The ergodic shadowing property for robust and generic volume-preserving diffeomorphisms

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Abstract. In this paper, we show the followings: (i) If a volume preserving diffeomorphism f belongs to the C^1-interior of the set of all volume preserving diffeomorphims having the ergodic shadowing property then it is transitive Anosov. Moreover, (ii) if a C^1-generic volume-preserving diffeomorphism f has the ergodic shadowing property then it is transitive Anosov.

1 Introduction

A main research of dynamical systems is the behavior of the orbits. It is very close to the shadowing theory. Roughly speaking, the shadowing theory means that for given a pseudo orbit, there is a true orbit. So, the notion used to study of the stability theory (see [23, 25]). From the fact, many researchers have been using the various shadowing properties to investigate for the stability properties, that is, structurally stable, hyperbolic, Axiom A, etc(see [6, 20, 21, 22]). For that, we consider the volume-preserving diffeomorphism case. Recently, we can found the results of the volume-preserving diffeomorphism which has the various shadowing properties(see [4, 8, 11, 12, 13]). It is a motivation of the paper. We consider the special shadowing property which is called the ergodic shadowing property. For the ergodic shadowing property, many results published in [3, 7, 9, 10, 14, 15, 16]. Therefore, in this paper, we study the relation between the ergodic shadowing and hyperbolcity.

The paper is constructed as follows: in section 2, we give the definitions and introduce main theorems. In section 3, under the robust condition, we show that if the system has the ergodic shadowing property then it is Anosov. In section 4, we show that C^1-generically, if the system has the ergodic shadowing property then it is Anosov.

2   Basic notions and main theorems

Let M be a d-dimensional ($d \geq 2$) Riemannian closed and connected manifold and let $d(\cdot, \cdot)$ denotes the distance on M inherited by the Riemannian structure. We endow M with a volume-form (cf. [18]) and let μ denote the Lebesgue measure related to it. Let $\mathrm{Diff}_\mu^1(M)$ denote the set of volume-preserving diffeomorphisms defined on M. Consider this space endowed with the C^1 Whitney topology. The Riemannian inner-product induces a norm $\|\cdot\|$ on the tangent bundle $T_x M$. We will use the usual uniform norm of a bounded linear map A given by $\|A\| = \sup_{\|v\|=1} \|Av\|$. We say that a closed f-invariant set Λ is *hyperbolic* if the tangent bundle $T_\Lambda M$ has a Df-invariant splitting $E^s \oplus E^u$ and there exist constants $C > 0$ and $0 < \lambda < 1$ such that

$$\|D_x f^n|_{E_x^s}\| \leq C\lambda^n \text{ and } \|D_x f^{-n}|_{E_x^u}\| \leq C\lambda^n$$

for all $x \in \Lambda$ and $n \geq 0$. If $\Lambda = M$ then f is Anosov.

For a point $x \in M$, we say that x is a *non-wandering point* if for any neighborhood U of x, there is $n \in \mathbb{Z}$ such that $f^n(U) \cap U \neq \varnothing$. Denote by $\Omega(f)$ the set of all non-wandering points of f. It is clear $P(f) \subset \Omega(f)$, where $P(f)$ is the set of periodic points of f, and $\overline{P(f)}$ is the closure of $P(f)$. We say that f satisfies *Axiom A* if $\Omega(f) = \overline{P(f)}$ is hyperbolic. In the volume preserving case, by Poincaré Recurrence Theorem, we have $\Omega(f) = M$. Thus if f satisfies the Axiom A then f is Anosov. Denote by $\mathcal{F}_\mu(M)$ the set of diffeomorphisms $f \in \mathrm{Diff}_\mu(M)$ which has a C^1-neighborhood $\mathcal{U}(f) \subset \mathrm{Diff}_\mu(M)$ such that if for any $g \in \mathcal{U}(f)$, every periodic point of g is hyperbolic. Note that $\mathcal{F}_\mu(M) \subset \mathcal{F}(M)$ (see [2, Corollary 1.2]). Arbieto and Catalan [2] proved that if a volume preserving diffeomorphism is contained in $\mathcal{F}_\mu(M)$ then it is Anosov. We can restate as follows.

