Small Ramsey Numbers

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ABSTRACT: We present data which, to the best of our knowledge, includes all known nontrivial values and bounds for specific graph, hypergraph and multicolor Ramsey numbers, where the avoided graphs are complete or complete without one edge. Many results pertaining to other more studied cases are also presented. We give references to all cited bounds and values, as well as to previous similar compilations. We do not attempt complete coverage of asymptotic behavior of Ramsey numbers, but concentrate on their specific values.

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1. Scope and Notation

There is a vast literature on Ramsey type problems starting in 1930 with the original paper of Ramsey [Ram]. Graham, Rothschild and Spencer in their book [GRS] present an exciting development of Ramsey Theory. The subject has grown amazingly, in particular with regard to asymptotic bounds for various types of Ramsey numbers (see the survey papers [GrRö, Neš, ChGra2]), but the progress on evaluating the basic numbers themselves has been very unsatisfactory for a long time. In the last two decades, however, considerable progress has been obtained in this area, mostly by employing computer algorithms. The few known exact values and several bounds for different numbers are scattered among many technical papers. This compilation is a fast source of references for the best results known for specific numbers. It is not supposed to serve as a source of definitions or theorems, but these can be easily accessed via the references gathered here.

Ramsey Theory studies conditions when a combinatorial object contains necessarily some smaller given objects. The role of Ramsey numbers is to quantify some of the general existential theorems in Ramsey Theory.

Let $G_1, G_2, \ldots, G_m$ be graphs or s-uniform hypergraphs (s is the number of vertices in each edge). $R(G_1, G_2, \ldots, G_m; s)$ denotes the m-color **Ramsey number** for s-uniform graphs/hypergraphs, avoiding G_i in color i for $1 \le i \le m$. It is defined as the least integer n such that, in any coloring with m colors of the s-subsets of a set of n elements, for some i the s-subsets of color i contain a sub-(hyper)graph isomorphic to G_i (not necessarily induced). The value of $R(G_1, G_2, \ldots, G_m; s)$ is fixed under permutations of the first m arguments.

If $s = 2$ (standard graphs) then s can be omitted. If G_i is a complete graph K_k, then we can write k instead of G_i, and if $G_i = G$ for all i we can use the abbreviation $R_m(G; s)$ or $R_m(G)$. For $s = 2$, $K_k - e$ denotes a K_k without one edge, and for $s = 3$, $K_k - t$ denotes a K_k without one triangle (hyperedge). P_i is a **path** on i vertices, C_i is a **cycle** of length i, and W_i is a **wheel** with $i-1$ spokes, i.e. a graph formed by some vertex x, connected to all vertices of some cycle C_{i-1}. $K_{n,m}$ is a complete n by m bipartite graph, in particular $K_{1,n}$ is a **star** graph. The **book** graph $B_i = K_2 + \overline{K}_i = K_1 + K_{1,i}$ has $i+2$ vertices, and can be seen as i triangular pages attached to a single edge. The **fan** graph F_n is defined by $F_n = K_1 + nK_2$. For a graph G, $n(G)$ and $e(G)$ denote the number of vertices and edges, respectively. Finally, let $\chi(G)$ be the chromatic number of G, and let nG denote n disjoint copies of G.

Section 2 contains the data for the classical two color Ramsey numbers $R(k, l)$ for complete graphs, and section 3 for the two color case when the avoided graphs are complete or have the form $K_k - e$, but not both are complete. Section 4 lists the most studied two color cases for other graphs. The multicolor and hypergraph cases are gathered in sections 5 and 6, respectively. Finally, section 7 gives pointers to cumulative data and to most of the previous surveys.

2. Classical Two Color Ramsey Numbers

2.1. Upper and lower bounds on $R(k,l)$

k \ l	3	4	5	6	7	8	9	10	11	12	13	14	15
3	6	9	14	18	23	28	36	40 43	46 51	52 59	59 69	66 78	73 88
4		18	25	35 41	49 61	56 84	69 115	92 149	97 191	128 238	133 291	141 349	153 417
5			43 49	58 87	80 143	101 216	121 316	141 442	157	181	205	233	261
6				102 165	111 298	127 495	169 780	178 1171	253	262	317		401
7					205 540	216 1031	232 1713	2826	405	416	511		
8						282 1870	317 3583	6090			817		861
9							565 6588	580 12677					
10								798 23556					1265

Table I. Known nontrivial values and bounds for two color Ramsey numbers $R(k,l) = R(k,l\,;2)$.

k \ l	4	5	6	7	8	9	10	11	12	13	14	15
3	GG	GG	Kéry	Ka2 GY	GR MZ	Ka2 GR	Ex5 RK2	Ka2 RK2	Ex12 Les	Piw1 RK2	Ex8 RK2	WW Les
4	GG	Ka1 MR4	Ex9 MR5	Ex3 Mac	Ex15 Mac	RK1 Mac	HaKr Mac	2.3.e Spe3	SLL2 Spe3	2.3.e Spe3	XXR Spe3	XXR Spe3
5		Ex4 MR5	Ex9 HZ1	CET Spe3	HaKr Spe3	Haa Mac	Ex12 Mac	XXER	Ex12	XXER	XXER	XXER
6			Ka1 Mac	2.3.e Mac	XXR Mac	XXER Mac	2.3.e Mac	XXR	2.3.e	XXER		2.3.h
7				She1 Mac	2.3.e Mac	2.3.g HZ1	Mac	XXER	2.3.e	XXR		
8					BR Mac	XXER Ea1	HZ1			XXER		2.3.h
9						She1 ShZ1	2.3.e Ea1					
10							She1 Shi2					2.3.h

References for Table I.

We split the data into the table of values and a table with corresponding references. In Table I, known exact values appear as centered entries, lower bounds as top entries, and upper bounds as bottom entries.

The task of proving $R(3,3) \le 6$ was the second problem in Part I of the William Lowell Putnam Mathematical Competition held in March 1953 [Bush].

All the critical graphs for the numbers $R(k,l)$ (graphs on $R(k,l)-1$ vertices without K_k and without K_l in the complement) are known for $k=3$ and $l=3$, 4, 5 [Kéry], 6 [Ka2], 7 [RK3, MZ], and there are 1, 3, 1, 7 and 191 of them, respectively. All $(3,k)$-graphs, for $k \le 6$, were enumerated in [RK3], and all (4,4)-graphs in [MR2]. There exists a unique critical graph for $R(4,4)$ [Ka2]. There are 430215 such graphs known for $R(3,8)$ [McK], 1 for $R(3,9)$ [Ka2] and 350904 for $R(4,5)$ [MR4], but there might be more of them. In [MR5] evidence is given for the conjecture that $R(5,5) = 43$ and that there exist 656 critical graphs on 42 vertices. The graphs constructed by Exoo in [Ex9, Ex12, Ex13, Ex14, Ex15, Ex16], and some others, are available electronically from http://ginger.indstate.edu/ge/RAMSEY.

The construction by Mathon [Mat] and Shearer [She1] (see also sections 2.3.i, 5.2.h and 5.2.i), using data obtained by Shearer [She1], gives the following lower bounds for higher diagonal numbers: $R(11,11) \ge 1597$, $R(13,13) \ge 2557$, $R(14,14) \ge 2989$, $R(15,15) \ge 5485$, and $R(16,16) \ge 5605$. Similarly, $R(17,17) \ge 8917$, $R(18,18) \ge 11005$ and $R(19,19) \ge 17885$ were obtained in [LSL]. The same approach does not improve on an easy bound $R(12,12) \ge 1637$ [XXR], which can be obtained by applying twice 2.3.e. Only some of the higher bounds implied by 2.3.* are shown, and more similar bounds could be easily derived. In general, we show bounds beyond the contiguous small values if they improve on results previously reported in this survey or published elsewhere. Some easy upper bounds implied by 2.3.a are marked as [Ea1].

Cyclic (or *circular*) graphs are often used for Ramsey graph constructions. Several cyclic graphs establishing lower bounds were given in the Ph.D. dissertation by J.G. Kalbfleisch in 1966, and many others were published in the next few decades. Only recently Harborth and Krause [HaKr] presented all best lower bounds up to 102 from cyclic graphs avoiding complete graphs. In particular, no lower bound in Table I can be improved with a cyclic graph on less than 102 vertices. See also item 2.3.k and section 4.16 [HaKr].

The claim that $R(5,5) = 50$ posted on the web [Stone] is in error, and despite being shown so more than once, this incorrect value is being cited by some authors. The bound $R(3,13) \ge 60$ [XZ] cited in the 1995 version of this survey was shown to be incorrect in [Piw1]. Another incorrect construction for $R(3,10) \ge 41$ was described in [DuHu].

There are really only two general upper bound inequalities useful for small parameters, namely 2.3.a and 2.3.b. Stronger upper bounds for specific parameters were difficult to obtain, and they often involved massive computations, like those for the cases of (3,8) [MZ], (4,5) [MR4], (4,6) and (5,5) [MR5]. The bound $R(6,6) \le 166$, only 1 more than the best known [Mac], is an easy consequence of a theorem in [Walk] (2.3.b) and $R(4,6) \le 41$. T. Spencer [Spe3], Mackey [Mac], and Huang and Zhang [HZ1], using the bounds for minimum and maximum number of edges in (4,5) Ramsey graphs listed in [MR3, MR5], were able to establish new upper bounds for several higher Ramsey numbers, improving on all of the

previous longstanding results by Giraud [Gi3, Gi5, Gi6].

We have recomputed the upper bounds in Table I marked [HZ1] using the method from the paper [HZ1], because the bounds there relied on an overly optimistic personal communication from T. Spencer. Further refinements of this method are studied in [HZ2, ShZ1, Shi2]. The paper [Shi2] subsumes the main results of the manuscripts [ShZ1, Shi2].

2.2. Lower bounds on $R(k,l)$, higher parameters

The lower bounds marked [XXR], [XXER], 2.3.e and 2.3.h need not to be cyclic, while all other lower bounds listed in Table II were obtained by construction of cyclic graphs.

k \ l	15	16	17	18	19	20	21	22	23
3	73 WW	79 WW	92 WWY1	98 WWY1	106 WWY1	109 WWY1	122 WWY1	125 WWY1	136 WWY1
4	153 XXR		182 LSS	187 2.3.e	198 LSZL	230 SLZL	242 SLZL	282 SL	
5	261 XXER	289 2.3.h	313 2.3.h	365 2.3.h	389 2.3.h	421 2.3.h	433 2.3.h	485 2.3.h	509 2.3.h
6	401 2.3.h	434 SLLL	548 SLLL	614 SLLL	710 SLLL	878 SLLL		1070 SLLL	
7			673 2.3.h	725 2.3.h	908 SLLL		1214 SLLL		
8	861 2.3.h		925 2.3.h		1054 XXR	1094 SLLL	1617 2.3.h		

Table II. Known nontrivial lower bounds for higher two color Ramsey numbers $R(k,l)$, with references.

Exoo in [Ex15] gives the bounds $R(3,27)\geq 158$ and $R(3,31)\geq 198$. The constructions establishing $R(3,26)\geq 150$, $R(3,29)\geq 174$, $R(3,31)\geq 198$ and $R(3,32)\geq 212$ are presented in [SLL1], [SLL3], [LSS] and [LSZL], respectively. Yu [Yu2] constructed a special class of triangle-free cyclic graphs establishing several lower bounds for $R(3,k)$, for $k\geq 61$. Only two of these bounds, $R(3,61)\geq 479$ and $R(3,103)\geq 955$, cannot be easily improved by the inequality $R(3,4k+1)\geq 6R(3,k+1)-5$ from [CCD] (2.3.c) and data from Tables I and II. Finally, for higher parameters we mention two more cases which improve on bounds listed in earlier revisions: $R(9,17)\geq 1411$ is given in [XXR] and $R(10,15)\geq 1265$ can be obtained by using 2.3.h.

In general, one can expect that the lower bounds in Table II are weaker than those in Table I, in the sense that with some work many of them should not be hard to improve, in contrast to the bounds in Table I, especially smaller ones.

2.3. Other results on $R(k,l)$

(a) $R(k,l) \le R(k-1,l)+R(k,l-1)$, with strict inequality when both terms on the right hand side are even [GG]. There are obvious generalizations of this inequality for avoiding graphs other than complete.

(b) $R(k,k) \le 4R(k,k-2)+2$ [Walk].

(c) Explicit construction for $R(3,4k+1) \ge 6R(3,k+1)-5$, for all $k \ge 1$ [CCD].

(d) Constructive results on triangle-free graphs in relation to the case of $R(3,k)$ [BBH1, BBH2, Fra1, Fra2, FrLo, Gri, KM1, Loc, RK3, RK4, Stat, Yu1].

(e) Bounds for the difference between consecutive Ramsey numbers, in particular the bound $R(k,l) \ge R(k,l-1)+2k-3$ for $k,l \ge 3$ [BEFS].

(f) By taking a disjoint union of two critical graphs one can easily see that $R(k,p) \ge s$ and $R(k,q) \ge t$ imply $R(k,p+q-1) \ge s+t-1$. Xu and Xie [XX1] improved this construction to yield better general lower bounds, in particular $R(k,p+q-1) \ge s+t+k-3$.

(g) For $2 \le p \le q$ and $3 \le k$, if (k,p)-graph G and (k,q)-graph H have a common induced subgraph on m vertices without K_{k-1}, then $R(k,p+q-1) > n(G)+n(H)+m$. In particular, this implies the bounds $R(k,p+q-1) \ge R(k,p)+R(k,q)+k-3$ and $R(k,p+q-1) \ge R(k,p)+R(k,q)+p-2$ [XX1, XXR].

(h) $R(2k-1,l) \ge 4R(k,l-1)-3$ for $l \ge 5$ and $k \ge 2$, and in particular for $k=3$ we obtain $R(5,l) \ge 4R(3,l-1)-3$ [XXER].

(i) If the quadratic residues Paley graph Q_p of prime order $p=4t+1$ contains no K_k, then $R(k,k) \ge p+1$ and $R(k+1,k+1) \ge 2p+3$ [She1, Mat]. Data for larger p was obtained in [LSL]. See also items 5.2.h and 5.2.i for similar multicolor results.

(j) Study of Ramsey numbers for large disjoint unions of graphs [Bu1, Bu9], in particular $R(nK_k, nK_l) = n(k+l-1)+R(K_{k-1},K_{l-1})-2$, for n large enough [Bu8].

(k) $R(k,l) \ge L(k,l)+1$, where $L(k,l)$ is the maximal order of any cyclic (k,l)-graph. A compilation of many best cyclic bounds was presented in [HaKr].

(l) Two-color lower bounds can be obtained by using items 5.2.k, 5.2.l and 5.2.m with $r=2$. Some generalizations of these were obtained in [ZLLS].

In the last six items of this section we only briefly mention some pointers to the literature dealing with asymptotics of Ramsey numbers. This survey was designed mostly for small, finite, and combinatorial results, but still we wish to give the reader some useful and representative references to more traditional papers looking first of all at the infinite.

(m) In a 1995 breakthrough Kim proved that $R(3,k) = \Theta(k^2/\log k)$ [Kim].

(n) Explicit triangle-free graphs with independence k on $\Omega(k^{3/2})$ vertices [Alon2, CPR].

(o) Other general and asymptotic results on triangle-free graphs in relation to the case of $R(3,k)$ [AKS, Alon2, CCD, CPR, Gri, FrLo, Loc, She2].

(p) In 1947, Erdös gave an amazingly simple probabilistic proof that $R(k,k) \geq c \cdot k 2^{k/2}$ [Erd1]. Spencer [Spe1] improved the constant in the last result. More probabilistic asymptotic lower bounds for other Ramsey numbers were obtained in [Spe1, Spe2, AlPu].

(q) Other asymptotic bounds for $R(k,k)$ can be found, for example, in [Chu3, McS] (lower bound) and [Tho] (upper bound), and for many other bounds in the general case of $R(k,l)$ consult [Spe2, GRS, GrRö, Chu4, ChGra2, LRZ, AlPu, Kriv].

(r) Explicit construction of a graph with clique and independence k on $2^{c \log^2 k / \log\log k}$ vertices by Frankl and Wilson [FraWi]. Further constructions by Chung [Chu3] and Grolmusz [Grol1, Grol2]. Explicit constructions like these are usually weaker than known probabilistic results.

3. Two Colors - Dropping One Edge from Complete Graph

G \ H	K_3-e	K_4-e	K_5-e	K_6-e	K_7-e	K_8-e	K_9-e	$K_{10}-e$	$K_{11}-e$
K_3-e	3	5	7	9	11	13	15	17	19
K_3	5	7	11	17	21	25	31	37 38	42 47
K_4-e	5	10	13	17	28	29 38	34	41	
K_4	7	11	19	27 36	37 52				
K_5-e	7	13	22	31 39	40 66				
K_5	9	16	30 34	43 67	112				
K_6-e	9	17	31 39	45 70	59 135				
K_6	11	21	37 55	119	205				
K_7-e	11	28	40 66	59 135	251				
K_7	13	28 34	51 88	204					

Table III. Two types of Ramsey numbers $R(G,H)$, includes all known nontrivial values.

The exact values in Table III involving K_3-e are trivial, since one can easily see that $R(K_3-e, K_k) = R(K_3-e, K_{k+1}-e) = 2k-1$, for all $k \geq 2$. Other bounds (not shown in Table III) can be obtained by using Table I, an obvious generalization of the inequality

$R(k,l) \le R(k-1,l) + R(k,l-1)$, and by monotonicity of Ramsey numbers, in this case $R(K_{k-1}, G) \le R(K_k - e, G) \le R(K_k, G)$. The upper bounds from the manuscripts [ShZ1, ShZ2] are subsumed by a later article [Shi2].

G \ H	K_4-e	K_5-e	K_6-e	K_7-e	K_8-e	K_9-e	$K_{10}-e$	$K_{11}-e$
K_3	CH2	Clan	FRS1	GH	Ra1	Ra1	MPR MPR	WWY2 MPR
K_4-e	CH1	FRS2	McR	McR	Ea1 HZ2	Ex14	Ex14	
K_4	CH2	EHM1	Ex11 Ea1	Ex14 HZ2				
K_5-e	FRS2	CEHMS	Ex14 Ea1	Ex14 HZ2				
K_5	BH	Ex8 Ex8	Ea1 HZ2	HZ2				
K_6-e	McR	Ex14 Ea1	Ex14 HZ2	Ex14 HZ2				
K_6	McN	Ex14 Ea1	ShZ2	ShZ2				
K_7-e	McR	Ex14 HZ2	Ex14 HZ2	ShZ1				
K_7	Ea1 Ea1	Ex14 ShZ2	ShZ2					

References for Table III.

All $(K_3, K_l - e)$-graphs for $l \le 6$ have been enumerated [Ra1]. For the following numbers it was established that the critical graphs are unique: $R(K_3, K_l - e)$ for $l = 3$ [Tr], 6 and 7 [Ra1], $R(K_4 - e, K_4 - e)$ [FRS2], $R(K_5 - e, K_5 - e)$ [Ra3] and $R(K_4 - e, K_7 - e)$ [McR]. The number of $R(K_3, K_l - e)$-critical graphs for $l = 4, 5$ and 8 is 4, 2 and 9, respectively [MPR], and there are at least 6 such graphs for $R(K_3, K_9 - e)$ [Ra1]. The bound $R(K_3, K_{12} - e) \ge 46$ is given in [MPR]. Wang, Wang and Yan in [WWY2] constructed cyclic graphs showing $R(K_3, K_{13} - e) \ge 54$, $R(K_3, K_{14} - e) \ge 59$ and $R(K_3, K_{15} - e) \ge 69$.

