\delta_{ik}\delta_{jl} - \delta_{il}\delta_{jk}) + (h_{ik}h_{jl} - h_{il}h_{jk}), \tag{2.9}$$

$$R_{ij} = -(n-1)\delta_{ij} + nHh_{ij} - \sum_k h_{ik}h_{kj}, \tag{2.10}$$

$$n(n-1)(R+1) = n^2H^2 - |h|^2, \tag{2.11}$$

where $n(n-1)R$ is the scalar curvature, H is the mean curvature and $|h|^2$ is the squared norm of the second fundamental form of M^n.
The Codazzi equation and the Ricci identity are

$$h_{ijk} = h_{ikj}, \tag{2.12}$$

$$h_{ijkl} - h_{ijlk} = \sum_m h_{mj}R_{mikl} + \sum_m h_{im}R_{mjkl}, \tag{2.13}$$

where h_{ijk} and h_{ijkl} denote the first and the second covariant derivatives of h_{ij}.

In order to represent our theorems, we need some notations, for details see Lawson [7], Ryan[12] or Liu[8]. First we give a description of the real hyperbolic space $H^{n+1}(c)$ of constant curvature $c(< 0)$.

For any two vectors x and y in R^{n+2}, we set

$$g(x, y) = x_1y_1 + \cdots + x_{n+1}y_{n+1} - x_{n+2}y_{n+2},$$

(R^{n+2}, g) is the so-called Minkowski space-time. Denote $\rho = \sqrt{-1/c}$. We define

$$H^{n+1}(c) = \{x \in R^{n+2} | g(x, x) = -\rho^2, x_{n+2} > 0\}.$$

Then $H^{n+1}(c)$ is a simply-connected hypersurface of R^{n+2}. Hence, we obtain a model of a real hyperbolic space.
We define

$$\begin{aligned} M_1 &= \{x \in H^{n+1}(c)|x_1 = 0\}, \\ M_2 &= \{x \in H^{n+1}(c)|x_1 = r > 0\}, \\ M_3 &= \{x \in H^{n+1}(c)|x_{n+2} = x_{n+1} + \rho\}, \\ M_4 &= \{x \in H^{n+1}(c)|x_1^2 + \cdots + x_{n+1}^2 = r^2 > 0\}, \\ M_5 &= \{x \in H^{n+1}(c)|x_1^2 + \cdots + x_{k+1}^2 = r^2 > 0, x_{k+2}^2 + \cdots + x_{n+1}^2 - x_{n+2}^2 = -\rho^2 - r^2\}. \end{aligned}$$

$M_1, \cdots, M_5$ are often called the standard examples of complete hypersurfaces in $H^{n+1}(c)$ with at most two distinct constant principal curvatures. It is obvious that $M_1, \cdots, M_4$ are totally umbilical. In the sense of Chen[5], they are called the hyperspheres of $H^{n+1}(c)$.M_3 is called the horosphere and M_4 the geodesic distance sphere of $H^{n+1}(c)$. Ryan[12] obtained the following:

Lemma 2.1([12]) *Let M^n be a complete hypersurface in $H^{n+1}(c)$. Suppose that, under a suitable choice of a local orthonormal tangent frame field of TM^n, the shape operator over TM^n is expressed as a matrix A. If M^n has at most two distinct constant principal curvatures, then it is congruent to one of the following:*

(1) M_1. In this case, $A = 0$, and M_1 is totally geodesic. Hence M_1 is isometric to $H^n(c)$;

(2) M_2. In this case, $A = \frac{1/\rho^2}{\sqrt{1/\rho^2+1/r^2}} I_n$, where I_n denotes the identity matrix of degree n, and M_2 is isometric to $H^n(-1/(r^2+\rho^2))$;

(3) M_3. In this case, $A = \frac{1}{\rho} I_n$, and M_3 is isometric to a Euclidean space R^n;

(4) M_4. In this case, $A = \sqrt{1/r^2 + 1/\rho^2} I_n$, M_4 is isometric to a round sphere $S^n(r)$ of radius r;

(5) M_5. In this case, $A = \lambda I_k \oplus \mu I_{n-k}$, where $\lambda = \sqrt{1/\rho^2 + 1/r^2}$, and $\mu = \frac{1/\rho^2}{\sqrt{1/r^2+1/\rho^2}}$, M_5 is isometric to $S^k(r) \times H^{n-k}(-1/(r^2+\rho^2))$.

3 Theorems and Their Proofs

In this section, we consider the hypersurface with constant scalar curvature and with two distinct principal curvatures in H^{n+1}. We firstly have the following Proposition 3.1 due to Otsuki[10].

