Bulletin of Mathematical analysis and Applications
ISSN: 1821-1291, URL: http://www.bmathaa.org
Volume 1, Issue 2, (2009), Pages 57-65

On the Level Spaces of Fuzzy Topological Spaces *

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Abstract

It is known that if (X, T) is a fuzzy topological space and $0 \leq \alpha < 1$ then the family $T_\alpha = \{\alpha(G) : G \in T\}$ where $\alpha(G) = \{x \in X : G(x) > \alpha\}$, forms a topology on X. In the present paper some level properties have been modified and it is proved that a fuzzy topological space (X, T) is α-compact (resp. α-Hausdorff, countably α-compact, α-Lindelöf, α-connected, locally α-compact) if and only if the corresponding α-level topological space (X, T_α) is compact (resp. Hausdorff, countably compact, Lindelöf, connected, locally compact). Some basic properties of α-level sets have also been obtained.

1 Introduction

The investigation of fuzzy topological spaces by considering the properties which a space may have to a certain degree or level was initiated by Gantner et. al [3]. This approach resulted into the investigation of α-Hausdorff axiom [10], countable α-compactness, α-Lindelöf property [6], local α-compactness [7], α-closure [4] etc. in fuzzy topological spaces.

Throughout this paper Chang's [1] definition of fuzzy topological space (abbreviated as fts) is used. If X is a set and T is a family of fuzzy subsets of X satisfying the following conditions (i) to (iii) then T is called a fuzzy topology on X ; (i) $X, \phi \in T$ (ii) arbitrary union of members of T is again a member of T and (iii) intersection of finitely many members of T is again a member of T. Further (X, T) is called a fuzzy topological space (fts). If (X, T) is a fts and $0 \leq \alpha < 1$ then the family $T_\alpha = \{\alpha(G) : G \in T\}$, of all subsets of X of the form $\alpha(G) = \{x \in X : G(x) > \alpha\}$ called α-level sets, forms a topology on X [4] and is called the α-level topology on X.

In this paper, some basic properties of α-level sets have been obtained. The α-Hausdorff axiom [10] and the local α-compactness of [7] have been modified. The α-connectedness has been proposed. It is proved that a fts (X, T) is α-compact (α-Hausdorff, countably α-compact, α-Lindelöf, α-connected, locally α-compact) if and only if the corresponding α-level topological space (X, T_α) is compact (resp. Hausdorff, countably compact, Lindelöf, connected, locally compact)

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* *Mathematics Subject Classifications 2000:* 54A40, 54D05, 54D20, 54D30, 54D45.2.

Key words: Fuzzy topological spaces, α-level sets, α-Hausdorff,α-compact, locally α-compact, α-Lindelöf, α-connected fts.

Submitted May, 2009. Published September, 2009.

2 α-Level Sets and Their Basic Properties

If G is any fuzzy set in a set X and $0 \leq \alpha < 1$ $(0 < \alpha \leq 1)$ then $\alpha(G) = \{x \in X : G(x) > \alpha\}$ (resp. $\alpha^*(G) = \{x \in X : G(x) \geq \alpha\}$) is called an α-level (resp. α^*-level) set in X.

The term crisp subset refers to an ordinary subset which is identified with its characteristic function as a fuzzy subset.

If $f : X \to Y$ is a function and A is a fuzzy subset of X then $f(A)$ is a fuzzy subset of Y defined by $f(A)(y) = sup\{A(x) : x \in f^{-1}(y)\}$ for each $y \in Y$. Further, if B is a fuzzy subset of Y then $f^{-1}(B)$ is a fuzzy subset of X defined by $f^{-1}(B)(x) = B(f(x))$ for each $x \in X$.

Some basic properties of α-level sets are given in the following.