The upper bounds in [HZ2] were obtained by a reasoning generalizing the bounds for classical numbers in [HZ1]. Several other results from section 2.3 apply, though checking in which situation they do may require looking inside the proofs whether they still hold for $K_n - e$.

4. General Graph Numbers in Two Colors

This section includes data with respect to general graph results. We tried to include all nontrivial values and identities regarding exact results (or references to them), but only those out of general bounds and other results which, in our opinion, have a direct connection to the evaluation of specific numbers. If some small value cannot be found below, it may be covered by the cumulative data gathered in section 7, or be a special case of a general result listed in this section. Note that $B_1 = F_1 = C_3 = W_3 = K_3$, $B_2 = K_4 - e$, $P_3 = K_3 - e$, $W_4 = K_4$ and $C_4 = K_{2,2}$ imply other identities not mentioned explicitly.

4.1. Paths

$R(P_n, P_m) = n + \lfloor m/2 \rfloor - 1$ for all $n \geq m \geq 2$ [GeGy]

4.2. Cycles

$R(C_3, C_3) = 6$ [GG]
$R(C_4, C_4) = 6$ [CH1]

Result obtained independently in [Ros] and [FS1], new simple proof in [KáRos]:

$$R(C_n, C_m) = \begin{cases} 2n-1 & \text{for } 3 \leq m \leq n,\ m \text{ odd}, (n,m) \neq (3,3) \\ n-1+m/2 & \text{for } 4 \leq m \leq n,\ m \text{ and } n \text{ even}, (n,m) \neq (4,4) \\ \max\{n-1+m/2, 2m-1\} & \text{for } 4 \leq m < n,\ m \text{ even and } n \text{ odd} \end{cases}$$

Unions of cycles, formulas and bounds for $R(nC_p, mC_q)$ [MS, Den]
$R(nC_3, mC_3) = 3n + 2m$ for $n \geq m \geq 1$, $n \geq 2$ [BES]
$R(nC_4, mC_4) = 2n + 4m - 1$ for $m \geq n \geq 1$, $(n,m) \neq (1,1)$ [LiWa1]
Formulas for $R(nC_4, mC_5)$ [LiWa2]

4.3. Wheels

$R(W_3, W_5) = 11$ [Clan]
$R(W_3, W_n) = 2n - 1$ for all $n \geq 6$ [BE2]
All critical colorings for $R(W_3, W_n)$ for all $n \geq 3$ [RaJi]
$R(W_4, W_5) = 17$ [He3]
$R(W_5, W_5) = 15$ [HaMe2, He2]
$R(W_4, W_6) = 19$, $R(W_5, W_6) = 17$ and $R(W_6, W_6) = 17$, and all critical colorings (2, 1 and 2) for these numbers [FM]. $R(W_6, W_6) = 17$ and $\chi(W_6) = 4$ gives a counterexample $G = W_6$ to the Erdös conjecture (see [GRS]) $R(G,G) \geq R(K_{\chi(G)}, K_{\chi(G)})$.

4.4. Books

$R(B_1, B_n) = 2n+3$ for all $n > 1$ [RS1]
$R(B_3, B_3) = 14$ [RS1, HaMe2]
$R(B_2, B_5) = 16$, $R(B_3, B_5) = 17$, $R(B_5, B_5) = 21$,
$R(B_4, B_4) = 18$, $R(B_4, B_6) = 22$, $R(B_6, B_6) = 26$ [RS1]
$254 \le R(B_{37}, B_{88}) \le 255$ [Par6]
$R(B_n, B_m) = 2n+3$ for all $n \ge cm$ for some c [NiRo1, NiRo2]
$R(B_n, B_n) = (4+o(1))n$ [RS1, NiRS]

In general, $R(B_n, B_n) = 4n+2$ for $4n+1$ a prime power, and
several other general equalities and bounds for $R(B_n, B_m)$ [RS1, FRS7, Par6, NiRS].

4.5. Complete bipartite graphs

HINT: This section gathers information on Ramsey numbers where specific bipartite graphs are avoided in a coloring of K_n (as everywhere in this survey), in contrast to often studied bipartite Ramsey numbers (not covered in this survey) where the initial coloring is of a bipartite graph $K_{n,m}$.

$R(K_{1,n}, K_{1,m}) = n+m-\varepsilon$, where $\varepsilon = 1$ if both n and m are even and $\varepsilon = 0$ otherwise [Har1]. It is also a special case of multicolor numbers for stars obtained in [BuRo1].

$R(nK_{1,3}, mK_{1,3}) = 4n+m-1$ for $n \ge m \ge 1$, $n \ge 2$ [BES]

$R(K_{2,3}, K_{2,3}) = 10$ [Bu4]
$R(K_{2,3}, K_{2,4}) = 12$ [ExRe]
$R(K_{2,3}, K_{1,7}) = 13$ [Par4]
$R(K_{2,3}, K_{3,3}) = 13$ and $R(K_{3,3}, K_{3,3}) = 18$ [HaMe3]
$R(K_{2,2}, K_{2,8}) = 15$ and $R(K_{2,2}, K_{2,11}) = 18$ [HaMe4]
$R(K_{2,2}, K_{1,15}) = 20$ [La2]
$R(K_{2,n}, K_{2,n}) \le 4n-2$ for all $n \ge 2$, exact values 6, 10, 14, 18, 21, 26, 30, 33, 38, 42, 46, 50, 54, 57 and 62 of $R(K_{2,n}, K_{2,n})$ for $2 \le n \le 16$, respectively.

The first open diagonal case is $65 \le R(K_{2,17}, K_{2,17}) \le 66$ [EHM2].

Conjecture that $4n-3 \le R(K_{2,n}, K_{2,n}) \le 4n-2$ for $n \ge 2$ [LorMe1].

Bounds and some values for the numbers of the form $R(K_{k,n}, K_{k,m})$ [LorMe1], and $R(K_{2,n-1}, K_{2,n})$ and $R(K_{2,n}, K_{2,n})$ [LorMe2].

The values of $R(K_{2,n}, K_{2,m})$ for all $2 \le n, m \le 10$ are gathered in [LorMe3] except 8 cases, for which lower and upper bounds are given. Several theorems giving exact formulas and bounds assuming special dependencies between n and m [LorMe3].

Asymptotics for $K_{2,m}$ versus K_n [CLRZ]

Upper bound asymptotics for $K_{k,m}$ versus K_n [LZ]

See section 4.10 for stars versus various bipartite graphs

4.6. Triangle versus other graphs

$R(3,k)=\Theta(k^2/\log k)$ [Kim]

Explicit construction for $R(3,4k+1) \geq 6R(3,k+1)-5$, for all $k \geq 1$ [CCD]

Explicit triangle-free graphs with independence k on $\Omega(k^{3/2})$ vertices [Alon2, CPR]

$R(K_3, K_7-2P_2) = R(K_3, K_7-3P_2) = 18$ [SchSch2]

$R(K_3, K_3+\overline{K}_m) = R(K_3, K_3+\overline{C}_m) = 2m+5$ for $m \geq 212$ [Zhou1]

$R(K_3, G)=2n(G)-1$ for any connected G on at least 4 vertices and with at most $(17n(G)+1)/15$ edges, in particular for $G=P_i$ and $G=C_i$, for all $i \geq 4$ [BEFRS1]

$R(K_3, G) \leq 2e(G)+1$ for any graph G without isolated vertices [Sid3, GK]

$R(K_3, G) \leq n(G)+e(G)$ for all G, a conjecture [Sid2]

$R(K_3, G)$ for all connected G up to 9 vertices BBH1, BBH2], see also section 7.1

$R(K_3, K_n)$, see section 2

$R(K_3, K_n-e)$, see section 3

Formulas for $R(nK_3, mG)$ for all G of order 4 without isolates [Zeng]

Since $B_1 = F_1 = C_3 = W_3 = K_3$, other sections apply

See also [AKS, BBH1, BBH2, FrLo, Fra1, Fra2, Gri, Loc, KM1, LZ, RK3, RK4, She2, Spe2, Stat, Yu1]

4.7. Paths versus other graphs

P_3 versus special graphs G [CH2]
Paths versus stars [Par2, BEFRS2]
Paths versus trees [FS4]
Paths versus books [RS2]
Paths versus cycles [FLPS, BEFRS2]
Paths versus K_n [Par1]
Paths versus $K_{n,m}$ [Häg]
Paths versus W_5 and W_6 [SuBa1]
Paths versus W_7 and W_8 [Bas]
Paths versus wheels [BaSu, ChenZZ1]
Paths and cycles versus trees [FSS1]
Sparse graphs versus paths and cycles [BEFRS2]
Graphs with long tails [Bu2, BG]
Unions of paths [BuRo2]

4.8. Cycles versus complete graphs

	C_3	C_4	C_5	C_6	C_7	C_8	...	C_n for $n \geq m$
K_3	6 GG	7 CS	9 CS	11 FS1	13 FS1	15 FS1	...	$2n-1$ FS1
K_4	9 GG	10 CH2	13 He2/JR4	16 JR2	19 YHZ1	22 YHZ1	...	$3n-2$ YHZ1
K_5	14 GG	14 Clan	17 He2/JR4	21 JR2	25 YHZ2	29 BJYHRZ	...	$4n-3$ BJYHRZ
K_6	18 Kéry	18 Ex2/RoJa1	21 JR5	26 Schi1	31 Schi1	36 Schi1	...	$5n-4$ Schi1
K_7	23 Ka2/GY	22 RT/JR1	25 Schi2		37 conj.	43 conj.	...	$6n-5$ conj.
K_8	28 GR/MZ	26 RT				50 conj.	...	$7n-6$ conj.
K_9	36 Ka2/GR	≥ 30 RT					...	$8n-7$ conj.
K_{10}	40 - 43 Ex5/RK2	≥ 34 RT					...	$9n-8$ conj.

Table IV. Known Ramsey numbers $R(C_n, K_m)$.

- The first column in Table IV gives data from the first row in Table I.

- Joint credit [He2/JR4] in Table IV refers to two cases in which Hendry [He2] announced the values without presenting the proofs, which later were given in [JR4]. For other joint credits in Table IV, the first reference is for the lower bound and the second for the upper bound. The special cases of $R(C_6, K_5) = 21$ [JR2] and $R(C_7, K_5) = 25$ were also solved independently in [YHZ2] and [BJYHRZ].

- Since 1976, it was conjectured that $R(C_n, K_m) = (n-1)(m-1)+1$ for all $n \geq m \geq 3$, except $n = m = 3$ [FS4, EFRS2]. The parts of this conjecture were proved as follows: for $n \geq m^2 - 2$ [BoEr], for $n > 3 = m$ [FS1], for $n \geq 4 = m$ [YHZ1], for $n \geq 5 = m$ [BJYHRZ], for $n \geq 6 = m$ [Schi1], for $n \geq m \geq 7$ with $n \geq m(m-2)$ [Schi1], and for $n \geq 4m+2$, $m \geq 3$ [Nik]. Still open conjectured cases are marked in Table IV by "conj."

- General study of cycles versus K_n numbers, including asymptotics [BoEr, Spe2, FS4, EFRS2, CLRZ, Sud1, ZaLi, AlRö].

4.9. Cycles versus other graphs

C_4 versus stars [Par3, Par5, BEFRS5, Chen, ChenJ, GoMC]
C_4 versus trees [EFRS4, Bu7, BEFRS5, Chen]
C_4 versus $K_{m,n}$ [HaMe4] and $K_{2,n}$ [LorMe3]
C_4 versus all graphs on six vertices [JR3]
$R(C_4, B_n) = 7, 9, 11, 12, 13$ and 16, for $2 \le n \le 7$, respectively [FRS6]
$R(C_4, B_n) = 17, 18, 19, 20$ and 21, for $8 \le n \le 12$, respectively [Tse1]
$R(C_4, B_{13}) = 22$ and $R(C_4, B_{14}) = 24$ [Tse2]
$R(C_4, W_n) = 10, 9, 10, 9, 11, 12, 13, 14, 16$ and 17, for $4 \le n \le 13$, respectively [Tse1]
$R(C_4, G) \le 2q + 1$ for any isolate-free graph G with q edges [RoJa2]
$R(C_4, G) \le p + q - 1$ for any connected graph G on p vertices and q edges [RoJa2]

$R(C_5, W_6) = 13$ [ChvS]
$R(C_5, K_6 - e) = 17$ [JR4]
$R(C_5, B_1) = R(C_5, B_2) = 9$ [CRSPS]
$R(C_5, B_3) = 10$, and in general $R(C_5, B_n) = 2n + 3$ for $n \ge 4$ [FRS8]
C_5 versus all graphs on six vertices [JR4]
$R(C_6, K_5 - e) = 17$ [JR2]
C_6 versus all graphs on five vertices [JR2]

$R(C_n, G) \le 2q + \lfloor n/2 \rfloor - 1$, for $3 \le n \le 5$, for any isolate-free graph G with $q > 3$ edges. It is conjectured that it also holds for other n [RoJa2].

Cycles versus paths [FLPS, BEFRS2]
Cycles versus stars [La1, Clark, see Par6]
Cycles versus trees [FSS1]
Cycles versus books [FRS6, FRS8, Zhou1]
Cycles versus $K_{n,m}$ [BoEr]
Cycles versus W_5 and W_6 [SuBB2]
Cycles versus wheels [Zhou2]
See also bipartite graphs for $K_{2,2} = C_4$

4.10. Stars versus other graphs

Stars versus C_4 [Par3, Par5, Chen, ChenJ, GoMC]
Stars versus W_5 and W_6 [SuBa1]
Stars versus wheels [ChenZZ2]
Stars versus paths [Par2, BEFRS2]
Stars versus cycles [La1, Clark, see Par6]
Stars versus books [CRSPS, RS2]
Stars versus $K_{2,n}$ [Par4, GoMC]
Stars versus $K_{n,m}$ [Stev, Par3]
Stars versus bipartite graphs [Par4, Stev]
Stars versus trees [Bu1, Coc, GV, ZZ]

Stars versus stripes [CL, Lor]
Stars versus $K_n - tK_2$ [Hua1, Hua2]
Stars versus $2K_2$ [MO]
Union of two stars [Gros2]

4.11. Books versus other graphs

$R(B_3, K_4) = 14$ [He3]
$R(B_3, K_5) = 20$ [He2][BaRT]
Books versus paths [RS2]
Books versus trees [EFRS7]
Books versus stars [CRSPS, RS2]
Books versus cycles [FRS6, FRS8, Zhou1, Tse1, Tse2]
Books versus K_n [LR1, Sud2]
Books versus wheels [Zhou3]
Books versus $K_2 + C_n$ [Zhou3]
Books and $(K_1 + tree)$ versus K_n [LR1]
Generalized books $K_r + qK_1$ versus K_n [NiRo3]

4.12. Wheels versus other graphs

$R(W_5, K_5 - e) = 17$ [He2][YH]
$R(W_5, K_5) = 27$ [He2][RST]
W_5 and W_6 versus stars and paths [SuBa1]
Wheels versus stars [ChenZZ2]
W_5 and W_6 versus trees [BSNM]
W_5 and W_6 versus cycles [SuBB2]
$R(W_6, C_5) = 13$ [ChvS]
W_7 and W_8 versus paths [Bas]
W_7 versus trees T with $\Delta(n(T)) \geq n(T) - 3$ [ChenZZ3]
Wheels versus paths [BaSu, ChenZZ1]
Odd wheels versus star-like trees [SuBB1]
Wheels versus C_4 [Tse1]
Wheels versus cycles [Zhou2]
Wheels versus books [Zhou3]
Wheels versus linear forests [SuBa2]

4.13. Trees and Forests

Trees, forests [Bu1, Bu7, CsKo, EFRS3, EG, FSS1, GeGy, GHK, GRS, GV, HaŁT]
Trees versus K_n [Chv]
Trees versus C_4 [EFRS4, Bu7, Chen]
Trees versus paths [FS4]
Trees versus paths and cycles [FSS1]
Trees versus stars [Bu1, Coc, GV, ZZ]

Trees versus books [EFRS7]
Trees versus W_5 and W_6 [BSNM]
Trees T with $\Delta(n(T)) \geq n(T)-3$ versus W_7 [ChenZZ3]
Star-like trees versus odd wheels [SuBB1, ChenZZ3]
Trees versus $K_n + \overline{K}_m$ [RS2, FSR]
Trees versus bipartite graphs [BEFRS5, EFRS6]
Trees versus almost complete graphs [GJ2]
Trees versus small ($n(G) \leq 5$) connected G [FRS4]
Trees versus multipartite complete graphs [EFRS8, BEFRSGJ]
Linear forests, forests [BuRo2, FS3, CsKo]
Linear forests versus wheels [SuBa2]
Forests versus K_n [Stahl]
Forests versus almost complete graphs [CGP]

4.14. Mixed special cases:

$R(C_5 + e, K_5) = 17$ [He5]
$R(W_5, K_5 - e) = 17$ [He2][YH]
$R(B_3, K_5) = 20$ [He2][BaRT]
$R(W_5, K_5) = 27$ [He2][RST]
$25 \leq R(K_5 - P_3, K_5) \leq 28$ [He2]
$26 \leq R(K_{2,2,2}, K_{2,2,2})$, $K_{2,2,2}$ is an octahedron [Ex8]

4.15. Mixed general cases

Unicyclic graphs [Gros1, Köh, KrRod]
$K_{2,m}$ and C_{2m} versus K_n [CLRZ]
$K_{2,n}$ versus any graph [RoJa2]
nK_3 versus mK_3, in particular $R(nK_3, nK_3) = 5n$ for $n \geq 2$ [BES]
nK_3 versus mK_4 [LorMu]
$R(nK_4, nK_4) = 7n + 4$ for large n [Bu8]
$2K_2$ versus K_n and general graphs G [CH2]
Variety of results on numbers $R(nG, mH)$ [Bu1]
Stripes [CL, Lor]
Union of two stars [Gros2]
Double stars* [GHK]
Graphs with bridge versus K_n [Li]
Fans $F_n = K_1 + nK_2$ versus K_m [LR2]
$R(F_1, F_n) = R(K_3, F_n) = 4n + 1$ for $n \geq 2$, and bounds for $R(F_m, F_n)$ [GGS]
Multipartite complete graphs [BEFRS3, EFRS4, FRS3, Stev]

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* double star is a union of two stars with their centers joined by an edge

Multipartite complete graphs versus trees [EFRS8, BEFRSGJ]
Disconnected graphs versus any graph [GJ1]
Graphs with long tails [Bu2, BG]
Brooms[+] [EFRS3]

4.16. Other general results

[Chv] $R(K_n, T_m) = (n-1)(m-1)+1$ for any tree T on m vertices.

[CH2] $R(G,H) \geq (\chi(G)-1)(c(H)-1)+1$, where $\chi(G)$ is the chromatic number of G, and $c(H)$ is the size of the largest connected component of H.

[BE1] $R(G,G) \geq \lfloor (4n(G)-1)/3 \rfloor$ for any connected G, and $R(G,G) \geq 2n-1$ for any connected nonbipartite G.

[BE2] Graphs yielding $R(K_n, G) = (n-1)(n(G)-1)+1$ and related results (see also [EFRS5]).