Proposition 3.1(Otsuki[10]). *Let M^n be a hypersurface in a hyperbolic space H^{n+1} such that the multiplicities of the principal curvatures are constant. Then the distribution of the space of the principal vectors corresponding to each principal curvature is completely integrable. In particular, if the multiplicity of a principal curvature is greater than 1, then this principal curvature is constant on each integral submanifold of the corresponding distribution of the space of the principal vectors.*

We may prove the following:

Theorem 3.1 *Let M^n be an n-dimensional complete hypersurface in H^{n+1} with constant scalar curvature $n(n-1)R$ and with two distinct principal curvatures. If the* *multiplicities of these two distinct principal curvatures are greater than 1, then M^n is isometric to the Riemannian product $S^k(r) \times H^{n-k}(-1/(r^2+1))$, for some $r > 0$.*

Proof. Let λ, μ be the principal curvatures of multiplicities k and $n-k$ respectively, where $1 < k < n-1$. By (2.11) we have

$$n(n-1)(R+1) = k(k-1)\lambda^2 + 2k(n-k)\lambda\mu + (n-k)(n-k-1)\mu^2. \tag{3.1}$$

Denote by D_λ and D_μ the integral submanifolds of the corresponding distribution of the space of principal vectors corresponding to the principal curvature λ and μ, respectively. From Proposition 3.1, we know that λ is constant on D_λ. Since the scalar curvature is constant, (3.1) implies that μ is constant on D_λ. Making use of Proposition 3.1 again, we have μ is constant on D_μ. Therefore, we know that μ is constant on M^n. By the same assertion we know that λ is constant on M^n. Therefore M^n is isoparametric. By Lemma 2.1, we know that M^n is isometric to $S^k(r) \times H^{n-k}(-1/(r^2+1)))$, for some $r > 0$. This completes the proof of Theorem 3.1.

From now on, we consider the complete hypersurfaces with constant scalar curvature and nonnegative sectional curvature. We obtain the following:

Theorem 3.2 *Let M^n be an n-dimensional complete hypersurface with constant scalar curvature $n(n-1)R$ in a hyperbolic space H^{n+1}. If $R+1 \geq 0$ and the sectional curvature of M^n is nonnegative, then M^n is a totally umbilical hypersurface; or M^n is isometric to $S^{n-1}(r) \times H^1(-1/(r^2+1))$, for some $r > 0$.*

In order to prove Theorem 3.2, we introduce an operator $\square$ due to Cheng-Yau[2] by

$$\square f = \sum_{i,j} (nH\delta_{ij} - h_{ij}) f_{ij}, \tag{3.2}$$

where f is a C^2-function on M^n, the gradient and Hessian (f_{ij}) are defined by

$$df = \sum_i f_i \omega_i, \quad \sum_j f_{ij}\omega_j = df_i + \sum_j f_j \omega_{ji}. \tag{3.3}$$

The Laplacian of f is defined by $\Delta f = \sum_i f_{ii}$.

We choose a local frame field $e_1, \cdots, e_n$ at each point of M^n, such that $h_{ij} = \lambda_i \delta_{ij}$. From (3.2) and (2.11), we have

$$\begin{aligned} \square(nH) &= nH\Delta(nH) - \sum_i \lambda_i (nH)_{ii} \\ &= \tfrac{1}{2}\Delta(nH)^2 - \sum_i (nH)_i^2 - \sum_i \lambda_i (nH)_{ii} \\ &= \tfrac{1}{2}\Delta|h|^2 - n^2|\bigtriangledown H|^2 - \sum_i \lambda_i (nH)_{ii}. \end{aligned} \tag{3.4}$$

From (2.12) and (2.13), by a standard and direct calculation, we have

$$\frac{1}{2}\Delta|h|^2 = \sum_{i,j,k} h_{ijk}^2 + \sum_i \lambda_i (nH)_{ii} + \frac{1}{2}\sum_{i,j} R_{ijij}(\lambda_i - \lambda_j)^2, \tag{3.5}$$

where $R_{ijij} = -1+\lambda_i\lambda_j (i \neq j)$ denotes the sectional curvature of the section spanned by $\{e_i, e_j\}$. From (3.4) and (3.5), we get

$$\Box(nH) = |\bigtriangledown h|^2 - n^2 |\bigtriangledown H|^2 + \frac{1}{2}\sum_{i,j}(-1+\lambda_i\lambda_j)(\lambda_i - \lambda_j)^2. \tag{3.6}$$

The following Lemma 3.1 due to [8] is useful in our proof.