Theorem 2.1. [2, Theorem 1.1] *If $f \in \mathcal{F}_\mu(M)$ then f is Anosov.*

For $\delta > 0$, a sequence of points $\{x_i\}_{i=a}^b (-\infty \leq a < b \leq \infty)$ in M is called a δ-*pseudo-orbit* of f if $d(f(x_i), x_{i+1}) < \delta$ for all $a \leq i \leq b-1$. We say that f has the *shadowing property* if for every $\epsilon > 0$ there is $\delta > 0$ such that for any δ-pseudo-orbit $\{x_i\}_{i=a}^b$ of $f(-\infty \leq a < b \leq \infty)$, there is a point $y \in M$ such that $d(f^i(y), x_i) < \epsilon$ for all $a \leq i \leq b-1$. Now, we introduce the notion of the ergodic shadowing property which was studied by [7]. For any $\delta > 0$, a sequence $\xi = \{x_i\}_{i \in \mathbb{Z}}$ is a δ-*ergodic pseudo orbit* of f if for $Np_n^+(\xi, f, \delta) = \{i : d(f(x_i), x_{i+1}) \geq \delta\} \cap \{0, 1, \ldots, n-1\}$, and $Np_n^-(\xi, f, \delta) = \{-i : d(f^{-1}(x_{-i}), x_{-i-1}) \geq \delta\} \cap \{-n+1, \ldots, -1, 0\}$

$$\lim_{n \to \infty} \frac{\#Np_n^+(\xi, f, \delta)}{n} = 0 \text{ and } \lim_{n \to -\infty} \frac{\#Np_n^-(\xi, f, \delta)}{n} = 0.$$

Here $\#A$ is the number of elements of the set A. We say that f has the *ergodic shadowing property* if for any $\epsilon > 0$, there is a $\delta > 0$ such that every δ-ergodic pseudo orbit $\xi = \{x_i\}_{i \in \mathbb{Z}}$ of f there is a point $z \in M$ such that for $Ns_n^+(\xi, f, z, \epsilon) = \{i : d(f^i(z), x_i) \geq \epsilon\} \cap \{0, 1, \ldots, n-1\}$, and $Ns_n^-(\xi, f, z, \epsilon) = \{-i : d(f^{-i}(z), x_{-i}) \geq \epsilon\} \cap \{-n+1, \ldots, -1, 0\}$,

$$\lim_{n \to \infty} \frac{\#Ns_n^+(\xi, f, z, \epsilon)}{n} = 0 \text{ and } \lim_{n \to -\infty} \frac{\#Ns_n^-(\xi, f, z, \epsilon)}{n} = 0.$$

We say that f is *transitive* if for any non-empty open sets U and V, there is $n > 0$ such that $f^n(U) \cap V \neq \varnothing$. Equivalently, there is $x \in M$ such that $\omega(x) = M$, where $\omega(x)$ is the omega limit set of x. We say that f is *mixing* if for any non-empty open sets U and V, there is $n > 0$ such that $f^i(U) \cap V \neq \varnothing$ for all $i \geq n$. Clearly, if f is mixing then it is transitive.

Note that if f is a Morse-Smale diffeomorphism then it has the shadowing property, and f has sinks and sources. But, if f has the ergodic shadowing property then it does not contain sinks nor sources (see [7, Corollary 3.5]). A transitive diffeomorphism has the shadowing property if and only if the diffeomorphism has the ergodic shadowing property (see [7, Theorem A]). For the ergodic shadowing property, Lee [15] showed that if the homoclinic class satisfies a local star condition and which is erogidc shadowing then it is hyperbolic. Here we say that a closed f-invariant set Λ satisfies the *local star condition* if there are a C^1-neighborhood $\mathcal{U}(f)$ and a neighborhood U of Λ such that for any $g \in \mathcal{U}(f)$, every periodic points in $\Lambda_g(U) = \bigcap_{n \in \mathbb{Z}} g^n(U)$ is hyperbolic.

Definition 2.1. We say that f has the C^1-*robustly ergodic shadowing property* if there is a C^1-neighborhood $\mathcal{U}(f)$ of f such that for any $g \in \mathcal{U}(f)$, g has the ergodic shadowing property.

Lee [14] showed that if f has the C^1-robustly erogodic shadowing property then it is structurally stable, Lee [16] and Barzanouni*et al* [3] showed that if f has the C^1-robustly erogodic shadowing property then it is transitive Anosov. For that, we have

Theorem A. Let $f \in \mathrm{Diff}_\mu(M)$. If f has the C^1-robustly ergodic shadowing property then it is Anosov.

