Gen. Math. Notes, Vol. 24, No. 1, September 2014, pp.17-24
ISSN 2219-7184; Copyright ©ICSRS Publication, 2014
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On the Numerical Solution of Tenth and Twelfth Order Boundary Value Problems Using Weighted Residual Method (WRM)

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(Received: 16-5-12 / Accepted: 24-2-14)

Abstract

In this paper, numerical solution of tenth and twelfth order linear and non linear boundary value problems are presented using weighted residual via partition method (WRM). A trial function is assumed which is made to satisfy the boundary conditions given, and used to generate the residual to be minimised. To investigate the effectiveness of the method, numerical examples were considered which were compared with both the exact solution and the solution obtained by other methods in the literature. The proposed method proves very accurate, better, efficient and appropriate.

Keywords: *Tenth and Twelfth order boundary value problems, Weighted residual method, Trial function Partition method.*

1 Introduction

Tenth and Twelfth order boundary value problems arise in the study of fluid dynamics, Hydromagnetic stability, beam and long wave theory, engineering and applied sciences. Owing to their mathematical significance and applications, several methods such as finite difference method, polynomial spline and decomposition method have been used to solve these set of problems.

In S.T Mohyud-din etal[7], Exp-function method which was developed by He and Wu was used to solve this class of problems, S.T. Mohyud-din[8] and H. Mirmoradi etal[5] used Homotopy perturbation method to solve Tenth order and Twelfth order boundary value problems respectively. Detail solutions of lower order boundary value problems are also found in [1,2,3,4] and other references therein.

In this article, a trial function of the form

$$y = \sum_{i=0}^{n} a_i x^i \tag{1}$$

is used in the weighted residual method[6] where the domain $[c-d]$ are subdivided into smaller sub domain within which the residual obtained is minimised using Simpson $\frac{1}{3}$ quadrature.

2 Analysis of the Method

Suppose we have a differential equation

$$L[u(x)] = f \quad in \quad the \quad domain \quad \Omega \tag{2}$$

$$B_\mu[u] = \Omega \quad on \quad \partial\Omega \tag{3}$$

where $L[u]$ denotes a general differential operator (linear or non-linear) involving spatial derivatives of dependent variable u, f is a known function of position, $B_\mu[u]$ represents the appropriate number of boundary conditions and Ω is the domain with the boundary $\partial\Omega$

The following steps are followed in solving this type of problems:

- We assumed a trial function of the form in equation (1).
- Substitute the trial function into the differential equation to generate the residual.
- The domain within $[0-1]$ is sundivided to $[0-0.1]$, $[0.1-0.2]$, $[0.2-0.3]$, ...$[0.9-1.0]$
- Impose the boundary conditions(3) on the trial function in step 1 to generate set of equations(10 equations for tenth order and 12 equations for twelfth order), including the condition that the residual should be zero at all points.

- Minimised the residual in step 2 by integrating it within the sub-division points in step 3 using Simpson $\frac{1}{3}$ rule which gives another sets of equations.

- Number of equations obtained equals the number of constants to be determined and so solve the equations to obtain the constants $a_i, \ \ i = 0..22$ which are substituted back into the trial function and hence the solution.

3 Numerical Examples

Example1[8]:

$$y^{(x)} = -8e^x + y''(x), \ \ 0 < x < 1 \tag{4}$$

subject to the boundary conditions $y(0) = 1, \ \ y'(0) = 0, \ \ y''(0) = -1, \ \ y'''(0) = -2, \ \ y^{(iv)} = -3, \ \ y(1) = 0, \ \ y'(1) = -e, \ \ y''(1) = -2e, \ \ y'''(1) = -3e, \ \ y^{(iv)} = -4e$

Exact solution is

$$y(x) = (1 - x)e^x$$

Using procedure itemised in section two we have

$$y = 1.0 - 0.500000000000000\, x^2 - 0.333333333333334\, x^3 - 0.125000000000000\, x^4 - 0.0333333333331242\, x^5 - 0.00694444444514969\, x^6 - 0.00119047618956445\, x^7 - 0.000173611111164485\, x^8 - 0.0000220458552624184\, x^9 - 0.00000248015873015873\, x^{10} - 0.0000002505209782\, x^{11} - 0.00000002296500179\, x^{12} - 0.000000001925770536\, x^{13} - 0.0000000001507279581\, x^{14} - 9.622351157 \times 10^{-12}\, x^{15} - 1.089487853 \times 10^{-12}\, x^{16}$$

Table 1 shows the results of weighted residual method and the error obtained for example 1 when compared with the exact solution.

