*Gen. Math. Notes, Vol. 3, No. 2, April 2011, pp. 123-129*
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# On Mellin Transform Involving the Product of a General Class of Polynomials, Struve's Function and H-Function of Two Variables

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(Received: 24-2-11/ Accepted: 25-3-11)

**Abstract**

*The object of this paper is to establish integrals involving the product of struve's function, general class of polynomials and H-function of two variables. Some special cases have also derived.*

**Keywords**: *Mellin transform, struve's function, general class of polynomials and H-function of two variables.*

## 1 Introduction

Recently, The Mellin transform of struve's function with H-function of two variables and Mellin transform of general class of polynomials with H-function of two variables [5] are evaluated. In the present paper we establish the Integral transform of H-function of two variables with general class of polynomials and struve's function.

We shall utilized the following formulae in the present investigation. The

H-function of two variables given by Prasad and Gupta [7].

(1.1) $$H[x , y] = H^{M,N:m,n;g,h}_{P,Q:p,q;u,v}\left[\begin{matrix}\gamma x^{\sigma} \\ \eta x^{\delta}\end{matrix}\middle|\begin{matrix}(a_j;\alpha_j,A_j)_{1,P}:(c_j,C_j)_{1,p};(e_j,E_j)_{1,u} \\ (b_j;\beta_j,B_j)_{1,Q}:(d_j,D_j)_{1,q};(f_j,F_j)_{1,v}\end{matrix}\right]$$

$$= \frac{1}{(2\pi i)^2}\int_{L_1}\int_{L_2}\phi_1(s)\phi_2(t)\psi(s,t)x^s y^t\,ds\,dt\,,\quad i=\sqrt{-1}$$

where x , y ≠ 0,

$$\phi_1(s) = \frac{\prod_{j=1}^{m}\Gamma(d_j - D_j s)\prod_{j=1}^{n}\Gamma(1-c_j+C_j s)}{\prod_{j=m+1}^{q}\Gamma(1-d_j+D_j s)\prod_{j=n+1}^{p}\Gamma(c_j - C_j s)}$$

$$\phi_2(t) = \frac{\prod_{j=1}^{g}\Gamma\left(f_j - F_j t\right)\prod_{j=1}^{h}\Gamma\left(1-e_j+E_j t\right)}{\prod_{j=g+1}^{v}\Gamma\left(1-f_j+F_j t\right)\prod_{j=h+1}^{u}\Gamma\left(e_j - E_j t\right)}$$

$$\psi(s,t) = \frac{\prod_{j=1}^{M}\Gamma\left(b_j - \beta_j s - B_j t\right)\prod_{j=1}^{N}\Gamma\left(1-a_j+\alpha_j s + A_j t\right)}{\prod_{j=M+1}^{Q}\Gamma\left(1-b_j+\beta_j s + B_j t\right)\prod_{j=N+1}^{P}\Gamma\left(a_j - \alpha_j s - A_j t\right)}$$

where M, N, P, Q, m, n, p, q, g, h, u, v are all non negative integers such that 0 ≤ N ≤ P, Q ≥ 0, 0 ≤ m ≤ q, 0 ≤ n ≤ q, 0 ≤ g ≤ v, 0 ≤ h ≤ u and $\alpha_j$, $\beta_j$, $A_j$, $B_j$, $C_j$, $D_j$, $E_j$, $F_j$ are all positive. The sequence of parameters $(a_P)$, $(b_Q)$, $(c_p)$, (dq), $(e_u)$ and $(f_v)$ are so restricted that none of the poles of the integrand coincide.

The contour $L_1$ lies in the complex s-plane and runs from -i∞ to +i∞ with loops, if necessary, to ensure that the poles of $\Gamma(d_j - D_js)$, (j = 1, 2, . . . , m), lie to the right of the path; and those of $\Gamma(1 - c_j + C_js)$, (j = 1, 2, . . . , n) and $\Gamma(1 - a_j + \alpha_js + A_jt)$, (j = 1, 2, . . . , N) lie to the left of the path.

Also the contour $L_2$ lies in the complex t-plane running from -i∞ to +i∞ with loops, if necessary, to ensure that the poles of $\Gamma(f_j - F_jt)$, (j = 1, 2, . . . , g), lie to the right of the path; and those of $\Gamma(1 - e_j + E_jt)$, (j = 1, 2, . . . , h) and $\Gamma(1 - a_j + \alpha_js + A_jt)$, (j = 1, 2, . . . , N) lie to the left of the path. All poles of the integrand are simple poles.

