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BEST PROXIMITY POINT RESULTS FOR THETA-CONTRACTION IN MODULAR METRIC AND FUZZY METRIC SPACES

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Abstract. The aim of this paper is to introduce new class of proximal contractions in non-Archimedean modular metric spaces and to prove some best proximity point theorems for such kind of mappings. As application we deduce best proximity results in fuzzy metric spaces. Consequently, some basic fixed point results in both modular and fuzzy metric spaces are obtained as corollaries of our work. Finally, an example is provided to illustrate the usability of our obtained results.

1. Introduction and Preliminaries

In 2010, the concept of modular metric space was introduced by Chistyakov [11, 12]. A metric $d : \chi \to \chi$, where χ is nonempty set, is a finite non-negative distance function between two elements $a, b \in \chi$. At a given time $\lambda > 0$, a modular metric function denoted by $\varpi_\lambda : \chi \times \chi \to [0, \infty]$, represents the absolute value of an average velocity(possibly infinite value), that cover the distance between $a, b \in \chi$ in a time λ.

Studying and solving differential and variational problems arising in applied science is a strong motivation for mathematicians and others to study fixed point problems in modular metric spaces [2, 3, 7, 10, 13, 27, 28, 30].

In 2010, Basha [9] introduced the notion of best proximity point of a non-self mappings. Zhang et al. [35] extended the notion of $P-$property by weak $P-$property. Jleli et al. [24] introduced the concept of $\alpha-$proximal admissible, and JS-contraction in [25].

In this paper, in the setting of Non-Archimedean modular metric spaces, we introduce the class of $(\alpha, \Theta) - \varpi$-contraction and we establish certain best proximity point results. As application of our results, we obtain some results of best proximity point results for non self-mappings defined on a Non-archimedean fuzzy metric spaces as consequence of those given for modular metric spaces, [14, 17, 18, 20]. Consequently, we get some fixed point results as corollaries in both modular and

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fuzzy metric influenced by Δ_Θ class functions. An example is furnished to demonstrate the validity of the obtained results. Let χ be a nonempty set and $\varpi : (0,+\infty) \times \chi \times \chi \to [0,+\infty]$ be a function, for simplicity, we will write

$$\varpi_{\lambda(a,b)} = \varpi(\lambda, a, b),$$

for all $\lambda > 0$ and $a, b \in \chi$.

Definition 1.1. [11, 12] *A function $\varpi : (0,+\infty) \times \chi \times \chi \to [0,+\infty]$ is called a modular metric on χ if the following axioms hold for all $\lambda_1, \lambda_2 > 0$ and $a, b, c \in \chi$;:*

(i) $a = b$ *if and only if* $\varpi_{\lambda_1}(a, b) = 0$;
(ii) $\varpi_{\lambda_1}(a, b) = \varpi_{\lambda_1}(b, a)$
(iii) $\varpi_{\lambda_1+\lambda_2}(a, b) \leq \varpi_{\lambda_1}(a, c) + \varpi_{\lambda_2}(c, b)$.

Note that: ϖ is called a pseudomodular metric if

(i') $\varpi_{\lambda_1}(a, a) = 0$ for all $\lambda_1 > 0$ and $a \in \chi$;

is used instead of (i) in the Definition 1.1.
ϖ is called regular if condition (i) is replaced by:

$$a = b \quad \text{if and only if} \quad \varpi_{\lambda_1}(b, a) = 0 \quad \text{for some} \quad \lambda_1 > 0.$$

In addition, if for $\lambda_1, \lambda_2 > 0$, and $a, b, c \in \chi$,

$$\varpi_{\{\lambda_1+\lambda_2\}}(a + b) \leq \varpi_{\lambda_1}(a + c)\varpi_{\lambda_2}(c + b),$$

then ϖ is called convex.

Remark 1.2. *The function ϖ_λ is non-Archimedean if the conditions (i) and (ii) of Definition 1.1 hold true, and replacing condition (iii) by*

(iii$'$) $\varpi_{max\{\lambda_1,\lambda_2\}}(a, b) \leq \varpi_{\lambda_1}(a, c) + \varpi_{\lambda_2}(c, b)$; *for all* $\lambda_1, \lambda_2 > 0$; $a, b, c \in \chi$.

Notice that condition (iii$'$) implies (iii) and so non-Archimedean modular metric is modular.

Remark 1.3. *The function $\lambda \to \varpi_\lambda(a, b)$ is nonincreasing on $(0,+\infty)$ for all $a, b \in \chi$, where ϖ is a pseudomodular. Indeed, if $0 < \lambda_1 < \lambda_2$, then*

$$\varpi_{\lambda_2}(a, b) \leq \varpi_{\lambda_2-\lambda_1}(a, a) + \varpi_{\lambda_1}(a, b) = \varpi_{\lambda_1}(a, b).$$

Definition 1.4. [11, 12] *Let ϖ be a pseudomodular on χ and $a_0 \in \chi$ fixed. Consider the two sets*

$$\chi_\varpi = \chi_\omega(a_0) = \{a \in \chi : \varpi_\lambda(a, a_0) \to 0 \quad as \quad \lambda \to +\infty\},$$

and

$$\chi^*_\varpi = \chi^*_\varpi(a_0) = \{a \in \chi : \exists\lambda = \lambda(a) > 0 \quad such\ that \quad \varpi_\lambda(a, a_0) < +\infty\}.$$

χ_ϖ and χ^*_ϖ are called modular spaces (around a_0).