A subset $\mathcal{R} \subset \mathrm{Diff}_\mu(M)$ is called *residual* if it contains a countable intersection of open and dense subsets of $\mathrm{Diff}_\mu(M)$. A dynamic property is called C^1*generic* if it holds in a residual subset of $\mathrm{Diff}_\mu(M)$. We use the terminology *for C^1-generic f* to express *there is a residual subset $\mathcal{R} \subset \mathrm{Diff}_\mu(M)$, and $f \in \mathcal{R}$*. Lee [16] showed that if C^1-generically, f has the ergodic shadowing property then it is Anosov. For that, we have

Theorem B. Let $\dim M \geq 3$. For C^1-generic $f \in \mathrm{Diff}_\mu(M)$, if f has the ergodic shadowing property, then f is mixing Anosov.

For any $p \in P(f)$, we have the followings: (i) p is hyperbolic saddle, (ii) p is an elliptic points, that is, nonreal eigenvalues are conjugated and of norm 1, and (iii) p is a parabolic point, that is, the eigenvalues equal 1 or -1. Robinson [24] showed that if $\dim M = 2$ then there is a residual set in $\mathrm{Diff}_\mu(M)$ such that any elementary in this residual displays all its elliptic points of elementary type. Here, we say that p is an elementary point if $D_p f$ has simple spectrum, and non of eigenvalues are root of unity or equal to 1. Newhouse [19] showed that C^1-generic volume-preserving diffeomorphisms in two dimensional manifold are either Anosov or the elliptic points are dense. For the results, we suggest the following problem: *For C^1-generic $f \in \mathrm{Diff}_\mu(M^2)$, if f has the ergodic shadowing property then is it Anosov?*

3   Proof of Theorem A

Let M be as before, and let $f \in \mathrm{Diff}_\mu(M)$. The following version of the Franks' lemma for the conservative case which is stated and proved in [5, Proposition 7.4].

Lemma 3.1. *Let $f \in \mathrm{Diff}_\mu(M)$, and $\mathcal{U}(f)$ be a C^1-neighborhood of f in $\mathrm{Diff}^1_\mu(M)$. Then there exist a C^1-neighborhood $\mathcal{U}_0(f) \subset \mathcal{U}(f)$ of f and $\epsilon > 0$ such that if $g \in \mathcal{U}_0(f)$, any finite f-invariant set $E = \{x_1, \ldots, x_m\}$, any neighborhood U of E and any volume-preserving linear maps $L_j : T_{x_j}M \to T_{g(x_j)}M$ with $\|L_j - D_{x_j}g\| \le \epsilon$ for all $j = 1, \ldots, m$, there is a conservative diffeomorphism $g_1 \in \mathcal{U}(f)$ coinciding with f on E and out of U, and $D_{x_j}g_1 = L_j$ for all $j = 1, \ldots, m$.*

Remark 3.1. By the definition of the ergodic shadowing property, we have the followings:

(a) The identity map does not have the ergodic shadowing property.

(b) Let $\Lambda \subset M$. If f has the ergodic shadowing property then f has the ergodic shadowing property on Λ.

From the Moser's Theorem *(see [18])*, there is a smooth conservative change of coordinates $\varphi_x : U(x) \to T_xM$ such that $\varphi_x(x) = \overrightarrow{0}$, where $U(x)$ is a small neighborhood of $x \in M$.

Lemma 3.2. *Suppose that f has the C^1-robustly ergodic shadowing property. Then there is a C^1-neighborhood $\mathcal{U}(f)$ of f such that for any $g \in \mathcal{U}(f)$, every periodic points of g is hyperbolic.*

Proof. Suppose that f has the C^1-robustly ergodic shadowing property. Let $\mathcal{U}(f) \subset \mathrm{Diff}_\mu(M)$ be a C^1-neighborhood of f. Then for any $g \in \mathcal{U}(f)$, g has the ergodic shadowing property. To derive a contradiction, we may assume that there is $g \in \mathcal{U}(f)$ such that g has a nonhyperbolic periodic point p. For simplicity, we assume that $g(p) = p$.

Then there is at least one eigenvalue λ of D_pg such that $|\lambda| = 1$, and $T_pM = E^s_p \oplus E^u_p \oplus E^c_p$, where E^s_p is the eigenspace corresponding to the eigenvalues of the smaller than 1, and E^u_p is the eigenspace corresponding to the eigenvalues of the greater than 1, and E^c_p the eigenspace corresponding to λ. Then we see that if $\lambda \in \mathbb{R}$ then $\dim E^c_p = 1$, and if $\lambda \in \mathbb{C}$ then $\dim E^c_p = 2$.

