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Intuitionistic Fuzzy GPR-Open and GPR-Closed Mappings

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Abstract

In this paper we introduced new type of closed and open mapping called intuitionistic fuzzy gpr-open and intuitionistic fuzzy gpr-closed mappings in intuitionistic fuzzy topology and obtain some of its basic properties and interrelation with such type of other function.

Keywords: *Intuitionistic fuzzy gpr-closed sets, Intuitionistic fuzzy gpr-open sets, Intuitionistic fuzzy gpr-continuous mappings, Intuitionistic fuzzy gpr-open mappings and intuitionistic fuzzy gpr-closed mappings.*

1 Introduction

Mappings play an important role in study of modern mathematics, especially in Topology and Functional Analysis. Closed and open mapping are one such mapping which are studied for different type of closed sets by various mathematicians for the past many years. The concept of intuitionistic fuzzy sets was introduced by Atanassov [1] as a generalization of fuzzy sets.

In 1997 Coker [5] introduced the concept of intuitionistic fuzzy topological spaces. Thakur and Chaturvedi [9] introduced the notion of intuitionistic fuzzy generalized closed sets in intuitionistic fuzzy topological spaces. Since then many mathematicians studied different forms of intuitionistic fuzzy g-closed sets and related topological properties. Recently the authors [17] of the paper introduced the concept of intuitionistic fuzzy gpr-closed sets. In the present paper we introduce weak form of iuntuitionistic fuzzy open and closed mappings called intuitionistic fuzzy gpr-open and intuitionistic fuzzy gpr-closed mappings and obtain some of their characterization and properties.

2 Preliminaries

Definition 2.1: *An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space $(X,\Im)$ is called:*

(a) Intuitionistic fuzzy g-closed if cl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy open.[9]
(b) Intuitionistic fuzzy rg-closed if cl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy regular open.[12]
(c) Intuitionistic fuzzy w-closed if cl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy semi open.[15]
(d) Intuitionistic fuzzy rw-closed if cl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy regular semi open.[16]
(e) Intuitionistic fuzzy gpr-closed if pcl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy regular open.[17]

The complements of the above mentioned closed set are their respective open sets.

Definition 2.2 [9]: *An intuitionistic fuzzy topological space $(X,\Im)$ is called intuitionistic fuzzy $T_{1/2}$ - Space if every intuitionistic fuzzy g- closed set in X is intuitionistic fuzzy closed in X.*

Definition 2.3 [12]: *An intuitionistic fuzzy topological space $(X,\Im)$ is called intuitionistic fuzzy $rT_{1/2}$ - Space if every intuitionistic fuzzy rg- closed set in X is intuitionistic fuzzy regular closed in X.*

Definition 2.4 [17]: *The gpr- interior and gpr- closure of an intuitionistic fuzzy set A of a intuitionistic fuzzy topological space $(X,\Im)$ respectively denoted by gprint(A) and gprcl(A) are defined as follows:*

$$\text{gprint}(A) = \cup \{ V : V\subseteq A , V \text{ is intuitionistic fuzzy gpr- open}\}$$
$$\text{gprcl}(A) = \cap \{ F : A\subseteq F, F \text{ is intuitionistic fuzzy gpr- closed}\}$$

Definition 2.5 [6]: *Let $(X,\Im)$ and (Y, σ) be two intuitionistic fuzzy topological spaces and let $f: X\to Y$ be a function. Then f is said to be*

(a) Intuitionistic fuzzy continuous if the pre image of each intuitionistic fuzzy open set of Y is an intuitionistic fuzzy open set in X.
(b) Intuitionistic fuzzy closed if the image of each intuitionisic fuzzy closed set in X is an intuitionistic fuzzy closed set in Y.
(c) Intuitionistic fuzzy open if the image of each intuitionisic fuzzy open set in X is an intuitionistic fuzzy open set in Y.

