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A Note on Carlitz's Twisted (h,q)-Euler Polynomials under Symmetric Group of Degree Five

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Abstract

In [12], Ryoo defined the Carlitz's twisted (h, q)-Euler numbers and polynomials. In this paper, we consider some new symmetric identities for Carlitz's twisted (h, q)-Euler polynomials arising from the fermionic p-adic integral over the p-adic numbers field under the symmetric group of degree five.

Keywords: *Carlitz's twisted (h, q)-Euler polynomials, Invariant under S_5, p-adic invariant integral on $\mathbb{Z}_p$, Symmetric identities.*

1 Introduction

In the Taylor expansion

$$\sum_{n=0}^{\infty} E_n(x)\frac{t^n}{n!} = \frac{2}{e^t+1}e^{xt} \text{ with } (|t| < \pi), \tag{1}$$

$E_n(x)$ denotes the n-th Euler polynomial. If we take $x = 0$ in the Eq. (1), we then have $E_n(0) := E_n$ that is commonly known as the n-th Euler number (see, e.g., [6, 8, 9, 11-13]).

Let $\mathbb{N} = \{1, 2, 3, \cdots\}$, $\mathbb{N}^* = \mathbb{N} \cup \{0\}$ and p be chosen as a fixed odd prime number. Along this paper $\mathbb{Z}_p$, $\mathbb{Q}$, $\mathbb{Q}_p$ and $\mathbb{C}_p$ shall denote topological closure of $\mathbb{Z}$, the field of rational numbers, topological closure of $\mathbb{Q}$ and the field of p-adic completion of an algebraic closure of $\mathbb{Q}_p$, respectively.

For d an odd positive number with $(d,p)=1$, let

$$X:=X_d=\lim_{\overleftarrow{N}}\mathbb{Z}/dp^N\mathbb{Z} \ \ and \ X_1=\mathbb{Z}_p$$

and

$$t+dp^N\mathbb{Z}_p=\left\{x\in X\mid x\equiv t\left(moddp^N\right)\right\}$$

in which $t\in\mathbb{Z}$ lies in $0\leq t<dp^N$. See, for more information, [1-13].

The normalized absolute value according to the theory of p-adic analysis is given by $|p|_p=p^{-1}$. The notation q can be considered as an indeterminate, a complex number $q\in\mathbb{C}$ with $|q|<1$, or a p-adic number $q\in\mathbb{C}_p$ with $|q-1|_p<p^{-\frac{1}{p-1}}$ and $q^x=\exp\left(x\log q\right)$ for $|x|_p\leq 1$.

For fixed x, let us introduce the following notation (see [1-13]):

$$[x]_q=\frac{1-q^x}{1-q} \tag{2}$$

which is known as q-number of x (or q-analogue of x). We note that as $q\to 1$, the notation $[x]_q$ reduces to the x.

For

$$f\in UD\left(\mathbb{Z}_p\right)=\left\{f\,|f:\mathbb{Z}_p\to\mathbb{C}_p \text{ is uniformly differentiable function}\right\},$$

Kim [8] defined the p-adic invariant integral on $\mathbb{Z}_p$ as follows:

$$I_{-1}(f)=\int_{\mathbb{Z}_p}f\left(x\right)d\mu_{-1}\left(x\right)=\lim_{N\to\infty}\sum_{x=0}^{p^N-1}f\left(x\right)\left(-1\right)^x. \tag{3}$$

From Eq. (3), we get

$$I_{-1}(f_n)=\left(-1\right)^n I_{-1}(f)+2\sum_{k=0}^{n-1}\left(-1\right)^{n-k-1}f(k)$$

where $f_n(x)$ implies $f(x+n)$. For more details about this topic, take a look at the references [2, 3, 8, 12, 13].