Theorem 2.1 *Let X, Y be any two sets and $0 \leq \alpha < 1$. The following statements are true.*

1. *If G is any fuzzy set in X then $G(x) \leq \alpha(G)(x)$ holds for all $x \in X$ with $G(x) > \alpha$.*
2. *If $G \leq H$ then $\alpha(G) \subset \alpha(H)$ for any two fuzzy sets G, H in X.*
3. *$\alpha(G) = G$ if and only if G is a crisp susbet of X.*
4. *$\alpha(\alpha(G)) = \alpha(G)$ for any fuzzy set G in X.*
5. *$\alpha(\bigvee_\lambda G_\lambda) = \bigcup_\lambda \alpha(G_\lambda)$ for any family $\{G_\lambda : \lambda \in \Lambda\}$ of fuzzy sets in X.*
6. *$\alpha(\bigwedge_\lambda G_\lambda) = \bigcap_\lambda \alpha(G_\lambda)$ for any family $\{G_\lambda : \lambda \in \Lambda\}$ of fuzzy sets in X.*
7. *If $f : X \to Y$, then $f(\alpha(G)) = \alpha(f(G))$ for each fuzzy set G in X.*
8. *If $f : X \to Y$, then $f^{-1}(\alpha(G)) = \alpha(f^{-1}(G))$ for each fuzzy set G in Y.*
9. *$\alpha(G \times H) = \alpha(G) \times \alpha(H)$ for any two fuzzy sets G, H in X where $G \times H$ is a fuzzy set in $X \times Y$ given by $(G \times H)(x, y) = G(x) \wedge H(y)$ for each $(x, y) \in X \times Y$.*

Proof. (1). Let $x \in X$ with $G(x) > \alpha$. Then $x \in \alpha(G)$ so that $(\alpha(G))(x) = 1 \geq G(x) > \alpha$ and therefore $G(x) \leq (\alpha(G))(x)$.

(2) If $x \in \alpha(G)$ then $G(x) > a$ and therefore $H(x) \geq G(x) > \alpha$. Consequently $x \in \alpha(H)$.

(3) If G is crisp and if $x \in X$ then $G(x) = 0$ or 1. If $G(x) = 0$ then $x \notin \alpha(G)$ and therefore $(\alpha(G))(x) = 0$ which proves $G(x) = \alpha(G(x))$. In case if $G(x) = 1$, then $G(x) = 1 > \alpha$ and therefore $x \in \alpha(G)$ which proves $(\alpha(G))(x) = 1 = G(x)$. The converse part follows as $\alpha(G)$ is crisp.

(4) Follows from (3) as $\alpha(G)$ is crisp.

(5) If $x \in \alpha(\bigvee_\lambda G_\lambda)$ then $Sup(G_\lambda(x)) > \alpha$. Consequently there exists a λ_o such that $G_{\lambda o}(x) > \alpha$ which implies $x \in \alpha(G_{\lambda o})$ and hence $x \in \bigcup_\lambda \alpha(G_\lambda)$. Therefore $\alpha(\bigvee_\lambda G_\lambda) \subset \bigcup_\lambda \alpha(G_\lambda)$. Similarly $\bigcup_\lambda \alpha(G_\lambda) \subset \alpha(\bigvee_\lambda G_\lambda)$ and hence the equality.

(6) If $x \in \alpha(\bigwedge_\lambda G_\lambda)$ then $(\bigwedge_\lambda G_\lambda)(x) > \alpha$ and therefore $G_\lambda(x) > \alpha$ for each λ. This implies that $x \in \alpha(G_\lambda)$ for each λ and therefore $x \in \bigwedge_\lambda \alpha(G_\lambda)$. Thus $\alpha(\bigwedge_\lambda G_\lambda) \subset \bigcap_\lambda \alpha(G_\lambda)$. Similarly $\bigcap_\lambda \alpha(G_\lambda) \subset \alpha(\bigwedge_\lambda G_\lambda)$.

(7) If $y \in f(\alpha(G))$ then there is an element $x \in \alpha(G)$ such that $y = f(x)$. Now $G(x) > \alpha$ and therefore $Sup\left\{G(x) : x \in f^{-1}(y)\right\} > \alpha$ which implies $(f(G))(y) > \alpha$. Then $y \in \alpha(f(G))$. Thus $f(\alpha(G)) \subset \alpha(f(G))$. Similarly it can be shown that $\alpha(f(G)) \subset f(\alpha(G))$ and hence the result follows.