Obviously $\chi_\varpi \subset \chi^*_\varpi$. Note that χ_ω can be endowed with the metric defined by

$$d_\varpi(a, b) = \inf\{\lambda > 0 : \varpi_\lambda(a, b) \leq \lambda\} \quad \text{for all} \quad a, b \in \chi_\varpi.$$

If ϖ is a convex, then $\chi^*_\varpi = \chi_\varpi$, and we can consider the metric d^*_ϖ defined by

$$d^*_\varpi(a, b) = \inf\{\lambda > 0 : \varpi_\lambda(a, b) \leq 1\} \quad \text{for all} \quad a, b \in \chi_\varpi;$$

Definition 1.5. [30] *Let χ_ϖ be a modular metric space and M a subset of χ_ϖ. Then*

(1) *the sequence* $\{a_n\} \in \chi_\varpi$ *is said to be a* ϖ*-convergent to some* $a \in \chi_\varpi$ *if* $\varpi_\lambda(a_n, a) \to 0$, *as* $n \to +\infty$. x *is said to be the* ϖ*-limit of* (a_n).
(2) $\{a_n\}$ *is called* ϖ*-Cauchy if* $\varpi_\lambda(a_m, a_n) \to 0$, *as* $m, n \to +\infty$.
(3) *For a* ϖ*-convergent* $\{a_n\} \in M$ *that converges to some* $a \in \chi_\varpi$. *If* $a \in M$, *then* M *is called* ω*-closed.*
(4) *For a* ϖ*-Cauchy sequence* $\{a_n\} \in M$. *If* $\{a_n\}$ *converges to some* $a \in M$, *then* M *is called* ϖ*-complete.*

In the next definitions, we use a function

$$\alpha : \chi \times \chi \to [0, \infty).$$

Definition 1.6. [34] *A self-mapping* g *on* χ *is said to be an* $\alpha-$*admissible mapping if*

$$a, b \in \chi, \quad \alpha(a, b) \geq 1 \quad \Longrightarrow \quad \alpha(ga, gb) \geq 1.$$

Definition 1.7. [21] *A self-mapping* g *on* χ, *where* (χ, d) *is a metric space is said to be an* $\alpha-$*continuous mapping if for any sequence*

$$\{a_n\} \in \chi \text{ such that } a_n \to a \text{ as } n \to +\infty,$$

with

$$\alpha(a_n, a_{n+1}) \geq 1 \quad \text{for all} n \in \mathbb{N} \Longrightarrow \; ga_n \to ga.$$

Definition 1.8. *A self-mapping* g *on* χ_ϖ *is said to be an* $\alpha - \varpi-$*continuous mapping, if for any sequence*

$$\{a_n\} \in \chi_\varpi \text{ such that } \varpi_\lambda(a_n, a) \to 0 \text{ as } n \to +\infty,$$

with

$$\alpha(a_n, a_{n+1}) \geq 1 \quad \text{for all} n \in \mathbb{N} \Longrightarrow \; \varpi_\lambda(ga_n, ga) \to 0.$$

Example 1.9. Let $\chi = [0, +\infty)$ and $\varpi_\lambda(a, b) = \frac{1}{\lambda}|a - b|$ be a modular metric on χ_ϖ. Assume that $g : \chi \to \chi$ and $\alpha : \chi \times \chi \to [0, +\infty)$ are defined by

$$gx = \begin{cases} a^7, & \text{if } a \in [0, 1] \\ \\ 10, & \text{if } (1, +\infty) \end{cases}, \quad \alpha(a, b) = \begin{cases} a^2 + b^2 + 1, & \text{if } a, b \in [0, 1] \\ \\ 0, & \text{otherwise.} \end{cases}$$

Then g is an $\alpha - \varpi-$continuous mapping but g is not $\varpi-$continuous.

2. Main results.

Let A_1 and A_2 be two non-empty subsets of a modular metric space χ_ω. We denote by ${A_1}_0^\lambda$ *and* ${A_2}_0^\lambda$ the following sets:

$${A_1}_0^\lambda = \{a \in A_1 : \varpi_\lambda(a, b) = \varpi_\lambda(A_1, A_2), \; for \; some \; b \in A_2\}$$

$${A_2}_0^\lambda = \{b \in A_2 : \varpi_\lambda(a, b) = \varpi_\lambda(A_1, A_2), \; for \; some \; a \in A_1\},$$

where $\varpi_\lambda(A_1, A_2) = \inf\{\varpi_\lambda(a, b) : a \in A_1 \; and \; b \in A_2\}$.
A point $a^* \in A_1$ is the best proximity point of the mapping g if

$$\varpi(a^*, ga^*) = \varpi(A_1, A_2).$$

Definition 2.1. *For a nonempty subset* ${A_1}_0^\lambda$ *and all* $\lambda > 0$, *the pair* (A_1, A_2) *has the weak* $P_\lambda-$*property if for* $a_1, a_2 \in {A_1}_0^\lambda$ *and* $b_1, b_2 \in {A_2}_0^\lambda$,

$$\varpi_\lambda(a_1, b_1) = \varpi_\lambda(A_1, A_2) \; and \; \varpi_\lambda(a_2, b_2) = \varpi_\lambda(A_1, A_2) \; \Longrightarrow \varpi_\lambda(a_1, a_2) \leq \omega_\lambda(b_1, b_2).$$

Definition 2.2. *Let $g : A_1 \to A_2$ and $\alpha : A_1 \times A_1 \to [0, \infty)$ be functions. g is an $\alpha-$proximal admissible if for all $a_1, a_2, b_1, b_2 \in A_1$,*

$$\begin{aligned} \alpha(a_1, a_2) &\geq 1 \\ \varpi_\lambda(b_1, ga_1) &= \varpi_\lambda(A_1, A_2) \\ \varpi_\lambda(b_2, ga_2) &= \varpi_\lambda(A_1, A_2) \end{aligned}$$

implies

$$\alpha(b_1, b_2) \geq 1.$$

Jleli and Samet [24], defined the class, Δ_Θ of all functions $\Theta : (0, +\infty) \to (1, +\infty)$ satisfying the following conditions:

(Θ_1) Θ is increasing;
(Θ_2) for all sequence $\{a_n\} \subseteq (0, +\infty)$, $\lim\limits_{n\to+\infty} a_n = 0$ if and only if $\lim\limits_{n\to+\infty} \Theta(a_n) = 1$;
(Θ_3) there exist $0 < r < 1$ and $\ell \in (0, +\infty]$ such that $\lim\limits_{t\to 0^+} \frac{\Theta(t)-1}{t^r} = \ell$.