[Bu2] Graphs H yielding $R(G,H) = (\chi(G)-1)(n(H)-1)+s(G)$, where $s(G)$ is a chromatic surplus of G, defined as the minimum number of vertices in some color class under all vertex colorings in $\chi(G)$ colors (such H's are called G-good). This idea, initiated in [Bu2], is a basis of a number of exact results for $R(G,H)$ for large and sparse graphs H [BG, BEFRS2, BEFRS4, Bu5, FS, EFRS4, FRS3, BEFSRGJ, BF, LR4]. A survey of this area appeared in [FRS5].

[BaLS] Graph G is Ramsey saturated if $R(G+e, G+e) > R(G,G)$ for every edge e in $\overline{G}$. Several theorems on Ramsey saturated and unsaturated graphs. A conjecture that almost all graphs are Ramsey unsaturated.

[Par3] Relations between some Ramsey graphs and block designs. See also [Par4].

[Bra3] $R(G,H) > h(G,d)n(H)$ for all nonbipartite G and almost every d-regular H, for some h unbounded in d.

[LZ] Lower bound asymptotics of $R(G,H)$ for large dense H [LZ].

[CSRT] $R(G,G) \leq c_d n(G)$ for all G, where constant c_d depends only on the maximum degree d in G. The constant was improved in [GRR1]. Tight lower and upper bounds for bipartite G [GRR2].

[ChenS] $R(G,G) \leq c_d n$ for all d-arrangeable graphs G on n vertices, in particular with the same constant for all planar graphs. The constant c_d was improved in [Eaton]. An extension to graphs not containing a subdivision of K_d [RöTh]. Progress towards a conjecture that the same inequality holds for all d-degenerate graphs G [KoRö1, KoRö2, KoSu].

[EFRS9] Study of graphs G, called *Ramsey size linear*, for which there exists a constant c_G such that for all H with no isolates $R(G,H) \leq c_G e(H)$. An overview and

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+ broom is a star with a path attached to its center

further results were given in [BaSS].

[LRS] $R(G,G) < 6n$ for all n-vertex graphs G, in which no two vertices of degree at least 3 are adjacent. This improves the result $R(G,G) \le 12n$ in [Alon1].

[AlKS] Discussion of a conjecture by Erdös that there exists a constant c such that $R(G,G) \le 2^{c\sqrt{e(G)}}$. Proof for bipartite graphs G and progress towards the conjecture in other cases.

[Kriv] Lower bound on $R(G,K_n)$ depending on the density of subgraphs of G. This construction for $G = K_m$ produces a bound similar to the best known probabilistic lower bound by Spencer [Spe2].

[NiRo3] $R(K_{p+1}, B_q^r) = p(q+r-1)+1$ for generalized books $B_q^r = K_r + qK_1$, for all sufficiently large q.

[Shi1] $R(Q_n, Q_n) \le 2^{(3+\sqrt{5})n/2+o(n)}$, for the n-dimensional cube Q_n with 2^n vertices. This bound can also be derived from a theorem in [KoRö1].

[Gros1] Conjecture that $R(G,G) = 2n(G)-1$ if G is unicyclic of odd girth. Further support for the conjecture was given in [Köh, KrRod].

[RoJa2] $R(K_{2,k}, G) \le kq+1$, for $k \ge 2$, for isolate-free graphs G with $q \ge 2$ edges.

[FSS1] Discussion of the conjecture that $R(T_1,T_2) \le n(T_1)+n(T_2)-2$ holds for all trees T_1, T_2. See also [Bu1, Bu7, CsKo, EFRS3, EG, GeGy, GHK, GRS, GV].

[HaŁT] If tree T is viewed as a bipartite graph with parts t_1 and t_2, $t_2 \ge t_1$, let $b(T) = \max(2t_1+t_2-1, 2t_2-1)$. Then the bound $R(T,T) \ge b(T)$ holds always, and $R(T,T) = b(T)$ holds for many classes of trees, and asymptotically.

[FM] $R(W_6, W_6) = 17$ and $\chi(W_6) = 4$. This gives a counterexample $G = W_6$ to the Erdös conjecture (see [GRS]) $R(G,G) \ge R(K_{\chi(G)}, K_{\chi(G)})$.

[LR3] Bounds on $R(H+\bar{K}_n, K_n)$ for general H. Also, for fixed k and m, as $n \to \infty$, $R(K_k+\bar{K}_m, K_n) \le (m+o(1))\, n^k/(\log n)^{k-1}$ [LRZ].

[Zeng] Formulas for $R(nK_3, mG)$ for all isolate-free graphs G on 4 vertices.

[BES] Study of Ramsey numbers for multiple copies of graphs.
See also [Bu1, Bu8, Bu9, LorMu].

[HaKr] Study of cyclic graphs yielding lower bounds for Ramsey numbers. Exact formulas for paths and cycles, small complete graphs and for graphs with up to five vertices.

[Bu6] Given integer m and graphs G and H, determining whether $R(G,H) \le m$ holds is NP-hard.

[-] Special cases of multicolor results listed in section 5.

[-] See also surveys listed in section 7.

5. Multicolor Graph Numbers

The only known value of a multicolor classical Ramsey number:

$R_3(3) = R(3,3,3) = R(3,3,3;2) = 17$ [GG]

2 critical colorings (on 16 vertices) [KaSt, LayMa]
2 colorings on 15 vertices [Hein]
115 colorings on 14 vertices [PR1]

General upper bound, implicit in [GG]:

$$R(k_1,\dots,k_r) \le 2 - r + \sum_{i=1}^{r} R(k_1,\dots,k_{i-1},k_i-1,k_{i+1},\dots,k_r) \qquad \text{(a)}$$

Inequality in (a) is strict if the right hand side is even, and at least one of the terms in the summation is even. It is suspected that this upper bound is never tight for $r \ge 3$ and $k_i \ge 3$, except for $r = k_1 = k_2 = k_3 = 3$. However, only two cases are known to improve over (a), namely $R_4(3) \le 62$ [FKR] and $R(3,3,4) \le 31$ [PR1, PR2], for which (a) produces only the bounds of 66 and 34, respectively.

5.1. Bounds for multicolor classical numbers

Diagonal Cases

r \ m	3	4	5	6	7	8	9
3	17 GG	128 HiIr	415 XXER	1070 Mat	3214 Xu	5384 XX2	13761 XXER
4	51 Chu1	634 XXER	3049 Xu	15202 XXER	62017 XXER		
5	162 Ex10	3416 XXER	26912 Xu				
6	538 FreSw						
7	1682 FreSw						

Table V. Known nontrivial lower bounds for diagonal multicolor Ramsey numbers $R_r(m)$, with references.

The best published bounds corresponding to the entries in Table V marked by personal communication [Xu] are: $3211 \le R_3(7)$ [Mat], $2721 \le R_4(5)$ [XXER] and $26082 \le R_5(5)$ [XXER].

The most studied and intriguing open case is

[Chu1] $51 \le R_4(3) = R(3,3,3,3) \le 62$ [FKR]

The inequality 5.a implies $R_4(3) \le 66$, Folkman [Fo] in 1974 improved this bound to 65, and Sánchez-Flores [San] in 1995 proved $R_4(3) \le 64$. The upper bounds in $162 \le R_5(3) \le 307$, $538 \le R_6(3) \le 1838$, $1682 \le R_7(3) \le 12861$, and $128 \le R(4,4,4) \le 236$ are implied by 5.(a) (we repeat lower bounds from Table V just to see easily the ranges).

Off-Diagonal Cases

Three colors:

k \ m	4	5	6	7	8	9	10	11	12	13	14
3	30 Ka2	45 Ex2	60 Rob3	79 Ex16	98 ZSL	110 SLZL	141 5.2.c	157 5.2.c	181 5.2.c	205 5.2.c	233 5.2.c
4	55 KLR	80 Ex12	99 5.2.g								
5	80 Ex12	123 5.2.g									

Table VI. Known nontrivial lower bounds for 3-color Ramsey numbers of the form $R(3,k,m)$, with references.

In addition, the bounds $303 \le R(3,6,6)$, $609 \le R(3,7,7)$ and $1689 \le R(3,9,9)$ were derived in [XXER] (used there for building other lower bounds for some diagonal cases).

The other most studied, and perhaps the only open case of a classical multicolor Ramsey number, for which we can anticipate exact evaluation in the not-too-distance future is

[Ka2] $30 \le R(3,3,4) \le 31$ [PR1, PR2]

In [PR1] it is conjectured that $R(3,3,4) = 30$, and the results in [PR2] eliminate some cases which could give $R(3,3,4) = 31$. The upper bounds in $45 \le R(3,3,5) \le 57$, $55 \le R(3,4,4) \le 79$, and $80 \le R(3,4,5) \le 160$ are implied by 5.(a) (we repeat lower bounds from the Table VI to show explicitly the current ranges).

Four colors:

$93 \le R(3,3,3,4) \le 153$	[Ex16, XXER], 5.(a)
$162 \le R(3,3,3,5)$	[XXER]
$171 \le R(3,3,4,4)$	[Ex16, XXER]
$561 \le R(3,3,3,11)$	[XX2, XXER]

Lower bounds for higher numbers can be obtained by using general constructive results from section 5.2 below. For example, the bounds $193 \le R(3,4,8)$, $261 \le R(3,3,15)$ and $241 \le R(3,3,3,7)$ were not published explicitly but are implied by 5.2.(c), 5.2.(c) and 5.2.(d), respectively.

5.2. General multicolor results for complete graphs

(b) $R_r(3) \ge 3R_{r-1}(3) + R_{r-3}(3) - 3$ [Chu1]

(c) $R(3,k,l) \ge 4R(k,l-1) - 3$, and in general for $r \ge 2$ and $k_i \ge 2$

$R(3,k_1,\dots,k_r) \ge 4R(k_1-1,k_2,\dots,k_r) - 3$ for $k_1 \ge 5$, and

$R(k_1,2k_2-1,k_3,\dots,k_r) \ge 4R(k_1-1,k_2,\dots,k_r) - 3$ for $k_1 \ge 5$ [XX2, XXER]

(d) $R(3,3,3,k_1,\dots,k_r) \ge 3R(3,3,k_1,\dots,k_r) + R(k_1,\dots,k_r) - 3$ [Rob2]

(e) Bounds for $R_k(3)$ [AbbH, Fre, Chu2, ChGri, GrRö, Wan]

(f) $R(k_1,\dots,k_r) \ge S(k_1,\dots,k_r) + 2$, where $S(k_1,\dots,k_r)$ is the generalized Schur number [AbbH, Gi1, Gi2]. In particular, the special case $k_1 = \dots = k_r = 3$ has been widely studied [Fre, FreSw, Ex10, Rob3].

(g) $R(k_1,\dots,k_r) \ge L(k_1,\dots,k_r) + 1$, where $L(k_1,\dots,k_r)$ is the maximal order of any cyclic $(k_1,\dots,k_r)$-coloring, which can be considered a special case of Schur partitions defining (symmetric) Schur numbers. Many lower bounds for Ramsey numbers were established by cyclic colorings. The following recurrence can be used to derive lower bounds for higher parameters. For $k_i \ge 3$

$L(k_1,\dots,k_r,k_{r+1}) \ge (2k_{r+1}-3)L(k_1,\dots,k_r) - k_{r+1} + 2$ [Gi2]

(h) $R_r(m) \ge p+1$ and $R_r(m+1) \ge r(p+1)+1$ if there exists a K_m-free cyclotomic $r-$class association scheme of order p [Mat].

(i) If the quadratic residues Paley graph Q_p of prime order $p = 4t+1$ contains no K_k, then $R(s,k+1,k+1) \ge 4ps - 6p + 3$ [XXER].

(j) $R_r(m) \ge c_m(2m-3)^r$, and some slight improvements of this bound for small values of m [AbbH, Gi1, Gi2, Song2].

(k) $R_r(pq+1) > (R_r(p+1)-1)(R_r(q+1)-1)$ [Abb1]

(l) $R_r(pq+1) > R_r(p+1)(R_r(q+1)-1)$ for $p \ge q$ [XXER]

(m) $R(p_1q_1+1,\dots,p_rq_r+1) > (R(p_1+1,\dots,p_r+1)-1)(R(q_1+1,\dots,q_r+1)-1)$ [Song3]

(n) $R_{r+s}(m) > (R_r(m)-1)(R_s(m)-1)$ [Song2]

(o) $R(k_1,k_2,\dots,k_r) > (R(k_1,\dots,k_i) - 1)(R(k_{i+1},\dots,k_r) - 1)$ in [Song1], see [XXER].

(p) $R(k_1,k_2,\dots,k_r) > (k_1+1)(R(k_2-k_1+1,k_3,\dots,k_r) - 1)$ [Rob4]

(q) Further lower bound constructions, though with more complicated assumptions, were presented in [XX2, XXER].

(r) Grolmusz [Grol1] generalized the classical constructive lower bound by Frankl and Wilson [FraWi] (section 2.3.r) to more colors and to hypergraphs [Grol3] (section 6).

All lower bounds in (b) through (r) above are constructive. (d) generalizes (b), (m) generalizes both (k) and (o), and (o) generalizes (n). (l) is stronger than (k). Finally observe that the construction (m) with $q_1 = \ldots = q_i = 1 = p_{i+1} = \ldots = p_r$ is the same as (o).

5.3. Special multicolor cases

$R_3(C_4) = 11$	[BS, see also Clap]
$R_3(C_5) = 17$	[YR1]
$R_3(C_6) = 12$	[YR2]
$R_3(C_7) = 25$	[FSS2]
$18 \le R_4(C_4) \le 19$	[Ex2] [Eng]
$27 \le R_5(C_4) \le 29$	[LaWo1]
$R(C_4, C_4, K_3) = 12$	[Schu]
$R(C_4, K_3, K_3) = 17$	[ExRe]
$13 \le R(C_3, C_4, C_5)$	[Rao]
$R(K_{1,3}, C_4, K_4) = 16$	[KM2]
$R(P_4, P_4, C_3) = 9$	[AKM]
$R(P_4, P_4, C_4) = 7$	[AKM]
$R(P_4, P_4, C_5) = 9$	[DzKu]
$R(K_4 - e, K_4 - e, P_3) = 11$	[Ex7]
$28 \le R_3(K_4 - e) \le 30$	[Ex7] [Piw2]
$R(C_4, C_4, C_4, T) = 16$ for $T = P_4$ and $T = K_{1,3}$	[ExRe]
$27 \le R(K_3, K_3, C_4, C_4)$	[Eng]
$86 \le R(K_4, K_4, C_4, C_4)$	[Bev], 5.2.(o)+

All colorings for $(K_4 - e, K_4 - e, P_3)$ were found in [Piw2].

5.4. General multicolor results for cycles and paths

- $R(C_n, C_n, C_n) \le (4 + o(1))n$, with equality for odd n [Łuc]. It was conjectured by Bondy and Erdös, see [Erd2], that $R(C_n, C_n, C_n) \le 4n - 3$ for $n \ge 4$. If true, then for all odd $n \ge 5$ we have $R(C_n, C_n, C_n) = 4n - 3$.
- Formulas for $R(C_n, C_m, C_k)$ and $R(C_n, C_m, C_k, C_l)$ for n sufficiently large [EFRS1].

- $R_k(C_4) \le k^2+k+1$ for all $k \ge 1$, $R_k(C_4) \ge k^2-k+2$ for all $k-1$ which is a prime power [Ir, Chu2, ChGra1], and $R_k(C_4) \ge k^2+2$ for odd prime power k [LaWo1]. The latter was extended to any prime power k in [Ling, LaMu].
- Bounds for $R_k(C_n)$ [Bu1, GRS].
- $R(P_3, C_n, C_n) = 2n-1$ $(= R(C_n, C_n))$ for odd $n \ge 5$ [DzKu].
- $R(P_4, P_4, C_n) = n+2$ for $n \ge 6$, and $R(P_3, P_5, C_n) = n+1$ for $n \ge 8$ [DzKu].
- Formulas for $R_k(P_3)$ for all k, and for $R_k(P_4)$ if k is not divisible by 3 [Ir]. Wallis [Wall] showed $R_6(P_4)=13$, which already implied $R_{3t}(P_4) = 6t+1$, for all $t \ge 2$. Independently, the case $R_k(P_4)$ for $k \ne 3^m$ was completed by Lindström in [Lind], and later Bierbrauer proved $R_{3^m}(P_4) = 2\cdot 3^m + 1$ for all $m \ge 1$.
- Monotone paths and cycles [Lef].
- Formulas for $R(P_{n_1}, \dots, P_{n_k})$, except few cases [FS2].
- Formulas for $R(n_1P_2, \dots, n_kP_2)$ [CL1].
- Formulas for $R(pP_3, qP_3, rP_3)$ and $R(pP_4, qP_4, rP_4)$ [Scob].
- See also sections 5.3 and 7.2, especially [AKM] for a number of small cases in three colors similar to those listed in section 5.3.
- Study of asymptotics for $R(C_m, \dots, C_m, K_n)$ [AlRö].
- Study of asymptotics for $R(C_{2m}, C_{2m}, K_n)$ for fixed m [ShiuLL, AlRö].

5.5. Other general multicolor results

- General bounds for $R_k(G)$ [CH3, Par6].
- Formulas for $R_k(G)$ for G being one of P_3, $2K_2$ and $K_{1,3}$ for all k, and for P_4 if k is not divisible by 3 [Ir].
- Bounds on $R_k(K_{s,t})$, in particular for $K_{2,2}=C_4$ and $K_{2,t}$ [ChGra1, AFM].
- $tk^2+1 \le R_k(K_{2,t+1}) \le tk^2+k+2$, where the upper bound is general, and the lower bound holds when both t and k are prime powers [ChGra1, LaMu].
- Bounds on $R_k(G)$ for unicyclic graphs G of odd girth. Some exact values for special graphs G, for $k=3$ and $k=4$ [KrRod].
- Formulas for $R(S_1, \dots, S_k)$, where S_i's are arbitrary stars [BuRo1].
- Formulas for $R(S_1, \dots, S_k, K_n)$, where S_i's are arbitrary stars [Jac].
- Formulas for $R(S_1, \dots, S_k, nP_2)$, where S_i's are arbitrary stars [CL2].
- Formulas for $R(S_1, \dots, S_k, T)$, where S_i's are stars and T is a tree [ZZ].
- Study of $R(G_1, \dots, G_k, G)$ for large sparse G [EFRS1, Bu3].
- Study of asymptotics for $R(C_n, \dots, C_n, K_m)$ [AlRö].
- Cockayne and Lorimer [CL1] found the exact formula for $R(n_1P_2, \dots, n_kP_2)$, and later Lorimer [Lor] extended it to a more general case of $R(K_m, n_1P_2, \dots, n_kP_2)$.

Still more general cases of the latter, with multiple copies of the complete graph and forests, were studied in [Stahl, LorSe, LorSo].

- If G is connected and $R(K_k, G) = (k-1)(n(G)-1)+1$, in particular if G is any tree, then $R(K_{k_1}, \dots, K_{k_r}, G) = (R(k_1, \dots, k_r)-1)(n(G)-1)+1$ [BE2]. A generalization for connected $G_1, \dots, G_n$ in place of G appeared in [Jac].
- If F, G, H are connected graphs then $R(F, G, H) \geq (R(F, G)-1)(\chi(H)-1)+ \min\{ R(F, G), s(H) \}$, where $s(G)$ is the chromatic surplus of G (see item [Bu2] in section 4.16). This leads to several formulas and bounds for F and G being stars and/or trees when $H = K_n$ [ShiuLL].
- $R(K_{k_1}, \dots, K_{k_r}, G_1, \dots, G_s) \geq (R(k_1, \dots, k_r)-1)(R(G_1, \dots, G_s)-1)$ for arbitrary graphs $G_1, \dots, G_s$ [Bev]. This generalizes 5.2.(o).
- Constructive bound $R(G_1, \dots, G_{t^{n-1}}) \geq t^n + 1$ for some families of decompositions of K_{t^n} [LaWo1, LaWo2].
- Bounds for trees $R_k(T)$ and forests $R_k(F)$ [EG, GRS, BB, GT, Bra1, Bra2, SwPr].
- Bounds on $R_k(G)$ for trees, forests, stars and cycles [Bu1].
- See also surveys listed in section 7.