First, we consider $\dim E^c_p = 1$. For simplicity, we may assume that $\lambda = 1$ (the other case is similar). By Lemma 3.1, we linearize g at p with respect to Moser's Theorem; that is, by choosing $\alpha > 0$ sufficiently small we construct g_1 C^1-nearby g such that

$$g_1(x) = \begin{cases} \varphi_p^{-1} \circ D_pg \circ \varphi_p(x) & \text{if} \quad x \in B_\alpha(p), \\ g(x) & \text{if} \quad x \notin B_{4\alpha}(p). \end{cases}$$

Then $g_1(p) = g(p) = p$. Since the eigenvalue λ of D_pg_1 is 1, we can take $\eta = \alpha/4$ such that $D_pg_1(v) = v$ for any $v \in E^c_p(\eta)$. Take $v_0 \in E^c_p(\eta)$ such that $\|v_0\| = \eta/4$. We set

$$\mathcal{I}_{v_0} = \{t \cdot v_0 : 1 \le t \le 1 + \eta/4\} \subset \varphi_p(B_\eta(p)),$$

and $\varphi_p^{-1}(\mathcal{I}_{v_0}) = \mathcal{J}_p$. Since $g_1(\mathcal{J}_p) = \mathcal{J}_p$ is the identity map, $\varphi_p^{-1}(\mathcal{I}_{v_0}) = \mathcal{J}_p$ is g_1-invariant and by the construction of $\mathcal{J}_p$ is normally hyperbolic. Since g_1 has the

ergodic shadowing property, by Remark 3.1(a) g_1 must have the ergodic shadowing property on $\mathcal{J}_p$. Since $g_1 : \mathcal{J}_p \to \mathcal{J}_p$ is the identity map, by Remark 3.1(b) g_1 does not have the ergodic shadowing property on $\mathcal{J}_p$. This is a contradiction.

Finally, if $\lambda \in \mathbb{C}$, then $\dim E_p^c = 2$. For simplicity, we may assume that $g(p) = p$. As in the first case, by Lemma 3.1, there are $\alpha > 0$ and $g_1 \in \mathcal{V}(f)$ such that $g_1(p) = g(p) = p$ and

$$g_1(x) = \begin{cases} \varphi_p^{-1} \circ D_p g \circ \varphi_p(x) & \text{if} \quad x \in B_\alpha(p), \\ g(x) & \text{if} \quad x \notin B_{4\alpha}(p). \end{cases}$$

With a C^1-small modification of the map $D_p g$, we may suppose that there is $l > 0$ (the minimum number) such that $D_p g^l(v) = v$ for any $v \in \varphi_p(B_\alpha(p)) \subset T_p M$. Take $v_0 \in \varphi_p(B_\alpha(p))$ such that $\|v_0\| = \alpha/4$, and set

$$\mathcal{L}_p = \varphi_p^{-1}(\{t \cdot v_0 : 1 \le t \le 1 + \alpha/4\}).$$

Then $\mathcal{L}_p$ is an arc such that

· $g_1^i(\mathcal{L}_p) \cap g_1^j(\mathcal{L}_p) = \varnothing$ for $0 \le i \neq j \le l - 1$,

· $g_1^l(\mathcal{L}_p) = \mathcal{L}_p$, and

· $g_1^l|_{\mathcal{L}_p}$ is the identity map.

Note that g_1 has the ergodic shadowing property if and only if g_1^k has the ergodic shadowing property, for all $k \in \mathbb{Z}$ (see [7, Proposition 3.3]). As in the previous arguments, we can show that g_1^l does not have the ergodic shadowing property on $\mathcal{L}_p$, which contradicts the fact that $g_1 \in \mathcal{U}(f)$. Thus, if f has the C^1-robustly ergodic shadowing property, every periodic point of f is hyperbolic. □

Proof of Theorem A. Since f has the C^1-robustly ergodic shadowing property, By Lemma 3.2, there is a C^1-neighborhood $\mathcal{U}(f)$ of f such that for any $g \in \mathcal{U}(f)$, every $p \in P(g)$ is hyperbolic. This means that $f \in \mathcal{F}_\mu(M)$. Thus by Theorem 2.1, f is Anosov. □

4 Proof of Theorem B.

Let $\dim M \ge 3$. Denote by $\mathcal{ES}_\mu(M) \subset \text{Diff}_\mu(M)$ the set of all volume preserving diffeomorphisms having the ergodic shadowing property.

