Hypersurfaces in a quasi Kähler manifold

J. H. Bae, J. H. Park, K. Sekigawa and W. M. Shin

Abstract. Okumura gave a necessary and sufficient condition for an oriented real hypersurface of a Kähler manifold to be a contact metric manifold with respect to the naturally induced almost contact metric structure. In this paper, we discuss an oriented hypersurface of a quasi Kähler manifold and give a necessary and sufficient condition for such a hypersurface to be a quasi contact metric manifold with respect to the naturally induced almost contact metric structure and we provide an application.

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1 Introduction

Let $M = (M, \phi, \xi, \eta, g)$ be a (2n+1)-dimensional *almost contact metric manifold* with almost contact metric structure (ϕ, ξ, η, g):

$$\phi^2 = -I + \eta \otimes \xi, \qquad (1.1)$$
$$\phi\xi = 0, \quad \eta \circ \phi = 0, \quad \eta(\xi) = 1,$$

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y), \quad \eta(X) = g(\xi, X), \qquad (1.2)$$

for any $X, Y \in \mathfrak{X}(M)$, where $\mathfrak{X}(M)$ denotes the Lie algebra of all smooth vector fields on M. An almost contact metric manifold $M = (M, \phi, \xi, \eta, g)$ is called a contact metric manifold if it satisfies

$$d\eta(X, Y) = g(X, \phi Y), \qquad (1.3)$$

for any $X, Y \in \mathfrak{X}(M)$. In [7], the authors defined a new class of almost contact metric manifold, say, the case of quasi contact metric manifold, which was originally introduced by Tashiro [10]. By the definition, it follows immediately that the class of quasi contact metric manifolds is a generalization of the contact metric manifold case. In [1, 7], basic properties have been discussed. For example, it is shown that a quasi contact metric manifold is a contact manifold, in addition many fundamental formulas have been known, which are common with the ones on contact metric manifolds.

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The following fundamental question was originally raised by two of the authors in [7].

Question. Does there exist a quasi contact metric manifold of dimension $(2n+1)(\geqq 5)$ which is not a contact metric manifold?

In the present paper, concerning the above question, we shall discuss oriented real hypersurfaces in a quasi Kähler manifold and we prove the following:

Theorem 1.1. Let $\bar{M} = (\bar{M}, \bar{J}, \bar{g})$ be a quasi Kähler manifold and $M = (M, \phi, \xi, \eta, g)$ be an oriented hypersurface of $\bar{M}$ with the naturally induced almost contact metric structure (ϕ, ξ, η, g). Then, $M = (M, \phi, \xi, \eta, g)$ is a quasi contact metric manifold if and only if the following equality:

$$g((A\phi + \phi A)X, Y) + \eta(X)\eta(A\phi Y) - \eta(Y)\eta(A\phi X) = -2g(\phi X, Y) \tag{1.4}$$

holds for any $X, Y \in \mathfrak{X}(M)$, where A is the shape operator with respect to the unit normal vector field ν corresponding to the orientation of the hypersurface M in $\bar{M}$.

2 Preliminaries

In this section, we prepare several basic terminologies and fundamental formulas. An almost Hermitian manifold is called a *quasi Kähler manifold* if $(\bar{\nabla}_{\bar{X}}\bar{J})\bar{Y} + (\bar{\nabla}_{\bar{J}\bar{X}}\bar{J})(\bar{J}\bar{Y}) = 0$ holds for any $\bar{X}, \bar{Y} \in \bar{\mathfrak{X}}(\bar{M})$, where $\bar{\mathfrak{X}}(\bar{M})$ denotes the Lie algebra of all smooth vector fields on $\bar{M}$ [6]. Now, let $\bar{M} = (\bar{M}, \bar{J}, \bar{g})$ be a (2n+2)-dimensional quasi Kähler manifold and $M = (M, g)$ be a hypersurface in $\bar{M}$ with the induced Riemannian metric g which is oriented by unit normal vector field ν. We here denote by $\bar{\nabla}$ (resp. ∇) the Levi-Civita connection of the Riemannian metric $\bar{g}$ (resp. g). Then, the Gauss formula and the Weingarten formula are given respectively by