6. Hypergraph Numbers

The only known value of a classical Ramsey number for hypergraphs:

$R(4,4;3) = 13$ [MR1]
more than 200000 critical colorings

Other hypergraph cases:

$33 \leq R(4,5;3)$	[Ex13]
$63 \leq R(5,5;3)$	[Ea1]
$56 \leq R(4,4,4;3)$	[Ex8]
$34 \leq R(5,5;4)$	[Ex11]
$R(K_4-t, K_4-t;3) = 7$	[Ea2]
$R(K_4-t, K_4;3) = 8$	[Sob, Ex1, MR1]
$14 \leq R(K_4-t, K_5;3)$	[Ex1]
$13 \leq R(K_4-t, K_4-t, K_4-t;3) \leq 17$	[Ex1] [Ea1]

The computer evaluation of $R(4,4;3)$ in [MR1] consisted of an improvement of the upper bound from 15 to 13, which followed an extensive theoretical study of this number in

[Gi4, Is1, Sid1]. Exoo in [Ex1] announced the bounds $R(4,5;3) \geq 30$ and $R(5,5;4) \geq 27$ without presenting the constructions. The bound of $R(4,5;3) \geq 24$ was obtained by Isbell [Is2]. Shastri in [Sha] shows a weak bound $R(5,5;4) \geq 19$ (now 34 in [Ex11]), nevertheless his lemmas and those in [Ka3, Abb2, GRS, HuSo] can be used to derive other lower bounds for higher numbers.

General hypergraph results:

- Several lower bound constructions for 3-uniform hypergraphs were presented in [HuSo]. Study of lower bounds on $R(p,q;4)$ can be found in [Song3] and [SYL, Song4] (the latter two papers are almost the same in contents). Most lower bounds in these papers can be easily improved by using the same techniques, but starting with better constructions for small parameters listed above.
- Let $H^{(r)}(s,t)$ be the complete r-partite r-uniform hypergraph with $r-2$ parts of size 1, one part of size s, and one part of size t (for example, for $r=2$ it is the same as $K_{s,t}$). For the multicolor numbers, Lazebnik and Mubayi [LaMu] proved that

  $$tk^2 - k + 1 \leq R_k(H^{(r)}(2, t+1)) \leq tk^2 + k + r,$$

  where the lower bound holds when both t and k are prime powers. For the general case of $H^{(r)}(s,t)$, more bounds are presented in [LaMu].
- Grolmusz [Grol1] generalized the classical constructive lower bound by Frankl and Wilson [FraWi] (section 2.3.r) to more colors and to hypergraphs [Grol3].
- Lower bounds on $R_m(k;s)$ are discussed in [DLR, AbbW]. In [AbbS], it is shown that for some values of a, b the numbers $R(m,a,b;3)$ are at least exponential in m.
- General lower bounds for large number of colors were given in an early paper by Hirschfeld [Hir], and some of them were later improved in [AbbL].
- Other theoretical results on hypergraph numbers are gathered in [GrRö, GRS].

7. Cumulative Data and Surveys

7.1. Cumulative data for two colors

[CH1] $R(G,G)$ for all graphs G without isolates on at most 4 vertices.

[CH2] $R(G,H)$ for all graphs G and H without isolates on at most 4 vertices.

[Clan] $R(G,H)$ for all graphs G on at most 4 vertices and H on 5 vertices, except five entries (now all solved).

[He4] All critical colorings for $R(G,H)$, for isolate-free graphs G and H as in [Clan] above.

[Bu4] $R(G,G)$ for all graphs G without isolates and with at most 6 edges.

[He1] $R(G,G)$ for all graphs G without isolates and with at most 7 edges.

[HaMe2] $R(G,G)$ for all graphs G on 5 vertices and with 7 or 8 edges.

[He2] $R(G,H)$ for all graphs G and H on 5 vertices without isolates, except 7 entries (3 still open, see the paragraph at the end of this section).

[HoMe] $R(G,H)$ for $G=K_{1,3}+e$ and $G=K_4-e$ versus all connected graphs H on 6 vertices, except $R(K_4-e,K_6)$. The result $R(K_4-e,K_6)=21$ was claimed by McNamara [McN, unpublished].

[FRS4] $R(G,T)$ for all connected graphs G on at most 5 vertices and all (except some cases) trees T.

[FRS1] $R(K_3,G)$ for all connected graphs G on 6 vertices.

[Jin] $R(K_3,G)$ for all connected graphs G on 7 vertices. Some errors in [Jin] were found by [SchSch1].

[Brin] $R(K_3,G)$ for all connected graphs G on at most 8 vertices. The numbers for K_3 versus sets of graphs with fixed number of edges, on at most 8 vertices, were presented in [KM1].

[BBH1] $R(K_3,G)$ for all connected graphs G on 9 vertices. See also [BBH2].

[JR3] $R(C_4,G)$ for all graphs G on at most 6 vertices.

[JR4] $R(C_5,G)$ for all graphs G on at most 6 vertices.

[JR2] $R(C_6,G)$ for all graphs G on at most 5 vertices.

[LorMe3] $R(K_{2,n},K_{2,m})$ for all $2\le n,m\le 10$ except 8 cases, for which lower and upper bounds are given.

[HaKr] All best lower bounds up to 102 from cyclic graphs. Formulas for best cyclic lower bounds for paths and cycles, small complete graphs and for graphs with up to five vertices.

Chvátal and Harary [CH1, CH2] formulated several simple but very useful observations how to discover values of some numbers. All five missing entries in the tables of Clancy [Clan] have been solved. Out of 7 open cases in [He2] 4 have been solved, namely $R(4,5) = R(G_{19},G_{23}) = 25$ and the items 2, 3 and 4 in section 4.14. The still open 3 cases are for K_5 versus the graphs K_5 (section 2.1), K_5-e (section 3), and K_5-P_3 (section 4.14).

7.2. Cumulative data for three colors

[YR3] $R_3(G)$ for all graphs G with at most 4 edges and no isolates.

[YR1] $R_3(G)$ for all graphs G with 5 edges and no isolates, except K_4-e.
The case of $R_3(K_4-e)$ remains open (see section 5.3).

[YY] $R_3(G)$ for all graphs G with 6 edges and no isolates, except 10 cases.

[AKM] $R(F,G,H)$ for most triples of isolate-free graphs with at most 4 vertices. Some of the missing cases completed in [KM2].

7.3. Surveys

[Bu1] A general survey of results in Ramsey graph theory by S. A. Burr (1974)

[Par6] A general survey of results in Ramsey graph theory by T. D. Parsons (1978)

[Har2] Summary of progress by Frank Harary (1981)

[ChGri] A general survey of bounds and values by F. R. K. Chung and C. M. Grinstead (1983)

[JGT] Special volume of the *Journal of Graph Theory* (1983)

[Rob1] A review of Ramsey graph theory for newcomers by F. S. Roberts (1984)

[Bu7] What can we hope to accomplish in generalized Ramsey Theory ? (1987)

[GrRö] Survey of asymptotic problems by R. L. Graham and V. Rödl (1987)

[GRS] An excellent book by R. L. Graham, B. L. Rothschild and J. H. Spencer, second edition (1990)

[FRS5] Survey by Faudree, Rousseau and Schelp of graph goodness results, i.e. conditions for the formula $R(G,H) = (\chi(G)-1)(n(H)-1)+s(G)$ (1991)

[Neš] A chapter in *Handbook of Combinatorics* by J. Nešetřil (1996)

[Caro] Survey of zero-sum Ramsey theory by Y. Caro (1996)

[Chu4] Among 114 open problems and conjectures of Paul Erdös, presented and commented by F. R. K. Chung, 31 are concerned directly with Ramsey numbers. 216 references are given (1997). An extended version of this work was prepared jointly with R. L. Graham [ChGra2]. (1998)

[CoPC] Special issue of *Combinatorics, Probability and Computing* (2003)

The surveys by S. A. Burr [Bu1] and T. D. Parsons [Par6] contain extensive chapters on general exact results in graph Ramsey theory. F. Harary presented the state of the theory in 1981 in [Har2], where he also gathered many references including seven to other early surveys of this area. More than two decades ago, Chung and Grinstead in their survey paper [ChGri] gave less data than in this work, but included a broad discussion of different methods used in Ramsey computations in the classical case. S. A. Burr, one of the most experienced researchers in Ramsey graph theory, formulated in [Bu7] seven conjectures on Ramsey numbers for sufficiently large and sparse graphs, and reviewed the evidence for them found in the literature. Three of them have been refuted in [Bra3].

For newer extensive presentations see [GRS, GrRö, FRS5, Neš, Chu4, ChGra2], though these focus on asymptotic theory not on the numbers themselves. Finally, this compilation could not pretend to be complete without mentioning special volumes of the *Journal of Graph Theory* [JGT, 1983] and *Combinatorics, Probability and Computing* [CoPC, 2003], dedicated entirely to Ramsey theory. Besides a number of research papers, they include historical notes and present to us Frank P. Ramsey (1903-1930) as a person.

8. Concluding Remarks

This compilation does not include information on numerous variations of Ramsey numbers, nor related topics, like size Ramsey numbers, zero-sum Ramsey numbers, irredundant Ramsey numbers, induced Ramsey numbers, local Ramsey numbers, connected Ramsey numbers, chromatic Ramsey numbers, avoiding sets of graphs in some colors, coloring graphs other than complete, or the so called Ramsey multiplicities. Interested reader can find such information in the surveys listed in section 7 here.

The author apologizes for any omissions or other errors in reporting results belonging to the scope of this work. Suggestions for any kind of corrections or additions will be greatly appreciated and considered for inclusion in the next revision of this survey.

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References

We mark the papers containing results obtained with the help of computer algorithms with stars. We identify two categories of such papers: marked with * involving some use of computers, where the results are easily verifiable with some computations, and those marked with **, where cpu intensive algorithms have to be implemented to replicate or verify the results. The first category contains mostly constructions done by algorithms, while the second mostly nonexistence results or claims of complete enumerations of special classes of graphs.

The references are ordered alphabetically by the last name of the first author, for the same first author by the last name of the second author, etc. We preferred that all work by the same author be in consecutive positions. Unfortunately, this causes that some of the abbreviations are not in alphabetical order, for example [BaRT] is earlier on the list than [BaLS].

[Abb1] H.L. Abbott, *Ph. D. thesis*, University of Alberta, Edmonton, 1965.

[Abb2] H.L. Abbott, A Theorem Concerning Higher Ramsey Numbers, in *Infinite and Finite Sets*, (A. Hajnal, R. Rado and V.T. Sós eds.) North Holland, (1975) 25-28.

[AbbH] H.L. Abbott and D. Hanson, A Problem of Schur and Its Generalizations, *Acta Arithmetica*, **20** (1972) 175-187.

[AbbL] H.L. Abbott and Andy Liu, Remarks on a Paper of Hirschfeld Concerning Ramsey Numbers, *Discrete Mathematics*, **39** (1982) 327-328.

[AbbS] H.L. Abbott and M.J. Smuga-Otto, Lower Bounds for Hypergraph Ramsey Numbers, *Discrete Applied Mathematics*, **61** (1995) 177-180.

[AbbW] H.L. Abbott and E.R. Williams, Lower Bounds for Some Ramsey Numbers, *Journal of Combinatorial Theory*, Series A, **16** (1974) 12-17.

[AKS] M. Ajtai, J. Komlós and E. Szemerédi, A Note on Ramsey Numbers, *Journal of Combinatorial Theory*, Series A, **29** (1980) 354-360.

[Alon1] N. Alon, Subdivided Graphs Have Linear Ramsey Numbers, *Journal of Graph Theory,* **18** (1994) 343-347.

[Alon2] N. Alon, Explicit Ramsey Graphs and Orthonormal Labelings, *Electronic Journal of Combinatorics,* http://www.combinatorics.org/, #R12, **1** (1994), 8 pages.

[AlKS] N. Alon, M. Krivelevich and B. Sudakov, Turán Numbers of Bipartite Graphs and Related Ramsey-Type Questions, *Combinatorics, Probability and Computing,* **12** (2003) 477-494.

[AlPu] N. Alon and P. Pudlák, Constructive Lower Bounds for off-diagonal Ramsey Numbers, *Israel Journal of Mathematics,* **122** (2001) 243-251.

[AlRö] N. Alon and V. Rödl, Asymptotically Tight Bounds for Some Multicored Ramsey Numbers, *to appear.*

[AKM] J. Arste, K. Klamroth and I. Mengersen, Three Color Ramsey Numbers for Small Graphs, *Utilitas Mathematica,* **49** (1996) 85-96.

[AFM] M. Axenovich, Z. Füredi and D. Mubayi, On Generalized Ramsey Theory: the Bipartite Case, *Journal of Combinatorial Theory,* Series B, **79** (2000) 66-86.

[BaRT]* A. Babak, S.P. Radziszowski and Kung-Kuen Tse, Computation of the Ramsey Number $R(B_3, K_5)$, *Bulletin of the Institute of Combinatorics and its Applications,* **41** (2004) 71-76.

[BaLS] P.N. Balister, J. Lehel and R.H. Schelp, Ramsey Unsaturated and Saturated Graphs, *manuscript,* (2004).

[BaSS] P.N. Balister, R.H. Schelp and M. Simonovits, A Note on Ramsey Size-Linear Graphs, *Journal of Graph Theory,* **39** (2002) 1-5.

[Bas] E.T. Baskoro, The Ramsey Number of Paths and Small Wheels, *Majalah Ilmiah Himpunan Matematika Indonesia,* MIHMI, **8** (2002) 13-16.

[BaSu] E.T. Baskoro and Surahmat, The Ramsey Number of Paths with respect to Wheels, *preprint,* (2002).

[BSNM] E.T. Baskoro, Surahmat, S.M. Nababan and M. Miller, On Ramsey Graph Numbers for Trees versus Wheels of Five or Six Vertices, *Graphs and Combinatorics,* **18** (2002) 717-721.

[-] E.T. Baskoro, see also [SuBa1, SuBa2, SuBB1, SuBB2].

[Bev] D. Bevan, *personal communication* (2002).

[BS] A. Bialostocki and J. Schönheim, On Some Turán and Ramsey Numbers for C_4, in *Graph Theory and Combinatorics* (ed. B. Bollobás), Academic Press, London, (1984) 29-33.

[Bier] J. Bierbrauer, Ramsey Numbers for the Path with Three Edges, *European Journal of Combinatorics,* **7** (1986) 205-206.

[BB] J. Bierbrauer and A. Brandis, On Generalized Ramsey Numbers for Trees, *Combinatorica,* **5** (1985) 95-107.

[BJYHRZ] B. Bollobás, C.J. Jayawardene, Yang Jian Sheng, Huang Yi Ru, C.C. Rousseau, and Zhang Ke Min, On a Conjecture Involving Cycle-Complete Graph Ramsey Numbers, *Australasian Journal of Combinatorics,* **22** (2000) 63-71.

[BH] R. Bolze and H. Harborth, The Ramsey Number $r(K_4 - x, K_5)$, in *The Theory and Applications of Graphs,* (Kalamazoo, MI, 1980), John Wiley & Sons, New York, (1981) 109-116.

[BoEr] J.A. Bondy and P. Erdös, Ramsey Numbers for Cycles in Graphs, *Journal of Combinatorial Theory,* Series B, **14** (1973) 46-54.

[-] A. Brandis, see [BB].

[Bra1] S. Brandt, Subtrees and Subforests in Graphs, *Journal of Combinatorial Theory,* Series B, **61** (1994) 63-70.

[Bra2] S. Brandt, Sufficient Conditions for Graphs to Contain All Subgraphs of a Given Type, *Ph.D. thesis,* Freie Universität Berlin, 1994.

[Bra3] S. Brandt, Expanding Graphs and Ramsey Numbers, *manuscript,* (1996).

[BBH1]** S. Brandt, G. Brinkmann and T. Harmuth, All Ramsey Numbers $r(K_3, G)$ for Connected Graphs of Order 9, *Electronic Journal of Combinatorics,* http://www.combinatorics.org/, #R7, **5** (1998), 20 pages.

[BBH2]** S. Brandt, G. Brinkmann and T. Harmuth, The Generation of Maximal Triangle-Free Graphs, *Graphs and Combinatorics,* **16** (2000) 149-157.

[Brin]** G. Brinkmann, All Ramsey Numbers $r(K_3, G)$ for Connected Graphs of Order 7 and 8, *Combinatorics, Probability and Computing,* **7** (1998) 129-140.

[-] G. Brinkmann, see also [BBH1, BBH2].

[-] H.J. Broersma, see [SuBB1, SuBB2].

[BR]* J.P. Burling and S.W. Reyner, Some Lower Bounds of the Ramsey Numbers $n(k, k)$, *Journal of Combinatorial Theory,* Series B, **13** (1972) 168-169.

[Bu1] S.A. Burr, Generalized Ramsey Theory for Graphs - a Survey, in *Graphs and Combinatorics* (R. Bari and F. Harary eds.), Springer LNM **406**, Berlin, (1974) 52-75.

[Bu2] S.A. Burr, Ramsey Numbers Involving Graphs with Long Suspended Paths, *Journal of the London Mathematical Society* (2), **24** (1981) 405-413.

[Bu3] S.A. Burr, Multicolor Ramsey Numbers Involving Graphs with Long Suspended Path, *Discrete Mathematics,* **40** (1982) 11-20.

[Bu4] S.A. Burr, Diagonal Ramsey Numbers for Small Graphs, *Journal of Graph Theory,* **7** (1983) 57-69.

[Bu5] S.A. Burr, Ramsey Numbers Involving Powers, *Ars Combinatoria,* **15** (1983) 163-168.

[Bu6] S.A. Burr, Determining Generalized Ramsey Numbers is NP-Hard, *Ars Combinatoria,* **17** (1984) 21-25.

[Bu7] S.A. Burr, What Can We Hope to Accomplish in Generalized Ramsey Theory?, *Discrete Mathematics,* **67** (1987) 215-225.

[Bu8] S.A. Burr, On the Ramsey Numbers $r(G, nH)$ and $r(nG, nH)$ When n Is Large, *Discrete Mathematics,* **65** (1987) 215-229.

[Bu9] S.A. Burr, On Ramsey Numbers for Large Disjoint Unions of Graphs, *Discrete Mathematics,* **70** (1988) 277-293.

[BE1] S.A. Burr and P. Erdös, Extremal Ramsey Theory for Graphs, *Utilitas Mathematica,* **9** (1976) 247-258.

[BE2] S.A. Burr and P. Erdös, Generalizations of a Ramsey-Theoretic Result of Chvátal, *Journal of Graph Theory,* **7** (1983) 39-51.

[BEFRS1] S.A. Burr, P. Erdös, R.J. Faudree, C.C. Rousseau and R.H. Schelp, An Extremal Problem in Generalized Ramsey Theory, *Ars Combinatoria,* **10** (1980) 193-203.