Definition 2.3. *Let χ_ϖ be a modular metric space, A_1 and A_2 are two non-empty subsets of χ_ω. Let $g : A_1 \to A_2$ and $\alpha : A_1 \times A_1 \to [0, +\infty)$ be functions. g is called an $(\alpha, \Theta) - \varpi-$contraction if for all $a, b \in A_1$ with $\alpha(a, b) \geq 1$*

$$\alpha(a, b)\Theta\big(\varpi_\lambda(ga, gb)\big) \leq \Big[\Theta\big(\varpi_\lambda(a, b)\big)\Big]^k, \tag{2.1}$$

for all $\lambda > 0$, whenever $\varpi_\lambda(ga, gb) > 0$, where $0 < k < 1$ and $\Theta \in \Delta_\Theta$.

Note that we shall assume ϖ to be regular in all next results.

Theorem 2.4. *Let χ_ϖ be a complete non-Archimedean modular metric space. Let A_1 and A_2 be two non-empty subsets of χ_ϖ; such that A_1 is closed and (A_1, A_2) has weak $P_\lambda-$ property. Assume that $g : A_1 \to A_2$ satisfy the following conditions:*

(i) *g is an $(\alpha, \Theta) - \varpi-$contraction;*
(ii) *g is an $\alpha-$ proximal admissible;*
(iii) *$g({A_1}_0^\lambda) \subseteq {A_2}_0^\lambda$;*
(iv) *there exist $a_0, a_1 \in {A_1}_0^\lambda$ such that $\varpi_\lambda(a_1, ga_0) = \varpi_\lambda(A_1, A_2)$, implies $\alpha(a_0, a_1) \geq 1$;*
(v) *g is an $\alpha - \varpi-$continuous.*

Then g has best proximity point.

Proof. Using condition (iv) together with condition (iii) to show that there exists an element $a_2 \in {A_1}_0^\lambda$ such that, $\varpi_\lambda(a_2, ga_1) = \omega_\lambda(A_1, A_2)$. Since g is an $\alpha-$proximal admissible, then $\alpha(a_1, a_2) \geq 1$. Again, relating these conditions (ii), (iii) and (iv) to show that, there exists $a_3 \in {A_1}_0^\lambda$ such that $\varpi_\lambda(a_3, ga_2) = \varpi_\lambda(A_1, A_2)$, $\alpha(a_2, a_3) \geq 1$. Continuing this process, we get

$$\varpi_\lambda(a_{n+1}, ga_n) = \varpi_\lambda(A_1, A_2),\ \alpha(a_{n-1}, a_n) \geq 1 \tag{2.2}$$

for all $n \in \mathbb{N} \cup \{0\}$, $\lambda > 0$. Using weak $P_\lambda-$ property for the pair (A_1, A_2), we get

$$\varpi_\lambda(a_n, a_{n+1}) \leq \varpi_\lambda(ga_{n-1}, ga_n), \tag{2.3}$$

for all $n \in \mathbb{N} \cup \{0\}$, $\lambda > 0$. If there exists $p \in \mathbb{N}$ such that $a_{p+1} = a_p$, by regularity of ϖ we get a_p is the best proximity point of g.

Therefore, we may assume that $\varpi_\lambda(a_n, a_{n+1}) > 0$ for all $n \in \mathbb{N} \cup \{0\}$.
Thus, from (i) we have

$$\begin{aligned} 1 &< \Theta(\varpi_\lambda(a_n, a_{n+1})) \\ &\leq \Theta(\varpi_\lambda(ga_{n-1}, ga_n)) \\ &\leq \alpha(a_{n-1}, a_n)\Theta(\varpi_\lambda(ga_{n-1}, ga_n)) \\ &\leq [\Theta(\varpi_\lambda(a_{n-1}, a_n))]^k. \end{aligned}$$

Therefore,

$$\begin{aligned} 1 &< \Theta\big(\varpi_\lambda a_n, a_{n+1})\big) \leq [\Theta\big(\omega_\lambda(x_{n-1}, x_n)\big)]^k \\ &\leq [\Theta\big(\varpi_\lambda(a_{n-2}, a_{n-1})\big)]^{k^2} \leq \cdots \leq [\Theta(\varpi_\lambda(a_0, a_1))]^{k^n}. \end{aligned} \tag{2.4}$$

Taking the limit as $n \to +\infty$ in (2.4), we get

$$\lim_{n\to+\infty} \Theta\big(\varpi_\lambda(a_n, a_{n+1})\big) = 1 \text{ for all } \lambda > 0,$$

and since $\Theta \in \Delta_\Theta$, we obtain

$$\lim_{n\to+\infty} \varpi_\lambda(a_n, a_{n+1}) = 0 \text{ for all } \lambda > 0. \tag{2.5}$$

Thus there exist $0 < r < 1$ and $0 < \ell \leq +\infty$ such that

$$\lim_{n\to+\infty} \frac{\Theta\big(\varpi_\lambda(a_n, a_{n+1})\big) - 1}{[\varpi_\lambda(a_n, a_{n+1})]^r} = \ell. \tag{2.6}$$

Now, let $B^{-1} \in (0, \ell)$. From the definition of limit, there exists $n_\lambda \in \mathbb{N}$ such that

$$\frac{\Theta\big(\varpi_\lambda(a_n, a_{n+1})\big) - 1}{[\varpi_\lambda(a_n, a_{n+1})]^r} \geq B^{-1} \quad \text{for all} \quad n \geq n_\lambda,$$

and so

$$n[\varpi_\lambda(a_n, a_{n+1})]^r \leq nB[\Theta\big(\varpi_\lambda(a_n, a_{n+1})\big) - 1] \quad \text{for all} \quad n \geq n_\lambda.$$

From (2.4), we deduce

$$n[\varpi_\lambda(a_n, a_{n+1})]^r \leq nB[(\Theta(\varpi_\lambda(a_0, a_1)))^{k^n} - 1] \quad \text{for all} \quad n \geq n_\lambda.$$