$$\bar{\nabla}_X Y = \nabla_X Y + \sigma(X, Y), \tag{2.1}$$

$$\bar{\nabla}_X \nu = -AX, \tag{2.2}$$

for $X, Y \in \mathfrak{X}(M)$, where σ is the second fundamental form and A is the shape operator with respect to the unit normal vector field ν and they are related by

$$\bar{g}(\sigma(X, Y), \nu) = g(AX, Y), \tag{2.3}$$

for any $X, Y \in \mathfrak{X}(M)$. We set

$$H(X, Y) = g(AX, Y), \tag{2.4}$$

for any $X, Y \in \mathfrak{X}(M)$. Then, we may check that H is a symmetric (0,2)-tensor field on M which is called the second fundamental tensor of the hypersurface M in $\bar{M}$. From (2.3) and (2.4), we have also

$$\sigma(X, Y) = H(X, Y)\nu, \tag{2.5}$$

for any $X, Y \in \mathfrak{X}(M)$. We denote by $\bar{R}$ (resp. R) the curvature tensor of $\bar{M}$ (resp. M), denoted respectively by $\bar{R}(\bar{X}, \bar{Y})\bar{Z} = [\bar{\nabla}_X, \bar{\nabla}_Y]Z - \bar{\nabla}_{[X,Y]}Z$ for $\bar{X}, \bar{Y}, \bar{Z} \in \mathfrak{X}(M)$ and $R(X,Y)Z = [\nabla_X, \nabla_Y]Z - \nabla_{[X,Y]}$.
Then, the Gauss and Codazzi equations are given respectively by

$$\bar{g}(\bar{R}(X,Y)Z,W) = g(R(X,Y)Z,W) + g(AX,Z)g(AY,W) - g(AY,Z)g(AX,W), \tag{2.6}$$

$$\bar{g}(\bar{R}(X,Y)Z,\nu) = g((\nabla_X A)Y,Z) - g((\nabla_Y A)X,Z), \tag{2.7}$$

for any $X, Y \in \mathfrak{X}(M)$. Now, we define vector field ξ, 1-form η and (1,1)-tensor field ϕ on M respectively by

$$\xi = -\bar{J}\nu, \quad \eta(X) = g(\xi, X), \tag{2.8}$$

$$\phi X = \bar{J}X - \eta(X)\nu, \tag{2.9}$$

for any $X \in \mathfrak{X}(M)$. Then, we may check that (ϕ, ξ, η, g) gives rise to an almost contact metric structure on M. We shall call it the naturally induced almost contact structure (with respect to the orientation defined by the unit normal vector field ν) on M. From (2.1)$\sim$(2.5), we have

$$\begin{aligned}(\bar{\nabla}_X\bar{J})Y &= \bar{\nabla}_X(\bar{J}Y) - \bar{J}(\bar{\nabla}_XY)\\ &= \bar{\nabla}_X(\phi Y + \eta(Y)\nu) - \bar{J}(\nabla_XY + g(AX,Y)\nu)\\ &= (\nabla_X\phi)Y + g(AX,Y)\xi - \eta(Y)AX + (g(AX,\phi Y) + (\nabla_X\eta)(Y))\nu,\end{aligned} \tag{2.10}$$

for any $X, Y \in \mathfrak{X}(M)$. Now, since $\bar{M}$ is a quasi Kähler manifold, we have

$$\begin{aligned}-(\bar{\nabla}_X\bar{J})Y &= (\bar{\nabla}_{\bar{J}X}\bar{J})\bar{J}Y\\ &= (\bar{\nabla}_{\phi X}\bar{J})\phi Y + \eta(Y)(\bar{\nabla}_{\phi X}\bar{J})\nu + \eta(X)(\bar{\nabla}_\nu\bar{J})\phi Y + \eta(X)\eta(Y)(\bar{\nabla}_\nu\bar{J})\nu,\end{aligned} \tag{2.11}$$

for any $X, Y \in \mathfrak{X}(M)$. Here, by making use of (2.10), we get

$$\begin{aligned}(\bar{\nabla}_{\phi X}\bar{J})\phi Y &= (\nabla_{\phi X}\phi)\phi Y + g(A\phi X,\phi Y)\xi + (g(A\phi X,\phi^2Y) + (\nabla_{\phi X}\eta)(\phi Y))\nu\\ &= (\nabla_{\phi X}\phi)\phi Y + g(A\phi X,\phi Y)\xi + (-g(A\phi X,Y) + \eta(Y)\eta(A\phi X) - \eta((\nabla_{\phi X}\phi)Y))\nu\end{aligned} \tag{2.12}$$