[BEFRS2] S.A. Burr, P. Erdös, R.J. Faudree, C.C. Rousseau and R.H. Schelp, Ramsey Numbers for the Pair Sparse Graph-Path or Cycle, *Transactions of the American Mathematical Society,* **269** (1982) 501-512.

[BEFRS3] S.A. Burr, P. Erdös, R.J. Faudree, C.C. Rousseau and R.H. Schelp, On Ramsey Numbers Involving Starlike Multipartite Graphs, *Journal of Graph Theory,* **7** (1983) 395-409.

[BEFRS4] S.A. Burr, P. Erdös, R.J. Faudree, C.C. Rousseau and R.H. Schelp, The Ramsey Number for the Pair Complete Bipartite Graph-Graph of Limited Degree, in *Graph Theory with Applications to Algorithms and Computer Science,* (Y. Alavi et al. eds.), John Wiley & Sons, New York, (1985) 163-174.

[BEFRS5] S.A. Burr, P. Erdös, R.J. Faudree, C.C. Rousseau and R.H. Schelp, Some Complete Bipartite Graph-Tree Ramsey Numbers, *Annals of Discrete Mathematics,* **41** (1989) 79-89.

[BEFRSGJ]S.A. Burr, P. Erdös, R.J. Faudree, C.C. Rousseau, R.H. Schelp, R.J. Gould and M.S. Jacobson, Goodness of Trees for Generalized Books, *Graphs and Combinatorics,* **3** (1987) 1-6.

[BEFS] S.A. Burr, P. Erdös, R.J. Faudree and R.H. Schelp, On the Difference between Consecutive Ramsey Numbers, *Utilitas Mathematica,* **35** (1989) 115-118.

[BES] S.A. Burr, P. Erdös and J.H. Spencer, Ramsey Theorems for Multiple Copies of Graphs, *Transactions of the American Mathematical Society*, **209** (1975) 87-99.

[BF] S.A. Burr and R.J. Faudree, On Graphs G for Which All Large Trees Are G-good, *Graphs and Combinatorics*, **9** (1993) 305-313.

[BG] S.A. Burr and J.W. Grossman, Ramsey Numbers of Graphs with Long Tails, *Discrete Mathematics*, **41** (1982) 223-227.

[BuRo1] S.A. Burr and J.A. Roberts, On Ramsey Numbers for Stars, *Utilitas Mathematica*, **4** (1973) 217-220.

[BuRo2] S.A. Burr and J.A. Roberts, On Ramsey Numbers for Linear Forests, *Discrete Mathematics*, **8** (1974) 245-250.

[Bush] L.E. Bush, The William Lowell Putnam Mathematical Competition, *American Mathematical Monthly*, **60** (1953) 539-542.

[CET]* N.J. Calkin, P. Erdös and C.A. Tovey, New Ramsey Bounds from Cyclic Graphs of Prime Order, *SIAM Journal of Discrete Mathematics*, **10** (1997) 381-387.

[Caro] Y. Caro, Zero-Sum Problems - A Survey, *Discrete Mathematics*, **152** (1996) 93-113.

[CLRZ] Y. Caro, Li Yusheng, C.C. Rousseau and Zhang Yuming, Asymptotic Bounds for Some Bipartite Graph - Complete Graph Ramsey Numbers, *Discrete Mathematics*, **220** (2000) 51-56.

[CGP] G. Chartrand, R.J. Gould and A.D. Polimeni, On Ramsey Numbers of Forests versus Nearly Complete Graphs, *Journal of Graph Theory*, **4** (1980) 233-239.

[CRSPS] G. Chartrand, C.C. Rousseau, M.J. Stewart, A.D. Polimeni and J. Sheehan, On Star-Book Ramsey Numbers, in *Proceedings of the Fourth International Conference on the Theory and Applications of Graphs*, (Kalamazoo, MI 1980), John Wiley & Sons, (1981) 203-214.

[CS] G. Chartrand and S. Schuster, On the existence of specified cycles in complementary graphs, *Bulletin of the American Mathematical Society*, **77** (1971) 995-998.

[Chen] Chen Guantao, A Result on C_4-Star Ramsey Numbers, *Discrete Mathematics*, **163** (1997) 243-246.

[ChenS] Chen Guantao and R.H. Schelp, Graphs with Linearly Bounded Ramsey Numbers, *Journal of Combinatorial Theory*, Series B, **57** (1993) 138-149.

[ChenJ] Chen Jie, The Lower Bound of Some Ramsey Numbers (in Chinese), *Journal of the Liaoning Normal University*, *Natural Science*, **25** (2002) 244-246.

[ChenZZ1] Chen Yaojun, Zhang Yunqing and Zhang Ke Min, The Ramsey Numbers of Paths versus Wheels, *preprint*, (2002).

[ChenZZ2] Chen Yaojun, Zhang Yunqing and Zhang Ke Min, The Ramsey Numbers of Stars versus Wheels, to appear in the *European Journal of Combinatorics*, (2004).

[ChenZZ3] Chen Yaojun, Zhang Yunqing and Zhang Ke Min, The Ramsey Number $R(T_n, W_6)$ for $\Delta(T_n) \geq n-3$, *Applied Mathematics Letters*, **17** (2004) 281-285.

[Chu1] F.R.K. Chung, On the Ramsey Numbers $N(3,3,...,3;2)$, *Discrete Mathematics*, **5** (1973) 317-321.

[Chu2] F.R.K. Chung, On Triangular and Cyclic Ramsey Numbers with k Colors, in *Graphs and Combinatorics* (R. Bari and F. Harary eds.), Springer LNM **406**, Berlin, (1974) 236-241.

[Chu3] F.R.K. Chung, A Note on Constructive Methods for Ramsey Numbers, *Journal of Graph Theory*, **5** (1981) 109-113.

[Chu4] F.R.K. Chung, Open problems of Paul Erdös in Graph Theory, *Journal of Graph Theory*, **25** (1997) 3-36.

[CCD] F.R.K. Chung, R. Cleve and P. Dagum, A Note on Constructive Lower Bounds for the Ramsey Numbers $R(3,t)$, *Journal of Combinatorial Theory*, Series B, **57** (1993) 150-155.

[ChGra1] F.R.K. Chung and R.L. Graham, On Multicolor Ramsey Numbers for Complete Bipartite Graphs, *Journal of Combinatorial Theory*, Series B, **18** (1975) 164-169.

[ChGra2] F.R.K. Chung and R.L. Graham, *Erdös on Graphs, His Legacy of Unsolved Problems*, A K Peters, Wellesley, Massachusetts (1998).

[ChGri] F.R.K. Chung and C.M. Grinstead, A Survey of Bounds for Classical Ramsey Numbers, *Journal of Graph Theory*, **7** (1983) 25-37.

[Chv] V. Chvátal, Tree-Complete Graph Ramsey Numbers, *Journal of Graph Theory*, **1** (1977) 93.

[CH1] V. Chvátal and F. Harary, Generalized Ramsey Theory for Graphs, II. Small Diagonal Numbers, *Proceedings of the American Mathematical Society*, **32** (1972) 389-394.

[CH2] V. Chvátal and F. Harary, Generalized Ramsey Theory for Graphs, III. Small Off-Diagonal Numbers, *Pacific Journal of Mathematics*, **41** (1972) 335-345.

[CH3] V. Chvátal and F. Harary, Generalized Ramsey Theory for Graphs, I. Diagonal Numbers, *Periodica Mathematica Hungarica*, **3** (1973) 115-124.

[CRST] V. Chvátal, V. Rödl, E. Szemerédi and W.T. Trotter Jr., The Ramsey Number of a Graph with Bounded Maximum Degree, *Journal of Combinatorial Theory*, Series B, **34** (1983) 239-243.

[ChvS] V. Chvátal and A. Schwenk, On the Ramsey Number of the Five-Spoked Wheel, in *Graphs and Combinatorics* (R. Bari and F. Harary eds.), Springer LNM **406**, Berlin, (1974) 247-261.

[Clan] M. Clancy, Some Small Ramsey Numbers, *Journal of Graph Theory*, **1** (1977) 89-91.

[Clap] C. Clapham, The Ramsey Number $r(C_4, C_4, C_4)$, *Periodica Mathematica Hungarica*, **18** (1987) 317-318.

[CEHMS] C. Clapham, G. Exoo, H. Harborth, I. Mengersen and J. Sheehan, The Ramsey Number of $K_5 - e$, *Journal of Graph Theory*, **13** (1989) 7-15.

[Clark] L. Clark, On Cycle-Star Graph Ramsey Numbers, *Congressus Numerantium*, **50** (1985) 187-192.

[-] R. Cleve, see [CCD].

[Coc] E.J. Cockayne, Some Tree-Star Ramsey Numbers, *Journal of Combinatorial Theory*, Series B, **17** (1974) 183-187.

[CL1] E.J. Cockayne and P.J. Lorimer, The Ramsey Number for Stripes, *Journal of the Australian Mathematical Society*, Series A, **19** (1975) 252-256.

[CL2] E.J. Cockayne and P.J. Lorimer, On Ramsey Graph Numbers for Stars and Stripes, *Canadian Mathematical Bulletin*, **18** (1975) 31-34.

[CPR] B. Codenotti, P. Pudlák and G. Resta, Some Structural Properties of Low-Rank Matrices Related to Computational Complexity, *Theoretical Computer Science*, **235** (2000) 89-107.

[CoPC] Special issue on Ramsey theory of *Combinatorics, Probability and Computing*, **12** (2003), Numbers 5 and 6.

[CsKo] R. Csákány and J. Komlós, The Smallest Ramsey Numbers, *Discrete Mathematics*, **199** (1999) 193-199.

[-] P. Dagum, see [CCD].

[Den] T. Denley, The Ramsey Numbers for Disjoint Unions of Cycles, *Discrete Mathematics*, **149** (1996) 31-44.

[DuHu] Duan Chanlun and Huang Wenke, Lower Bound of Ramsey Number $r(3, 10)$ (in Chinese), *Acta Scientiarum Naturalium Universitatis NeiMongol*, **31** (2000) 468-470.

[DLR] D. Duffus, H. Lefmann and V. Rödl, Shift Graphs and Lower Bounds on Ramsey Numbers $r_k(l; r)$, *Discrete Mathematics*, **137** (1995) 177-187.

[DzKu] T. Dzido and M. Kubale, On Some Ramsey and Turán-type Numbers for Paths and Cycles, *manuscript*, (2004).

[Ea1] Easy to obtain by simple combinatorics from other results, in particular by using graphs establishing lower bounds with smaller parameters.

[Ea2] Unique 2-(6,3,2) design gives lower bound 7, upper bound is easy.

[Eaton] N. Eaton, Ramsey Numbers for Sparse Graphs, *Discrete Mathematics*, **185** (1998) 63-75.

[Eng] A. Engström, A Note on Two Multicolor Ramsey Numbers, *manuscript*, (2003).

[Erd1] P. Erdös, Some Remarks on the Theory of Graphs, *Bulletin of the American Mathematical Society*, **53** (1947) 292-294.

[Erd2] P. Erdös, On the Combinatorial Problems Which I Would Most Like to See Solved, *Combinatorica*, **1** (1981) 25-42.

[EFRS1] P. Erdös, R.J. Faudree, C.C. Rousseau and R.H. Schelp, Generalized Ramsey Theory for Multiple Colors, *Journal of Combinatorial Theory*, Series B, **20** (1976) 250-264.

[EFRS2] P. Erdös, R.J. Faudree, C.C. Rousseau and R.H. Schelp, On Cycle-Complete Graph Ramsey Numbers, *Journal of Graph Theory*, **2** (1978) 53-64.

[EFRS3] P. Erdös, R.J. Faudree, C.C. Rousseau and R.H. Schelp, Ramsey Numbers for Brooms, *Congressus Numerantium*, **35** (1982) 283-293.

[EFRS4] P. Erdös, R.J. Faudree, C.C. Rousseau and R.H. Schelp, Multipartite Graph-Sparse Graph Ramsey Numbers, *Combinatorica*, **5** (1985) 311-318.

[EFRS5] P. Erdös, R.J. Faudree, C.C. Rousseau and R.H. Schelp, A Ramsey Problem of Harary on Graphs with Prescribed Size, *Discrete Mathematics*, **67** (1987) 227-233.

[EFRS6] P. Erdös, R.J. Faudree, C.C. Rousseau and R.H. Schelp, Extremal Theory and Bipartite Graph-Tree Ramsey Numbers, *Discrete Mathematics*, **72** (1988) 103-112.

[EFRS7] P. Erdös, R.J. Faudree, C.C. Rousseau and R.H. Schelp, The Book-Tree Ramsey Numbers, *Scientia*, Series A: Mathematical Sciences, Valparaíso, Chile, **1** (1988) 111-117.

[EFRS8] P. Erdös, R.J. Faudree, C.C. Rousseau and R.H. Schelp, Multipartite Graph-Tree Graph Ramsey Numbers, in *Graph Theory and Its Applications: East and West, Proceedings of the First China-USA International Graph Theory Conference*, Annals of the New York Academy of Sciences, **576** (1989) 146-154.

[EFRS9] P. Erdös, R.J. Faudree, C.C. Rousseau and R.H. Schelp, Ramsey Size Linear Graphs, *Combinatorics, Probability and Computing*, **2** (1993) 389-399.

[EG] P. Erdös and R.L. Graham, On Partition Theorems for Finite Sets, in *Infinite and Finite Sets*, (A. Hajnal, R. Rado and V.T. Sós eds.) North Holland, (1975) 515-527.

[-] P. Erdös, see also [BoEr, BE1, BE2, BEFRS1, BEFRS2, BEFRS3, BEFRS4, BEFRS5, BEFRSGJ, BEFS, BES, CET].

[Ex1]* G. Exoo, Ramsey Numbers of Hypergraphs, *Journal of Combinatorial Mathematics and Combinatorial Computing*, **2** (1987) 5-11.

[Ex2]* G. Exoo, Constructing Ramsey Graphs with a Computer, *Congressus Numerantium*, **59** (1987) 31-36.

[Ex3]* G. Exoo, Applying Optimization Algorithm to Ramsey Problems, in *Graph Theory, Combinatorics, Algorithms, and Applications* (Y. Alavi ed.), SIAM Philadelphia, (1989) 175-179.

[Ex4]* G. Exoo, A Lower Bound for $R(5,5)$, *Journal of Graph Theory*, **13** (1989) 97-98.

[Ex5]* G. Exoo, On Two Classical Ramsey Numbers of the Form $R(3,n)$, *SIAM Journal of Discrete Mathematics*, **2** (1989) 488-490.

[Ex6]* G. Exoo, A Lower Bound for $r(K_5-e,K_5)$, *Utilitas Mathematica*, **38** (1990) 187-188.

[Ex7]* G. Exoo, On the Three Color Ramsey Number of K_4-e, *Discrete Mathematics*, **89** (1991) 301-305.

[Ex8]* G. Exoo, Indiana State University, *personal communication* (1992).

[Ex9]* G. Exoo, Announcement: On the Ramsey Numbers $R(4,6)$, $R(5,6)$ and $R(3,12)$, *Ars Combinatoria*, **35** (1993) 85. The construction of a graph proving $R(4,6)\geq 35$ is presented in detail at http://ginger.indstate.edu/ge/RAMSEY (2001).

[Ex10]* G. Exoo, A Lower Bound for Schur Numbers and Multicolor Ramsey Numbers of K_3, *Electronic Journal of Combinatorics*, http://www.combinatorics.org/, #R8, **1** (1994), 3 pages.

[Ex11]* G. Exoo, Indiana State University, *personal communication* (1997).

[Ex12]* G. Exoo, Some New Ramsey Colorings, *Electronic Journal of Combinatorics*, http://www.combinatorics.org/, #R29, **5** (1998), 5 pages. The constructions are available electronically

from http://ginger.indstate.edu/ge/RAMSEY.

[Ex13]* G. Exoo, Indiana State University, *personal communication* (1998). Constructions available at http://ginger.indstate.edu/ge/RAMSEY.

[Ex14]* G. Exoo, Indiana State University, *New Lower Bounds for Table III*, (2000). Constructions available at http://ginger.indstate.edu/ge/RAMSEY.

[Ex15]* G. Exoo, Some Applications of pq-groups in Graph Theory, *Discussiones Mathematicae Graph Theory*, **24** (2004) 109-114. Constructions available at http://ginger.indstate.edu/ge/RAMSEY.

[Ex16]* G. Exoo, Indiana State University, *personal communication* (2002-2004). Constructions available at http://ginger.indstate.edu/ge/RAMSEY.

[EHM1] G. Exoo, H. Harborth and I. Mengersen, The Ramsey Number of K_4 versus K_5-e, *Ars Combinatoria*, **25A** (1988) 277-286.

[EHM2] G. Exoo, H. Harborth and I. Mengersen, On Ramsey Number of $K_{2,n}$, in *Graph Theory, Combinatorics, Algorithms, and Applications* (Y. Alavi, F.R.K. Chung, R.L. Graham and D.F. Hsu eds.), SIAM Philadelphia, (1989) 207-211.

[ExRe]* G. Exoo and D.F. Reynolds, Ramsey Numbers Based on C_5-Decompositions, *Discrete Mathematics*, **71** (1988) 119-127.

[-] G. Exoo, see also [CEHMS, XXER].

[FLPS] R.J. Faudree, S.L. Lawrence, T.D. Parsons and R.H. Schelp, Path-Cycle Ramsey Numbers, *Discrete Mathematics*, **10** (1974) 269-277.

[FM]** R.J. Faudree and B.D. McKay, A Conjecture of Erdös and the Ramsey Number $r(W_6)$, *Journal of Combinatorial Mathematics and Combinatorial Computing*, **13** (1993) 23-31.

[FRS1] R.J. Faudree, C.C. Rousseau and R.H. Schelp, All Triangle-Graph Ramsey Numbers for Connected Graphs of Order Six, *Journal of Graph Theory*, **4** (1980) 293-300.

[FRS2] R.J. Faudree, C.C. Rousseau and R.H. Schelp, Studies Related to the Ramsey Number $r(K_5-e)$, in *Graph Theory and Its Applications to Algorithms and Computer Science*, (Y. Alavi et al. eds.), John Wiley and Sons, New York, (1985) 251-271.

[FRS3] R.J. Faudree, C.C. Rousseau and R.H. Schelp, Generalizations of the Tree-Complete Graph Ramsey Number, in *Graphs and Applications*, (F. Harary and J.S. Maybee eds.), John Wiley and Sons, New York, (1985) 117-126.

[FRS4] R.J. Faudree, C.C. Rousseau and R.H. Schelp, Small Order Graph-Tree Ramsey Numbers, *Discrete Mathematics*, **72** (1988) 119-127.

[FRS5] R.J. Faudree, C.C. Rousseau and R.H. Schelp, A Good Idea in Ramsey Theory, in *Graph Theory, Combinatorics, Algorithms, and Applications* (San Francisco, CA 1989), SIAM Philadelphia, PA (1991) 180-189.

[FRS6] R.J. Faudree, C.C. Rousseau and J. Sheehan, More from the Good Book, in *Proceedings of the Ninth Southeastern Conference on Combinatorics, Graph Theory, and Computing*, Utilitas Mathematica Publ., *Congressus Numerantium* **XXI** (1978) 289-299.

[FRS7] R.J. Faudree, C.C. Rousseau and J. Sheehan, Strongly Regular Graphs and Finite Ramsey Theory, *Linear Algebra and its Applications*, **46** (1982) 221-241.