Let $h\in\mathbb{Z}$ and

$$T_p=_{N\geq 1}C_{p^N}=\lim_{N\to\infty}C_{p^N},$$

where $C_{p^N}=\left\{w:w^{p^N}=1\right\}$ is the cyclic group of order p^N. For $w\in T_p$, we show by $\phi_w:\mathbb{Z}_p\to C_p$ the locally constant function $x\to w^x$. For $q\in C_p$ with $|1-q|_p<1$ and $w\in T_p$, the Carlitz's twisted (h,q)-Euler polynomials are defined by the following p-adic fermionic integral on $\mathbb{Z}_p$ in [12]:

$$\mathcal{E}_{n,q,w}^{(h)}(x)=\int_{\mathbb{Z}_p}w^y q^{hy}\left[x+y\right]_q^n d\mu_{-1}\left(y\right)\quad\left(n\geq 0\right). \tag{4}$$

If we let $x = 0$ into the Eq. (4), we get $\mathcal{E}_{n,q,w}^{(h)}(0) := \mathcal{E}_{n,q,w}^{(h)}$ called n-th Carlitz's twisted (h, q)-Euler number.

Taking $w = 1$ and $q \to 1$ in the Eq. (4) yields to

$$\mathcal{E}_{n,q,w}^{(h)}(x) \to E_n(x) := \int_{\mathbb{Z}_p} (x+y)^n d\mu_{-1}(y).$$

Recently, symmetric identities of some special polynomials, such as q-Genocchi polynomials of higher order under third Dihedral group D_3 in [1], q-Genocchi polynomials under the symmetric group of degree four in [4], weighted q-Genocchi polynomials under the symmetric group of degree four in [5], q-Frobenious-Euler polynomials under symmetric group of degree five in [3], Carlitz's-type q-Euler polynomials invariant under the symmetric group of degree five in [9], higher-order Carlitz's q-Bernoulli polynomials under the symmetric group of degree five in [10], have been studied by many mathematicians.

In the following section, we investigate some new symmetric identities for Carlitz's twisted (h, q)-Euler polynomials arising from the p-adic invariant integral on $\mathbb{Z}_p$ under the symmetric group of degree five denoted by S_5.

2 Symmetric Identities for $\mathcal{E}_{n,q,w}^{(h)}(x)$ under S_5

Let $h \in \mathbb{Z}$, $w_i \in \mathbb{N}$ be a natural number which satisfies the condition $w_i \equiv 1(mod2)$, in which $i \in \mathbb{Z}$ lies in $1 \leq i \leq 5$. By the Eqs. (3) and (4), we acquire

$$\begin{aligned} &\int_{\mathbb{Z}_p} w^{w_1w_2w_3w_4y} q^{hw_1w_2w_3w_4y} \\ &\times e^{[w_1w_2w_3w_4y+w_1w_2w_3w_4w_5x+w_5w_4w_2w_3i+w_5w_4w_1w_3j+w_5w_4w_1w_2k+w_5w_3w_1w_2s]_q t} d\mu_{-1}(y) \\ &= \lim_{N\to\infty} \sum_{y=0}^{p^N-1} (-1)^y w^{w_1w_2w_3w_4y} q^{hw_1w_2w_3w_4y} \\ &\times e^{[w_1w_2w_3w_4y+w_1w_2w_3w_4w_5x+w_5w_4w_2w_3i+w_5w_4w_1w_3j+w_5w_4w_1w_2k+w_5w_3w_1w_2s]_q t} \\ &= \lim_{N\to\infty} \sum_{l=0}^{w_5-1} \sum_{y=0}^{p^N-1} (-1)^{l+y} w^{w_1w_2w_3w_4(l+w_5y)} q^{hw_1w_2w_3w_4(l+w_5y)} \\ &\times e^{[w_1w_2w_3w_4(l+w_5y)+w_1w_2w_3w_4w_5x+w_5w_4w_2w_3i+w_5w_4w_1w_3j+w_5w_4w_1w_2k+w_5w_3w_1w_2s]_q t}. \end{aligned} \tag{5}$$