(8) Let $x \in f^{-1}(\alpha(G))$. Then $f(x) = y \in \alpha(G)$ so that $G(y) = G(f(x)) > \alpha$. Therefore $\left[f^{-1}(G)\right](x) > \alpha$ which implies $x \in \alpha\left[f^{-1}(G)\right]$ and hence it follows that $f^{-1}(\alpha(G)) \subset \alpha(f^{-1}(G))$. Similarly $\alpha(f^{-1}(G)) \subset f^{-1}(\alpha(G))$ and hence the equality.

(9) If $(x, y) \in \alpha(G \times H)$ then $(G \times H)(x, y) > \alpha$ and therefore $x \in \alpha(G)$ and $y \in \alpha(H)$. So $(x, y) \in \alpha(G) \times \alpha(H)$. Thus $\alpha(G \times H) \subset \alpha(G) \times \alpha(H)$. Similarly it can be shown that $\alpha(G) \times \alpha(H) \subset \alpha(G \times H)$ and hence the equality follows.

3 Level Spaces and Main Results

In the beginning of this section we deal with Rodabaugh's [10] α-Hausdorff fts.

Definition 3.1 *Let $0 \le \alpha < 1$ ($0 < \alpha \le 1$). A fts (X, T) is said to be α-Hausdorff (resp. α^*-Hausdorff) if for each x, y in X with $x \ne y$, there exist G, H in T such that $G(x) > \alpha$ (resp. $G(x) \ge \alpha$), $H(y) > \alpha$ (resp. $H(y) \ge \alpha$) and $G \wedge H = 0$.*

We have the following

Theorem 3.2 *Let $0 \le \alpha < 1$. If a fts (X, T) is α-Hausdorff, then (X, T_α) is Hausdorff topological space.*

Proof. Let $x, y \in X$ with $x \neq y$. Then there are G, H in T such that $G(x) > \alpha$, $H(y) > \alpha$ and $G \wedge H = 0$. Then $\alpha(G)$ and $\alpha(H)$ are open sets in (X, T_α) and $x \in \alpha(G)$, $y \in \alpha(H)$. Also $\alpha(G) \cap \alpha(H) = \phi$ since $G \wedge H = 0$. Hence (X, T_α) is Hausdorff topological space.

The converse of the above theorem holds for the case of $\alpha = 0$, which is given in the following.

Theorem 3.3 *Let (X,T) be a fts. If (X, T_0) is Hausdorff topological space, then (X,T) is 0-Hausdorff fts.*

Proof. Let $x, y \in X$ with $x \neq y$. Then there are open sets U, V in (X, T_0) such that $x \in U$, $y \in V$ and $U \cap V = \phi$. Let $U = 0(G), V = 0(H)$ for some G, H in T. Then it follows that $G(x) > 0$ and $H(y) > 0$. Further $G \wedge H = 0$ as $U \cap V = \phi$. Hence (X,T) is 0-Hausdorff.

Definition 3.4 *Let X be a set and $0 \leq \alpha < 1$ $(0 < \alpha \leq 1)$. A family $\{G_\lambda\}_\lambda$ of fuzzy sets in X is said to be α-disjoint (resp. α^*-disjoint) if $\bigwedge_\lambda G_\lambda \leq \alpha$ (resp. $\bigwedge_\lambda G_\lambda < \alpha$).*

It is evident that two fuzzy sets G, H in X are α-disjoint (α^*-disjoint) if and only if for each x in X either $G(x) \leq \alpha$ (resp. $G(x) < \alpha$) or $H(x) \leq \alpha$ (resp.$H(x) < \alpha$).

Rodabaugh's definition is suitably modified in the following.

Definition 3.5 *Let $0 \leq \alpha < 1$ $(0 < \alpha \leq 1)$. A fts (X,T) is said to be α-Hausdorff (resp. α^*-Hausdorff) if for each x, y in X with $x \neq y$, there exist G, H in T such that $G(x) > \alpha$ (resp. $G(x) \geq \alpha$), $H(y) > \alpha$ (resp. $H(y) \geq \alpha$) and G, H are α-disjoint (resp. α^*-disjoint).*

For the modified class of α-Hausdorff fuzzy topological spaces we have the following.