[FRS8] R.J. Faudree, C.C. Rousseau and J. Sheehan, Cycle-Book Ramsey Numbers, *Ars Combinatoria*, **31** (1991) 239-248.

[FS1] R.J. Faudree and R.H. Schelp, All Ramsey Numbers for Cycles in Graphs, *Discrete Mathematics*, **8** (1974) 313-329.

[FS2] R.J. Faudree and R.H. Schelp, Path Ramsey Numbers in Multicolorings, *Journal of Combinatorial Theory*, Series B, **19** (1975) 150-160.

[FS3] R.J. Faudree and R.H. Schelp, Ramsey Numbers for All Linear Forests, *Discrete Mathematics*, **16** (1976) 149-155.

[FS4] R.J. Faudree and R.H. Schelp, Some Problems in Ramsey Theory, in *Theory and Applications of Graphs*, (conference proceedings, Kalamazoo, MI 1976), Lecture Notes in Mathematics **642**, Springer, Berlin, (1978) 500-515.

[FSR] R.J. Faudree, R.H. Schelp and C.C. Rousseau, Generalizations of a Ramsey Result of Chvátal, in *Proceedings of the Fourth International Conference on the Theory and Applications of Graphs*, (Kalamazoo, MI 1980), John Wiley & Sons, (1981) 351-361.

[FSS1] R.J. Faudree, R.H. Schelp and M. Simonovits, On Some Ramsey Type Problems Connected with Paths, Cycles and Trees, *Ars Combinatoria*, **29A** (1990) 97-106.

[FSS2] R.J. Faudree, A. Schelten and I. Schiermeyer, The Ramsey Number $r(C_7, C_7, C_7)$, *Discussiones Mathematicae Graph Theory*, **23** (2003) 141-158.

[FS] R.J. Faudree and M. Simonovits, Ramsey Problems and Their Connection to Turán-Type Extremal Problems, *Journal of Graph Theory*, **16** (1992) 25-50.

[-] R.J. Faudree, see also [BEFRS1, BEFRS2, BEFRS3, BEFRS4, BEFRS5, BEFRSGJ, BEFS, BF, EFRS1, EFRS2, EFRS3, EFRS4, EFRS5, EFRS6, EFRS7, EFRS8, EFRS9].

[FKR]** S. Fettes, R.L. Kramer and S.P. Radziszowski, An Upper Bound of 62 on the Classical Ramsey Number $R(3,3,3,3)$, to appear in *Ars Combinatoria*, (2004).

[Fo] J. Folkman, Notes on the Ramsey Number $N(3,3,3,3)$, *Journal of Combinatorial Theory*, Series A, **16** (1974) 371-379.

[FraWi] P. Frankl and R.M. Wilson, Intersection Theorems with Geometric Consequences, *Combinatorica*, **1** (1981) 357-368.

[Fra1] K. Fraughnaugh Jones, Independence in Graphs with Maximum Degree Four, *Journal of Combinatorial Theory*, Series B, **37** (1984) 254-269.

[Fra2] K. Fraughnaugh Jones, Size and Independence in Triangle-Free Graphs with Maximum Degree Three, *Journal of Graph Theory*, **14** (1990) 525-535.

[FrLo] K. Fraughnaugh and S.C. Locke, Finding Independent Sets in Triangle-free Graphs, *SIAM Journal of Discrete Mathematics*, **9** (1996) 674-681.

[Fre] H. Fredricksen, Schur Numbers and the Ramsey Numbers $N(3,3,...,3;2)$, *Journal of Combinatorial Theory*, Series A, **27** (1979) 376-377.

[FreSw]* H. Fredricksen and M.M. Sweet, Symmetric Sum-Free Partitions and Lower Bounds for Schur Numbers, *Electronic Journal of Combinatorics*, http://www.combinatorics.org/, #R32, **7** (2000), 9 pages.

[-] Z. Füredi, see [AFM].

[GeGy] L. Gerencsér and A. Gyárfás, On Ramsey-Type Problems, *Annales Universitatis Scientiarum Budapestinensis, Eötvös Sect. Math.*, **10** (1967) 167-170.

[Gi1] G. Giraud, Une généralisation des nombres et de l'inégalité de Schur, *C.R. Acad. Sc. Paris*, Séries A-B, **266** (1968) A437-A440.

[Gi2] G. Giraud, Minoration de certains nombres de Ramsey binaires par les nombres de Schur généralisés, *C.R. Acad. Sc. Paris*, Séries A-B, **266** (1968) A481-A483.

[Gi3] G. Giraud, Nouvelles majorations des nombres de Ramsey binaires-bicolores, *C.R. Acad. Sc. Paris*, Séries A-B, **268** (1969) A5-A7.

[Gi4] G. Giraud, Majoration du nombre de Ramsey ternaire-bicolore en (4,4), *C.R. Acad. Sc. Paris*, Séries A-B, **269** (1969) A620-A622.

[Gi5] G. Giraud, Une minoration du nombre de quadrangles unicolores et son application à la majoration des nombres de Ramsey binaires-bicolores, *C.R. Acad. Sc. Paris*, Séries A-B, **276** (1973) A1173-A1175.

[Gi6] G. Giraud, Sur le problème de Goodman pour les quadrangles et la majoration des nombres de Ramsey, *Journal of Combinatorial Theory*, Series B, **27** (1979) 237-253.

[-] A.M. Gleason, see [GG].

[GK] W. Goddard and D.J. Kleitman, An upper bound for the Ramsey numbers $r(K_3, G)$, *Discrete Mathematics*, **125** (1994) 177-182.

[GoMC] A. Gonçalves and E.L. Monte Carmelo, Some Geometric Structures and Bounds for Ramsey Numbers, *Discrete Mathematics*, **280** (2004) 29-38.

[GJ1] R.J. Gould and M.S. Jacobson, Bounds for the Ramsey Number of a Disconnected Graph Versus Any Graph, *Journal of Graph Theory*, **6** (1982) 413-417.

[GJ2] R.J. Gould and M.S. Jacobson, On the Ramsey Number of Trees Versus Graphs with Large Clique Number, *Journal of Graph Theory*, **7** (1983) 71-78.

[-] R.J. Gould, see also [BEFRSGJ, CGP].

[GrRö] R.L. Graham and V. Rödl, Numbers in Ramsey Theory, in *Surveys in Combinatorics*, (ed. C. Whitehead), Cambridge University Press, 1987.

[GRR1] R.L. Graham, V. Rödl and A. Ruciński, On Graphs with Linear Ramsey Numbers, *Journal of Graph Theory*, **35** (2000) 176-192.

[GRR2] R.L. Graham, V. Rödl and A. Ruciński, On Bipartite Graphs with Linear Ramsey Numbers, Paul Erdös and his mathematics, *Combinatorica*, **21** (2001) 199-209.

[GRS] R.L. Graham, B.L. Rothschild and J.H. Spencer, *Ramsey Theory*, John Wiley & Sons, 1990.

[-] R.L. Graham, see also [ChGra1, ChGra2, EG].

[GY] J.E. Graver and J. Yackel, Some Graph Theoretic Results Associated with Ramsey's Theorem, *Journal of Combinatorial Theory*, **4** (1968) 125-175.

[GG] R.E. Greenwood and A.M. Gleason, Combinatorial Relations and Chromatic Graphs, *Canadian Journal of Mathematics*, **7** (1955) 1-7.

[GH] U. Grenda and H. Harborth, The Ramsey Number $r(K_3, K_7 - e)$, *Journal of Combinatorics, Information & System Sciences*, **7** (1982) 166-169.

[Gri] J.R. Griggs, An Upper Bound on the Ramsey Numbers $R(3,k)$, *Journal of Combinatorial Theory*, Series A, **35** (1983) 145-153.

[GR]** C. Grinstead and S. Roberts, On the Ramsey Numbers $R(3,8)$ and $R(3,9)$, *Journal of Combinatorial Theory*, Series B, **33** (1982) 27-51.

[-] C. Grinstead, see also [ChGri].

[Grol1] V. Grolmusz, Superpolynomial Size Set-Systems with Restricted Intersections mod 6 and Explicit Ramsey Graphs, *Combinatorica*, **20** (2000) 73-88.

[Grol2] V. Grolmusz, Low Rank Co-Diagonal Matrices and Ramsey Graphs, *Electronic Journal of Combinatorics*, http://www.combinatorics.org/, #R15, **7** (2000) 7 pages.

[Grol3] V. Grolmusz, Set-Systems with Restricted Multiple Intersections, *Electronic Journal of Combinatorics*, http://www.combinatorics.org/, #R8, **9** (2002) 10 pages.

[Gros1] J.W. Grossman, Some Ramsey Numbers of Unicyclic Graphs, *Ars Combinatoria*, **8** (1979) 59-63.

[Gros2] J.W. Grossman, The Ramsey Numbers of the Union of Two Stars, *Utilitas Mathematica*, **16** (1979) 271-279.

[GHK] J.W. Grossman, F. Harary and M. Klawe, Generalized Ramsey Theory for Graphs, X: Double Stars, *Discrete Mathematics*, **28** (1979) 247-254.

[-] J.W. Grossman, see also [BG].

[GV] Guo Yubao and L. Volkmann, Tree-Ramsey Numbers, *Australasian Journal of Combinatorics*, **11** (1995) 169-175.

[-] L. Gupta, see [GGS].

[GGS] S.K. Gupta, L. Gupta and A. Sudan, On Ramsey Numbers for Fan-Fan Graphs, *Journal of Combinatorics, Information & System Sciences*, **22** (1997) 85-93.

[GT] A. Gyárfás and Z. Tuza, An Upper Bound on the Ramsey Number of Trees, *Discrete Mathematics,* **66** (1987) 309-310.

[-] A. Gyárfás, see also [GeGy].

[Haa]* H. Haanpää, A Lower Bound for a Ramsey Number, *Congressus Numerantium,* **144** (2000) 189-191.

[Häg] R. Häggkvist, On the Path-Complete Bipartite Ramsey Number, *Discrete Mathematics,* **75** (1989) 243-245.

[Han]* D. Hanson, Sum-free Sets and Ramsey Numbers, *Discrete Mathematics,* **14** (1976) 57-61.

[-] D. Hanson, see also [AbbH].

[Har1] F. Harary, Recent Results on Generalized Ramsey Theory for Graphs, in *Graph Theory and Applications,* (Y. Alavi et al. eds.) Springer, Berlin (1972) 125-138.

[Har2] F. Harary, Generalized Ramsey Theory I to XIII: Achievement and Avoidance Numbers, in *Proceedings of the Fourth International Conference on the Theory and Applications of Graphs,* (Kalamazoo, MI 1980), John Wiley & Sons, (1981) 373-390.

[-] F. Harary, see also [CH1, CH2, CH3, GHK].

[HaKr]** H. Harborth and S. Krause, Ramsey Numbers for Circulant Colorings, *Congressus Numerantium,* **161** (2003) 139-150.

[HaMe1] H. Harborth and I. Mengersen, An Upper Bound for the Ramsey Number $r(K_5-e)$, *Journal of Graph Theory,* **9** (1985) 483-485.

[HaMe2] H. Harborth and I. Mengersen, All Ramsey Numbers for Five Vertices and Seven or Eight Edges, *Discrete Mathematics,* **73** (1988/89) 91-98.

[HaMe3] H. Harborth and I. Mengersen, The Ramsey Number of $K_{3,3}$, in *Combinatorics, Graph Theory, and Applications,* Vol. **2** (Y. Alavi, G. Chartrand, O.R. Oellermann and J. Schwenk eds.), John Wiley & Sons, (1991) 639-644.

[-] H. Harborth, see also [BH, CEHMS, EHM1, EHM2, GH].

[HaMe4] M. Harborth and I. Mengersen, Some Ramsey Numbers for Complete Bipartite Graphs, *Australasian Journal of Combinatorics,* **13** (1996) 119-128.

[-] T. Harmuth, see [BBH1, BBH2].

[HaŁT] P.E. Haxell, T. Łuczak and P.W. Tingley, Ramsey Numbers for Trees of Small Maximum Degree, *Combinatorica,* **22** (2002) 287-320.

[Hein] K. Heinrich, Proper Colourings of K_{15}, *Journal of the Australian Mathematical Society,* Series A, **24** (1977) 465-495.

[He1] G.R.T. Hendry, Diagonal Ramsey Numbers for Graphs with Seven Edges, *Utilitas Mathematica,* **32** (1987) 11-34.

[He2] G.R.T. Hendry, Ramsey Numbers for Graphs with Five Vertices, *Journal of Graph Theory,* **13** (1989) 245-248.

[He3] G.R.T. Hendry, The Ramsey Numbers $r(K_2+\overline{K}_3,K_4)$ and $r(K_1+C_4,K_4)$, *Utilitas Mathematica,* **35** (1989) 40-54, addendum in **36** (1989) 25-32.

[He4] G.R.T. Hendry, Critical Colorings for Clancy's Ramsey Numbers, *Utilitas Mathematica,* **41** (1992) 181-203.

[He5] G.R.T. Hendry, Small Ramsey Numbers II. Critical Colorings for $r(C_5+e,K_5)$, *Quaestiones Mathematica,* **17** (1994) 249-258.

[-] G.R.T. Hendry, see also [YH].

[HiIr]* R. Hill and R.W. Irving, On Group Partitions Associated with Lower Bounds for Symmetric Ramsey Numbers, *European Journal of Combinatorics,* **3** (1982) 35-50.

[Hir] J. Hirschfeld, A Lower Bound for Ramsey's Theorem, *Discrete Mathematics,* **32** (1980) 89-91.

[HoMe] M. Hoeth and I. Mengersen, Ramsey Numbers for Graphs of Order Four versus Connected Graphs of Order Six, *Utilitas Mathematica,* **57** (2000) 3-19.

[HuSo] Huang Da Ming and Song En Min, Properties and Lower Bounds of the Third Order Ramsey Numbers (in Chinese), *Mathematica Applicata*, **9** (1996) 105-107.

[Hua1] Huang Guotai, Some Generalized Ramsey Numbers (in Chinese), *Mathematica Applicata*, **1** (1988) 97-101.

[Hua2] Huang Guotai, An Unsolved Problem of Gould and Jacobson (in Chinese), *Mathematica Applicata*, **9** (1996) 234-236.

[-] Huang Wenke, see [DuHu].

[HZ1] Huang Yi Ru and Zhang Ke Min, An New Upper Bound Formula for Two Color Classical Ramsey Numbers, *Journal of Combinatorial Mathematics and Combinatorial Computing*, **28** (1998) 347-350.

[HZ2] Huang Yi Ru and Zhang Ke Min, New Upper Bounds for Ramsey Numbers, *European Journal of Combinatorics*, **19** (1998) 391-394.

[-] Huang Yi Ru, see also [BJYHRZ, YHZ1, YHZ2].

[Ir] R.W. Irving, Generalised Ramsey Numbers for Small Graphs, *Discrete Mathematics*, **9** (1974) 251-264.

[-] R.W. Irving, see also [HiIr].

[Is1] J.R. Isbell, $N(4,4;3) \geq 13$, *Journal of Combinatorial Theory*, **6** (1969) 210.

[Is2] J.R. Isbell, $N(5,4;3) \geq 24$, *Journal of Combinatorial Theory*, Series A, **34** (1983) 379-380.

[Jac] M.S. Jacobson, On the Ramsey Number for Stars and a Complete Graph, *Ars Combinatoria*, **17** (1984) 167-172.

[-] M.S. Jacobson, see also [BEFRSGJ, GJ1, GJ2].

[JR1] C.J. Jayawardene and C.C. Rousseau, An Upper Bound for the Ramsey Number of a Quadrilateral versus a Complete Graph on Seven Vertices, *Congressus Numerantium*, **130** (1998) 175-188.

[JR2] C.J. Jayawardene and C.C. Rousseau, Ramsey Numbers $r(C_6, G)$ for All Graphs G of Order Less than Six, *Congressus Numerantium*, **136** (1999) 147-159.

[JR3] C.J. Jayawardene and C.C. Rousseau, The Ramsey Numbers for a Quadrilateral vs. All Graphs on Six Vertices, *Journal of Combinatorial Mathematics and Combinatorial Computing*, **35** (2000) 71-87.

[JR4] C.J. Jayawardene and C.C. Rousseau, Ramsey Numbers $r(C_5, G)$ for All Graphs G of Order Six, *Ars Combinatoria*, **57** (2000) 163-173.

[JR5] C.J. Jayawardene and C.C. Rousseau, The Ramsey Number for a Cycle of Length Five vs. a Complete Graph of Order Six, *Journal of Graph Theory*, **35** (2000) 99-108.

[-] C.J. Jayawardene, see also [BJYHRZ, RoJa1, RoJa2].

[Jin]** Jin Xia, Ramsey Numbers Involving a Triangle: Theory & Applications, *Technical Report RIT-TR-93-019*, MS thesis, Department of Computer Science, Rochester Institute of Technology, 1993.

[-] Jin Xia, see also [RaJi].

[JGT] Special volume on Ramsey theory of *Journal of Graph Theory*, Volume **7**, Number 1, (1983).

[Ka1] J.G. Kalbfleisch, Construction of Special Edge-Chromatic Graphs, *Canadian Mathematical Bulletin*, **8** (1965) 575-584.

[Ka2]* J.G. Kalbfleisch, Chromatic Graphs and Ramsey's Theorem, *Ph.D. thesis*, University of Waterloo, January 1966.

[Ka3] J.G. Kalbfleisch, On Robillard's Bounds for Ramsey Numbers, *Canadian Mathematical Bulletin*, **14** (1971) 437-440.

[KaSt] J.G. Kalbfleisch and R.G. Stanton, On the Maximal Triangle-Free Edge-Chromatic Graphs in Three Colors, *Journal of Combinatorial Theory*, **5** (1968) 9-20.

[KáRos] G. Károlyi and V. Rosta, Generalized and Geometric Ramsey Numbers for Cycles, *Theoretical Computer Science*, **263** (2001) 87-98.

[Kéry] G. Kéry, On a Theorem of Ramsey (in Hungarian), *Matematikai Lapok*, **15** (1964) 204-224.

[Kim] J.H. Kim, The Ramsey Number $R(3,t)$ has Order of Magnitude $t^2/\log t$, *Random Structures and Algorithms*, **7** (1995) 173-207.

[KM1] K. Klamroth and I. Mengersen, Ramsey Numbers of K_3 versus (p,q)-Graphs, *Ars Combinatoria,* **43** (1996) 107-120.

[KM2] K. Klamroth and I. Mengersen, The Ramsey Number of $r(K_{1,3}, C_4, K_4)$, *Utilitas Mathematica,* **52** (1997) 65-81.

[-] K. Klamroth, see also [AKM].

[-] M. Klawe, see [GHK].

[-] D.J. Kleitman, see [GK].

[Köh] W. Köhler, On a Conjecture by Grossman, *Ars Combinatoria,* **23** (1987) 103-106.

[-] J. Komlós, see [CsKo, AKS].

[KoRö1] A.V. Kostochka and V. Rödl, On Graphs with Small Ramsey Numbers, *Journal of Graph Theory,* **37** (2001) 198-204.

[KoRö2] A.V. Kostochka and V. Rödl, On Graphs with Small Ramsey Numbers, II, to appear in *Combinatorica,* (2004).

[KoSu] A.V. Kostochka and B. Sudakov, On Ramsey Numbers of Sparse Graphs, *Combinatorics, Probability and Computing,* **12** (2003) 627-641.

[-] R.L. Kramer, see [FKR].