Lemma 3.1([8]) *Let M^n be an n-dimensional hypersurface in H^{n+1}. Suppose that the scalar curvature $n(n-1)R$ is constant and $R+1 \geq 0$. Then $|\bigtriangledown h|^2 \geq n^2 |\bigtriangledown H|^2$.*

From Lemma 3.1 and (3.6) we get

$$\Box(nH) \geq \frac{1}{2}\sum_{i,j}(-1+\lambda_i\lambda_j)(\lambda_i - \lambda_j)^2. \tag{3.7}$$

On the other hand,

$$\begin{aligned} \Box(nH) &= \sum_{i,j}(nH\delta_{ij} - h_{ij})(nH)_{ij} \\ &= \sum_{i}(nH - h_{ii})(nH)_{ii} = n\sum_i H(nH)_{ii} - \sum_i \lambda_i (nH)_{ii} \\ &\leq (n|H|_{\max} - C)\Delta(nH), \end{aligned} \tag{3.8}$$

where $|H|_{\max}$ is the maximum of the mean curvature H and C is the minimum of the principal curvatures $\{\lambda_i\}_{i=1}^n$ of M^n.

Proof of the Theorem 3.2. We need the Generalized Maximum principle due to Omori [11] and Yau[14].

Lemma 3.2([11][14]) *Let M^n be complete Riemannian manifold whose Ricci curvature is bounded from below. If F is a C^2- function bounded from above on M^n, then for any $\varepsilon > 0$, there is a point $x \in M^n$ such that*

$$\sup F - \varepsilon < F(x), \quad \|gradF\|(x) < \varepsilon, \quad \Delta F(x) < \varepsilon. \tag{3.9}$$

We consider the following smooth function on M^n defined by $F = -(f^2+a)^{-1/2}$, where $a(>0)$ is a real number, f is a nonnegative C^2-function on M^n. Since M^n has nonnegative sectional curvature, this implies the Ricci curvature of M^n is bounded from below by zero. Obviously, F is bounded from above, so we can apply Lemma 3.2 to F. For any $\varepsilon > 0$, there is a point $x \in M^n$, such that at which F satisfies the properties (3.9) in Lemma 3.2. By a simple and direct calculation, we have

$$F \bigtriangleup F = 3\|dF\|^2 - \frac{1}{2}F^4 \Delta f^2. \tag{3.10}$$

From (3.9) and (3.10)

$$\frac{1}{2}F^4(x)\Delta f^2(x) = 3\|dF\|^2(x) - F(x)\Delta F(x) < 3\varepsilon^2 - \varepsilon F(x). \tag{3.11}$$

Therefore, for any convergent $\{\varepsilon_m\}$, with $\varepsilon_m > 0$ and $\lim_{m\to\infty} \varepsilon_m = 0$, there exists a point sequence $\{x_m\}$ such that the sequence $\{F(x_m)\}$ converges to F (we can take a subsequence if necessary) and satisfies (3.9). Thus $\lim_{m\to\infty}\varepsilon_m[3\varepsilon_m - F(x_m)] = 0$. From the definition of supremum and (3.9), we have $\lim_{m\to\infty} F(x_m) = F_0 = \sup F$ and hence the definition of F give rise to $\lim_{m\to\infty} f(x_m) = f_0 = \sup f$.

Now we set $f = \sqrt{nH}$. So $\lim_{m\to\infty}(nH)(x_m) = \sup(nH)$, thus, by (2.11) $\lim_{m\to\infty}|h(x_m)|^2 = \sup|h|^2$. Since $|h|^2 = \sum_i \lambda_i^2$ is bounded, any principal curvature λ_i is bounded and hence so is any sequence $\{\lambda_i(x_m)\}$. Then there exists a subsequence $\{x_{m'}\}$ of $\{x_m\}$ such that for some λ_{i0} and any i

$$\lim_{m'\to\infty}\lambda_i(x_{m'}) = \lambda_{i0}. \tag{3.12}$$

In fact, since a sequence $\{\lambda_1(x_m)\}$ is bounded, it converges to some λ_{10} by taking a subsequence $\{x_{m_1}\}$ if necessary. For the point sequence $\{x_{m_1}\}$, a sequence $\{\lambda_2(x_{m_1})\}$ is also bounded and hence there is a subsequence $\{x_{m_2}\}$ of $\{x_{m_1}\}$ such that $\{\lambda_2(x_{m_2})\}$ converges to some λ_{20} as m_2 tends to infinity. Thus we can inductively show that there exists a point sequence $\{x_{m'}\}$ of $\{x_m\}$ such that the property (3.12) holds. Hence for the subsequence $\{x_{m'}\}$ of $\{x_m\}$, by (3.7), (3.8) and (3.11) we have

$$\begin{aligned} 0 &\le \tfrac{1}{4}F^4(x_{m'})\sum_{i,j}[-1+\lambda_i(x_{m'})\lambda_j(x_{m'})][\lambda_i(x_{m'})-\lambda_j(x_{m'})]^2 \\ &\le \tfrac{1}{2}F^4(x_{m'})\Box[nH(x_{m'})] \le (n|H|_{\max}-C)\tfrac{1}{2}F^4(x_{m'})\Delta(nH)(x_{m'}) \\ &\le (n|H|_{\max}-C)[3\varepsilon_{m'}^2 - \varepsilon_{m'}F(x_{m'})]. \end{aligned} \tag{3.13}$$