Theorem 3.6 *Let $0 \leq \alpha < 1$. A fts (X,T) is a α-Hausdorff if and only if (X, T_α) is Hausdorff topological space.*

Proof. Let (X,T) be α-Hausdorff. Let $x, y \in X$ with $x \neq y$. Then there exist G, H in T with $G(x) > \alpha$, $H(y) > \alpha$ and $G \wedge H \leq \alpha$. Then $\alpha(G)$, $\alpha(H)$ are open sets in (X, T_α) such that $x \in \alpha(G)$, $y \in \alpha(H)$ and $\alpha(G) \cap \alpha(H) = \alpha(G \cap H) = \{x \in X : (G \wedge H)(x) > \alpha\} = \phi$ as $G \wedge H \leq \alpha$. Therefore (X, T_α) is α-Hausdorff.

Conversely, suppose (X, T_α) is α-Hausdorff. Let $x, y \in X$ with $x \neq y$. Then there exist open sets U, V in (X, T_α) such that $x \in U$, $y \in V$ and $U \cap V = \phi$. Let $U = \alpha(G)$ and $V = \alpha(H)$ for some $G, H \in T$. Then $x \in \alpha(G)$ and $y \in \alpha(H)$. Therefore $G(x) > \alpha$ and $H(y) > \alpha$. Further $G \wedge H \leq \alpha$ as $U \cap V = \phi$. Hence (X,T) is α-Hausdorff.

Let $0 \leq \alpha < 1$ $(0 < \alpha \leq 1)$. A family $\{G_\lambda : \lambda \in \Lambda\}$ of fuzzy subsets of a fts (X, T) is said to be an α-shading (α^*-shading) of X if for each $x \in X$, there exists a G_{λ_o} in $\{G_\lambda : \lambda \in \Lambda\}$ such that $G_{\lambda_o}(x) > \alpha$ $(\geq \alpha)$.

The following definition is due to Gantner et. al [3].

Definition 3.7 *Let $0 \leq \alpha < 1$ $(0 < \alpha \leq 1)$. A fts (X, T) is said to be α-compact (resp. α^*-compact) if each α-shading (resp. α^*-shading) of X by open fuzzy sets has a finite α-subshading (resp. α^*-subshading).*

We have the following

Theorem 3.8 *Let $0 \leq \alpha < 1$. A fts (X, T) is α-compact if and only if (X, T_α) is compact topological space.*

Proof. Let (X, T) be α-compact. Let $U = \{U_\lambda : \lambda \in \Lambda\}$ be an open cover of (X, T_α). Then, since for each U_λ, there exists a G_λ in T such that $U_\lambda = \alpha(G_\lambda)$, we have $U = \{\alpha(G_\lambda) : \lambda \in \Lambda\}$. Then the family $V = \{G_\lambda : \lambda \in \Lambda\}$ is an α-shading of (X, T). To see this, let $x \in X$. Since U is an open cover of (X, T_α), there is an $U_{\lambda_o} \in U$ such that $x \in U_{\lambda_o}$. But $U_{\lambda_o} = \alpha(G_{\lambda_o})$, for some $G_{\lambda_o} \in T$. Therefore $x \in \alpha(G_{\lambda_o})$ which implies that $G_{\lambda_o}(x) > \alpha$. By α-compactness of (X, T), V has a finite α-subshading say $\{G_{\lambda_i}\}_{i=1}^{k}$. Then $\{\alpha(G_{\lambda_i})\}_{i=1}^{k}$ forms a finite subcover of U and thus (X, T_α) is compact.

