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Generalized Jordan Triple Higher $*-$Derivations on Semiprime Rings[1]

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Abstract

The concepts of generalized Jordan higher $*-$derivations and generalized Jordan triple higher $*-$derivations are introduced and it is shown that they coincide on 6-torsion free semiprime $*-$rings.

Keywords: *Semiprime rings, derivations, higher derivations, generalized Jordan higher $*-$derivations.*

1 Introduction

Let R be an associative ring not necessarily with identity element. For any $x, y \in R$. Recall that R is *prime* if $xRy = 0$ implies $x = 0$ or $y = 0$, and is *semiprime* if $xRx = 0$ implies $x = 0$. Given an integer $n \geq 2$, R is said to be $n-$torsion free if for $x \in R, nx = 0$ implies $x = 0$.An additive mapping $d : R \to R$ is called a *derivation* if $d(xy) = d(x)y + yd(x)$ holds for all $x, y \in R$, and it is called a *Jordan derivation* if $d(x^2) = d(x)x + xd(x)$ for all $x \in R$. Every derivation is obviously a Jordan derivation and the converse is in general not true [1, Example 3.2.1]. A classical Herstein theorem [12] shows that any Jordan derivation on a 2-torsion free prime ring is a derivation. Later on Brešar [2] has extended Herstein's theorem to 2-torsion free semiprime ring. A *Jordan triple derivation* is an additive mapping $d : R \to R$ satisfying

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$d(xyx) = d(x)yx + xd(y)x + xyd(x)$ for all $x, y \in R$. Any derivation is obviously a Jordan triple derivation. It is also easy to see that every Jordan derivation of a 2-torsion free ring is a Jordan triple derivation [13, Lemma 3.5]. Brešar [3] has proved that any Jordan derivation of a 2-torsion free prime ring is a derivation. Generalized derivations have been primarily defined by Brešar [5]. An additive mapping $f : R \to R$ is said to be a *generalized derivation*(resp. *generalized Jordan derivation*) if there exists a derivation (resp. Jordan derivation) $d : R \to R$ such that $f(xy) = f(x)y + yd(x)$ (resp. $f(x^2) = f(x)x + xd(x)$) holds for all $x, y \in R$. Hvala [15] has initiated the algebraic study of generalized derivations and extended some results concerning derivation to generalized derivation. Jing and Lu [16] have introduced the notion of generalized Jordan triple derivation as an additive mapping $f : R \to R$ with an associated Jordan triple derivation $d : R \to R$ such that $f(xyx) = f(x)yx + yd(x)x + xyd(x)$ holds for all $x, y \in R$. They have proved that every generalized Jordan triple derivaation on a 2-torsion free prime ring is a generalized derivation.

An additive mapping $x \to x^*$ satisfying $(xy)^* = y^*x^*$ and $(x^*)^* = x$ for all $x, y \in R$ is called an *involution* and R is called a $*-$ *ring*. Let R be a $*-$ring. An additive mapping $d : R \to R$ is called a $*-$*derivation* if $d(xy) = d(x)y^* + xd(y)$ holds for all $x, y \in R$; and it is called a *Jordan* $*-$*derivation* if $d(x^2) = d(x)x^* + xd(x)$ holds for all $x \in R$. The reader might guess that any Jordan $*-$derivation of a 2-torsion free prime $*-$ring is a $*-$derivation, but this is not the case. It was proved in [4] that a noncommutative prime $*-$ring does not admit a non-trivial $*-$derivation. A *Jordan triple* $*-$*derivation* is an additive mapping $d : R \to R$ with the property $d(xyx) = d(x)y^*x^* + xd(y)x^* + xyd(x)$ for all $x, y \in R$. It could easily be seen that any Jordan $*-$derivation on a 2-torsion free $*-$ring is a Jordan triple $*-$derivation [4, Lemma 2]. Vukman [19] has proved that any Jordan triple $*-$derivation on a 6-torsion free semiprime $*-$ring is a Jordan $*-$derivation. Following Daif and El-Sayiad [7], An additive mapping $F : R \to R$ is said to be a *generalized* $*-$*derivation* (resp. *generalized Jordan* $*-$*derivation*) if there exists a $*-$derivation (resp. Jordan $*-$derivation) $d : R \to R$ such that $F(xy) = F(x)y^* + xd(y)$ (resp. $F(x^2) = d(x)x^* + xd(x)$)holds for all $x, y \in R$. They also have introduced the notion of *generalized Jordan triple* $*-$*derivation* as an additive mapping $F : R \to R$ associated with a Jordan triple $*-$derivation $d : R \to R$ with the property $F(xyx) = F(x)y^*x^* + xd(y)x^* + xyd(x)$ for all $x, y \in R$. They have proved that every generalized Jordan triple $*-$derivation on a 6-torsion free semiprime ring is a generalized Jordan $*-$derivation. This extended the above Vukman's main theorem [19].