Table 1

x	Exact	WRM	WRM error
0.0	1.0	1.0	0
0.1	0.994653826268085	0.994653826268084	$1. * 10^{-15}$
0.2	0.977122206528136	0.977122206528137	$1. * 10^{-15}$
0.3	0.944901165303200	0.944901165303202	$2. * 10^{-15}$
0.4	0.895094818584762	0.895094818584764	$2. * 10^{-15}$
0.5	0.824360635350065	0.824360635350065	0
0.6	0.728847520156204	0.728847520156205	$1. * 10^{-15}$
0.7	0.604125812241144	0.604125812241145	$1. * 10^{-15}$
0.8	0.445108185698494	0.445108185698494	0
0.9	0.245960311115695	0.245960311115697	$2. * 10^{-15}$
1.0	0	$-6.892119075 * 10^{-17}$	$6.892119075 * 10^{-17}$

Example2 [7,8]:

$$y^{(x)}(x) = e^{-x}y^2(x), \quad 0 < x < 1 \tag{5}$$

subject to the boundary conditions

$$y(0) = 1, \quad y''(0) = y^{(iv)}(0) = y^{(vi)}(0) = y^{(viii)}(0) = 1,$$

$$y(1) = e, \quad y''(1) = y^{(iv)}(1) = y^{(vi)}(1) = y^{(viii)}(1) = e$$

Exact solution is

$$y(x) = e^x$$

Using procedure itemised in section two we have

$$y = 1.0+0.999999999999329\,x+0.500000000000000\,x^2+0.166666666667767\,x^3+ 0.0416666666666667\,x^4+0.00833333333281123\,x^5+0.00138888888888889\,x^6+ 0.000198412698519703\,x^7+0.0000248015873015873\,x^8+0.00000275573191352611\,x^9+ 0.0000002755731922\,x^{10}+0.00000002505210192\,x^{11}+0.000000002087710502\,x^{12}+ 0.0000000001605099908\,x^{13}+1.156925448\times 10^{-11}\,x^{14}+6.982160464\times 10^{-13}\,x^{15}+ 7.074894973\times 10^{-14}\,x^{16}$$

Table 2 shows the results of weighted residual method and the error obtained for example 2 when compared with the exact solution.

Table 2

x	Exact	WRM	WRM error
0.0	1.0	1.0	0
0.1	1.10517091807565	1.10517091807558	$7. * 10^{-14}$
0.2	1.22140275816017	1.22140275816005	$1.2 * 10^{-13}$
0.3	1.34985880757600	1.34985880757583	$1.7 * 10^{-13}$
0.4	1.49182469764127	1.49182469764109	$1.8 * 10^{-13}$
0.5	1.64872127070013	1.64872127069991	$2.2 * 10^{-13}$
0.6	1.82211880039051	1.82211880039032	$1.9 * 10^{-13}$
0.7	2.01375270747048	2.01375270747031	$1.7 * 10^{-13}$
0.8	2.22554092849247	2.22554092849235	$1.2 * 10^{-13}$
0.9	2.45960311115695	2.45960311115689	$6. * 10^{-14}$
1.0	2.71828182845905	2.71828182845905	0

Example3 [5]:

$$y^{(xii)}(x) + xy(x) = -(120 + 23x + x^3)e^x \tag{6}$$

subject to the conditions

$y(0) = 0,\ \ y'(0) = 1,\ \ y''(0) = 0,\ \ y'''(0) = -3,\ \ y(iv)(0) = -8,\ \ y^{(v)}(0) = -15,$

$y(1) = 0,\ \ y'(1) = -e,\ \ y''(1) = -4e,\ \ y'''(1) = -9e,\ \ y(iv)(1) = -16e,\ \ y^{(v)}(1) = -25e$

Exact solution is

$$y(x) = x(1-x)e^x$$

Using procedure itemised in section two we have

$y = 1.0\,x - 0.500000000000001\,x^3 - 0.333333333333334\,x^4 - 0.125000000000000\,x^5 -$