and similarly

$$\begin{aligned}(\bar{\nabla}_{\phi X}\bar{J})\nu &= -(\bar{\nabla}_X\bar{J})\xi + \eta(X)(\bar{\nabla}_\xi\bar{J})\xi\\ &= \phi\nabla_X\xi - \eta(AX)\xi + AX - \eta(X)\phi\nabla_\xi\xi + \eta(X)\eta(A\xi)\xi - \eta(X)A\xi,\end{aligned} \tag{2.13}$$

$$\begin{aligned}(\bar{\nabla}_\nu\bar{J})\phi Y &= -(\bar{\nabla}_{\bar{J}\nu}\bar{J})\bar{J}\phi Y\\ &= -(\bar{\nabla}_\xi\bar{J})Y + \eta(Y)(\bar{\nabla}_\xi\bar{J})\xi\\ &= -(\nabla_\xi\phi)Y - \eta(AY)\xi + \eta(Y)(\nabla_\xi\phi)\xi + \eta(Y)\eta(A\xi)\xi - (\eta(A\phi Y) + (\nabla_\xi\eta)(Y))\nu,\end{aligned} \tag{2.14}$$

$$(\bar{\nabla}_\nu\bar{J})\nu = -(\bar{\nabla}_{\bar{J}\nu}\bar{J})\bar{J}\nu = -(\nabla_\xi\phi)\xi - \eta(A\xi)\xi + A\xi. \tag{2.15}$$

Thus, from (2.10)∼(2.15), we get the following:

(2.16)
$$(\nabla_X\phi)Y+(\nabla_{\phi X}\phi)\phi Y+g(AX,Y)\xi+g(A\phi X,\phi Y)\xi+\eta(Y)\phi\nabla_X\xi-\eta(Y)\eta(AX)\xi$$
$$-\eta(X)\eta(Y)\phi\nabla_\xi\xi-\eta(X)(\nabla_\xi\phi)Y-\eta(X)\eta(AY)\xi+\eta(X)\eta(Y)\eta(A\xi)\xi=0,$$

$$\begin{aligned}(2.17)\qquad & g(X,(A\phi+\phi A)Y)+(\nabla_X\eta)(Y)+\eta(Y)\eta(A\phi X)\\ &-\eta((\nabla_{\phi X}\phi)Y)-\eta(Y)\eta(A\phi Y)-\eta(X)(\nabla_\xi\eta)(Y)=0,\end{aligned}$$

for any $X,Y\in\mathfrak{X}(M)$. Summing up the above arguments, we have the following:

Theorem 2.1 Let $\bar{M}=(\bar{M},\bar{J},\bar{g})$ be a quasi Kähler manifold and $M=(M,\phi,\xi,\eta,g)$ be an oriented hypersurface of $\bar{M}$ with the naturally induced almost contact metric structure (ϕ,ξ,η,g). Then, the equation (2.16) and (2.17) hold on M.

3 Proof of Theorem 1.1

In this section, we shall prove Theorem 1.1 based on the discussions in the previous sections. First, we recall the (1,1)-tensor field h on any almost contact metric manifold $M=(M,\phi,\xi,\eta,g)$ given by

$$(3.1)\qquad h=\frac{1}{2}\pounds_\xi\phi,$$

where $\pounds_\xi$ denotes the Lie derivative with respect to the characteristic vector field ξ. From (3.1), we may check that the following equalities hold on M

$$(3.2)\qquad h\xi=0,\quad trh=0.$$

We here recall the definition of the quasi contact metric manifold. An almost contact metric manifold $M=(M,\phi,\xi,\eta,g)$ is said to be a *quasi contact metric manifold* if the corresponding almost Hermitian cone to be a quasi Kähler manifold([6], Remark 1.1). The notion of quasi contact metric manifold was introduced by Tashiro([10] as an O*-contact metric manifold). A quasi contact metric manifold is a generalization of the contact metric manifold, and is characterized by the following.