[KrRod] I. Krasikov and Y. Roditty, On Some Ramsey Numbers of Unicyclic Graphs, *Bulletin of the Institute of Combinatorics and its Applications,* **33** (2001) 29-34.

[-] S. Krause, see [HaKr].

[KLR]* D.L. Kreher, Li Wei and S.P. Radziszowski, Lower Bounds for Multi-Colored Ramsey Numbers From Group Orbits, *Journal of Combinatorial Mathematics and Combinatorial Computing,* **4** (1988) 87-95.

[-] D.L. Kreher, see also [RK1, RK2, RK3, RK4].

[Kriv] M. Krivelevich, Bounding Ramsey Numbers through Large Deviation Inequalities, *Random Structures and Algorithms,* **7** (1995) 145-155.

[-] M. Krivelevich, see also [AlKS].

[-] M. Kubale, see [DzKu].

[-] P.C.B. Lam, see [ShiuLL].

[La1] S.L. Lawrence, Cycle-Star Ramsey Numbers, *Notices of the American Mathematical Society,* **20** (1973) Abstract A- 420.

[La2] S.L. Lawrence, Bipartite Ramsey Theory, *Notices of the American Mathematical Society,* **20** (1973) Abstract A- 562.

[-] S.L. Lawrence, see also [FLPS].

[LayMa] C. Laywine and J.P. Mayberry, A Simple Construction Giving the Two Non-isomorphic Triangle-Free 3-Colored K_{16}'s, *Journal of Combinatorial Theory,* Series B, **45** (1988) 120-124.

[LaMu] F. Lazebnik and D. Mubayi, New Lower Bounds for Ramsey Numbers of Graphs and Hypergraphs, *Advances in Applied Mathematics,* **28** (2002) 544-559.

[LaWo1] F. Lazebnik and A. Woldar, New Lower Bounds on the Multicolor Ramsey Numbers $r_k(C_4)$, *Journal of Combinatorial Theory,* Series B, **79** (2000) 172-176.

[LaWo2] F. Lazebnik and A. Woldar, General Properties of Some Families of Graphs Defined by Systems of Equations, *Journal of Graph Theory,* **38** (2001) 65-86.

[Lef] H. Lefmann, Ramsey Numbers for Monotone Paths and Cycles, *Ars Combinatoria,* **35** (1993) 271-279.

[-] H. Lefmann, see also [DLR].

[-] J. Lehel, see [BaLS].

[Les]* A. Lesser, Theoretical and Computational Aspects of Ramsey Theory, *Examensarbeten i Matematik*, Matematiska Institutionen, Stockholms Universitet, **3** (2001).

[LiWa1] Li Da Yong and Wang Zhi Jian, The Ramsey Number $r(mC_4, nC_4)$ (in Chinese), *Journal of Shanghai Tiedao University*, **20** (1999) 66-70, 83.

[Liwa2] Li Da Yong and Wang Zhi Jian, The Ramsey Numbers $r(mC_4, nC_5)$, *Journal of Combinatorial Mathematics and Combinatorial Computing*, **45** (2003) 245-252.

[-] Li Guiqing, see [LSZL, SLLL, SLL1, SLZL].

[-] Li Jinwen, see [ZLLS].

[-] Li Qiao, see [SLLL, SLL2, SLL3].

[-] Li Wei, see [KLR].

[Li] Li Yusheng, Some Ramsey Numbers of Graphs with Bridge, *Journal of Combinatorial Mathematics and Combinatorial Computing*, **25** (1997) 225-229.

[LR1] Li Yusheng and C.C. Rousseau, On Book-Complete Graph Ramsey Numbers, *Journal of Combinatorial Theory*, Series B, **68** (1996) 36-44.

[LR2] Li Yusheng and C.C. Rousseau, Fan-Complete Graph Ramsey Numbers, *Journal of Graph Theory*, **23** (1996) 413-420.

[LR3] Li Yusheng and C.C. Rousseau, On the Ramsey Number $r(H + \overline{K}_n, K_n)$, *Discrete Mathematics*, **170** (1997) 265-267.

[LR4] Li Yusheng and C.C. Rousseau, A Ramsey Goodness Result for Graphs with Many Pendant Edges, *Ars Combinatoria*, **49** (1998) 315-318.

[LRS] Li Yusheng, C.C. Rousseau and L. Soltés, Ramsey Linear Families and Generalized Subdivided Graphs, *Discrete Mathematics*, **170** (1997) 269-275.

[LRZ] Li Yusheng, C.C. Rousseau and Zang Wenan, Asymptotic Upper Bounds for Ramsey Functions, *Graphs and Combinatorics*, **17** (2001) 123-128.

[LZ] Li Yusheng and Zang Wenan, Ramsey Numbers Involving Large Dense Graphs and Bipartite Turán Numbers, *Journal of Combinatorial Theory*, Series B, **87** (2003) 280-288.

[-] Li Yusheng, see also [CLRZ, ShiuLL, ZaLi].

[-] Li Zhenchong, see [LSL].

[Lind] B. Lindström, Undecided Ramsey-Numbers for Paths, *Discrete Mathematics*, **43** (1983) 111-112.

[Ling] A.C.H. Ling, Some Applications of Combinatorial Designs to Extremal Graph Theory, *Ars Combinatoria*, **67** (2003) 221-229.

[-] Andy Liu, see [AbbL].

[-] Liu Linzhong, see [ZLLS].

[-] Liu Yanwu, see [SYL].

[Loc] S.C. Locke, Bipartite Density and the Independence Ratio, *Journal of Graph Theory*, **10** (1986) 47-53.

[-] S.C. Locke, see also [FrLo].

[Lor] P.J. Lorimer, The Ramsey Numbers for Stripes and One Complete Graph, *Journal of Graph Theory*, **8** (1984) 177-184.

[LorMu] P.J. Lorimer and P.R. Mullins, Ramsey Numbers for Quadrangles and Triangles, *Journal of Combinatorial Theory*, Series B, **23** (1977) 262-265.

[LorSe] P.J. Lorimer and R.J. Segedin, Ramsey Numbers for Multiple Copies of Complete Graphs, *Journal of Graph Theory*, **2** (1978) 89-91.

[LorSo] P.J. Lorimer and W. Solomon, The Ramsey Numbers for Stripes and Complete Graphs 1, *Discrete Mathematics*, **104** (1992) 91-97. Corrigendum in *Discrete Mathematics*, **131** (1994) 395.

[-] P.J. Lorimer, see also [CL1, CL2].

[LorMe1] R. Lortz and I. Mengersen, On the Ramsey Numbers $r(K_{2,n-1}, K_{2,n})$ and $r(K_{2,n}, K_{2,n})$, *Utilitas Mathematica*, **61** (2002) 87-95.

[LorMe2] R. Lortz and I. Mengersen, Bounds on Ramsey Numbers of Certain Complete Bipartite Graphs, *Results in Mathematics*, **41** (2002) 140-149.

[LorMe3]* R. Lortz and I. Mengersen, Off-Diagonal and Asymptotic Results on the Ramsey Number $r(K_{2,m}, K_{2,n})$, *Journal of Graph Theory*, **43** (2003) 252-268.

[Łuc] T. Łuczak, $R(C_n, C_n, C_n) \le (4+o(1))n$, *Journal of Combinatorial Theory*, Series B, **75** (1999) 174-187.

[-] T. Łuczak, see also [HaŁT].

[LSL]* Luo Haipeng, Su Wenlong and Li Zhenchong, The Properties of Self-Complementary Graphs and New Lower Bounds for Diagonal Ramsey Numbers, *Australasian Journal of Combinatorics*, **25** (2002) 103-116.

[LSS]* Luo Haipeng, Su Wenlong and Shen Yun-Qiu, New Lower Bounds of Ten Classical Ramsey Numbers, *Australasian Journal of Combinatorics*, **24** (2001) 81-90.

[LSZL]* Luo Haipeng, Su Wenlong, Zhang Zhengyou and Li Guiqing, New Lower Bounds for Twelve Classical 2-Color Ramsey Numbers $R(k,l)$ (in Chinese), *Guangxi Sciences*, **7**, 2 (2000) 120-121.

[-] Luo Haipeng, see also [SL, SLLL, SLL1, SLL2, SLL3, SLZL, ZSL].

[Mac]* J. Mackey, Combinatorial Remedies, *Ph.D. Thesis*, Department of Mathematics, University of Hawaii, 1994.

[Mat]* R. Mathon, Lower Bounds for Ramsey Numbers and Association Schemes, *Journal of Combinatorial Theory*, Series B, **42** (1987) 122-127.

[-] J.P. Mayberry, see [LayMa].

[McS] C. McDiarmid and A. Steger, Tidier Examples for Lower Bounds on Diagonal Ramsey Numbers, *Journal of Combinatorial Theory*, Series A, **74** (1996) 147-152.

[McK]** B.D. McKay, Australian National University, *personal communication* (2003). Graphs available at http://cs.anu.edu.au/people/bdm/data/ramsey.html.

[MPR]** B.D. McKay, K. Piwakowski and S.P. Radziszowski, Ramsey Numbers for Triangles versus Almost-Complete Graphs, to appear in *Ars Combinatoria*, (2004).

[MR1]** B.D. McKay and S.P. Radziszowski, The First Classical Ramsey Number for Hypergraphs is Computed, *Proceedings of the Second Annual ACM-SIAM Symposium on Discrete Algorithms*, SODA'91, San Francisco, (1991) 304-308.

[MR2]* B.D. McKay and S.P. Radziszowski, A New Upper Bound for the Ramsey Number $R(5,5)$, *Australasian Journal of Combinatorics*, **5** (1992) 13-20.

[MR3]** B.D. McKay and S.P. Radziszowski, Linear Programming in Some Ramsey Problems, *Journal of Combinatorial Theory*, Series B, **61** (1994) 125-132.

[MR4]** B.D. McKay and S.P. Radziszowski, $R(4,5) = 25$, *Journal of Graph Theory*, **19** (1995) 309-322.

[MR5]** B.D. McKay and S.P. Radziszowski, Subgraph Counting Identities and Ramsey Numbers, *Journal of Combinatorial Theory*, Series B, **69** (1997) 193-209.

[MZ]** B.D. McKay and Zhang Ke Min, The Value of the Ramsey Number $R(3,8)$, *Journal of Graph Theory*, **16** (1992) 99-105.

[-] B.D. McKay, see also [FM].

[McN]** J. McNamara, SUNY Brockport, *personal communication* (1995).

[McR]** J. McNamara and S.P. Radziszowski, The Ramsey Numbers $R(K_4-e, K_6-e)$ and $R(K_4-e, K_7-e)$, *Congressus Numerantium*, **81** (1991) 89-96.

[MO] I. Mengersen and J. Oeckermann, Matching-Star Ramsey Sets, *Discrete Applied Mathematics,* **95** (1999) 417-424.

[-] I. Mengersen, see also [AKM, CEHMS, EHM1, EHM2, HoMe, HaMe1, HaMe2, HaMe3, HaMe4, KM1, KM2, LorMe1, LorMe2, LorMe3].

[-] M. Miller, see [BSNM].

[MS] H. Mizuno and I. Sato, Ramsey Numbers for Unions of Some Cycles, *Discrete Mathematics,* **69** (1988) 283-294.

[-] E.L. Monte Carmelo, see [GoMC].

[-] D. Mubayi, see [AFM, LaMu].

[-] P.R. Mullins, see [LorMu].

[-] S.M. Nababan, see [BSNM].

[Neš] J. Nešetřil, Ramsey Theory, chapter 25 in *Handbook of Combinatorics,* ed. R.L. Graham, M. Grötschel and L. Lovász, The MIT-Press, Vol. II, 1996, 1331-1403.

[Nik] V. Nikiforov, The Cycle-Complete Graph Ramsey Numbers, *to appear*.

[NiRo1] V. Nikiforov and C.C. Rousseau, A Note on Ramsey Numbers for Books, *manuscript,* (2003).

[NiRo2] V. Nikiforov and C.C. Rousseau, Book Ramsey Numbers I, *manuscript,* (2003).

[NiRo3] V. Nikiforov and C.C. Rousseau, Large Generalized Books Are p-Good, to appear in the *Journal of Combinatorial Theory,* Series B, (2004).

[NiRS] V. Nikiforov, C.C. Rousseau and R.H. Schelp, Book Ramsey Numbers and Quasi-Randomness, *manuscript,* (2004).

[-] J. Oeckermann, see [MO].

[Par1] T.D. Parsons, The Ramsey Numbers $r(P_m, K_n)$, *Discrete Mathematics,* **6** (1973) 159-162.

[Par2] T.D. Parsons, Path-Star Ramsey Numbers, *Journal of Combinatorial Theory,* Series B, **17** (1974) 51-58.

[Par3] T.D. Parsons, Ramsey Graphs and Block Designs, I, *Transactions of the American Mathematical Society,* **209** (1975) 33-44.

[Par4] T.D. Parsons, Ramsey Graphs and Block Designs, *Journal of Combinatorial Theory,* Series A, **20** (1976) 12-19.

[Par5] T.D. Parsons, Graphs from Projective Planes, *Aequationes Mathematica,* **14** (1976) 167-189.

[Par6] T.D. Parsons, Ramsey Graph Theory, in *Selected Topics in Graph Theory,* (L.W. Beineke and R.J. Wilson eds.), Academic Press, (1978) 361-384.

[-] T.D. Parsons, see also [FLPS].

[Piw1]* K. Piwakowski, Applying Tabu Search to Determine New Ramsey Graphs, *Electronic Journal of Combinatorics,* http://www.combinatorics.org/, #R6, **3** (1996), 4 pages.

[Piw2]** K. Piwakowski, A New Upper Bound for $R_3(K_4-e)$, *Congressus Numerantium,* **128** (1997) 135-141.

[PR1]** K. Piwakowski and S.P. Radziszowski, $30 \le R(3,3,4) \le 31$, *Journal of Combinatorial Mathematics and Combinatorial Computing,* **27** (1998) 135-141.

[PR2]** K. Piwakowski and S.P. Radziszowski, Towards the Exact Value of the Ramsey Number $R(3,3,4)$, *Congressus Numerantium,* **148** (2001) 161-167.

[-] K. Piwakowski, see also [MPR].

[-] A.D. Polimeni, see [CGP, CRSPS].

[-] L.M. Pretorius, see [SwPr].

[-] P. Pudlák, see [AlPu, CPR].

[Ra1]** S.P. Radziszowski, The Ramsey Numbers $R(K_3, K_8-e)$ and $R(K_3, K_9-e)$, *Journal of Combinatorial Mathematics and Combinatorial Computing,* **8** (1990) 137-145.

[Ra2] S.P. Radziszowski, Small Ramsey Numbers, *Technical Report RIT-TR-93-009*, Department of Computer Science, Rochester Institute of Technology (1993).

[Ra3]** S.P. Radziszowski, On the Ramsey Number $R(K_5-e, K_5-e)$, *Ars Combinatoria*, **36** (1993) 225-232.

[RaJi] S.P. Radziszowski and Jin Xia, Paths, Cycles and Wheels in Graphs without Antitriangles, *Australasian Journal of Combinatorics*, **9** (1994) 221-232.

[RK1]* S.P. Radziszowski and D.L. Kreher, Search Algorithm for Ramsey Graphs by Union of Group Orbits, *Journal of Graph Theory*, **12** (1988) 59-72.

[RK2]** S.P. Radziszowski and D.L. Kreher, Upper Bounds for Some Ramsey Numbers $R(3,k)$, *Journal of Combinatorial Mathematics and Combinatorial Computing*, **4** (1988) 207-212.

[RK3]** S.P. Radziszowski and D.L. Kreher, On $R(3,k)$ Ramsey Graphs: Theoretical and Computational Results, *Journal of Combinatorial Mathematics and Combinatorial Computing*, **4** (1988) 37-52.

[RK4] S.P. Radziszowski and D.L. Kreher, Minimum Triangle-Free Graphs, *Ars Combinatoria*, **31** (1991) 65-92.

[RT]* S.P. Radziszowski and Kung-Kuen Tse, A Computational Approach for the Ramsey Numbers $R(C_4, K_n)$, *Journal of Combinatorial Mathematics and Combinatorial Computing*, **42** (2002) 195-207.

[RST]* S.P. Radziszowski, J. Stinehour and Kung-Kuen Tse, Computation of the Ramsey Number $R(W_5, K_5)$, *in preparation.*

[-] S.P. Radziszowski, see also [BaRT, FKR, KLR, MPR, MR1, MR2, MR3, MR4, MR5, McR, PR1, PR2, XXER, XXR].

[Ram] F.P. Ramsey, On a Problem of Formal Logic, *Proceedings of the London Mathematical Society*, **30** (1930) 264-286.

[Rao]* S. Rao, Applying a Genetic Algorithm to Improve the Lower Bounds of Multi-Color Ramsey Numbers, *MS thesis*, Department of Computer Science, Rochester Institute of Technology, 1997.

[-] G. Resta, see [CPR].

[-] S.W. Reyner, see [BR].

[-] D.F. Reynolds, see [ExRe].

[Rob1] F.S. Roberts, *Applied Combinatorics*, Prentice-Hall, Englewood Cliffs, 1984.

[-] J.A. Roberts, see [BuRo1, BuRo2].

[-] S. Roberts, see [GR].

[Rob2]* A. Robertson, New Lower Bounds for Some Multicolored Ramsey Numbers, *Electronic Journal of Combinatorics*, http://www.combinatorics.org/, #R12, **6** (1999), 6 pages.

[Rob3]* A. Robertson, Difference Ramsey Numbers and Issai Numbers, *Advances in Applied Mathematics*, **25** (2000) 153-162.

[Rob4] A. Robertson, New Lower Bounds Formulas for Multicolored Ramsey Numbers, *Electronic Journal of Combinatorics*, http://www.combinatorics.org/, #R13, **9** (2002), 6 pages.

[-] Y. Roditty, see [KrRod].

[RöTh] V. Rödl and R. Thomas, Arrangeability and Clique Subdivisions, in *The Mathematics of Paul Erdös II*, 236-239, Algorithms and Combinatorics **14**, Springer, Berlin, 1997.

[-] V. Rödl, see also [AlRö, CRST, DLR, GrRö, GRR1, GRR2, KoRö1, KoRö2].

[Ros] V. Rosta, On a Ramsey Type Problem of J.A. Bondy and P. Erdös, I & II, *Journal of Combinatorial Theory*, Series B, **15** (1973) 94-120.

[-] V. Rosta, see also [KR].

[-] B.L. Rothschild, see [GRS].

[RoJa1] C.C. Rousseau and C.J. Jayawardene, The Ramsey Number for a Quadrilateral vs. a Complete Graph on Six Vertices, *Congressus Numerantium*, **123** (1997) 97-108.

[RoJa2] C.C. Rousseau and C.J. Jayawardene, Harary's Problem for $K_{2,k}$, *unpublished manuscript*, (1999).

[RS1] C.C. Rousseau and J. Sheehan, On Ramsey Numbers for Books, *Journal of Graph Theory*, **2** (1978) 77-87.

[RS2] C.C. Rousseau and J. Sheehan, A Class of Ramsey Problems Involving Trees, *Journal of the London Mathematical Society (2)*, **18** (1978) 392-396.