Taking

$$\sum_{i=0}^{w_1-1} \sum_{j=0}^{w_2-1} \sum_{k=0}^{w_3-1} \sum_{s=0}^{w_4-1} (-1)^{i+j+k+s} w^{w_5w_4w_2w_3i+w_5w_4w_1w_3j+w_5w_4w_1w_2k+w_5w_3w_1w_2s}$$
$$\times q^{h(w_5w_4w_2w_3i+w_5w_4w_1w_3j+w_5w_4w_1w_2k+w_5w_3w_1w_2s)}$$

on the both sides of Eq. (5) gives

$$\sum_{i=0}^{w_1-1}\sum_{j=0}^{w_2-1}\sum_{k=0}^{w_3-1}\sum_{s=0}^{w_4-1}(-1)^{i+j+k+s}w^{w_5w_4w_2w_3i+w_5w_4w_1w_3j+w_5w_4w_1w_2k+w_5w_3w_1w_2s} \tag{6}$$
$$\times q^{h(w_5w_4w_2w_3i+w_5w_4w_1w_3j+w_5w_4w_1w_2k+w_5w_3w_1w_2s)}\int_{\mathbb{Z}_p} w^{w_1w_2w_3w_4y}q^{hw_1w_2w_3w_4y}$$
$$\times e^{[w_1w_2w_3w_4y+w_1w_2w_3w_4w_5x+w_5w_4w_2w_3i+w_5w_4w_1w_3j+w_5w_4w_1w_2k+w_5w_3w_1w_2s]_q t}d\mu_{-1}(y)$$
$$=\lim_{N\to\infty}\sum_{i=0}^{w_1-1}\sum_{j=0}^{w_2-1}\sum_{k=0}^{w_3-1}\sum_{s=0}^{w_4-1}\sum_{l=0}^{w_5-1}\sum_{y=0}^{p^N-1}(-1)^{i+j+k+s+y+l}$$
$$\times w^{w_1w_2w_3w_4(l+w_5y)+w_5w_4w_2w_3i+w_5w_4w_1w_3j+w_5w_4w_1w_2k+w_5w_3w_1w_2s}$$
$$\times q^{h(w_1w_2w_3w_4(l+w_5y)+w_5w_4w_2w_3i+w_5w_4w_1w_3j+w_5w_4w_1w_2k+w_5w_3w_1w_2s)}$$
$$\times e^{[w_1w_2w_3w_4(l+w_5y)+w_1w_2w_3w_4w_5x+w_5w_4w_2w_3i+w_5w_4w_1w_3j+w_5w_4w_1w_2k+w_5w_3w_1w_2s]_q t}.$$

Observe that the equation (6) is invariant for any permutation $\sigma\in S_5$. Therefore, we obtain the following theorem.

Let $h\in\mathbb{Z}$, $w_i\in\mathbb{N}$ be a natural number which satisfies the condition $w_i\equiv 1(mod 2)$, in which $i\in\mathbb{Z}$ lies in $1\le i\le 5$ and $n\ge 0$. Then the following

$$\sum_{i=0}^{w_{\sigma(1)}-1}\sum_{j=0}^{w_{\sigma(2)}-1}\sum_{k=0}^{w_{\sigma(3)}-1}\sum_{s=0}^{w_{\sigma(4)}-1}(-1)^{i+j+k+s}$$
$$\times w^{w_{\sigma(5)}w_{\sigma(4)}w_{\sigma(2)}w_{\sigma(3)}i+w_{\sigma(5)}w_{\sigma(4)}w_{\sigma(1)}w_{\sigma(3)}j+w_{\sigma(5)}w_{\sigma(4)}w_{\sigma(1)}w_{\sigma(2)}k+w_{\sigma(5)}w_{\sigma(3)}w_{\sigma(1)}w_{\sigma(2)}s}$$
$$\times q^{h\left(w_{\sigma(5)}w_{\sigma(4)}w_{\sigma(2)}w_{\sigma(3)}i+w_{\sigma(5)}w_{\sigma(4)}w_{\sigma(1)}w_{\sigma(3)}j+w_{\sigma(5)}w_{\sigma(4)}w_{\sigma(1)}w_{\sigma(2)}k+w_{\sigma(5)}w_{\sigma(3)}w_{\sigma(1)}w_{\sigma(2)}s\right)}$$
$$\times\int_{\mathbb{Z}_p} w^{w_{\sigma(1)}w_{\sigma(2)}w_{\sigma(3)}w_{\sigma(4)}\left(l+w_{\sigma(5)}y\right)}q^{hw_{\sigma(1)}w_{\sigma(2)}w_{\sigma(3)}w_{\sigma(4)}\left(l+w_{\sigma(5)}y\right)}$$
$$\times\exp\left([w_{\sigma(1)}w_{\sigma(2)}w_{\sigma(3)}w_{\sigma(4)}y+w_{\sigma(1)}w_{\sigma(2)}w_{\sigma(3)}w_{\sigma(4)}w_{\sigma(5)}x\right.$$
$$+w_{\sigma(5)}w_{\sigma(4)}w_{\sigma(2)}w_{\sigma(3)}i+w_{\sigma(5)}w_{\sigma(4)}w_{\sigma(1)}w_{\sigma(3)}j$$
$$\left.+w_{\sigma(5)}w_{\sigma(4)}w_{\sigma(1)}w_{\sigma(2)}k+w_{\sigma(5)}w_{\sigma(3)}w_{\sigma(1)}w_{\sigma(2)}s]_q t\right)d\mu_{-1}(y)$$