Mellin transform of the H-function is defined as follows [12]

(1.2) $$\int_0^\infty z^{s-1} H_{P,Q}^{M,N}\left[az\middle|\begin{matrix}(c_j,\gamma_j)_{1,P}\\(d_j,\delta_j)_{1,Q}\end{matrix}\right]dx = a^{-s}\theta(-s)$$

where

$$\theta(-s)=\frac{\prod_{j=1}^{N}\Gamma((1-c_j)-\gamma_j s)\prod_{j=1}^{M}\Gamma(1-(1-d_j)+\delta_j s)}{\prod_{j=N+1}^{P}\Gamma(1-(1-c_j)+\gamma_j s)\prod_{j=M+1}^{Q}\Gamma((1-d_j)-\delta_j s)}$$

Provided the corresponding conditions stated in [12]
According Erdely [1, p.307]

(1.3) $$\int_0^\infty x^{s-1}\left[\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} g(s)x^{-s}ds\right]dx = g(s)$$

The Struve's function is defined as [3],

(1.4) $$H_{v,y,u}^{\lambda,k}[z]=\sum_{m=0}^{\infty}\frac{(-1)^m(z/2)^{v+2m+1}}{\Gamma(km+y)\Gamma(v+\lambda m+u)}$$

Re (k) > 0, Re (λ) > 0, Re (y ) > 0, Re (v + u) > 0
The class of polynomials [10]

(1.5)

$$S_n^m[x] = \sum_{k=0}^{[n/m]}\frac{(-n)_{mk}}{k!}A_{n,k}\,x^k \quad n = 0, 1, 2, \ldots$$

where m is an arbitrary positive integer and the coefficients $A_{n,k}$ (n, k ≥ 0) are arbitrary constants.

## 2 Main Result

(2.1) $$\int_0^\infty x^{s-1}\,\mathrm{H}_{v,y,u}^{\lambda,k}[ax^h]S_n^m[bx^h]H_{P,Q:p_1,q_1;p_2,q_2}^{M,N:m_1,n_1;m_2,n_2}\left[\begin{matrix}\gamma x^{\sigma}\\ \eta x^{\delta}\end{matrix}\middle|\begin{matrix}(a_j;\alpha_j,A_j)_{1,P}:(c_j,C_j)_{1,p_1};(e_j,E_j)_{1,p_2}\\(b_j;\beta_j,B_j)_{1,Q}:(d_j,D_j)_{1,q_1};(f_j,F_j)_{1,q_2}\end{matrix}\right]dx$$

$$= \frac{1}{\delta}\sum_{l=0}^{\infty} G(l) \sum_{r=0}^{[n/m]} F(r)\eta^{-\frac{(s+\phi(l,r))}{\delta}} H^{M+m_1+n_2,N+n_1+m_2}_{P+p_1+q_2,Q+q_1+p_2}\left[\eta^{-\frac{\sigma}{\delta}}\gamma \middle| \begin{matrix} (a_j+\left(\frac{s+\phi(l,r)}{\delta}\right)A_j,\alpha_j-(\sigma/\delta)A_j)_{1,N},(c_j,C_j)_{1,n_1}, \\ (b_j+\left(\frac{s+\phi(l,r)}{\delta}\right)B_j,\beta_j-(\sigma/\delta)B_j)_{1,M},(d_j,D_j)_{1,m_1}, \end{matrix}\right.$$

$$\left. \begin{matrix} (1-f_j-\left(\frac{s+\phi(l,r)}{\delta}\right)F_j,(\sigma/\delta)F_j)_{1,q_2},(a_j+\left(\frac{s+\phi(l,r)}{\delta}\right)A_j,\alpha_j-(\sigma/\delta)A_j)_{N+1,P},(c_j,C_j)_{n_1+1,p_1} \\ (1-e_j-\left(\frac{s+\phi(l,r)}{\delta}\right)E_j,(\sigma/\delta)E_j)_{1,p_2},(b_j+\left(\frac{s+\phi(l,r)}{\delta}\right)B_j,\beta_j-(\sigma/\delta)B_j)_{M+1,Q},(d_j,D_j)_{m_1+1,q_1} \end{matrix}\right]$$

where

$$G(l) = \frac{(-1)^l a^{v+2l+1}}{\Gamma(kl+y)\Gamma(v+\lambda l+u)}$$

$$F(r) = \frac{(-n)_{mr}}{r!} A_{n,r}\, b^r$$

$$\phi(l,r) = g(v+2l+1)+hr$$

Provided $\sigma > 0$, $\delta > 0$, $\lambda > 0$, $h > 0$,

$\alpha_j - (\sigma/\delta)\, A_j > 0$ for j = 1, 2,. . . , P
$\beta_j - (\sigma/\delta)\, B_j > 0$ for j = 1, 2, . . . , Q