If f is transitive, f does not contains sinks nor sources. Thus every $p \in P(f)$ is saddle. Let p be a hyperbolic periodic point of f. Then there are a C^1-neighborhood $\mathcal{U}(f)$ of f and a neighborhood U of p such that for any $g \in \mathcal{U}(f)$, there is an unique p_g, where p_g called the continuation of p. Let $p \in P(f)$ be a hyperbolic saddle with period $\pi(p) > 0$, then there are the local stable manifold $W_\epsilon^s(p)$ and the local unstable manifold $W_{\epsilon(p)}^u(p)$ for some $\epsilon = \epsilon(p) > 0$. Then we see that if $x \in W_\epsilon^s(p)$, then $d(f^i(x), f^i(p)) \le \epsilon$, for $i \ge 0$ and if $x \in W_\epsilon^u(p)$ then $d(f^{-i}(x), f^{-i}(p)) \le \epsilon$ for $i \ge 0$. The stable manifold $W^s(p)$ and the unstable manifold $W^u(p)$ of p are defined as usual. The dimension of the stable manifold $W^s(p)$ is called *index of p*, and we denote it by $\text{index}(p)$. The following was proved by [16, Lemma 2.4] for diffeomorphisms. For

the volume-preserving diffeomorphisms, the proof is analogue. By [7, Corollary 3.5], If $f \in \mathcal{ES}_\mu(M)$ then it is mixing. Then, we have the following.

Lemma 4.1. *Let $p, q \in P(f)$ be hyperbolic. If $f \in \mathcal{ES}_\mu(M)$ then $W^s(p) \cap W^u(q) \neq \varnothing$, and $W^u(p) \cap W^s(q) \neq \varnothing$.*

A diffeomorphism $f \in \mathrm{Diff}_\mu(M)$ is said to be *Kupka-Smale* if any element of $P(f)$ is hyperbolic, and its invariant manifolds intersect transversely. The Kupka-Smale volume preserving diffeomorphisms given by Robinson's theorem (see [24]). Denote by $\mathcal{KS}_\mu$ the set of all Kupka-Smale volume preserving diffeomorphisms.

Lemma 4.2. *There is a residual set $\mathcal{R}_1 \subset \mathrm{Diff}_\mu(M)$ such that any $f \in \mathcal{R}_1$, if $f \in \mathcal{ES}_\mu(M)$ then for any hyperbolic $p, q \in P(f)$, index$(p) =$ index(q).*

Proof. Let $f \in \mathcal{R}_1 = \mathcal{KS}_\mu$ have the ergodic shadowing property, and let $p, q \in P(f)$ be hyperbolic. Suppose, by contradiction, that index$(p) \neq$ index(q). Then we have

$$\dim W^s(p) + \dim W^u(q) < \dim M \text{ or } \dim W^u(p) + \dim W^s(q) < \dim M.$$

Without loss of generality, we assume that $\dim W^s(p) + \dim W^u(q) < \dim M$ (other case is similar). Since $f \in \mathcal{R}_1$, we can see $W^s(p) \cap W^u(q) = \varnothing$. This is a contradiction by Lemma 4.1. $\qquad\square$

The following was proved by [4]. However, the paper is still not published yet. For convenience, we give a sketch of proof.

Lemma 4.3. *Let $\mathcal{U}(f) \subset \mathrm{Diff}_\mu(M)$ be a C^1-neighborhood of f. If $p \in P(f)$ is not hyperbolic then there is $g \in \mathcal{U}(f)$ such that g has two hyperbolic periodic points q, r with index$(q) \neq$ index(r).*

Proof. Let $p \in P(f)$ be the non-hyperbolic with the period $\pi(p)$. Then we have $T_p M = E_p^s \oplus E_p^c \oplus E_p^u$, where E_p^s is the eigenspace corresponding to the eigenvalues with modulus less than 1, E_p^c is the eigenspace corresponding to the eigenvalues with modulus equal 1, and E_p^u is the eigenspace corresponding to the eigenvalues with modulus grater than 1. Using Lemma 3.1, there is g C^1-close to f such that an $g^{\pi(p)}$-invariant small curve $\mathcal{L}_p$. Take two points $q, r \in \mathcal{L}_p$ such that the points q, r are the endpoints of the curve $\mathcal{L}_p$. Since p is not hyperbolic,

$$D_q g_{|E_p^c}^{\pi(p)} = D_r g_{|E_p^c}^{\pi(p)} = D_p f_{|E_p^c}^{\pi(p)} = 1.$$