Let m' tends to infinity in (3.13), we have

$$(-1+\lambda_{i0}\lambda_{j0})(\lambda_{i0}-\lambda_{j0})^2 = 0, \tag{3.14}$$

for any distinct indices i and j. By a simple algebraic calculation it is easily seen that the number of distinct limits in $\{\lambda_{i0}\}$ is at most two.

Case(i). If all limits λ_{i0} coincide with each other, we set $\lambda_{i0} = \lambda_0$ for all i. Because $|h|^2 - nH^2 = \sum_i \lambda_i^2 - \frac{1}{n}(\sum_i \lambda_i)^2 = \frac{1}{n}\sum_{i<j}(\lambda_i-\lambda_j)^2$, then $\lim_{m'\to 0}(|h|^2 - nH^2)(x_{m'}) = 0$. On the other hand, by (2.11) we have

$$|h|^2 - nH^2 = \frac{n-1}{n}[|h|^2 - n(R+1)], \tag{3.15}$$

Hence, $0 = \lim_{m'\to 0}(|h|^2 - nH^2)(x_{m'}) = \frac{n-1}{n}[\sup|h|^2 - n(R+1)]$, we get $\sup|h|^2 = n(R+1)$. From (3.15) we have $\sup(|h|^2 - nH^2) = \frac{n-1}{n}[\sup|h|^2 - n(R+1)] = 0$ i.e. $|h|^2 = nH^2$, M is a totally umbilical hypersurface.

Case(ii) If $\{\lambda_{i0}\}$ has exactly two distinct elements, without loss of the generality, we may suppose that

$$\lambda_{10} = \cdots = \lambda_{l0} = \lambda, \quad \lambda_{l+10} = \cdots = \lambda_{n0} = \mu, \quad \lambda \ne \mu,$$

for some $l = 1, 2, \cdots, n-1$. From (3.14) we have

$$\lambda\mu = 1. \tag{3.16}$$

If $l \ge 2, n-l \ge 2$. From $R_{ijij}(x_{m'}) = (-1+\lambda_i\lambda_j)(x_{m'}) \ge 0$, we have

$$-1+\lambda^2 \geq 0, \qquad -1+\mu^2 \geq 0. \tag{3.17}$$

By (3.16), (3.17) we get $1=\lambda^2\mu^2 \geq \mu^2$ and $1=\lambda^2\mu^2 \geq \lambda^2$. Hence, from (3.17) again we have $\lambda^2=\mu^2=1$. Since $\lambda \neq \mu$, we have $\lambda=-\mu$. Taking this into (3.16), we know that $-\mu^2=1$, this is a contradiction. Therefore, we must have $l=1$, or $n-l=1$. If $l=1$, by (2.11) we have

$$\begin{aligned} n(n-1)(R+1) &= \lim_{m'\to\infty}[n^2H^2(x_{m'})-|h|^2(x_{m'})] \\ &= \lim_{m'\to\infty}\{[\sum_i \lambda_i(x_{m'})]^2-\sum_i \lambda_i^2(x_{m'})\} \\ &= [\lambda+(n-1)\mu]^2-[\lambda^2+(n-1)\mu^2] \\ &= 2(n-1)\lambda\mu+(n-1)(n-2)\mu^2. \end{aligned} \tag{3.18}$$

From (3.16), (3.18) we have $\mu^2=\frac{n(R+1)-2}{n-2}, \lambda^2=\frac{n-2}{n(R+1)-2}$. Hence

$$\begin{aligned} \sup|h|^2 &= \lim_{m'\to\infty}|h|^2=\lim_{m'\to\infty}[\sum_i \lambda_i^2(x_{m'})]=\lambda^2+(n-1)\mu^2 \\ &= \frac{n-2}{n(R+1)-2}+(n-1)\frac{n(R+1)-2}{n-2}. \end{aligned} \tag{3.19}$$

We set $\bar{R}=R+1$. Then

$$\sup|h|^2=\frac{n}{(n-2)(n\bar{R}-2)}[n(n-1)\bar{R}^2-4(n-1)\bar{R}+n]. \tag{3.20}$$

If $n-l=1$, by making use of the similar methods above, we know that (3.20) holds. Therefore, by the result due to Liu and Su[8], we have M^n is isometric to $S^{n-1}(r)\times H^1(-1/(r^2+1))$, for some $r>0$. This completes the proof of Theorem 3.2.

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