$|\arg \gamma| < (1/2)\, \pi\, \Delta_1$, $|\arg \eta| < (1/2)\, \pi\, \Delta_2$

where $\Delta_1 = \sum_{j=1}^{N}\alpha_j - \sum_{j=N+1}^{P}\alpha_j + \sum_{j=1}^{M}\beta_j - \sum_{j=M+1}^{Q}\beta_j + \sum_{j=1}^{m_1}D_j - \sum_{j=m_1+1}^{q_1}D_j + \sum_{j=1}^{n_1}C_j - \sum_{j=n_1+1}^{p_1}C_j$ ,

$$\Delta_2 = \sum_{j=1}^{N}A_j - \sum_{j=N+1}^{P}A_j + \sum_{j=1}^{M}B_j - \sum_{j=M+1}^{Q}B_j + \sum_{j=1}^{m_2}F_j - \sum_{j=m_2+1}^{q_2}F_j + \sum_{j=1}^{n_2}E_j - \sum_{j=n_2+1}^{p_2}E_j$$

and

$$\mathrm{Re}\left(s+gv+g+\frac{\delta(a_j-1)}{A_j}\right) < 0 \qquad \text{for } j = 1,2,\ldots, N$$

$$\mathrm{Re}\left(s+gv+g+\frac{\delta b_j}{B_j}\right) > 0 \qquad \text{for } j = 1,2,\ldots, M$$

$$\mathrm{Re}\left(s+gv+g+\frac{\delta f_j}{F_j}+\frac{\sigma d_i}{D_i}\right) > 0 \qquad \text{for } j = 1,2,\ldots, m_2;\ \text{for } i = 1,2,\ldots, m_1$$

$$\mathrm{Re}\left(s+gv+g+\frac{\delta(e_j-1)}{E_j}+\frac{\sigma(c_i-1)}{C_i}\right) > 0 \qquad \text{for } j = 1,2,\ldots, n_2;\ \text{for } i = 1,2,\ldots, n_1$$

**Proof.**

We have

$$H_{v,y,u}^{\lambda,k}\left[ax^{g}\right] S_{n}^{m}[bx^{h}] = \sum_{l=0}^{\infty} \frac{(-1)^{m}(ax^{g}/2)^{v+2l+1}}{\Gamma(kl+y)\Gamma(v+\lambda l+u)} \sum_{r=0}^{[n/m]} \frac{(-n)_{mr}}{r!} A_{n,r} \ (bx^{h})$$

Multiply both sides with

$$x^{s-1} H_{P,Q:p_1,q_1;p_2,q_2}^{M,N:m_1,n_1;m_2,n_2}\left[\begin{matrix}\gamma x^{\sigma}\\ \eta x^{\delta}\end{matrix}\middle| \begin{matrix}(a_j;\alpha_j,A_j)_{1,P}:(c_j,C_j)_{1,p_1};(e_j,E_j)_{1,p_2}\\ (b_j;\beta_j,B_j)_{1,Q}:(d_j,D_j)_{1,q_1};(f_j,F_j)_{1,q_2}\end{matrix}\right]$$

and integrate with respect to x from 0 to ∞. By using (1.1) , represent H-function in integral form and put $\delta t_2 = -w$. Interchange the order of integration then use result (1.3) and (1.2) to get the result. Change of order of integration is justifiable due to convergence of integrals.

# 3 Special Cases

Put h = 0, b = 1 in (2.1) we get Mellin transform of product of Struve's function with H-function of two variables

$$(3.1)\ \int_0^{\infty} x^{s-1}\mathbf{H}_{v,y,u}^{\lambda,k}[ax^{h}] H_{P,Q:p_1,q_1;p_2,q_2}^{M,N:m_1,n_1;m_2,n_2}\left[\begin{matrix}\gamma x^{\sigma}\\ \eta x^{\delta}\end{matrix}\middle| \begin{matrix}(a_j;\alpha_j,A_j)_{1,P}:(c_j,C_j)_{1,p_1};(e_j,E_j)_{1,p_2}\\ (b_j;\beta_j,B_j)_{1,Q}:(d_j,D_j)_{1,q_1};(f_j,F_j)_{1,q_2}\end{matrix}\right]dx$$