Proposition 3.1 An almost contact metric manifold $M=(M,\phi,\xi,\eta,g)$ is called a quasi contact metric manifold if and only if it satisfies the following equality:

$$(3.3)\qquad (\nabla_X\phi)Y+(\nabla_{\phi X}\phi)\phi Y=2g(X,Y)\xi-\eta(Y)X-\eta(X)\eta(Y)\xi-\eta(Y)hX$$

for any $X,Y\in\mathfrak{X}(M)$.

Further, a quasi contact metric manifold $M=(M,\phi,\xi,\eta,g)$ is a contact metric manifold if and only if M satisfies the equality (3.3) and the tensor field h is symmetric with respect to the Riemannian metric g ([7]).
We may also note that the 1-form η of a quasi contact metric manifold is a contact 1-form [9]. Now let $M=(M,\phi,\xi,\eta,g)$ be a (2n+1)-dimensional quasi contact metric

manifold. Then, the following equalities in addition to the equalities (3.2) and (3.3) hold on M [3, 4]:

$$(\nabla_X \eta)(Y) + (\nabla_{\phi X}\phi)(\phi Y) + 2g(\phi X, Y) = 0, \tag{3.4}$$

$$\nabla_\xi \xi = 0, \quad \nabla_\xi \phi = 0, \tag{3.5}$$

$$\nabla_X \xi = -\phi X - \phi h X, \tag{3.6}$$

$$\eta \circ h = 0, \quad \phi h + h\phi = 0, \tag{3.7}$$

$$(\nabla_\xi h)X = \phi X - h^2\phi X - \phi R(X,\xi)\xi, \tag{3.8}$$

for $X \in \mathfrak{X}(M)$. We here note that the equalities (3.4)∼(3.8) can be derived from the equality (3.3). In the reminder of this section, we assume that $M = (M, \phi, \xi, \eta, g)$ is a $(2n+1)$-dimensional contact metric manifold and $\bar{M} = (\bar{M}, \bar{J}, \bar{g})$ is a $(2n+2)$-dimensional quasi Kähler manifold and $M = (M, \phi, \xi, \eta, g)$ be a quasi contact metric hypersurface of $\bar{M}$ with the naturally induced almost contact metric structure (ϕ, ξ, η, g) with respect to the unit normal vector field ν. Then, from (2.16) and (2.17), by making use of the equalities (3.2)∼(3.8), we have

$$\begin{aligned} 2g(X,Y) - 2\eta(X)\eta(Y) + g(AX,Y) + g(A\phi X, \phi Y) \\ - \eta(X)\eta(AY) - \eta(Y)\eta(AX) + \eta(X)\eta(Y)\eta(A\xi) = 0 \end{aligned} \tag{3.9}$$

and

$$g((A\phi + \phi A)X, Y) + \eta(X)\eta(A\phi Y) - \eta(Y)\eta(A\phi X) + 2g(\phi X, Y) = 0, \tag{3.10}$$

for any $X, Y \in \mathfrak{X}(M)$. Now, replacing X by ϕX in (3.9), we get

$$2g(\phi X, Y) + g(A\phi X, Y) - g(AX, \phi Y) + \eta(X)\eta(A\phi Y) - \eta(Y)\eta(A\phi X) = 0,$$

and hence

$$2g(\phi X, Y) + g((A\phi + \phi A)X, Y) + \eta(X)\eta(A\phi Y) - \eta(Y)\eta(A\phi X) = 0, \tag{3.11}$$

for any $X, Y \in \mathfrak{X}(M)$. (3.9) is nothing but (3.11). Similarly, replacing Y by ϕY in (3.10), we may derive (3.9). Namely, (3.9) and (3.10) are equivalent. Therefore, we have Theorem 1.1.