Definition 2.6: *Let (X,ℑ) and (Y, σ) be two intuitionistic fuzzy topological spaces and let f: X→Y be a function. Then f is said to be*

(a) Intutionistic fuzzy g-continuous if the pre image of every intuitionistic fuzzy closed set in Y is intuitionistic fuzzy g –closed in X.[10]
(b) Intuitionistic fuzzy g-open if image of every open set of X is intuitionistic fuzzy g-open in Y.[14]
(c) Intuitionistic fuzzy g-closed if image of every closed set of X is intuitionistic fuzzy g-closed in Y.[14]
(d) Intutionistic fuzzy w-continuous if the pre image of every intuitionistic fuzzy closed set in Y is intuitionistic fuzzy w –closed in X.[15]
(e) Intuitionistic fuzzy w-open if image of every open set of X is intuitionistic fuzzy w-open in Y.[15]
(f) Intuitionistic fuzzy w-closed if image of every closed set of X is intuitionistic fuzzy w-closed in Y.[15]
(g) Intutionistic fuzzy rw-continuous if the pre image of every intuitionistic fuzzy closed set in Y is intuitionistic fuzzy rw –closed in X.[16]
(h) Intuitionistic fuzzy rw-open if image of every open set of X is intuitionistic fuzzy rw-open in Y.[16]
(i) Intuitionistic fuzzy rw-closed if image of every closed set of X is intuitionistic fuzzy rw-closed in Y.[16]
(j) Intutionistic fuzzy rg-continuous if the pre image of every intuitionistic fuzzy closed set in Y is intuitionistic fuzzy rg –closed in X.[13]
(k) Intuitionistic fuzzy rg-open if image of every open set of X is intuitionistic fuzzy rg-open in Y.[13]
(l) Intuitionistic fuzzy rg-closed if image of every closed set of X is intuitionistic fuzzy w-closed in Y.[13]
(m) intuitionistic fuzzy R mapping if the pre image of each intuitionistic fuzzy regular open set of Y is an intuitionistic fuzzy regular open set in X.[12]
(n) Intutionistic fuzzy gpr-continuous if the pre image of every intuitionistic fuzzy closed set in Y is intuitionistic fuzzy gpr –closed in X.[17]
(o) Intuitionistic fuzzy gpr- irresolute if pre image of every intuitionistic fuzzy gpr-closed set in Y is intuitionistic fuzzy gpr –closed in X.[17]

Remark 2.1 [14]: *Every intuitionistic fuzzy open (intuitionistic fuzzy closed) mapping is intuitionistic fuzzy g-open (intuitionistic fuzzy g-closed), but the converse may not be true.*

Remark 2.2 [15]: *Every intuitionistic fuzzy w- open (intuitionistic fuzzy w-closed) mapping is intuitionistic fuzzy g-open (intuionstic fuzzy g-closed), but the converse may not be true.*

Remark 2.3 [16]: *Every intuitionistic fuzzy w- open (intuitionistic fuzzy w-closed) mapping is intuitionistic fuzzy rw-open (intuionstic fuzzy rw-closed), but the converse may not be true.*

Remark 2.4 [16]: *Every intuitionistic fuzzy rw- open (intuitionistic fuzzy rw-closed) mapping is intuitionistic fuzzy rg-open (intuionstic fuzzy rg-closed), but the converse may not be true.*

Remark 2.5 [14]: *Every intuitionistic fuzzy g- open (intuitionistic fuzzy g-closed) mapping is intuitionistic fuzzy rg-open (intuionstic fuzzy rg-closed), but the converse may not be true.*

3 Intuitionistic Fuzzy GPR-Open Mappings

Definition 3.1: *A mapping f: (X, $\Im$)→(Y, σ) is intuitionistic fuzzy gpr-open if image of every intuitionistic fuzzy open set of X is intuitionistic fuzzy gpr-open set in Y.*

Remark 3.1: *Every intuitionistic fuzzy open mapping is intuitionistic fuzzy gpr-open but converse may not be true. For,*