$$= \frac{1}{\delta}\sum_{l=0}^{\infty} G(l)\eta^{-\frac{(s+\phi(l,0))}{\delta}} H_{P+p_1+q_2,Q+q_1+p_2}^{m_1+n_2,N+n_1+m_2}\left[\eta^{-\frac{\sigma}{\delta}}\gamma \middle| \begin{matrix}(a_j+\left(\frac{s+\phi(l,0)}{\delta}\right)A_j,\alpha_j-(\sigma/\delta)A_j)_{1,N},(c_j,C_j)_{1,n_1},\\ (b_j+\left(\frac{s+\phi(l,0)}{\delta}\right)B_j,\beta_j-(\sigma/\delta)B_j)_{1,M},(d_j,D_j)_{1,m_1},\end{matrix}\right.$$

$$\left.\begin{matrix}(1-f_j-\left(\frac{s+\phi(l,0)}{\delta}\right)F_j,(\sigma/\delta)F_j)_{1,q_2},(a_j+\left(\frac{s+\phi(l,0)}{\delta}\right)A_j,\alpha_j-(\sigma/\delta)A_j)_{N+1,P},(c_j,C_j)_{n_1+1,p_1}\\ (1-e_j-\left(\frac{s+\phi(l,0)}{\delta}\right)E_j,(\sigma/\delta)E_j)_{1,p_2},(b_j+\left(\frac{s+\phi(l,0)}{\delta}\right)B_j,\beta_j-(\sigma/\delta)B_j)_{M+1,Q},(d_j,D_j)_{m_1+1,q_1}\end{matrix}\right]$$

Put g = 0, a = 1 in (2.1) we get Mellin transform of product of generals class of polynomials with H-function of two variables [5, Result 2.1, p.45]

Put M = 0, we obtain

$$(3.2)\ \int_0^{\infty} x^{s-1}\mathbf{H}_{v,y,u}^{\lambda,k}[ax^{h}]S_n^m[bx^h] H_{P,Q:p_1,q_1;p_2,q_2}^{0,N:m_1,n_1;m_2,n_2}\left[\begin{matrix}\gamma x^{\sigma}\\ \eta x^{\delta}\end{matrix}\middle| \begin{matrix}(a_j;\alpha_j,A_j)_{1,P}:(c_j,C_j)_{1,p_1};(e_j,E_j)_{1,p_2}\\ (b_j;\beta_j,B_j)_{1,Q}:(d_j,D_j)_{1,q_1};(f_j,F_j)_{1,q_2}\end{matrix}\right]dx$$

$$=\frac{1}{\delta}\sum_{l=0}^{\infty}G(l)\sum_{r=0}^{[n/m]}F(r)\eta^{-\frac{(s+\phi(l,r))}{\delta}}H^{m_1+n_2,N+n_1+m_2}_{P+p_1+q_2,Q+q_1+p_2}\left[\eta^{-\frac{\sigma}{\delta}}\gamma\left|\begin{matrix}(a_j+\left(\frac{s+\phi(l,r)}{\delta}\right)A_j,\alpha_j-(\sigma/\delta)A_j)_{1,N},(c_j,C_j)_{1,n_1},\\(b_j+\left(\frac{s+\phi(l,r)}{\delta}\right)B_j,\beta_j-(\sigma/\delta)B_j)_{1,M},(d_j,D_j)_{1,m_1},\end{matrix}\right.\right.$$

$$\left.\begin{matrix}(1-f_j-\left(\frac{s+\phi(l,r)}{\delta}\right)F_j,(\sigma/\delta)F_j)_{1,q_2},(a_j+\left(\frac{s+\phi(l,r)}{\delta}\right)A_j,\alpha_j-(\sigma/\delta)A_j)_{N+1,P},(c_j,C_j)_{n_1+1,p_1}\\(1-e_j-\left(\frac{s+\phi(l,r)}{\delta}\right)E_j,(\sigma/\delta)E_j)_{1,p_2},(b_j+\left(\frac{s+\phi(l,r)}{\delta}\right)B_j,\beta_j-(\sigma/\delta)B_j)_{M+1,Q},(d_j,D_j)_{m_1+1,q_1}\end{matrix}\right]$$