4 An application of Theorem 1.1

In this section, we provide a result obtained through discussions around Theorem 1.1 from the previous sections. Let $\bar{M} = (\bar{M}, \bar{J}, \bar{g})$ be a (2n+2)-dimensional nearly Kähler manifold and $M = (M, \phi, \xi, \eta, g)$ be an oriented hypersurface of $\bar{M}$ with the naturally induced contact metric structure (ϕ, ξ, η, g). Here, we assume that $M = (M, \phi, \xi, \eta, g)$

is a quasi contact metric manifold. By the hypothesis, taking account of (2.10) and (3.6), we have

$$(\nabla_X\phi)Y + (\nabla_Y\phi)X + 2g(AX,Y)\xi - \eta(Y)AX - \eta(X)AY = 0, \tag{4.1}$$

$$g(\phi AX,Y) + g(\phi AY,X) - (\nabla_X\eta)(Y) - (\nabla_Y\eta)X = 0, \tag{4.2}$$

for any $X, Y \in \mathfrak{X}(M)$. From (4.2), taking account of (3.6), we get

$$-g(\phi AX,Y) + g(A\phi X,Y) + g(({}^th\phi - \phi h)X,Y) = 0, \tag{4.3}$$

for any $X, Y \in \mathfrak{X}(M)$. Thus, from (4.3), we have

$$A\phi - \phi A +^t h\phi - \phi h = 0. \tag{4.4}$$

Thus, from (4.4), it follows that

$$\phi A\xi = 0$$

and there exists a smooth function λ on M such that

$$A\xi = \lambda\xi \tag{4.5}$$

holds. It is well-known that a nearly Kähler manifold is a quasi Kähler manifold. Thus, from (1.4) in Theorem and (4.5), it follows that

$$g((A\phi + \phi A)X,Y) = -2g(\phi X,Y) \tag{4.6}$$

holds for any $X, Y \in \mathfrak{X}(M)$. On the other hand, by setting $X = \xi$ in (4.1), taking account of (3.5), (3.6) and (4.5), we have

$$-Y - AY + (\lambda+1)\eta(Y)\xi - hY = 0, \tag{4.7}$$

for any $Y \in \mathfrak{X}(M)$. From (4.7), we have also

$$g(hY,Z) = -g(Y,Z) - g(AY,Z) + (\lambda+1)\eta(Y)\eta(Z), \tag{4.8}$$

for any $Y, Z \in \mathfrak{X}(M)$. Thus, from (4.8), it follows that h is symmetric with respect to the Riemannian metric g. Therefore, from Proposition 3.1, we see that $M = (M,\phi,\xi,\eta,g)$ is a contact metric manifold. Summing up the above arguments, we finally get the following:

Theorem 4.1 Let $\bar{M} = (\bar{M},\bar{J},\bar{g})$ be a nearly Kähler manifold and M be a hypersurface of $\bar{M}$ oriented by a unit normal vector field ν. Then $M = (M,\phi,\xi,\eta,g)$ is a quasi contact metric manifold with respect to the naturally induced almost contact metric structure (ϕ,ξ,η,g) if and only if it satisfies the equality:

$$g((A\phi + \phi A)X,Y) = -2g(\phi X,Y),$$

for any $X, Y \in \mathfrak{X}(M)$. Then ξ is an eigenvector field of the shape operator A with respect to the unit normal vector field ν and $M = (M,\phi,\xi,\eta,g)$ is a contact metric manifold.

5 Remarks

We here recall the following result by Okumura ([8], Theorem 3.1).

Theorem 5.1 Let $\bar{M} = (\bar{M}, \bar{J}, \bar{g})$ be a Kähler manifold and $M = (M, \phi, \xi, \eta, g)$ be a real hypersurface of $\bar{M}$ oriented by a unit normal vector field ν. Then $M = (M, \phi, \xi, \eta, g)$ is a contact metric manifold with respect to the naturally induced almost contact metric structure (ϕ, ξ, η, g) if and only if it satisfies the equality:

$$g((A\phi + \phi A)X, Y) = -2g(\phi X, Y),$$

for any $X, Y \in \mathfrak{X}(M)$, where A is the shape operator with respect to the unit normal vector field ν.