Conversely, let (X, T_α) be compact and $U = \{G_\lambda : \lambda \in \Lambda\}$ be an open α-shading of (X, T). Then the family $V = \{\alpha(G_\lambda) : \lambda \in \Lambda\}$ is an open cover of (X, T_α). For, let $x \in X$. Then there exists a G_{λ_o} in U such that $G_{\lambda_o}(x) > \alpha$. Therefore $x \in \alpha(G_{\lambda_o})$ and $(G_{\lambda_o}) \in V$. By compactness of (X, T_α), V has a finite subcover say $\{\alpha(G_{\lambda_i})\}_{i=1}^{n}$. Then the family $\{G_{\lambda_i}\}_{i=1}^{n}$ forms a finite α-subshading of U and hence (X, T) is α-compact.

Countable compact fts have been studied in [6, 11-13].

Definition 3.9 *Let $0 \leq \alpha < 1$ $(0 < \alpha \leq 1)$. A fts (X, T) is said to be countably α-compact (resp. countably α^*-compact) if every countable open α-shading (resp. countable open α^*-shading) of X has a finite α-subshading (resp. finite α^*-subshading).*

It is easy to verify the following

Theorem 3.10 *Let $0 \leq \alpha < 1$. A fts (X, T) is countably α-compact if and only if (X, T_α) is countably compact topological space.*

Lindelöf fuzzy topological spaces were studied in [6, 8, and 12]. Lindelöf fuzzy topological spaces, using shading families, were introduced in [16].

Definition 3.11 *Let $0 \leq \alpha < 1$ $(0 < \alpha \leq 1)$. A fts (X, T) is said to be α-Lindelöf (resp. α^*-Lindelöf) if and only if every open α-shading (resp. open α^*-shading) of X has a countable α-subshading (resp. countable α^*-subshading).*

Again it is easy to verify the following.

Theorem 3.12 *Let $0 \leq \alpha < 1$. A fts (X, T) is α-Lindelöf if and only if (X, T_α) is Lindelöf topological space.*

Definition 3.13 *Let $0 \leq \alpha < 1$ $(0 < \alpha \leq 1)$. Let X be a non-empty set. A fuzzy set A in X is said to be an empty fuzzy set of order α (resp. order α^*) if $A(x) \leq \alpha$ (resp. $A(x) < \alpha$) for each $x \in X$.*

A fuzzy set A in X is said to be non-empty of order α (resp. order α^*) if there exists $x_o \in X$ such that $A(x_o) > \alpha$ (resp. $A(x_o) \geq \alpha$).

Connectedness in fuzzy topological spaces was studied in [5, 9].

Connectedness, using shading families, is given in the following.

Definition 3.14 *Let $0 \leq \alpha < 1$ $(0 < \alpha \leq 1)$. A fts (X, T) is said to be α-disconnected (resp. α^*-disconnected) if there exists an α-shading (resp. α^*-shading) family of two open fuzzy sets in X which are non-empty of order α (resp. order α^*) and α-disjoint (resp. α^*-disjoint).*

Definition 3.15 *Let $0 \leq \alpha < 1$ $(0 < \alpha \leq 1)$. A fts (X, T) is said to be α-connected (resp. α^*-connected) if there does not exist an α-shading (resp. α^*-shading) family of two open fuzzy sets in X which are non-empty of order α (resp. order α^*) and α-disjoint (resp. α^*-disjoint).*

We now prove the following

Theorem 3.16 *Let $0 \leq \alpha < 1$. A fts (X, T) is α-connected if and only if (X, T_α) is connected topological space.*

Proof. Let (X, T) be α-connected. Suppose (X, T_α) is disconnected. Then there exist non-empty disjoint open sets U, V in (X, T_α) such that $U \cup V = X$. Now $U = \alpha(G)$, $V = \alpha(H)$ for some $G, H \in T$. Since U, V are non-empty sets it follows that G and H are non-empty fuzzy sets of order α. Further $\{G, H\}$ is an α-shading of X: For if $x \in X$ then $x \in U$ or $x \in V$ and therefore $x \in \alpha(G)$ or $x \in \alpha(H)$ which implies that $G(x) > \alpha$ or $H(x) > \alpha$. Also G, H are α-disjoint: For, $U \cap V = \phi$ implies that $\alpha(G) \cap \alpha(H) = \phi$. Therefore $\alpha(G \wedge H) = \phi$. That is $\{x \in X : (G \wedge H)(x) > \alpha\} = \phi$. Therefore for each $x \in X$,$(G \wedge H)(x) \leq \alpha$ and so G, H are α-disjoint. Thus it follows that $\{G, H\}$ is an α-shading of open fuzzy sets which are non-empty of order α and are α-disjoint. Therefore (X, T) is α-disconnected, which contradicts the hypothesis. Hence (X, T_α) is connected topological space.