[-] C.C. Rousseau, see also [BJYHRZ, BEFRS1, BEFRS2, BEFRS3, BEFRS4, BEFRS5, BEFRSGJ, CLRZ, CRSPS, EFRS1, EFRS2, EFRS3, EFRS4, EFRS5, EFRS6, EFRS7, EFRS8, EFRS9, FRS1, FRS2, FRS3, FRS4, FRS5, FRS6, FRS7, FRS8, FSR, JR1, JR2, JR3, JR4, JR5, LR1, LR2, LR3, LR4, LRS, LRZ, NiRo1, NiRo2, NiRo3, NiRS].

[-] P. Rowlinson, see [YR1, YR2, YR3].

[-] A. Ruciński, see [GRR1, GRR2].

[San] A. Sánchez-Flores, An Improved Bound for Ramsey Number $N(3,3,3,3;2)$, *Discrete Mathematics*, **140** (1995) 281-286.

[-] I. Sato, see [MS].

[-] R.H. Schelp, see [BaLS, BaSS, BEFRS1, BEFRS2, BEFRS3, BEFRS4, BEFRS5, BEFRSGJ, BEFS, ChenS, EFRS1, EFRS2, EFRS3, EFRS4, EFRS5, EFRS6, EFRS7, EFRS8, EFRS9, FLPS, FRS1, FRS2, FRS3, FRS4, FRS5, FS1, FS2, FS3, FS4, FSR, FSS1, NiRS].

[-] J. Schönheim, see [BS].

[SchSch1]* A. Schelten and I. Schiermeyer, Ramsey Numbers $r(K_3, G)$ for Connected Graphs G of Order Seven, *Discrete Applied Mathematics*, **79** (1997) 189-200.

[SchSch2] A. Schelten and I. Schiermeyer, Ramsey Numbers $r(K_3, G)$ for $G \cong K_7 - 2P_2$ and $G \cong K_7 - 3P_2$, *Discrete Mathematics*, **191** (1998) 191-196.

[-] A. Schelten, see also [FSS2].

[Schi1] I. Schiermeyer, All Cycle-Complete Graph Ramsey Numbers $r(C_m, K_6)$, *Journal of Graph Theory*, **44** (2003) 251-260.

[Schi2] I. Schiermeyer, The Cycle-Complete Graph Ramsey Number $r(C_5, K_7)$, *manuscript*, (2003).

[-] I. Schiermeyer, see also [FSS2, SchSch1, SchSch2].

[Schu] C. -U. Schulte, Ramsey-Zahlen für Bäume und Kreise, *Ph.D. thesis*, Heinrich-Heine-Universität Düsseldorf, (1992).

[-] S. Schuster, see[CS].

[-] A. Schwenk, see [ChvS].

[Scob] M.W. Scobee, On the Ramsey Number $R(m_1P_3, m_2P_3, m_3P_3)$ and Related Results, ..., *MA thesis*, University of Louisville (1993).

[-] R.J. Segedin, see [LorSe].

[Sha] A. Shastri, Lower Bounds for Bi-Colored Quaternary Ramsey Numbers, *Discrete Mathematics*, **84** (1990) 213-216.

[She1]* J.B. Shearer, Lower Bounds for Small Diagonal Ramsey Numbers, *Journal of Combinatorial Theory*, Series A, **42** (1986) 302-304.

[She2] J.B. Shearer, A Note on the Independence Number of Triangle-free Graphs II, *Journal of Combinatorial Theory*, Series B, **53** (1991) 300-307.

[-] J. Sheehan, see [CRSPS, CEHMS, FRS6, FRS7, FRS8, RS1, RS2].

[-] Shen Yun-Qiu, see [LSS].

[Shi1] Shi Ling Sheng, Cube Ramsey Numbers Are Polynomial, *Random Structures & Algorithms*, **19** (2001) 99--101.

[Shi2] Shi Ling Sheng, Upper Bounds for Ramsey Numbers, *Discrete Mathematics*, **270** (2003) 251-265.

[ShZ1] Shi Ling Sheng and Zhang Ke Min, An Upper Bound Formula for Ramsey Numbers, *manuscript*, (2001).

[ShZ2] Shi Ling Sheng and Zhang Ke Min, A Sequence of Formulas for Ramsey Numbers, *manuscript*, (2001).

[ShiuLL] Shiu Wai Chee, Peter Che Bor Lam and Li Yusheng, On Some Three-Color Ramsey Numbers, *Graphs and Combinatorics*, **19** (2003) 249-258.

[Sid1] A.F. Sidorenko, On Turán Numbers $T(n,5,4)$ and Number of Monochromatic 4-cliques in 2-colored 3-graphs (in Russian), *Voprosy Kibernetiki*, **64** (1980) 117-124.

[Sid2] A.F. Sidorenko, An Upper Bound on the Ramsey Number $R(K_3,G)$ Depending Only on the Size of the Graph G, *Journal of Graph Theory*, **15** (1991) 15-17.

[Sid3] A.F. Sidorenko, The Ramsey Number of an N-Edge Graph Versus Triangle Is at Most $2N+1$, *Journal of Combinatorial Theory*, Series B, **58** (1993) 185-196.

[-] M. Simonovits, see [BaSS, FSS1, FS].

[-] M.J. Smuga-Otto, see [AbbS].

[Sob] A. Sobczyk, Euclidian Simplices and the Ramsey Number $R(4,4;3)$, *Technical Report #10, Clemson University* (1967).

[-] W. Solomon, see [LorSo].

[-] L. Soltés, see [LRS].

[Song1] Song En Min, Study of Some Ramsey Numbers (in Chinese), a note (announcement of results without proofs), *Mathematica Applicata*, **4**(2) (1991) 6.

[Song2] Song En Min, New Lower Bound Formulas for the Ramsey Numbers $N(k,k,...,k;2)$ (in Chinese), *Mathematica Applicata*, **6** (1993) suppl., 113-116.

[Song3] Song En Min, An Investigation of Properties of Ramsey Numbers (in Chinese), *Mathematica Applicata*, **7** (1994) 216-221.

[Song4] Song En Min, Properties and New Lower Bounds of the Ramsey Numbers $R(p,q;4)$ (in Chinese), *Journal of Huazhong University of Science and Technology*, **23** (1995) suppl. II, 1-4.

[SYL] Song En Min, Ye Weiguo and Liu Yanwu, New Lower Bounds for Ramsey Number $R(p,q;4)$, *Discrete Mathematics*, **145** (1995) 343-346.

[-] Song En Min, see also [HuSo, ZLLS].

[Spe1] J.H. Spencer, Ramsey's Theorem - A New Lower Bound, *Journal of Combinatorial Theory*, Series A, **18** (1975) 108-115.

[Spe2] J.H. Spencer, Asymptotic Lower Bounds for Ramsey Functions, *Discrete Mathematics*, **20** (1977/1978) 69-76.

[-] J.H. Spencer, see also [BES, GRS].

[Spe3]* T. Spencer, University of Nebraska at Omaha, *personal communication* (1993), and, Upper Bounds for Ramsey Numbers via Linear Programming, *preprint*, (1994).

[Stahl] S. Stahl, On the Ramsey Number $R(F,K_m)$ where F is a Forest, *Canadian Journal of Mathematics*, **27** (1975) 585-589.

[-] R.G. Stanton, see [KaSt].

[Stat] W. Staton, Some Ramsey-type Numbers and the Independence Ratio, *Transactions of the American Mathematical Society*, **256** (1979) 353-370.

[-] A. Steger, see [McS].

[-] J. Stinehour, see [RST].

[Stev] S. Stevens, Ramsey Numbers for Stars Versus Complete Multipartite Graphs, *Congressus Numerantium*, **73** (1990) 63-71.

[-] M.J. Stewart, see [CRSPS].

[Stone] J.C. Stone, Utilizing a Cancellation Algorithm to Improve the Bounds of $R(5,5)$, (1996), http://oas.okstate.edu/ojas/jstone.htm. This paper claims incorrectly that $R(5,5)=50$.

[SL]* Su Wenlong and Luo Haipeng, Prime Order Cyclic Graphs and New Lower Bounds for Three Classical Ramsey Numbers $R(4,n)$ (in Chinese), *Journal of Mathematical Study*, **31**, 4 (1998) 442-446.

[SLLL]* Su Wenlong, Luo Haipeng, Li Guiqing and Li Qiao, Lower Bounds of Ramsey Numbers Based on Cubic Residues, *Discrete Mathematics*, **250** (2002) 197-209.

[SLL1]* Su Wenlong, Luo Haipeng and Li Guiqing, Two Lower Bounds of Classical 2-color Ramsey Numbers $R(3,q)$ (in Chinese), *Journal of Guangxi University for Nationalities*, **5**, 1 (1999) 1-4.

[SLL2]* Su Wenlong, Luo Haipeng and Li Qiao, New Lower Bounds of Classical Ramsey Numbers $R(4,12)$, $R(5,11)$ and $R(5,12)$, *Chinese Science Bulletin*, **43**, 6 (1998) 528.

[SLL3]* Su Wenlong, Luo Haipeng and Li Qiao, New Lower Bounds for Seven Classical Ramsey Numbers $R(k,l)$ (in Chinese), *Journal of Systems Science and Mathematical Sciences*, **20**, 1 (2000) 55-57.

[SLZL]* Su Wenlong, Luo Haipeng, Zhang Zhengyou and Li Guiqing, New Lower Bounds of Fifteen Classical Ramsey Numbers, *Australasian Journal of Combinatorics*, **19** (1999) 91-99.

[-] Su Wenlong, see also [LSL, LSS, LSZL, ZSL].

[Sud1] B. Sudakov, A Note on Odd Cycle-Complete Graph Ramsey Numbers, *Electronic Journal of Combinatorics*, http://www.combinatorics.org/, #N1, **9** (2002), 4 pages.

[Sud2] B. Sudakov, Large K_r-free Subgraphs in K_s-free Graphs and Some Other Ramsey-Type Problems, *preprint.*

[-] B. Sudakov, see also [AlKS, KoSu].

[-] A. Sudan, see [GGS].

[SuBa1] Surahmat and E.T. Baskoro, On the Ramsey Number of a Path or a Star versus W_4 or W_5, *Proceedings of the 12-th Australasian Workshop on Combinatorial Algorithms*, Bandung, Indonesia, July 14-17 (2001) 174-179.

[SuBa2] Surahmat and E.T. Baskoro, The Ramsey Number of Linear Forest versus Wheel, paper presented at the *13-th Australasian Workshop on Combinatorial Algorithms*, Fraser Island, Queensland, Australia, July 7-10, 2002.

[SuBB1] Surahmat, E.T. Baskoro and H.J. Broersma, The Ramsey Numbers of Large Star-like Trees versus Large Odd Wheels, *Technical Report* #1621, Faculty of Mathematical Sciences, University of Twente, The Netherlands, (2002).

[SuBB2] Surahmat, E.T. Baskoro and H.J. Broersma, The Ramsey Numbers of Large Cycles versus Small Wheels, *Technical Report* #1634, Faculty of Mathematical Sciences, University of Twente, The Netherlands, (2002).

[-] Surahmat, see also [BaSu, BSNM].

[SwPr] C.J. Swanepoel and L.M. Pretorius, Upper Bounds for a Ramsey Theorem for Trees, *Graphs and Combinatorics*, **10** (1994) 377-382.

[-] M.M. Sweet, see [FreSw].

[-] E. Szemerédi, see [AKS, CRST].

[-] R. Thomas, see [RöTh].

[Tho] A. Thomason, An Upper Bound for Some Ramsey Numbers, *Journal of Graph Theory*, **12** (1988) 509-517.

[-] P.W. Tingley, see [HaŁT].

[-] C.A. Tovey, see [CET].

[Tr] Trivial results.

[-] W.T. Trotter Jr., see [CRST].

[Tse1]* Kung-Kuen Tse, On the Ramsey Number of the Quadrilateral versus the Book and the Wheel, *Australasian Journal of Combinatorics*, **27** (2003) 163-167.

[Tse2]* Kung-Kuen Tse, Kean University, *personal communication,* (2004).

[-] Kung-Kuen Tse, see also [BaRT, RST, RT].

[-] Z. Tuza, see [GT].

[-] L. Volkmann, see [GV].

[Walk] K. Walker, Dichromatic Graphs and Ramsey Numbers, *Journal of Combinatorial Theory,* **5** (1968) 238-243.

[Wall] W.D. Wallis, On a Ramsey Number for Paths, *Journal of Combinatorics, Information & System Sciences,* **6** (1981) 295-296.

[Wan] Wan Honghui, Upper Bounds for Ramsey Numbers $R(3,3,\cdots,3)$ and Schur Numbers, *Journal of Graph Theory,* **26** (1997) 119-122.

[-] Wang Gongben, see [WW, WWY1, WWY2].

[WW]* Wang Qingxian and Wang Gongben, New Lower Bounds of Ramsey Numbers $r(3,q)$ (in Chinese), *Acta Scientiarum Naturalium, Universitatis Pekinensis,* **25** (1989) 117-121.

[WWY1]* Wang Qingxian, Wang Gongben and Yan Shuda, A Search Algorithm And New Lower Bounds for Ramsey Numbers $r(3,q)$, *preprint,* (1994).

[WWY2]* Wang Qingxian, Wang Gongben and Yan Shuda, The Ramsey Numbers $R(K_3,K_q-e)$ (in Chinese), *Beijing Daxue Xuebao Ziran Kexue Ban,* **34** (1998) 15-20.

[-] Wang Zhi Jian, see [LiWa1, LiWa2].

[Wh] E.G. Whitehead, The Ramsey Number $N(3,3,3,3;2)$, *Discrete Mathematics,* **4** (1973) 389-396.

[-] E.R. Williams, see [AbbW].

[-] R.M. Wilson, see [FraWi].

[-] A. Woldar, see [LaWo1, LaWo2].

[XZ]* Xie Jiguo and Zhang Xiaoxian, A New Lower Bound for Ramsey Number $r(3,13)$ (in Chinese), *Journal of Lanzhou Railway Institute,* **12** (1993) 87-89.

[-] Xie Zheng, see [XX1, XX2, XXER, XXR].

[Xu] Xu Xiaodong, *personal communication,* (2004).

[XX1]* Xu Xiaodong and Xie Zheng, A Constructive Approach for the Lower Bounds on the Ramsey Numbers $r(k,l)$, *manuscript,* (2002).

[XX2] Xu Xiaodong and Xie Zheng, A Constructive Approach for the Lower Bounds on Multicolor Ramsey Numbers, *manuscript,* (2002).

[XXER]* Xu Xiaodong, Xie Zheng, G. Exoo and S.P. Radziszowski, Constructive Lower Bounds on Classical Multicolor Ramsey Numbers, *Electronic Journal of Combinatorics,* http://www.combinatorics.org/, #R35, **11** (2004), 24 pages.

[XXR] Xu Xiaodong, Xie Zheng and S.P. Radziszowski, A Constructive Approach for the Lower Bounds on the Ramsey Numbers $R(s,t)$, to appear in the *Journal of Graph Theory,* (2004).

[-] J. Yackel, see [GY].

[-] Yan Shuda, see [WWY1, WWY2].

[YHZ1] Yang Jian Sheng, Huang Yi Ru and Zhang Ke Min, The Value of the Ramsey Number $R(C_n,K_4)$ is $3(n-1)+1$ $(n\geq 4)$, *Australasian Journal of Combinatorics,* **20** (1999) 205-206.

[YHZ2] Yang Jian Sheng, Huang Yi Ru and Zhang Ke Min, $R(C_6,K_5)=21$ and $R(C_7,K_5)=25$, *European Journal of Combinatorics,* **22** (2001) 561-567.

[-] Yang Jian Sheng, see also [BJYHRZ].

[YY]** Yang Yuansheng, On the Third Ramsey Numbers of Graphs with Six Edges, *Journal of Combinatorial Mathematics and Combinatorial Computing,* **17** (1995) 199-208.

[YH]* Yang Yuansheng and G.R.T. Hendry, The Ramsey Number $r(K_1+C_4,K_5-e)$, *Journal of Graph Theory,* **19** (1995) 13-15.

[YR1]** Yang Yuansheng and P. Rowlinson, On the Third Ramsey Numbers of Graphs with Five Edges, *Journal of Combinatorial Mathematics and Combinatorial Computing*, **11** (1992) 213-222.

[YR2]* Yang Yuansheng and P. Rowlinson, On Graphs without 6-Cycles and Related Ramsey Numbers, *Utilitas Mathematica*, **44** (1993) 192-196.

[YR3]* Yang Yuansheng and P. Rowlinson, The Third Ramsey Numbers for Graphs with at Most Four Edges, *Discrete Mathematics*, **125** (1994) 399-406.

[-] Ye Weiguo, see [SYL].

[Yu1]* Yu Song Nian, A Computer Assisted Number Theoretical Construction of $(3,k)$-Ramsey Graphs, *Annales Universitatis Scientiarum Budapestinensis, Sect. Comput.*, **10** (1989) 35-44.

[Yu2]* Yu Song Nian, Maximal Triangle-free Circulant Graphs and the Function $K(c)$ (in Chinese), *Journal of Shanghai University, Natural Science*, **2** (1996) 678-682.

[Zali] Zang Wenan and Li Yusheng, The Independence Number of Graphs with a Forbidden Cycle and Ramsey Numbers, *Journal of Combinatorial Optimization*, **7** (2003) 353-359.

[-] Zang Wenan, see [LRZ, LZ].

[Zeng] Zeng Wei Bin, Ramsey Numbers for Triangles and Graphs of Order Four with No Isolated Vertex, *Journal of Mathematical Research & Exposition*, **6** (1986) 27-32.

[ZZ] Zhang Ke Min and Zhang Shu Sheng, Some Tree-Stars Ramsey Numbers, *Proceedings of the Second Asian Mathematical Conference* 1995, 287-291, World Sci. Publishing, River Edge, NJ, 1998.

[-] Zhang Ke Min, see also [BJYHRZ, ChenZZ1, ChenZZ2, ChenZZ3, HZ1, HZ2, MZ, ShZ1, ShZ2, YHZ1, YHZ2].

[-] Zhang Shu Sheng, see [ZZ].

[-] Zhang Xiaoxian, see [XZ].

[-] Zhang Yuming, see [CLRZ].

[-] Zhang Yunqing, see [ChenZZ1, ChenZZ2, ChenZZ3].

[ZSL] Zhang Zhengyou, Su Wenlong and Luo Haipeng, Edge Coloration of Complete Graph K_{97} and New Lower Bounds of Four Ramsey Numbers (in Chinese), *Application Research of Computers*, **18**, 6 (2001) 29-31, 57.

[-] Zhang Zhengyou, see also [LSZL, SLZL].

[ZLLS] Zhang Zhongfu, Liu Linzhong, Li Jinwen and Song En Min, Some Properties of Ramsey Numbers, *Applied Mathematics Letters*, **16** (2003) 1187-1193.

[Zhou1] Zhou Huai Lu, Some Ramsey Numbers for Graphs with Cycles (in Chinese), *Mathematica Applicata*, **6** (1993) 218.

[Zhou2] Zhou Huai Lu, The Ramsey Number of an Odd Cycle with Respect to a Wheel (in Chinese), *Journal of Mathematics, Shuxue Zazhi* (Wuhan), **15** (1995) 119-120.

[Zhou3] Zhou Huai Lu, On Book-Wheel Ramsey Number, *Discrete Mathematics*, **224** (2000) 239-249.

Out of 403 references gathered above, 327 appeared in 58 different periodicals, among which most articles were published in: *Discrete Mathematics* 49, *Journal of Combinatorial Theory* (old, Series A and B) 42, *Journal of Graph Theory* 41, and *Ars Combinatoria* 19. The results of 93 references depend on computer algorithms.