Let $\mathbb{N}_0$ be the set of all nonnegative integers and $D = \{d_i\}_{i \in \mathbb{N}_0}$ be a family of additive mappings of a ring R such that $d_0 = id_R$. Then D is said to be a *higher derivation*, (resp. a *Jordan higher derivation*) of R if for each $n \in \mathbb{N}_0, d_n(xy) = \sum_{i+j=n} d_i(x)d_j(y)$ (resp. $d_n(x^2) = \sum_{i+j=n} d_i(x)d_j(x)$) holds for all $x, y \in R$. The concept of higher derivations was introduced by Hasse and Schmidt [11]. This interesting notion of higher derivations has been studied in both commutative and noncommutative rings, see e,g., [18], [14], [20] and [9]. Clearly, every higher derivation is a Jordan higher derivation. Ferrero and Haetinger [9] extended Herstein's theorem [12] for higher derivations on 2-torsion free semiprime rings. For an account of higher derivations the reader is referred to [10]. A family $D = (d_i)_{i\in\mathbb{N}_0}$ of additive mappings of a ring R,where $d_0 = id_R$, is called a *Jordan triple higher derivation* if $d_n(xyx) = \sum_{i+j+k=n} d_i(x)d_j(y^i)d_k(x^{i+j})$ holds for all $x, y \in R$. Ferrero and Haetinger [9] have proved that every Jordan higher derivation of a 2-torsion free ring is a Jordan triple higher derivation. They also have proved that every Jordan triple higher derivation of a 2-torsion free semiprime ring is a higher derivation. Later on, Cortes and Haetinger [6] have defined the concept of generalized higher derivations. A family $F = \{f_i\}_{i\in\mathbb{N}_0}$ of additive mappings of a ring R such that $f_0 = id_R$ is said to be a *generalized higher derivation*, (resp. a *generalized Jordan higher derivation*) of R if there exists a higher derivation (resp. Jordan higher derivation) $D = (d_i)_{i\in\mathbb{N}_0}$ and for each $n \in \mathbb{N}_0, f_n(xy) = \sum_{i+j=n} f_i(x)d_j(y)$ (resp. $f_n(x^2) = \sum_{i+j=n} f_i(x)d_j(x)$) holds for all $x, y \in R$. They have proved that if R is a 2-torsion free ring which has a commutator right nonzero divisor and U is a square closed Lie ideal of R, then every generalized higher derivation of U into R is a generalized higher derivation of U into R. A family $F = (d_i)_{i\in\mathbb{N}_0}$ of additive mappings of a ring R, where $f_0 = id_R$, is called a *generalized Jordan triple higher derivation* if $f_n(xyx) = \sum_{i+j+k=n} f_i(x)d_j(y^i)d_k(x^{i+j})$ holds for all $x, y \in R$. Jung [17] has proved that every generalized Jordan triple higher derivation on a 2-torsion free semiprime ring is a generalized Jordan higher derivation.