Example 3.1: *Let X = {a, b}, Y = {x, y} and the intuitionistic fuzzy set U and V are defined as follows:*

U= {< a, 0.3, 0.6 >, <b, 0.4, 0.6 >}
V = {< x, 0.5. 0.4 >, < y ,0.6,0.3>}

Then $\Im = \{\tilde{0}, U, \tilde{1}\}$ and $\sigma = \{\tilde{0}, V, \tilde{1}\}$ be intuitionistic fuzzy topologies on *X* and *Y* respectively. Then the mapping $f : (X, \Im) \to (Y, \sigma)$ defined by $f(a) = x$ and $f(b) = y$ is intuitionistic fuzzy gpr-open but it is not intuitionistic fuzzy open.

Remark 3.2: *Every intuitionistic fuzzy w-open mapping is intuitionistic fuzzy gpr-open but converse may not be true. For,*

Example 3.2: *Let X = {a, b}, Y = {x, y} and the intuitionistic fuzzy set U and V are defined as follows:*

U= {< a, 0.7, 0.2 >, <b, 0.8, 0.1 >}
V = {< x, 0.7. 0.2 >, < y ,0.6,0.3>}

Then $\Im = \{\tilde{0}, U, \tilde{1}\}$ and $\sigma = \{\tilde{0}, V, \tilde{1}\}$ be intuitionistic fuzzy topologies on X and Y respectively. Then the mapping f: $(X, \Im) \rightarrow (Y, \sigma)$ defined by $f(a)$ = x and $f(b) = y$ is intuitionistic fuzzy gpr-open but it is not intuitionistic fuzzy w-open.

Remark 3.3: Every intuitionistic fuzzy rw-open mapping is intuitionistic fuzzy gpr-open but converse may not be true. For,

Example 3.3: *Let X = {a, b, c, d, e} Y = { p, q ,r, s, t} and intuitionistic fuzzy sets O, U, V, W defined as follows*

O = { < a,0.9,0.1> ,<b,0.8,0.1>,<c,0,1>,<d,0,1> <e,0,1>}

U = { < a,0,1,> ,<b,0,1>,<c,0.8,0.1>,<d,0.7,0.2> <e,0,1>}

V = { < a,0.9,0.1> ,<b,0.8,0.1>,<c,0.8,0.1>, <d,0.7,0.2>,<e,0,1>}

W = {<p, 0.9, 0.1 >, <q ,0, 1 >, <r ,0, 1>, <s, 0, 1>, <t ,0,1>}

Let $\Im = \{\tilde{0}, O, U, V, \tilde{1}\}$ and $\sigma = \{\tilde{0}, W, \tilde{1}\}$ be an intuitionistic fuzzy topology on X and Y respectively. Then the mapping f: $(X, \Im) \rightarrow (Y, \sigma)$ defined by $f(a) = p$, $f(b) = q$, $f(c)$ = r, $f(d) = s$, $f(e) = t$ is intuitionistic fuzzy gpr-open but it is not intuitionistic fuzzy rw-open.

Remark 3.4: *Every intuitionistic fuzzy rg-open mapping is intuitionistic fuzzy gpr-open but converse may not be true. For,*

Example 3.4: *Let X = {a, b, c, d, e} Y = {p, q, r, s, t} and intuitionistic fuzzy sets O, U, V, W defined as follows*