Taking the limit as $n \to +\infty$ in the above inequality, we have

$$\lim_{n\to+\infty} n[\varpi_\lambda(a_n, a_{n+1})]^r = 0 \text{ for all } \lambda > 0. \tag{2.7}$$

From (2.7), it follows that for all $\lambda > 0$ there exists $N_\lambda \in \mathbb{N}$ such that

$$n[\varpi_\lambda(a_n, a_{n+1})]^r \leq 1 \quad \text{for all} \quad n \geq N_\lambda.$$

Thus

$$\varpi_\lambda(a_n, a_{n+1}) \leq \frac{1}{n^{1/r}} \quad \text{for all} \quad n \geq N_\lambda,\ \lambda > 0. \tag{2.8}$$

By regularity of ϖ and χ_ϖ is non-Archimedean, then for $m > n \geq N_\lambda$, by (2.8), we get

$$\omega_1(a_n, a_m) \leq \sum_{i=n}^{m-1} \varpi_1(a_i, a_{i+1}) \leq \sum_{i=n}^{m-1} \frac{1}{i^{1/r}}.$$

Since $0 < r < 1$, then

$$\lim_{n\to+\infty} \sum_{i=n}^{\infty} \frac{1}{i^{1/r}} = 0$$

and hence $\varpi_1(a_n, a_m) \to 0$ as $m, n \to +\infty$. Thus, we have proved that $\{a_n\}$ is a $\varpi-$Cauchy sequence in A. Closeness of A_1 implies that A_1 is complete. So, there exists $a^* \in A_1$ such that $\omega_1(a_n, a^*) \to 0$ as $n \to \infty$. Using (2.2), since g is an $\alpha - \varpi-$continuous mapping, $\varpi_1(ga_n, ga^*) \to 0$ as $n \to +\infty$.
Now,

$$\begin{aligned} \varpi_1(a^*, ga^*) &\leq \varpi_1(a^*, a_{n+1}) + \varpi_1(a_{n+1}, ga_n) + \varpi_1(ga_n, ga^*) \\ &= \varpi_1(a^*, a_{n+1}) + \varpi_1(A_1, A_2) + \varpi_1(ga_n, ga^*), \end{aligned}$$

taking limit as $n \to +\infty$, we get $\varpi_1(a^*, ga^*) = \varpi_1(A_1, A_2)$ and hence a^* is best proximity point of g. □

If in above theorem, we set $\alpha(a, b) = 1$, for all $a, b \in A_1$, we get the following corollary.

Corollary 2.5. *Let χ_ω be a complete non-Archimedean modular metric space. Let A_1 and A_2 be two non-empty subsets of χ_ω; such that A_1 is closed and (A_1, A_2) has weak $P_\lambda-$ property. Let $g : A_1 \to A_2$ satisfies the following conditions:*

(i) *g is a $\Theta - \omega-$contraction mapping;*
(ii) *$g(A^\lambda_{1_0}) \subseteq A^\lambda_{2_0}$;*
(iii) *there exist $a_0, a_1 \in A^\lambda_0$ such that $\omega_\lambda(a_1, ga_0) = \omega_\lambda(A_1, A_2)$;*
(iv) *g is a $\varpi-$continuous mapping.*

Then g has a unique best proximity point $a^* \in A_1$.

Theorem 2.6. *Under the hypotheses of Theorem 2.4, without the continuity assumption of g, assume that*
for a $\varpi-$convergent sequence $\{a_n\} \in A_1$ to some $a^* \in A$, such that $\alpha(a_n, a_{n+1}) \geq 1$, then $\alpha(a_n, a^*) \geq 1$. Then g has best proximity point.

Proof. As in the proof of Theorem 2.4, we deduce that there exists a $\varpi-$Cauchy sequence $\{a_n\}$ in A_1, which converges to some $a^* \in A_1$. By condition (iv), we have $\alpha(a_n, a^*) \geq 1$.
Now, by regularity of ϖ and condition (i),

$$\begin{aligned} \Theta(\varpi_1(ga_n, ga^*)) &\leq \alpha(a_n, a^*)\Theta(\varpi_1(ga_n, ga^*)) \\ &\leq [\Theta(\varpi_1(a_n, a^*))]^k \\ &< \Theta(\varpi_1(a_n, a^*). \end{aligned}$$

Since Θ is increasing, then we have

$$\varpi_1(ga_n, ga^*) < \varpi_1(a_n, a^*).$$

Taking limit as $n \to \infty$

$$\varpi_1(ga_n, ga^*) \to 0.$$

So,

$$\begin{aligned} \varpi_1(a^*, ga^*) &\leq \varpi_1(a^*, a_{n+1}) + \varpi_1(a_{n+1}, ga_n) + \varpi_1(ga_n, ga^*) \\ &= \varpi_1(a^*, a_{n+1}) + \varpi_1(A_1, A_2) + \varpi_1(ga_n, ga^*). \end{aligned}$$

Taking limit as $n \to \infty$, we have $\varpi_1(a^*, ga^*) = \varpi_1(A_1, A_2)$ and hence a^* is best proximity point of g. □

To prove uniqueness of best proximity point of g, we introduce the following condition.
Condition $(\mathbb{B})$:
For any distinct best proximity points $a^*,\ b^*$, we have $\alpha(a^*, b^*) \geq 1$.