Motivated by the notions of generalized $*-$derivations and generalized higher derivations, we introduce the notions of *generalized higher $*-$derivations*, *generalized Jordan higher $*-$derivations* and *generalized Jordan triple higher $*-$derivations*. Our main objective is to show that every generalized Jordan triple higher $*-$derivations of a 6-torsion free semiprime $*-$ring is a generalized Jordan higher $*-$derivations. This result extends the main results of [7] and [19]. It is also shown that every generalized Jordan higher $*-$derivations of a 2-torsion free $*-$ring is a generalized Jordan triple higher $*-$derivations. So we can conclude that the notions of generalized Jordan triple higher $*-$derivations and generalized Jordan higher $*-$derivations are coincident on 6-torsion free semiprime $*-$rings.

2 Preliminaries and Main Results

We begin by the following definition

Definition 2.1. *Let $\mathbb{N}_0$ be the set of all nonnegative integers and let $F = \{f_i\}_{i\in\mathbb{N}_0}$ be a family of additive mappings of a $*-$ring R such that $f_0 = id_R$. F is called:*

(a) a generalized higher $*-$derivation *of R if for each $n \in \mathbb{N}_0$ there exists a higher $*-$derivation $D = \{d_i\}_{i\in\mathbb{N}_0}$ such that*

$$f_n(xy) = \sum_{i+j=n} f_i(x)d_j(y^{*^i}) \text{ for all } x, y \in R;$$

(b) a generalized Jordan higher $*-$derivation *of R if for each $n \in \mathbb{N}_0$ there exists a Jordan higher $*-$derivation $D = \{d_i\}_{i\in\mathbb{N}_0}$ such that*

$$f_n(x^2) = \sum_{i+j=n} f_i(x)d_j(x^{*^i}) \text{ for all } x \in R;$$

(c) a generalized Jordan triple higher $*-$derivation *of R if for each $n \in \mathbb{N}_0$ there exists a Jordan tipple higher $*-$derivation $D = \{d_i\}_{i\in\mathbb{N}_0}$ such that*

$$f_n(xyx) = \sum_{i+j+k=n} f_i(x)d_j(y^{*^i})d_k(x^{*^{i+j}}) \text{ for all } x, y \in R.$$

Throughout this section, we will use the following notation:

Notation. Let $F = \{f_i\}_{i\in\mathbb{N}_0}$ be a generalized Jordan triple higher $*-$derivation of a $*-$ring R with an associated Jordan triple higher $*-$derivation $D = \{d_i\}_{i\in\mathbb{N}_0}$. For every fixed $n \in \mathbb{N}_0$ and each $x, y \in R$, we denote by $A_n(x)$ and $B_n(x, y)$ the elements of R defined by:

$$A_n(x) = f_n(x^2) - \sum_{i+j=n} f_i(x)d_j(x^{*^i}),$$
$$B_n(x, y) = f_n(xy + yx) - \sum_{i+j=n} f_i(x)d_j(y^{*^i}) - \sum_{i+j=n} f_i(y)d_j(x^{*^i}).$$

It can easily be seen that $A_n(-x) = A_n(x)$, $B_n(-x, y) = -B_n(x, y)$ and $A_n(x + y) = A_n(x) + A_n(y) + B_n(x, y)$ for each pair $x, y \in R$. The following lemmas are crucial in developing the proof of the main results.

Linearizing the last definition the following lemma can be obtained directly.

Lemma 2.1. *Let $F = \{f_i\}_{i\in\mathbb{N}_0}$ be a generalized Jordan triple higher $*-$derivation with an associated Jordan triple higher $*-$derivation $D = \{d_i\}_{i\in\mathbb{N}_0}$. Then we have for all $x, y, z \in R$ and each $n \in \mathbb{N}_0$,*

$$f_n(xyz + zyx) = \sum_{i+j+k=n} f_i(x)d_j(y^{*^i})d_k(z^{*^{i+j}}) + f_i(z)d_j(y^{*^i})d_k(x^{*^{i+j}}).$$

Lemma 2.2. *Let $F = \{f_i\}_{i\in\mathbb{N}_0}$ be a generalized Jordan triple higher $*-$derivation of a 6-torsion free semiprime $*-$ring R with an associated Jordan triple higher $*-$derivation $D = \{d_i\}_{i\in\mathbb{N}_0}$. If $A_m(x) = 0$ for all $x \in R$ and for each $m < n$, then $A_n(x)y^{*^n}x^{*^n} = 0$ for each $n \in \mathbb{N}_0$ and for every $x, y \in R$.*