O = { < a,0.9,0.1> ,<b,0.8,0.1>,<c,0,1>,<d,0,1> <e,0,1>}

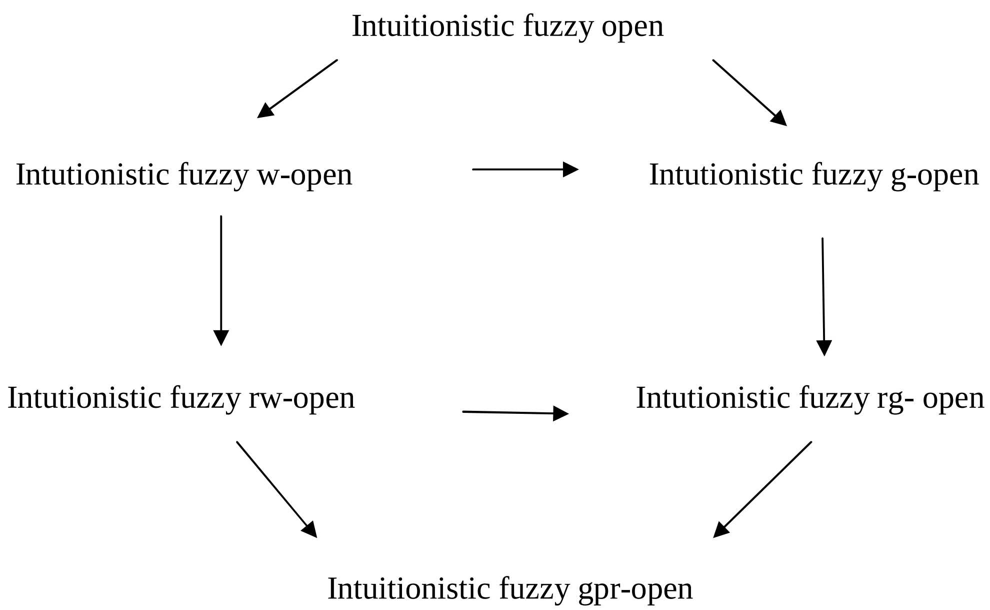
U = { < a,0,1,> ,<b,0,1>,<c,0.8,0.1>,<d,0.7,0.2> <e,0,1>}

V = { < a,0.9,0.1> ,<b,0.8,0.1>,<c,0.8,0.1>, <d,0.7,0.2>,<e,0,1>}

W = {<p, 0.9, 0.1 > ,<q ,0, 1 >, <r ,0, 1>, <s, 0, 1>, <t ,0,1>}

Let $\Im = \{ \tilde{\mathbf{0}}, O, U, V, \tilde{1}\}$ and $\sigma = \{\tilde{0}, W, \tilde{1}\}$ be an intuitionistic fuzzy topology on X and Y respectively. Then the mapping f: $(X, \Im) \rightarrow (Y, \sigma)$ defined by $f(a) = p$, $f(b) = q$, $f(c) = r$, $f(d) = s$, $f(e) = t$ is intuitionistic fuzzy gpr-open but it is not intuitionistic fuzzy rg-open.

Remark 3.5: *From the above discussion and known results we have the following diagram of implication:*



Theorem 3.1: *A mapping* $f : (X, \Im) \rightarrow (Y, \sigma)$ *is intuitionistic fuzzy gpr-open if and only if for every intuitionisic fuzzy set* U *of* X $f(int(U)) \subseteq gprint(f(U))$.

Proof:

Necessity: Let f be an intuitionistic fuzzy gpr-open mapping and U is an intuitionistic fuzzy open set in X. Now $\text{int}(U) \subseteq U$ which implies that $f\,(\text{int}(U) \subseteq f(U)$. Since f is an intuitionistic fuzzy gpr-open mapping, $f(\text{int}(U)$ is intuitionistic fuzzy gpr-open set in Y such that $f(\text{int}(U) \subseteq f(U)$ therefore $f(\text{int}(U) \subseteq \text{gprint}\, f(U)$.

Sufficiency: For the converse suppose that U is an intuitionistic fuzzy open set of X. Then $f\,(U) = f(\text{int}(U)) \subseteq \text{gprint}\, f(U)$. But $\text{gprint}\,(f(U)) \subseteq f(U)$. Consequently $f(U) = \text{gprint}\,(U)$ which implies that $f(U)$ is an intuitionistic fuzzy gpr-open set of Y and hence f is an intuitionistic fuzzy gpr-open.