holds true for any $\sigma\in S_5$.

By using Eq. (2), we have

$$[w_1w_2w_3w_4y+w_1w_2w_3w_4w_5x+w_5w_4w_2w_3i \tag{7}$$
$$+w_5w_4w_1w_3j+w_5w_4w_1w_2k+w_5w_3w_1w_2s]_q$$
$$=[w_1w_2w_3w_4]_q\left[y+w_5x+\frac{w_5}{w_1}i+\frac{w_5}{w_2}j+\frac{w_5}{w_3}k+\frac{w_5}{w_4}s\right]_{q^{w_1w_2w_3w_4}}.$$

From Eqs. (5) and (7), we derive

$$\int_{\mathbb{Z}_p} e^{[w_1w_2w_3w_4y+w_1w_2w_3w_4w_5x+w_5w_4w_2w_3i+w_5w_4w_1w_3j+w_5w_4w_1w_2k+w_5w_3w_1w_2s]_q t} \tag{8}$$
$$\times w^{w_1w_2w_3w_4y} q^{hw_1w_2w_3w_4y} d\mu_{-1}(y)$$
$$= \sum_{n=0}^{\infty} [w_1w_2w_3w_4]_q^n$$
$$\times \int_{\mathbb{Z}_p} w^{w_1w_2w_3w_4y} q^{hw_1w_2w_3w_4y}$$
$$\times \left[y + w_5x + \frac{w_5}{w_1}i + \frac{w_5}{w_2}j + \frac{w_5}{w_3}k + \frac{w_5}{w_4}s\right]_{q^{w_1w_2w_3ww_4}}^n d\mu_{-1}(y)\frac{t^n}{n!}$$
$$= \sum_{n=0}^{\infty} [w_1w_2w_3w_4]_q^n$$
$$\times \mathcal{E}_{n,q^{w_1w_2w_3w_4},w^{w_1w_2w_3w_4}}^{(h)} \left(w_5x + \frac{w_5}{w_1}i + \frac{w_5}{w_2}j + \frac{w_5}{w_3}k + \frac{w_5}{w_4}s\right)\frac{t^n}{n!}.$$

By Eq. (8), for $n \geq 0$,we have

$$\int_{\mathbb{Z}_p} [w_1w_2w_3w_4y + w_1w_2w_3w_4w_5x + w_5w_4w_2w_3i \tag{9}$$
$$\times +w_5w_4w_1w_3j + w_5w_4w_1w_2k + w_5w_3w_1w_2s]_q^n$$
$$\times w^{w_1w_2w_3w_4y} q^{hw_1w_2w_3w_4y} d\mu_{-1}(y)$$
$$= [w_1w_2w_3w_4]_q^n \mathcal{E}_{n,q^{w_1w_2w_3w_4},w^{w_1w_2w_3w_4}}^{(h)} \left(w_5x + \frac{w_5}{w_1}i + \frac{w_5}{w_2}j + \frac{w_5}{w_3}k + \frac{w_5}{w_4}s\right).$$

Thus, from Theorem 2 and Eq. (9), we have the following theorem.