$0.0333333333323412\,x^6 - 0.00694444444887511\,x^7 - 0.00119047618246354\,x^8 -$
$0.000173611118420438\,x^9 - 0.00002204585202134\,x^{10} - 0.0000024801593513851 2\,x^{11} -$

$0.0000002505210839\,x^{12} - 0.0000000229644187 7\,x^{13} - 0.000000001927149499\,x^{14} -$

$0.0000000001489911209\,x^{15} - 1.084355228 \times 10^{-11}\,x^{16} - 6.352723637 \times 10^{-13}\,x^{17} -$

$6.985565274 \times 10^{-14}\,x^{18}$

Table 3 shows the results of weighted residual method and the error obtained for example 3 when compared with the exact solution.

Table 3

x	Exact	WRM	WRM error
0	0	0	0
0.1	0.0994653826268085	0.0994653826268084	$1. * 10^{-16}$
0.2	0.195424441305627	0.195424441305628	$1. * 10^{-15}$
0.3	0.283470349590960	0.283470349590960	0
0.4	0.358037927433905	0.358037927433907	$2. * 10^{-15}$
0.5	0.412180317675032	0.412180317675033	$1. * 10^{-15}$
0.6	0.437308512093722	0.437308512093723	$1. * 10^{-15}$
0.7	0.422888068568801	0.422888068568802	$1. * 10^{-15}$
0.8	0.356086548558795	0.356086548558796	$1. * 10^{-15}$
0.9	0.221364280004126	0.221364280004127	$1. * 10^{-15}$
1.0	0	$6.33029146870 * 10^{-17}$	$6.33029146870 * 10^{-17}$

Example 4 [7]:

$$y^{(xii)}(x) = \frac{1}{2}e^{-x}y(x)^2 \tag{7}$$

subject to the conditions

$$y(0) = y''(0) = y^{(iv)}(0) = y^{(vi)}(0) = y^{(xiii)}(0) = y^{(x)}(0) = 2$$

$$y(1) = y''(1) = y^{(iv)}(1) = y^{(vi)}(1) = y^{(xiii)}(1) = y^{(x)}(1) = 2e$$

Exact solution is

$$y(x) = 2e^x$$

Using procedure itemised in section two we have

$y = 2.0+2.00000000000001\,x+1.0\,x^2+0.333333333333335\,x^3+0.0833333333333333\,x^4+0.0166666666666667\,x^5+0.00277777777777778\,x^6+0.000396825396825398\,x^7+0.0000496031746031746\,x^8+0.00000551146384479720\,x^9+0.0000005511463845\,x^{10}+0.00000005010421677\,x^{11}+0.000000004175351398\,x^{12}+0.0000000003211808767\,x^{13}+2.294149120\times10^{-11}\,x^{14}+1.529432729\times10^{-12}\,x^{15}+9.558958162\times10^{-14}\,x^{16}+5.622865620\times10^{-15}\,x^{17}+3.124328437\times10^{-16}\,x^{18}+1.640662945\times10^{-17}\,x^{19}+8.392600419\times10^{-19}\,x^{20}+3.351645334\times10^{-20}\,x^{21}+2.846489658\times10^{-21}\,x^{22}$

Table 4 shows the results of weighted residual method and the error obtained for example 4 when compared with the exact solution.

Table 4

x	Exact	WRM	WRM error
0	2.0	2.0	0
0.1	2.21034183615130	2.21034183615130	0
0.2	2.44280551632034	2.44280551632034	0
0.3	2.69971761515200	2.69971761515199	$1. * 10^{-14}$
0.4	2.98364939528254	2.98364939528254	0
0.5	3.29744254140026	3.29744254140025	$1. * 10^{-14}$
0.6	3.64423760078102	3.64423760078104	$2. * 10^{-14}$
0.7	4.02750541494096	4.02750541494097	$1. * 10^{-14}$
0.8	4.45108185698494	4.45108185698495	$1. * 10^{-14}$
0.9	4.91920622231390	4.91920622231390	0
1.0	5.43656365691810	5.43656365691810	0

4 Conclusion

This paper presents an account of how weighted residual via partition methos is used to solve tenth and twelfth order two point boundary value problems with a single trial function for different problems. Computational procedure and results of numerical examples considered shows that the method is simple, effective and straightforward, and hence make the method suitable for this class of problems.

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