Proof. The substitution $(xy + yx)$ for y in the definition of generalized Jordan triple higher $*-$derivation gives

$$\begin{aligned}
f_n(x(xy+yx)x) &= \sum_{i+j+k=n} f_i(x)d_j((xy+yx)^{*^i})d_k(x^{*^{i+j}}) \\
&= \sum_{i+j+k=n} f_i(x)\Big(\sum_{p+q=j} d_p(x^{*^i})d_q(y^{*^{i+p}}) + d_p(y^{*^i})d_q(x^{*^{i+p}})\Big)d_k(x^{*^{i+j}}) \\
&= \sum_{i+p+q+k=n} f_i(x)d_p(x^{*^i})d_q(y^{*^{i+p}})d_k(x^{*^{i+p+q}}) \\
&\quad + \sum_{i+p+q+k=n} f_i(x)d_p(y^{*^i})d_q(x^{*^{i+p}})d_k(x^{*^{i+p+q}}) \\
&= \sum_{i+p=n} f_i(x)d_p(x^{*^i})y^{*^n}x^{*^n} \\
&\quad + \sum_{\substack{i+p+q+k=n \\ i+p\neq n}} f_i(x)d_p(x^{*^i})d_q(y^{*^{i+p}})d_k(x^{*^{i+p+q}}) \\
&\quad + \sum_{i+p+q+k=n} f_i(x)d_p(y^{*^i})d_q(x^{*^{i+p}})d_k(x^{*^{i+p+q}}).
\end{aligned}$$

On the other hand the substitution x^2 for x in Lemma 2.1 shows, using the assumption on $A_m(x), m < n$ and the fact that $D = \{d_i\}$ turns to be a Jordan higher $*-$derivation by [8, Theorem 2.1], that

$$\begin{aligned}
f_n(xyx^2 + x^2yx) &= \sum_{i+j+k=n} f_i(x)d_j(y^{*^i})d_k(x^{2^{*^{i+j}}}) + f_i(x^2)d_j(y^{*^i})d_k(x^{*^{i+j}}) \\
&= \sum_{i+j+k=n} f_i(x)d_j(y^{*^i})\Big(\sum_{s+t=k} d_s(x)d_t(x^{*^s})\Big) \\
&\quad + \sum_{\substack{i+j+k=n \\ i\neq n}} \Big(\sum_{l+r=i} f_l(x)d_r(x^{*^l})\Big)d_j(y^{*^i})d_k(x^{*^{i+j}}) \\
&\quad + f_n(x^2)y^{*^n}x^{*^n} \\
&= \sum_{i+j+s+t=n} f_i(x)d_j(y^{*^i})d_s(x)d_t(x^{*^s}) \\
&\quad + \sum_{\substack{l+r+j+k=n \\ l+r\neq n}} f_l(x)d_r(x^{*^l})d_j(y^{*^{l+r}})d_k(x^{*^{l+r+j}})
\end{aligned}$$

$$+ f_n(x^2)y^{*^n}x^{*^n}.$$

Now, subtracting the two relations so obtained we find that

$$\Big(f_n(x^2) - \sum_{i+p=n} f_i(x)d_p(x^{*^i})\Big)y^{*^n}x^{*^n} = 0.$$

Using our notation, the last relation reduces to the required result □

Now, we are ready to prove our main results.