By taking M = N = P = Q = 0, we have

(3.3)

$$\int_0^{\infty}x^{s-1}\mathbf{H}^{\lambda,k}_{v,y,u}[ax^h]S_n^m[bx^h]H^{0,0:m_1,n_1;m_2,n_2}_{0,0:p_1,q_1;p_2,q_2}\left[\begin{matrix}\gamma x^{\sigma}\\\eta x^{\delta}\end{matrix}\left|\begin{matrix}(a_j;\alpha_j,A_j)_{1,P}:(c_j,C_j)_{1,p_1};(e_j,E_j)_{1,p_2}\\(b_j;\beta_j,B_j)_{1,Q}:(d_j,D_j)_{1,q_1};(f_j,F_j)_{1,q_2}\end{matrix}\right.\right]dx$$

$$=\int_0^{\infty}x^{s-1}\mathrm{H}^{\lambda,k}_{v,y,u}[ax^h]S_n^m[bx^h]H^{m_1,n_1}_{p_1,q_1}\left[\gamma x^{\sigma}\left|\begin{matrix}(c_j,C_j)_{1,p_1}\\(d_j,D_j)_{1,q_1}\end{matrix}\right.\right]H^{m_2,n_2}_{p_2,q_2}\left[\eta x^{\delta}\left|\begin{matrix}(e_j,E_j)_{1,p_2}\\(f_j,F_j)_{1,q_2}\end{matrix}\right.\right]dx$$

$$=\frac{1}{\delta}\sum_{l=0}^{\infty}G(l)\sum_{r=0}^{[n/m]}F(r)\eta^{-\frac{(s+\phi(l,r))}{\delta}}H^{m_1+n_2,n_1+m_2}_{p_1+q_2,q_1+p_2}\left[\eta^{-\frac{\sigma}{\delta}}\gamma\left|\begin{matrix}(a_j+\left(\frac{s+\phi(l,r)}{\delta}\right)A_j,\alpha_j-(\sigma/\delta)A_j)_{1,N},(c_j,C_j)_{1,n_1},\\(b_j+\left(\frac{s+\phi(l,r)}{\delta}\right)B_j,\beta_j-(\sigma/\delta)B_j)_{1,M},(d_j,D_j)_{1,m_1},\end{matrix}\right.\right.$$

$$\left.\begin{matrix}(1-f_j-\left(\frac{s+\phi(l,r)}{\delta}\right)F_j,(\sigma/\delta)F_j)_{1,q_2},(a_j+\left(\frac{s+\phi(l,r)}{\delta}\right)A_j,\alpha_j-(\sigma/\delta)A_j)_{N+1,P},(c_j,C_j)_{n_1+1,p_1}\\(1-e_j-\left(\frac{s+\phi(l,r)}{\delta}\right)E_j,(\sigma/\delta)E_j)_{1,p_2},(b_j+\left(\frac{s+\phi(l,r)}{\delta}\right)B_j,\beta_j-(\sigma/\delta)B_j)_{M+1,Q},(d_j,D_j)_{m_1+1,q_1}\end{matrix}\right]$$

Put M = N = P = Q = 0 and (α)j = (β)j = (A)j = (B)j = (C)j = (D)j = (E)j = (F)j = 1, we obtain

(3.4)

$$\int_0^{\infty}x^{s-1}\mathbf{H}^{\lambda,k}_{v,y,u}[ax^h]S_n^m[bx^h]G^{m_1,n_1}_{p_1,q_1}\left[\gamma x^{\sigma}\left|\begin{matrix}(c_j,C_j)_{1,p_1}\\(d_j,D_j)_{1,q_1}\end{matrix}\right.\right]G^{m_2,n_2}_{p_2,q_2}\left[\eta x^{\delta}\left|\begin{matrix}(e_j,E_j)_{1,p_2}\\(f_j,F_j)_{1,q_2}\end{matrix}\right.\right]dx$$

$$=\frac{1}{\delta}\sum_{l=0}^{\infty}G(l)\sum_{r=0}^{[n/m]}F(r)\eta^{-\frac{(s+\phi(l,r))}{\delta}}G^{m_1+n_2,n_1+m_2}_{p_1+q_2,q_1+p_2}\left[\eta^{-\frac{\sigma}{\delta}}\gamma\left|\begin{matrix}(c_j)_{1,n_1}, & (1-f_j-\left(\frac{s+\phi(l,r)}{\delta}\right),(\sigma/\delta))_{1,q_2},(c_j)_{n_1+1,p_1}\\(d_j)_{1,m_1}, & (1-e_j-\left(\frac{s+\phi(l,r)}{\delta}\right),(\sigma/\delta))_{1,p_2},(d_j)_{m_1+1,q_1}\end{matrix}\right.\right]$$

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