Conversely, suppose (X, T_α) is connected. Let (X, T) be α-disconnected. Then there exist an α-shading $\{G, H\}$ of two open fuzzy sets in X which are non-empty of order α and α-disjoint. Clearly $\alpha(G)$, $\alpha(H)$ are open sets in (X, T_α). Further $\alpha(G)$, $\alpha(H)$ are non-empty as G, H are non-empty of order α. Also $\alpha(G) \cap \alpha(H) = \alpha(G \wedge H) = \{x \in X : (G \wedge H)(x) > \alpha\} = \phi$ since $(G \wedge H)(x) \leq \alpha$ as G, H are α-disjoint. Finally $\alpha(G) \cup \alpha(H) = X$: For if $x \in X$ then either $G(x) > \alpha$ or $H(x) > \alpha$ as $\{G, H\}$ is an α-shading of X. Therefore $x \in \alpha(G)$ or $x \in \alpha(H)$ and therefore $x \in \alpha(G) \cup \alpha(H)$. Thus $X \subset \alpha(G) \cup \alpha(H)$. Also $\alpha(G) \cup \alpha(H) \subset X$ is obvious. Therefore $\alpha(G) \cup \alpha(H) = X$. Hence it follows that X is the union of two non-empty disjoint open sets in (X, T_α) and therefore (X, T_α) is disconnected, which contradicts the hypothesis. Hence (X, T) is α-connected fts.

Local compactness in fuzzy topological spaces was studied in [2, 3, 7, 14]. The definition of local compactness in [7] is modified in the following.

Definition 3.17 *Let $0 \leq \alpha < 1$ $(0 < \alpha \leq 1)$. A fts (X, T) is said to be locally α-compact (resp. locally α^*-compact) if for each $p \in X$ there exists an open fuzzy set N such that $N(p) > \alpha$ (resp. $N(p) \geq \alpha$) and $\overline{\alpha(N)}$ (resp. $\overline{\alpha^*(N)}$) is α-compact (resp. α^*-compact).*

We prove the following

Theorem 3.18 *Let $0 \leq \alpha < 1$. A fts (X, T) is locally α-compact if and only if (X, T_α) is locally compact topological space.*

Proof. Let (X, T) be locally α-compact. Let $x \in X$. There exists an open fuzzy set N in (X, T) such that $N(x) > \alpha$ and $\overline{\alpha(N)}$ is α-compact. Therefore $\alpha(N)$ is an open set in (X, T_α) containing x such that $\overline{\alpha(N)}$ is compact subset in (X, T_α): For if $\{U_\lambda = \alpha(G_\lambda) : \lambda \in \Lambda, G_\lambda \in T\}$ is an open cover of $\overline{\alpha(N)}$ in (X, T_α) then the family $\{G_\lambda : \lambda \in \Lambda\}$ is an open α-shading of $\overline{\alpha(N)}$ in (X, T). Since $\overline{\alpha(N)}$ is α-compact $\{G_\lambda : \lambda \in \Lambda\}$ has a finite α-subshading say $\{G_{\lambda_i}\}_{i=1}^k$. Then $\{\alpha(G_{\lambda_i}) = U_{\lambda_i} : i = 1, 2, ..., k\}$ is a finite subcover of $\{U_\lambda : \lambda \in \Lambda\}$ for $\overline{\alpha(N)}$. So $\overline{\alpha(N)}$ is a compact subset of (X, T_α). Thus for each $x \in X$, there exists an open set $\alpha(N)$ in (X, T_α) such that $x \in \alpha(N)$ and $\overline{\alpha(N)}$ is compact. Hence (X, T_α) is locally compact topological space.

