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Erratum: A note on Kesten's Choquet-Deny lemma

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Abstract

There is an error in Proposition 3.1 in ECP volume 18 paper 65 (2013): Condition (C) does not imply that the set $\Lambda(\Gamma)$ generates a dense subgroup of $\mathbb{R}$. This has to be made an assumption. Alternatively, one can assume that the matrices are invertible.

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Proposition 3.1 in [4], which was quoted from an earlier version ([1]) of [2], does not hold true. In fact, consider the matrices

$$\mathbf{a} := \begin{pmatrix} 3 & 3 \\ 4 & 4 \end{pmatrix} \text{ and } \mathbf{b} := \begin{pmatrix} 4 & 4 \\ 3 & 3 \end{pmatrix},$$

having dominant eigenvalue $\lambda_{\mathbf{a}} = \lambda_{\mathbf{b}} = 7$ and corresponding eigenvectors

$$w_{\mathbf{a}} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \text{ resp. } w_{\mathbf{b}} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}.$$

The measure $\mu = \frac{1}{2}\delta_{\mathbf{a}} + \frac{1}{2}\delta_{\mathbf{b}}$ satisfies Condition (C) of Definition 2.1, but

$$\Lambda(\Gamma) = \{\log \lambda_{\mathbf{a}} \,:\, \mathbf{a} \in [\operatorname{supp} \mu] \cap \operatorname{int}(\mathcal{M}_+)\} = \{\log 7\},$$

thus the first assertion of Proposition 3.1, i.e. that $\Lambda(\Gamma)$ generates a dense subgroup of $\mathbb{R}$, does not hold.

But this last assertion and the derived aperiodicity is crucial for the main result, Theorem 2.2. in [4] to hold: In the example described above, the function

$$L(x, s) := \sin\left(\frac{2\pi}{\log 7}\, s\right)$$

(not depending on x) satisfies assumptions (a) and (b) of Theorem 2.2 in [4], but it is not constant.

Therefore, aperiodicity has to be assumed, while it is not neccessary to assume that there is no invariant subspace. Namely, the correct definition of condition (C) is as follows.

Definition 0.1 (Replacing Definition 2.1 in [4])**.** *A subsemigroup $\Gamma \subset \mathcal{M}_+$ is said to satisfy condition (C), if*

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1. *every* $\mathbf{a} \in \Gamma$ *is allowable and*

2. $\Lambda(\Gamma) = \{\log \lambda_{\mathbf{a}} \ : \ \mathbf{a} \in \Gamma \cap \mathrm{int}(\mathcal{M}_+)\}$ *generates a dense subgroup of* $\mathbb{R}$.

Observe that this new set of assumptions coincides with the one imposed by Kesten in [3], which makes the discussion in Section 5 in [4] meaningless.

A sufficient condition for (2) to hold is that $\mathrm{supp}\,\mu$ consists only of invertible matrices and that no subspace $W \subsetneq \mathbb{R}^d$ with $W \cap \mathbb{R}^d_{\geq} \neq \{0\}$ satisfies $\Gamma W \subset W$, see the discussion in [2].

References

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