Theorem 2.7. *Applying condition $(\mathbb{B})$ in Theorem 2.4 (Theorem 2.6 respectively), then the best proximity point a^* is unique.*

Proof. Let b^* be another best proximity points in A_1, such that $a^* \neq b^*$, $\varpi_1(a^*, ga^*) = \varpi_1(A_1, A_2)$ and $\varpi_1(b^*, gb^*) = \varpi_1(A_1, A_2)$ with $\alpha(a^*, b^*) \geq 1$. Then by weak $P_\lambda-$ property and condition (i), we have

$$\begin{aligned} \Theta(\varpi_1(a^*, b^*)) &\leq \alpha(x^*, u^*)\Theta(\varpi_1(ga^*, gb^*) \\ &\leq [\Theta(\varpi_1(a^*, b^*)]^k \\ &< \Theta(\varpi_1(a^*, b^*) \end{aligned}$$

which is contradiction, and hence $a^* = b^*$. □

Example 2.8. *Let $(\mathbb{R}^2, \varpi)$ be a complete non-Archimedean modular metric space, where $\varpi_\lambda((a_1, a_2), (b_1, b_2)) = \frac{1}{\lambda}(|a_1 - b_1| + |a_2 - b_2|)$ for all $\lambda > 0$.*
Define the sets $A_1 = \{(1, 0), (4, 5), (5, 4)\} \cup [-\infty, -1] \times [-\infty, -1]$ and $A_2 = \{(0, 0), (2, 0), (0, 2)\} \cup [10, \infty) \times [10, \infty)$.
Clearly A_1 and A_2 are nonempty closed subsets of χ, $\varpi_\lambda(A_1, A_2) = \frac{1}{\lambda}$, $A_{1}{}_0^\lambda = \{(1, 0)\}$, $A_{2}{}_0^\lambda = \{(0, 0), (2, 0)\}$ and the pair (A_1, A_2) has the weak property P_λ.
Define $g : A_1 \to A_2$ by

$$g(a) = \begin{cases} (10a_1^2, 10a_2^4), & \text{if } a_1, a_2 \in [-\infty, -1] \\ (\frac{a_1}{2}, 0), & \text{if } a_1, a_2 \notin [-\infty, -1] \text{ with } a_1 \leq a_2 \\ (0, \frac{a_2}{2}), & \text{if } a_1, a_2 \notin [-\infty, -1] \text{ with } a_1 > a_2 \end{cases}$$

Notice that $gA_{1}{}_0^\lambda \subseteq A_{2}{}_0^\lambda$, for all $\lambda > 0$.
Define the function $\alpha : A_1 \times A_1 \to [0, \infty)$ by:

$$\alpha((a_1, a_2), (b_1, b_2)) = \begin{cases} 2, & \text{if } (a_1, a_1), (b_1, b_2) \in \{(1, 0), (4, 5), (5, 4) : \text{ with } a_1 \neq b_2\} \\ \frac{1}{4}, & \text{if otherwise} \end{cases}$$

If $\alpha((a_1, a_2), (b_1, b_2) \geq 1$, then

$$(a_1, a_2), (b_1, b_2) \in \{(1, 0), (4, 5), (5, 4) : \text{ with } a_1 \neq b_2\}.$$

So,

$$(ga_1, ga_2), (gb_1, gb_2) \in \{(0, 0), (2, 0), (0, 2)\}.$$

For

$$\varpi_\lambda((u_1, u_2), (ga_1, ga_2)) = \frac{1}{\lambda}$$
$$\varpi_\lambda((v_1, v_2), (gy_1, gb_2)) = \frac{1}{\lambda}.$$

Then,$(u_1, u_2) = (v_1, v_2) = (1, 0) \Longrightarrow \alpha((u_1, u_2), (v_1, v_2)) = 2 > 1$ and hence g is an $\alpha-$proximal admissible.
Let $\Theta(t) = e^{\sqrt{t}}$, and for $((x_1, x_1), (y_1, y_2)) \in A$ with $\alpha((a_1, a_1), (b_1, b_2)) \geq 1$ we have

cases.
We can assume that

$$((a_1,a_2),(b_1,b_2)) = ((1,0),(4,5)) \text{ or } ((a_1,a_2),(b_1,b_2)) = ((1,0),(5,4)).$$

So, these two cases will be distinguished as follows:

1. *if* $((a_1,a_2),(b_1,b_2)) = ((1,0),(4,6))$, *then*

$$\begin{aligned}\varpi_\lambda(g(ab_1,a_2),g(b_{1,2})) &= varpi_\lambda((0,0),(2,0)) \\ &= \frac{1}{\lambda}(|0-2|+|0-0|) \\ &= \frac{1}{\lambda}(|0-\frac{4}{2}|+|0-0|) \\ &= \frac{1}{2}.\frac{1}{\lambda}(|4|) \\ &< \frac{1}{2}.\frac{1}{\lambda}(|1-4|+|0-5|) \\ &= \frac{1}{2}.\varpi_\lambda((a_1,a_2),(b_1,b_2)). \qquad (2.9)\end{aligned}$$

2. *if* $(a_1,a_2),(b_1,b_2) = (1,0),(6,4)$,
   we get similarly same result as in (2.9).

In both cases we have $\varpi_\lambda(g(a_1,a_2),g(b_1,b_2)) > 0$,
and hence we get,

$$\begin{aligned}\Theta(\varpi_\lambda(g(a_1,a_2),g(b_1,b_2))) &= e^{\sqrt{\varpi_\lambda(g(a_1,a_2),g(b_1,b_2))}} \\ &< e^{\sqrt{\frac{1}{2}\varpi_\lambda((a_1,a_2),(b_1,b_2))}} \\ &= e^{\sqrt{\varpi_\lambda((a_1,a_2),(b_1,b_2))}\frac{1}{\sqrt{2}}} \\ &= [e^{\sqrt{\varpi_\lambda((a_1,a_2),(b_1,b_2))}}]^{\frac{1}{\sqrt{2}}} \\ &= [\Theta(\varpi_\lambda((a_1,a_2),(b_1,b_2)))]^{\frac{1}{\sqrt{2}}}.\end{aligned}$$

Hence g *is an* $(\alpha,\Theta)-\varpi-$*contraction with* $k=\frac{1}{\sqrt{2}}$.
Moreover, if $\{x_n\}$ *is a sequence in* A_1 *with* $\alpha(a_n,a_{n+1}) \geq 1$ *for all* $n \in \mathbb{N}\cup\{0\}$, *and* $a_n \to a$ *as* $n \to \infty$, *then* $\{a_n\} \subseteq \{(1,0),(4,5),(5,4): \text{ with } a_1 \neq b_2\}$. *This implies that* $a \in \{(1,0),(4,5),(5,4): \text{ with } x_1 \neq y_2\}$, *and hence* $\alpha(x_n,x) \geq 1$.
All hypothesis of Theorem 2.6 are hold true, and hence T *has best proximity point* $(0,1)$ *which is unique ,that is*

$$\varpi_\lambda((0,1),g(0,1)) = \frac{1}{\lambda} = \varpi_\lambda(A_1,A_2).$$

If $A_1 = A_2 = \chi_\varpi$ in both Theorems 2.4 and 2.6 and $\alpha(a,b)=1$ for all $a,b \in \chi_\varpi$, this leads us to the next corollary.