Conversely, suppose (X, T_α) is locally compact. Let $p \in X$. Then there exists an open set $\alpha(G)$ in (X, T_α), where $G \in T$, such that $p \in \alpha(G)$ and $\overline{\alpha(G)}$ is compact set in (X, T_α). Now $G \in T$ and $G(p) > \alpha$. Further $\overline{\alpha(G)}$ is α-compact in (X, T): For if $\{H_\lambda\}_{\lambda \in \Lambda}$ is an open α-shading of $\overline{\alpha(G)}$ in (X, T), then $\{\alpha(H_\lambda) : \lambda \in \Lambda\}$ is an open cover of $\overline{\alpha(G)}$. Since $\overline{\alpha(G)}$ is compact in (X, T_α), $\{\alpha(H_\lambda) : \lambda \in \Lambda\}$ has a finite subcover say $\{\alpha(H_{\lambda_i}) : i = 1, 2, ..., k\}$. Then $\{H_{\lambda_i} : i = 1, 2, ..., k\}$ is a finite α-subshading of $\{H_\lambda\}_{\lambda \in \Lambda}$ for $\overline{\alpha(G)}$. Therefore every open α-shading for $\overline{\alpha(G)}$ has a finite α-subshading and therefore $\overline{\alpha(G)}$ is α-compact. Thus for each $p \in X$ there exists an open fuzzy set G in (X, T) such that $G(p) > \alpha$ and $\overline{\alpha(G)}$ is α-compact in (X, T). Hence (X, T) is locally α-compact .

Acknowledgements. The authors are grateful to the referee for his comments which improved the readability of the paper. The authors are also grateful to the University Grants Commission, New Delhi for its financial support under UGC SAP I to the Department of Mathematics, Karnatak University, Dharwad India.

References

[1] C.L.Chang, Fuzzy Topological Spaces, *J. Math.Anal.Appl.*, **24** (1968), 182-190.

[2] F. T. Christoph, Quotient Fuzzy Topology and local compactness, *J. Math. Anal. Appl.*, **57** (1977), 497-504.

[3] T. E. Gantner, R. C. Steinlage and R. H. Warren, Compactness in Fuzzy Topological Spaces *J. Math. Anal. Appl.* **62** (1978), 547-562.

[4] A. J. Klein, α- Closure in Fuzzy Topology, *Rocky Mount. J. Math.*, **11** (1981), 553-560.

[5] R. Lowen, Connectedness in Fuzzy Topological Spaces, *Rocky Mount. J. Math.*, **11** (1981), 427-433.

[6] S. R. Malghan and S. S. Benchalli, On Fuzzy Topological Spaces, *Glasnik Mat.* **16 (36)** (1981) 313-325.

[7] S.R.Malghan and S.S.Benchalli, Open Maps, Closed Maps and Local Compactness in Fuzzy topological spaces, *J. Math. Anal. Appl.*, **99 No.2** (1984) 338-349.

[8] P. P. Ming and L. Y. Ming, Fuzzy Topology I, Neighbourhood Structure of fuzzy point and Moore-Smith convergence, *J. Math. Anal. Appl.*, **76** (1980), 571-599.

[9] K. S. Rajasethupathy and S. Laxmivarahan, Connectedness in Fuzzy Topology, *Kybernetika*, **13** (1977), 190-193.

[10] S. E. Rodabaugh, The Hausdorff Separation Axiom for Fuzzy Topological Spaces, *Topology and its Appl.*, **11** (1980), 319-334.

[11] M. Sarkar, On Fuzzy Topological Spaces , *J. Math. Anal. Appl.*, **79** (1981), 384-394.

[12] C. K. Wong, Covering Properties of Fuzzy Topological Spaces , *J. Math. Anal. Appl.*, **43** (1973), 697-704.

[13] C. K. Wong, Fuzzy Topology: Product and Quotient Theorems, *J. Math. Anal. Appl.*, **45** (1974), 512-521.

[14] C. K. Wong, Fuzzy Points and Local properties of Fuzzy Topology, *J. Math. Anal. Appl.*, **46** (1974), 316-328.

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