Theorem 2.1. *Let R be a 6-torsion free semiprime $*-$ring. Then every generalized Jordan triple higher $*-$derivation $F = \{f_i\}_{i\in\mathbb{N}_0}$ of R is a generalized Jordan higher $*-$derivation of R.*

Proof. By [8, Theorem 2.1] we can conclude that the associated D of F turns to be a Jordan higher $*-$derivation. We intend to show that $A_n(x) = 0$ for all $x \in R$. In case $n = 0$, we get trivially $A_0(x) = 0$ for all $x \in R$. If $n = 1$, then it follows from [7, Theorem 2.1] that $A_1(x) = 0$ for all $x \in R$. Thus we assume that $A_m(x) = 0$ for all $x \in R$ and $m < n$. From Lemma 2.2, we see that

$$A_n(x)y^{*^n}x^{*^n} = 0 \text{ for all } x \in R. \tag{2.1}$$

In case n is even (2.1) reduces to $A_n(x)yx = 0$. Now, replacing y by $xyA_n(x) = 0$, we have $A_n(x)xyA_n(x)x = 0$ for all $y \in R$. By the semiprimeness of R, we get

$$A_n(x)x = 0 \quad \text{for all } x \in R. \tag{2.2}$$

On the other hand, multiplying $A_n(x)yx = 0$ by $A(x)$ from right and by x from left we get $xA_n(x)yxA_n(x) = 0$ for all $x, y \in R$. Again, by the semiprimeness of R we get

$$xA_n(x) = 0 \quad \text{for all } x \in R. \tag{2.3}$$

Linearizing (2.2) we get

$$A_n(x)y + B_n(x,y)x + A_n(y)x + B_n(x,y)y = 0 \text{ for all } x, y \in R. \tag{2.4}$$

Putting $-x$ for x in (2.4) we get

$$A_n(x)y + B_n(x,y)x - A_n(y)x - B_n(x,y)y = 0 \text{ for all } x, y \in R. \tag{2.5}$$

Adding (2.4) and (2.5) we get since R is 2-torsion free

$$A_n(x)y + B_n(x,y)x = 0 \text{ for all } x, y \in R. \tag{2.6}$$

Multiplying (2.6) by $A_n(x)$ from right and using (2.3) we get $A_n(x)yA_n(x) = 0$ for all $x, y \in R$. By the semiprimeness of R, we get $A_n(x) = 0$ for all $x \in R$.

In case n is odd (2.1) reduces to $A_n(x)y^*x^* = 0$. By the surjectiveness of the involution we obtain $A_n(x)yx^* = 0$. Now, replacing y by $x^*yA_n(x) = 0$, we have $A_n(x)x^*yA_n(x)x^* = 0$ for all $y \in R$, By the semiprimeness of R, we get

$$A_n(x)x^* = 0 \quad \text{for all } x \in R. \tag{2.7}$$

On the other hand multiplying $A_n(x)yx^* = 0$ by $A(x)$ from right and by x^* from left we get $x^*A_n(x)yx^*A_n(x) = 0$ for all $x, y \in R$. Again by the semiprimeness of R gives

$$x^*A_n(x) = 0 \quad \text{for all } x \in R. \tag{2.8}$$

Linearizing (2.7) we get

$$A_n(x)y^* + B_n(x,y)x^* + A_n(y)x^* + B_n(x,y)y^* = 0 \text{ for all } x, y \in R. \tag{2.9}$$

Putting $-x$ for x in (2.9) we get

$$A_n(x)y^* + B_n(x,y)x^* - A_n(y)x^* - B_n(x,y)y^* = 0 \text{ for all } x, y \in R. \tag{2.10}$$

Adding (2.9) and (2.10) we get since R is 2-torsion free that

$$A_n(x)y^* + B_n(x,y)x^* = 0 \text{ for all } x, y \in R. \tag{2.11}$$

Multiplying by $A_n(x)$ from right and using (2.8) we get $A_n(x)y^*A_n(x) = 0$, by the surjectiveness of the involution we get $A_n(x)yA_n(x) = 0$ for all $x, y \in R$. By the semiprimeness of R, we get $A_n(x) = 0$ for all $x \in R$. So in either cases we reach to our intended result. This completes the proof of the theorem. □

Corollary 2.1 ([8, Theorem 2.1]). *Every Jordan triple higher $*-$derivation of a 6-torsion free semiprime $*-$ring is a Jordan higher $*-$derivation.*

Corollary 2.2 ([7, Theorem 2.1]). *Every generalized Jordan triple $*-$derivation of a 6-torsion free semiprime $*-$ring is a generalized Jordan $*-$derivation.*