Corollary 2.9. *Let* χ_ϖ *be a complete non-Archimedean modular metric space, and let* g *be a continuous self-mapping on* χ_ϖ. *Suppose that there exists a function* $\Theta \in \Delta_\Theta$ *such that*

$$\Theta(\varpi_\lambda(ga,gb)) \leq [\Theta(\varpi_\lambda(a,b))]^k;$$

whenever $\varpi_\lambda(ga,gb) > 0$ *and* $0<k<1$. *Then* g *has a unique fixed point.*

3. Results in partially ordered modular metric space

Let $(\chi_{\varpi}, \preceq)$ be a partially ordered modular metric space. Let A_1 and A_2 be two non-empty subsets of χ_{ϖ}. Best proximity point results in partially ordered metric space have been discussed by many authors (see [1], [8], [15], [19], [22], [31]). In this section, we will introduce some new best proximity and fixed point results for such mappings in partially ordered non-Archimedean modular metric space influenced by Δ_Θ class functions.

Definition 3.1. *A mapping $g : A_1 \to A_2$ is said to be a proximally order-preserving, if for all $a_1, a_2, b_1, b_2 \in A_1$, $\lambda > 0$,*

$$\begin{cases} a_1 \preceq a_2 \\ \varpi_\lambda(b_1, ga_1) = \varpi_\lambda(A_1, A_2) \\ \varpi_\lambda(b_2, ga_2) = \varpi_\lambda(A_1, A_2) \end{cases} \Longrightarrow b_1 \preceq b_2$$

If $A_1 = A_2 = \chi_{\varpi}$, then the map g is called a non-decreasing map.

Theorem 3.2. *Let $(\chi_{\varpi}, \preceq)$ be a partially ordered complete modular metric space. Let A_1 and A_2 be two nonempty subsets of χ_{ϖ} such that A_1 is closed and (A_1, A_2) has weak $P_\lambda-$property. Let $g : A_1 \to A_2$ be a non-self mapping. Suppose that there exists a function $\Theta \in \Delta_\Theta$ such that the following conditions are satisfied:*

(i) *for all $a, b \in A_1$ with $a \preceq b$*

$$\Theta\big(\varpi_\lambda(ga, gb)\big) \leq \Big[\Theta\big(\varpi_\lambda(a, b)\big)\Big]^k, \tag{3.1}$$

whenever $\varpi_\lambda(ga, gb) > 0$, where $0 < k < 1$;

(ii) *g is proximally order-preserving;*

(iii) *$g(A^\lambda_{1_0}) \subseteq A^\lambda_{2_0}$;*

(iv) *there exist a_0, a_1 in A^λ_0 such that, $\varpi_\lambda(a_1, ga_0) = \varpi_\lambda(A_1, A_2)$ implies $a_0 \preceq a_1$;*

(v) *g is continuous.*

Then g has best proximity point $a^* \in A_1$.

Proof. Define $\alpha : A_1 \times A_1 \times (0, \infty) \to [0, +\infty)$ by

$$\alpha(a, b) = \begin{cases} 2, & \text{if } a \preceq b \\ \\ \frac{1}{2}, & \text{otherwise.} \end{cases}$$

At first we prove that, g is an α-proximal admissible mapping. For this we may assume that if for $a, b, u, v \in A_1$,

$$\begin{cases} \alpha(a, b) \geq 1 \\ \varpi_\lambda(u, ga) = \varpi_\lambda(A_1, A_2) \\ \varpi_\lambda(v, gb) = \varpi_\lambda(A_1, A_2). \end{cases}$$

Then

$$\begin{cases} a \preceq b \\ \varpi_\lambda(u, ga) = \varpi_\lambda(A_1, A_2) \\ \varpi_\lambda(v, gb) = \varpi_\lambda(A_1, A_2). \end{cases}$$

Now, since, g is proximally order-preserving so, $u \preceq v$; and hence $\alpha(u, v) \geq 1$. It is obvious that g is an $(\alpha, \Theta) - \omega-$ contraction non-self mapping. Consequently, all conditions of Theorem 2.4 hold true, and hence g has best proximity point $a^* \in A_1$. $\square$

Similarly, the next theorem follows from Theorem 2.6, and get a result of best proximity point in partially ordered modular spaces.

Theorem 3.3. *Under the hypotheses of Theorem 3.2, without the continuity assumption of g, assume that*
for a $\varpi-$convergent sequence $\{a_n\} \in A_1$ to some $a^* \in A_1$, such that $\alpha(a_n, a_{n+1}) \geq 1$, then $\alpha(a_n, a^*) \geq 1$. Then g has best proximity point $a^* \in A_1$.

Using the following condition for uniqueness of best proximity point in partially ordered modular metric space. Condition $(\mathbb{B}^{'})$:
For any distinct best proximity points $a^*,\ b^* \in (X_\omega, \preceq)$, we have $a^* \preceq b^*$.

Theorem 3.4. *Applying condition $(\mathbb{B}^{'})$ in Theorem 3.2, (Theorem 3.3 respectively), then the best proximity point a^* of g is unique.*

If $A_1 = A_2 = \chi_\varpi$, in Theorem 3.2 (Theorem 3.3 respectively), we have the new fixed point result.