Thus, we see that Theorem 4.1 is generalization of the result of Okumura. From Theorem 4.1, we see that there does not exist an oriented totally geodesic hypersurface $M = (M, \phi, \xi, \eta, g)$ in a nearly Kähler manifold $\bar{M} = (\bar{M}, \bar{J}, \bar{g})$ which is a quasi contact metric manifold with respect to the naturally induced almost contact metric structure (ϕ, ξ, η, g). We shall also remark that there does not exist an oriented totally umbilic hypersurface in the nearly Kähler 6-sphere which is a contact metric manifold with respect to the naturally induced almost contact metric structure (ϕ, ξ, η, g). We now suppose that there exists an oriented totally umbilic hypersurface $M = (M, \phi, \xi, \eta, g)$ in the nearly Kähler unit 6-sphere $S^6 = (S^6, \bar{J}, \bar{g})$ such that (ϕ, ξ, η, g) is a quasi contact metric structure. Then, the shape operator A takes of the form $A = \lambda I$ for some smooth function λ on M. Then, taking account of (2.7), we see also that λ is constant on M, and hence $\lambda = -1$ by Theorem 4.1. Thus, we see that the hypersurface $M = (M, \phi, \xi, \eta, g)$ under consideration is a contact metric manifold, and hence the tensor field h is symmetric with respect to the Riemannian metric g by Proposition 3.1. Thus, the equality (4.4) reduces to the equality:

$$h\phi - \phi h = 0. \tag{5.1}$$

Thus, from (3.7) and (5.1), we also have

$$\phi h = h\phi = 0$$

and hence

$$h = 0 \tag{5.2}$$

by (3.2) and (3.7). Since $M = (M, \phi, \xi, \eta, g)$ is a contact metric manifold, we see that it is a K-contact manifold from (5.2). It is well-known that the sectional curvature of the 2-plane containing the vector ξ in the tangent space at each point of M is equal to 1. On the other hand, taking account of (2.6), we see that $M = (M, \phi, \xi, \eta, g)$ is a space of constant sectional curvature 2. Thus, this is a contradiction. On the other hand, it is also known that there exists a contact metric manifold which is a totally geodesic hypersurface in the nearly Kähler 6-sphere ([3], p64). Let M be an oriented hypersurface in a quasi Kähler manifold $\bar{M} = (\bar{M}, \bar{J}, \bar{g})$ which is a quasi contact metric manifold with respect to the naturally induced almost contact metric structure $M = (M, \phi, \xi, \eta, g)$. Then $M = (M, \phi, \xi, \eta, g)$ is a Hopf hypersurface of the quasi Kähler manifold $\bar{M} = (\bar{M}, \bar{J}, \bar{g})$. Our discussion in this paper is related to the ones in [2, 5].

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References

[1] J. H. Bae, J. H. Park and W. M. Shin, *Curvature identities on contact metric manifolds and its applications*, Adv. Studies Contemp. Math., 25 (2015), 423-435.

[2] J. Berndt, J. Bolton and L.M. Woodward, *Almost complex curves and Hopf hypersurfaces in the nearly Kähler 6-sphere*, Geom. Dedicata, 56 (1995), 237-247.

[3] D. E. Blair, *Riemannian geometry of contact and symplectic manifolds*, Second edition, Progress in Math. 203 (2010), Birkhäuser, Boston.

[4] Y. D. Chai, J. H. Kim, J. H. Park, K. Sekigawa and W. M. Shin, *Notes on quasi contact metric manifolds*, An. Ştiinţ. Univ. Al. I. Cuza Iasi Mat. (N.S.) 62 (2016), 349-360.

[5] S. Deshmukh and F. R. Al-Solamy, *Hopf hypersurfaces in nearly Kähler 6-sphere*, Balkan J. Geom. Appl. 13 (2008), 38-46.

[6] A. Gray, and L. Hervella, *The sixteen classes of almost Hermitian manifolds and their linear invariants*, Ann. Mat. Pura. Appl. (4) 123 (1980), 35-58.

[7] J. H. Kim, J. H. Park and K. Sekigawa, *A generalization of contact metric manifolds*, Balkan J. Geom. Appl. 19 (2014) 94-105.

[8] *M. Okumura, Contact hypersurfaces in certain Kaehlerian manifolds*, Tohoku Math. J. 18 (1966), 74-102.

[9] J. H. Park, K. Sekigawa and W. M. Shin, *A remark on quasi contact metric manifolds*, Bull. Korean Math Soc. 52(3) (2015), 1027-1034.

[10] Y. Tashiro, *On contact structure of hypersurfaces in complex manifolds. I, II* Tohoku Math. J. 15 (1963), 62-78 and 167-175.

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