Again use Lemma 3.1, there is g_1 C^1-close to f (also, g_1 C^1-close to g) such that g_1 has two hyperbolic periodic points q_{g_1}, r_{g_1} with index$(q_{g_1}) \neq$ index(r_{g_1}). $\qquad\square$

The following due to [17, Lemma 2.2] for diffeomorphisms case and [12, Lemma 8] for conservative systems case.

Lemma 4.4. *There is a residual set $\mathcal{R}_2 \subset \mathrm{Diff}(M)$ such that for any $f \in \mathcal{R}_2$, if for any C^1-neighborhood $\mathcal{U}(f)$ of f, there exists $g \in \mathcal{U}(f)$ such that two hyperbolic periodic points $p_g, q_g \in P(g)$ with index$(p_g) \neq$ index(q_g), then f has two hyperbolic periodic points $p, q \in P(f)$ with index$(p) \neq$ index(q).*

Let p be a periodic point of f. For any $\delta \in (0,1)$, we say that p has a δ-*weak eigenvalue* if $D_p f^{\pi(p)}$ has an eigenvalue λ such that $(1-\delta)^{\pi(p)} < |\lambda| < (1+\delta)^{\pi(p)}$. The following is due to Arbieto [1, Lemma 5.1]

Lemma 4.5. *There is a residual set $\mathcal{R}_3 \subset \mathrm{Diff}_\mu(M)$ such that for any $f \in \mathcal{R}_3$, for any $\delta > 0$, if for any C^1-neighborhood $\mathcal{U}(f)$ there is $g \in \mathcal{U}(f)$ such that g has a hyperbolic periodic point p_g with a δ-weak eigenvalue then f has a hyperbolic periodic point p with a 2δ-weak eigenvalue.*

Lemma 4.6. *There is a residual set $\mathcal{R}_4 \subset \mathrm{Diff}_\mu(M)$ such that for any $f \in \mathcal{R}_4$, if f has the ergodic shadowing property then there is $\delta > 0$ such that for any $p \in P(f)$, p does not have a δ-weak eigenvalue.*

Proof. Let $\mathcal{R}_4 = \mathcal{R}_1 \cap \mathcal{R}_2$ and let f has the ergodic shadowing property. To derive a contradiction, we assume that for any $\delta > 0$ there is $p \in P(f)$ such that p has a δ-weak eigenvalue. By lemma 3.1, there is g C^1-close to f such that g has a non-hyperbolic periodic point q. Then by Lemma 4.3, there is g_1 C^1-close to g(also, C^1-close to f) such that g_1 has tow hyperbolic periodic points r, s with index$(r) \neq$ index(s). Since $f \in \mathcal{R}_2$, by Lemma 4.4, f has two hyperbolic periodic points r_f, s_f with index$(r_f) \neq$ index(s_f). This is a contradiction by Lemma 4.2. $\square$
as To prove Theorem B, it is enough to show that $f \in \mathcal{F}_\mu(M)$.

Lemma 4.7. *There is a residual set $\mathcal{R}_5 \subset \mathrm{Diff}_\mu(M)$ such that for any $f \in \mathcal{R}_5$, if f has the ergodic shadowing property, then $f \in \mathcal{F}_\mu(M)$.*

Proof. Let $\mathcal{R}_5 = \mathcal{R}_3 \cap \mathcal{R}_4$ have the erogodic shadowing property. To derive a contradiction, we assume that $f \notin \mathcal{F}_\mu(M)$. Then for any $\delta > 0$ there is g C^1-close to f such that p has a $\delta/2$-weak eigenvalue. By Lemma 4.6, f has a periodic point p_f with a δ-weak eigenvalue. This is a contradiction by Lemma 4.6. $\square$

Proof of Theorem B. Let $f \in \mathcal{R}_5$ have the ergodic shadowing property. Then by Lemma 4.7, $f \in \mathcal{F}_\mu(M)$. By Theorem 2.1, f is transitive Anosov. $\square$

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