Corollary 3.5. *Let $(\chi_\varpi, \preceq)$ be a partially ordered complete modular metric space. Let g be a non-decreasing self mapping on χ_ϖ satisfying 3.1 for all $a, b \in \chi_\varpi$ such that $a \preceq b$. Suppose following conditions hold true:*

(i) *there exist a_0 in χ_ϖ such that $a_1 \preceq ga_0$;*
(ii) *either g is continuous or for $\{a_n\}$ is a sequence in χ_ϖ such that $a_n \preceq a_{n+1}$ with $\varpi_\lambda(a_n, a^*) \to 0$, as $n \to +\infty$, $\lambda > 0$, then $a_n \preceq a^*$.*

Then g has a fixed point $a^* \in \chi_\varpi$.
If for any distinct fixed points a^*, b^*, we have $a^* \preceq b^*$, then the fixed point is unique.

4. Modular Metric Spaces to Fuzzy Metric Spaces

In this section, we show that best proximity point results in fuzzy metric spaces can be easily derived from corresponding results in modular metric spaces.

Definition 4.1. *A binary operation $* : [0,1] \times [0,1] \to [0,1]$ is called a continuous t-norm if it satisfies the following assertions:*

(CTN1) *$*$ is commutative and associative;*
(CTN1) *$*$ is continuous;*
(CTN1) *$x * 1 = x$ for all $x \in [0,1]$;*
(CTN1) *$x_1 * y_1 \leq x^{'} * y^{'}$ when $x_1 \leq x^{'}$ and $y_1 \leq y^{'}$ and $x_1, y_1, x^{'}, y^{'} \in [0,1]$.*

Examples of t-norm are $x * y = \min\{x, y\}$, $x * y = xy$ and $x * y = max\{0, x + y - 1\}$.

Definition 4.2. *([18]) For a nonempty set χ and a continuous t-norm $*$ and a fuzzy set $\mu : \chi \times \chi \times (0, +\infty)$, satisfying the following conditions, for all $a, b, c \in \chi$ and $t_1, t_2 > 0$:*

(FM1) *$\mu(a, b, t_1) > 0$;*
(FM2) *$\mu(a, b, t_1) = 1$ if and only if a, b;*
(FM3) *$\mu(a, b, t_1) = \mu(b, a, t_1)$;*
(FM4) *$\mu(a, b, t_1) * \mu(b, c, t_2) \leq \mu(a, c, t_1 + t_2)$;*
(FM5) *$\mu(a, b, \cdot) : (0, +\infty) \to (0, 1]$ is left continuous.*

Then the triple $(\chi, \mu, *)$ is called a fuzzy metric space.

If condition $(FM2)$ is replaced by

$$\mu(a, b, t_1) = 1 \ if \ and \ only \ if \ a = b, \ for \ some \ t_1 > 0,$$

then μ is said to be regular.

$$If\ \mu(a,b,t_1)*\mu(b,c,t_2)\leq\mu(a,c,\max\{t_1,t_2\}),$$

is instead of condition $(FM4)$, then μ is non-Archimedean.

Definition 4.3 ([16])**.** *Let $(\chi,\mu,*)$ be a fuzzy metric space. The fuzzy metric μ is called triangular whenever*

$$\frac{1}{\mu(a,b,t_1)}-1\leq\frac{1}{\mu(a,c,t_1)}-1+\frac{1}{\mu(c,b,t_1)}-1$$

for all $a,b,c\in\chi$ and all $t_1>0$.

In the definition that follows, we give some notions of convergence and continuity.

Definition 4.4. *Let $(\chi,\mu,*)$ be a fuzzy metric and $g:\chi\to\chi$ and $\alpha:\chi\times\chi\to[0,+\infty)$ be functions. Then:*

(i) *The sequence $\{a_n\}$ is said to be μ^t-Cauchy sequence if for all $0<\epsilon<1$, $\lim_{m,n\to\infty}\mu(a_n,a_m,t)=1$, for all $m>n$ $t>0$;*
(ii) *The sequence $\{a_n\}$ is said to convergent to $a\in\chi$, if $\lim_{n\to\infty}\mu(a_n,a,t)=1$ for all $t>0$;*
(iii) *$(\chi,\mu,*)$ is called complete if for every μ^t-Cauchy sequence is convergent in χ.*
(iv) *g is an $\mu^t\alpha$-continuous mapping, if $\lim\limits_{n\to+\infty}\mu(a_n,a,t)=1$ with $\alpha(a_n,a_{n+1})\geq1$ implies $\lim\limits_{n\to+\infty}\mu(ga_n,ga,t)=1$, for all $t>0$;*
(v) *g is an μ^t-continuous mapping, if $\lim\limits_{n\to+\infty}\mu(a_n,a,t)=1$ implies $\lim\limits_{n\to+\infty}\mu(ga_n,ga,t)=1$.*

Let A_1 and A_2 be two nonempty subsets of $(\chi,\mu,*)$. We introduce the following definitions:

Definition 4.5.

$$A_{10}(t)=\{a\in A_1:\mu(a,b,t)=\mu(A_1,A_2,t)\ for\ some\ b\ \in\ A_2\};$$
$$A_{20}(t)=\{b\in A_2:\mu(a,b,t)=\mu(A_1,A_2,t)\ for\ some\ a\ \in\ A_1\};$$

where $\mu(A_1,A_2,t)=\sup\{\mu(a,b,t):a\in A_1,\ b\in A_2\}$.
A point $a^*\in A_1$ is called best proximity point in $(\chi,\mu,*)$ if

$$\mu(a^*,ga^*,t)=\mu(A_1,A_2,t),$$

for all $t>0$.

Very recently, Hussain and Salimi in [23] proved the following useful lemma, which establishes a relation between fuzzy metric and modular metric.

Lemma 4.6. [23] *Let $(\chi,\mu,*)$ be a triangular fuzzy metric space. Define*

$$\varpi_\lambda(a,b)=\frac{1}{\mu(a,b,\lambda)}-1 \tag{4.1}$$

for all $a,b\in\chi$ and all $\lambda>0$. Then ϖ_λ is a modular metric on χ.