Let $h \in \mathbb{Z}$, $w_i \in \mathbb{N}$ be a natural number which satisfies the condition $w_i \equiv 1(mod2)$, in which $i \in \mathbb{Z}$ lies in $1 \leq i \leq 5$ and $n \geq 0$. Hence, the following

$$\left[w_{\sigma(1)}w_{\sigma(2)}w_{\sigma(3)}w_{\sigma(4)}\right]_q^n \sum_{i=0}^{w_{\sigma(1)}-1} \sum_{j=0}^{w_{\sigma(2)}-1} \sum_{k=0}^{w_{\sigma(3)}-1} \sum_{s=0}^{w_{\sigma(4)}-1} (-1)^{i+j+k+s}$$
$$\times w^{w_{\sigma(5)}w_{\sigma(4)}w_{\sigma(2)}w_{\sigma(3)}i+w_{\sigma(5)}w_{\sigma(4)}w_{\sigma(1)}w_{\sigma(3)}j+w_{\sigma(5)}w_{\sigma(4)}w_{\sigma(1)}w_{\sigma(2)}k+w_{\sigma(5)}w_{\sigma(3)}w_{\sigma(1)}w_{\sigma(2)}s}$$
$$\times q^{h\left(w_{\sigma(5)}w_{\sigma(4)}w_{\sigma(2)}w_{\sigma(3)}i+w_{\sigma(5)}w_{\sigma(4)}w_{\sigma(1)}w_{\sigma(3)}j+w_{\sigma(5)}w_{\sigma(4)}w_{\sigma(1)}w_{\sigma(2)}k+w_{\sigma(5)}w_{\sigma(3)}w_{\sigma(1)}w_{\sigma(2)}s\right)}$$
$$\times \mathcal{E}_{n,q^{w_{\sigma(1)}w_{\sigma(2)}w_{\sigma(3)}w_{\sigma(4)}},w^{w_{\sigma(1)}w_{\sigma(2)}w_{\sigma(3)}w_{\sigma(4)}}}^{(h)}$$
$$\left(w_{\sigma(5)}x + \frac{w_{\sigma(5)}}{w_{\sigma(1)}}i + \frac{w_{\sigma(5)}}{w_{\sigma(2)}}j + \frac{w_{\sigma(5)}}{w_{\sigma(3)}}k + \frac{w_{\sigma(5)}}{w_{\sigma(4)}}s\right)$$

holds true for any $\sigma \in S_5$.

It is easily shown, by using the definition of $[x]_q$, that

$$\left[y+w_5x+\frac{w_5}{w_1}i+\frac{w_5}{w_2}j+\frac{w_5}{w_3}k+\frac{w_5}{w_4}s\right]^n_{q^{w_1w_2w_3w_4}} \tag{10}$$
$$=\sum_{m=0}^{n}\binom{n}{m}\left(\frac{[w_5]_q}{[w_1w_2w_3w_4]_q}\right)^{n-m}$$
$$\times\,[w_2w_3w_4i+w_1w_3w_4j+w_1w_2w_4k+w_1w_2w_3s]^{n-m}_{q^{w_5}}$$
$$\times\,q^{m(w_5w_4w_2w_3i+w_5w_4w_1w_3j+w_5w_4w_1w_2k+w_5w_3w_1w_2s)}\,[y+w_5x]^m_{q^{w_1w_2w_3w_4}}\,.$$

Taking $\int_{\mathbb{Z}_p}w^{w_1w_2w_3w_4y}q^{hw_1w_2w_3w_4y}d\mu_{-1}(y)$ on the both sides of Eq. (10) yields

$$\int_{\mathbb{Z}_p}w^{w_1w_2w_3w_4y}q^{hw_1w_2w_3w_4y} \tag{11}$$
$$\times\left[y+w_5x+\frac{w_5}{w_1}i+\frac{w_5}{w_2}j+\frac{w_5}{w_3}k+\frac{w_5}{w_4}s\right]^n_{q^{w_1w_2w_3w_4}}d\mu_{-1}(y)$$
$$=\sum_{m=0}^{n}\binom{n}{m}\left(\frac{[w_5]_q}{[w_1w_2w_3w_4]_q}\right)^{n-m}$$
$$\times\,[w_2w_3w_4i+w_1w_3w_4j+w_1w_2w_4k+w_1w_2w_3s]^{n-m}_{q^{w_5}}$$
$$\times\,q^{m(w_5w_4w_2w_3i+w_5w_4w_1w_3j+w_5w_4w_1w_2k+w_5w_3w_1w_2s)}\mathcal{E}^{(h)}_{m,q^{w_1w_2w_3w_4},w^{w_1w_2w_3w_4}}\left(w_5x\right).$$