Theorem 2.2. *Let R be a 6-torsion free semiprime $*-$ring. Then every generalized Jordan higher $*-$derivation $F = \{f_i\}_{i\in\mathbb{N}_0}$ of R is a generalized Jordan triple higher $*-$derivation of R.*

Proof. In view of [8, Theorem 2.2], the associated derivation D of F turns to be a Jordan triple higher $*-$derivation. By definition we have

$$f_n(x^2) = \sum_{i+j=n} f_i(x)d_j(x^{*^i}). \tag{2.12}$$

Putting $v = x + y$ and using (2.12) we obtain

$$\begin{aligned} f_n(v^2) &= \sum_{i+j=n} f_i(x+y)d_j((x+y)^{*^i}) \\ &= \sum_{i+j=n} f_i(x)d_j(x^{*^i}) + f_i(y)d_j(y^{*^i}) + f_i(x)d_j(y^{*^i}) + f_i(y)d_j(x^{*^i}). \end{aligned}$$

and

$$\begin{aligned} f_n(v^2) &= f_n(x^2 + xy + yx + y^2) \\ &= f_n(x^2) + f_n(y^2) + f_n(xy + yx) \\ &= \sum_{l+m=n} f_l(x)d_m(x^{*^l}) + \sum_{r+s=n} f_r(y)d_s(y^{*^r}) + f_n(xy+yx). \end{aligned}$$

Comparing the last two forms of $f_n(v^2)$ gives

$$f_n(xy+yx) = \sum_{i+j=n} f_i(x)d_j(y^{*^i}) + f_i(y)d_j(x^{*^i}). \tag{2.13}$$

Now put $w = x(xy + yx) + (xy + yx)x$. Using (2.13) we get

$$\begin{aligned} f_n(w) &= \sum_{i+j=n} f_i(x)d_j((xy+yx)^{*^i}) + \sum_{i+j=n} f_i(xy+yx)d_j(x^{*^i}) \\ &= \sum_{i+j=n}\sum_{r+s=j} f_i(x)d_r(x^{*^i})d_s(y^{*^{i+r}}) + \sum_{i+j=n}\sum_{r+s=j} f_i(x)d_r(y^{*^i})d_s(x^{*^{i+r}}) \\ &\quad + \sum_{i+j=n}\sum_{k+l=i} f_k(x)d_l(y^{*^k})d_j(x^{*^{k+l}}) + \sum_{i+j=n}\sum_{k+l=i} f_k(y)d_l(x^{*^k})d_j(x^{*^{k+l}}) \\ &= \sum_{i+r+s=n} f_i(x)d_r(x^{*^i})d_s(y^{*^{i+r}}) + 2\sum_{i+j+k=n} f_i(x)d_j(y^{*^i})d_k(x^{*^{i+j}}) \\ &\quad + \sum_{k+l+j=n} f_k(y)d_l(x^{*^k})d_j(x^{*^{k+l}}). \end{aligned}$$

Also,

$$\begin{aligned} f_n(w) &= f_n((x^2y + yx^2) + 2xyx) \\ &= f_n(x^2y + yx^2) + 2f_n(xyx) \\ &= 2f_n(xyx) + \sum_{r+s+j=n} f_r(x)d_s(x^{*^r})d_j(y^{*^{r+s}}) \\ &\quad + \sum_{i+k+l=n} f_i(y)d_k(x^{*^i})d_l(x^{*^{i+k}}). \end{aligned}$$

Comparing the last two forms of $f_n(w)$ and using the fact that R is 2-torsion free we obtain the required result □

By Theorem 2.1 and Theorem 2.2, we can state the following.

Theorem 2.3. *The notions of a generalized Jordan higher $*-$derivation and a generalized Jordan triple higher $*-$derivation on a 6-torsion free semiprime $*-$ring are equivalent.*

Corollary 2.3 ([8, Theorem 2.3])**.** *The notions of a Jordan higher $*-$derivation and a Jordan triple higher $*-$derivation on a 6-torsion free semiprime $*-$ring are equivalent.*

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