We combine Lemma 4.6 and our previous theorems, and deduce the following new results in triangular non-Archimedean fuzzy metric spaces.
Note that: in all next results:

(.) μ is supposed to be triangular and regular.

(.) α is a function defined as $\alpha : A \times A \to [0, +\infty)$, and $\Theta \in \Delta_\Theta$.

Theorem 4.7. *Let $(\chi, \mu, *)$ be a complete non-Archimedean fuzzy metric space . Let A_1 and A_2 be two nonempty subsets of χ, where A_1 is closed and (A_1, A_2) has weak P^t-property. Let $g : A_1 \to A_2$ satisfies the following conditions:*

(i) *there exists $k \in (0,1)$ such that for $a, b \in A_1$ with $\alpha(a,b) \geq 1$, we have*

$$\alpha(a,b)\Theta\big(\tfrac{1}{\alpha(a,b)\mu(ga,gb,t)} - 1\big) \leq \Big[\Theta\big(\tfrac{1}{\mu(a,b,t)} - 1\big)\Big]^k;$$

whenever $\mu(ga, gb, t) < 1$, and $t > 0$;

(ii) *g is an α-proximal admissible mapping;*

(iii) *$g(A_{10}(t)) \subseteq A_{20}(t)$ for all $t > 0$;*

(iv) *there exist elements a_0 and a_1 in $A_{10}(t)$ with $\alpha(a_0, a_1) \geq 1$, such that $\mu(a_1, ga_0, t) = \mu(A_1, A_2, t)$; for all $t > 0$;*

(v) *g is an $\alpha - \mu^t$-continuous mapping.*

If there exists $a_0 \in \chi$ such that $\lim_{t\to\infty} \mu(a, a_0, t) = 1$ implies $\lim_{t\to\infty} \mu(ga, a_0, t) = 1$, then g has best proximity point $a^* \in A_1$.

Proof. Let χ_ϖ be the modular metric space around a_0 induced by the modular metric ϖ_λ defined as in Lemma 4.6, that is

$$\chi_\varpi = \{a \in \chi : \lim_{t\to\infty} \varpi_t(a, a_0) = 0\},$$

or equivalently

$$X_\varpi = \{a \in \chi : \lim_{t\to\infty} \mu(a, a_0, t) = 1\}.$$

Trivially $\chi_\varpi \neq \phi$, since $a_0 \in \chi_\varpi$. Now, we show that χ_ϖ is closed in $(\chi, \mu, *)$. Let $\{a_n\}$ be a sequence in χ_ϖ converges to some $a \in \chi$, then for each $\epsilon \in (0,1)$ and $t_0 > 0$, there exists $n_0 \in \mathcal{N}$ such that $\mu(a_{n_0}, a, t_0) > 1 - \epsilon$. From $(FM4)$ in Definition 4.2, we have

$$\begin{aligned} \mu(a_0, a, t) &= \mu(a_0, a, (t - t_0 + t_0)) \\ &\geq \mu(a_0, a_{n_0}, t - t_0) * \mu(a_{n_0}, a, t_0) \\ &> \mu(a_0, a_{n_0}, t - t_0) * (1 - \epsilon). \end{aligned}$$

Taking limits as $t \to \infty$ in above inequalities, we get

$$\lim_{n\to\infty} \mu(a_0, a, t) \geq 1 - \epsilon \ for\ all\ \epsilon > 0,$$

and hence $a \in \chi_\varpi$, that is, χ_ϖ is closed in $(\chi, \mu, *)$ and so complete. All hypotheses of Theorem 2.4 are hold true, so we get the conclusion.

□

A similar remark to the one above and Theorem 2.6 yield the following result.

Theorem 4.8. *Under the hypotheses of Theorem 4.7, without the continuity assumption of g, assume that*
for a convergent sequence $\{a_n\} \in A_1$ to some $a^* \in A_1$, such that $\alpha(a_n, a_{n+1}) \geq 1$, then $\alpha(a_n, a^*) \geq 1$. Then g has best proximity point $a^* \in A_1$.

If $A_1 = A_2 = \chi$, we get a fixed point theorem as a corollary as following

Corollary 4.9. *Let $(\chi, \mu, *)$ be a complete non-Archimedean fuzzy metric space. Let g be a self mapping on χ satisfies the following conditions:*

(i) *there exists* $k \in (0,1)$ *such that for* $a, b \in A_1$ *with* $\alpha(a,b) \geq 1$*, we have*

$$\alpha(a,b)\Theta\big(\tfrac{1}{\mu(ga,gb,t)} - 1\big) \leq \Big[\Theta\big(\tfrac{1}{\mu(a,b,t)} - 1\big)\Big]^k;$$

whenever $\mu(ga, gb, t) < 1$*, and* $t > 0$;

(ii) g *is an* α*-admissible mapping;*

(iii) *there exists* $a_0 \in \chi$ *such that* $\alpha(a_1, ga_0) \geq 1$;

(iv) *either* g *is an*$\alpha - \mu^t-$*continuous mapping, or for a sequence* $\{a_n\} \subseteq \chi$ *with* $\alpha(a_n, a_{n+1}) \geq 1$ *such that* $a_n \to a$*, then* $\alpha(a_n, a) \geq 1$.

If there exists $a_0 \in \chi$ *such that* $\lim_{t\to\infty} \mu(a, a_0, t) = 1$ *implies* $\lim_{t\to\infty} \mu(ga, a_0, t) = 1$*,, then* g *has fixed point* $a^* \in \chi$.

If $\alpha(a,b) = 1$, then we get the following corollary.

Corollary 4.10. *Let* $(\chi, \mu*)$ *be a complete non-Archimedean fuzzy metric space. Let* g *be an* μ^t-*continuous self mapping on* χ*, such that*

$$\Theta\big(\tfrac{1}{\mu(ga,gb,t)} - 1\big) \leq \Big[\Theta\big(\tfrac{1}{\mu(a,b,t)} - 1\big)\Big]^k;$$

whenever $\mu(ga, gb, t) < 1$*, and* $0 < k < 1$ *,* $t > 0$*. Then* g *has a unique fixed point.*

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