In view of the Eq. (11), we acquire

$$[w_1w_2w_3w_4]^n_q\sum_{i=0}^{w_1-1}\sum_{j=0}^{w_2-1}\sum_{k=0}^{w_3-1}\sum_{s=0}^{w_4-1}(-1)^{i+j+k+s} \tag{12}$$
$$\times\,w^{w_5w_4w_2w_3i+w_5w_4w_1w_3j+w_5w_4w_1w_2k+w_5w_3w_1w_2s}$$
$$\times\,q^{h(w_5w_4w_2w_3i+w_5w_4w_1w_3j+w_5w_4w_1w_2k+w_5w_3w_1w_2s)}$$
$$\times\int_{\mathbb{Z}_p}w^{w_1w_2w_3w_4y}q^{hw_1w_2w_3w_4y}$$
$$\times\left[y+w_5x+\frac{w_5}{w_1}i+\frac{w_5}{w_2}j+\frac{w_5}{w_3}k+\frac{w_5}{w_4}s\right]^n_{q^{w_1w_2w_3w_4}}d\mu_{-1}(y)$$
$$=\sum_{m=0}^{n}\binom{n}{m}[w_1w_2w_3w_4]^m_q\,[w_5]^{n-m}_q$$
$$\times\,\mathcal{E}^{(h)}_{m,q^{w_1w_2w_3w_4},w^{w_1w_2w_3w_4}}\left(w_5x\right)U_{n,q^{w_5},w^{w_5}}(w_1,w_2,w_3,w_4\mid m),$$

where

$$U_{n,q,w}(w_1, w_2, w_3, w_4 \mid m) \tag{13}$$
$$= \sum_{i=0}^{w_1-1} \sum_{j=0}^{w_2-1} \sum_{k=0}^{w_3-1} \sum_{s=0}^{w_4-1} (-1)^{i+j+k+s} w^{w_2w_3w_4i+w_1w_3w_4j+w_1w_2w_4k+w_1w_2w_3s}$$
$$\times q^{(m+h)(w_2w_3w_4i+w_1w_3w_4j+w_1w_2w_4k+w_1w_2w_3s)}$$
$$\times [w_2w_3w_4i + w_1w_3w_4j + w_1w_2w_4k + w_1w_2w_3s]_q^{n-m}.$$

Hereby, by Eq. (13), we arrive at the following theorem.

Let $h \in \mathbb{Z}$, $w_i \in \mathbb{N}$ be a natural number which satisfies the condition $w_i \equiv 1(mod2)$, in which $i \in \mathbb{Z}$ lies in $1 \le i \le 5$. For $n \ge 0$, the following

$$\sum_{m=0}^{n} \binom{n}{m} \left[w_{\sigma(1)}w_{\sigma(2)}w_{\sigma(3)}w_{\sigma(4)}\right]_q^m \left[w_{\sigma(5)}\right]_q^{n-m}$$
$$\times \mathcal{E}^{(h)}_{m,q^{w_{\sigma(1)}w_{\sigma(2)}w_{\sigma(3)}w_{\sigma(4)}},w^{w_{\sigma(1)}w_{\sigma(2)}w_{\sigma(3)}w_{\sigma(4)}}}\left(w_{\sigma(5)}x\right)$$
$$\times U_{n,q^{w_{\sigma(5)}},w^{w_{\sigma(5)}}}(w_{\sigma(1)}, w_{\sigma(2)}, w_{\sigma(3)}, w_{\sigma(4)} \mid m)$$

holds true for some $\sigma \in S_5$.

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