Theorem 3.2: *If* $f : (X, \Im) \rightarrow (Y, \sigma)$ *is an intuitionistic fuzzy gpr-open mapping then* $int\,(f^{-1}(G)) \subseteq f^{-1}(gprint(G))$ *for every intuitionistic fuzzy set* G *of* Y.

Proof: Let G is an intuitionistic fuzzy set of Y. Then $\text{int}\, f^{-1}(G)$ is an intuitionistic fuzzy open set in X. Since f is intuitionistic fuzzy gpr-open $f(\text{int}\, f^{-1}(G))$ is intuitionistic fuzzy gpr-open in Y and hence $f\,(\text{int}\, f^{-1}(G)\,) \subseteq \text{gprint}(f(\,f^{-1}(G)) \subseteq \text{gprint}\,(G)$. Thus $\text{int}(\,f^{-1}(G\,)) \subseteq f^{-1}(\text{gprint}(G))$.

Theorem 3.3: *A mapping* $f : (X, \Im) \rightarrow (Y, \sigma)$ *is intuitionistic fuzzy gpr-open if and only if for each intuitionistic fuzzy set* S *of* Y *and for each intuitionistic fuzzy closed set* U *of* X *containing* $f^{-1}(S)$ *there is an intuitionistic fuzzy gpr-closed* V *of* Y *such that* $S \subseteq V$ *and* $f^{-1}(V) \subseteq U$.

Proof:

Necessity: Suppose that f is an intuitionistic fuzzy gpr- open mapping. Let S be the intuitionistic fuzzy closed set of Y and U is an intuitionistic fuzzy closed set of X such that $f^{-1}(S) \subseteq U$. Then $V = (f^{-1}(U^c))^c$ is intuitionistic fuzzy gpr- closed set of Y such that $f^{-1}(V) \subseteq U$.

Sufficiency: For the converse suppose that F is an intuitionistic fuzzy open set of X. Then $f^{-1}((f(F))^c \subseteq F^c$ and F^c is intuitionistic fuzzy closed set in X. By hypothesis there is an intuitionistic fuzzy gpr-closed set V of Y such that $(f(F))^c \subseteq V$ and $f^{-1}(V) \subseteq F^c$. Therefore $F \subseteq (f^{-1}(V))^c$. Hence $V^c \subseteq f(F) \subseteq f((f^{-1}(V))^c) \subseteq V^c$ which implies $f(F) = V^c$. Since V^c is intuitionistic fuzzy gpr-open set of Y. Hence $f(F)$ is intuitionistic fuzzy gpr-open in Y and thus f is intuitionistic fuzzy gpr-open mapping.

Theorem 3.4: *A mapping $f : (X, \Im) \to (Y, \sigma)$ is intuitionistic fuzzy gpr-open if and only if $f^{-1}(gprcl(B) \subseteq cl(f^{-1}(B))$ for every intuitionistic fuzzy set B of Y.*

Proof:

Necessity: Suppose that f is an intuitionistic fuzzy gpr-open mapping. For any intuitionistic fuzzy set B of Y, $f^{-1}(B) \subseteq \mathrm{cl}(f^{-1}(B))$. Therefore by theorem 3.3 there exists an intuitionistic fuzzy gpr-closed set F in Y such that $B \subseteq F$ and $f^{-1}(F) \subseteq \mathrm{cl}(f^{-1}(B))$. Therefore we obtain that $f^{-1}(\mathrm{gprcl}(B)) \subseteq f^{-1}(\mathrm{F}) \subseteq \mathrm{cl}(f^{-1}(B))$.

Sufficiency: For the converse suppose that B is an intuitionistic fuzzy set of Y and F is an intuitionistic fuzzy closed set of X containing $f^{-1}(\mathrm{B})$. Put W= cl(B), then we have B $\subseteq$ W and W is gpr-closed and $f^{-1}(\mathrm{W}) \subseteq \mathrm{cl}(f^{-1}(\mathrm{B})) \subseteq \mathrm{F}$. Then by theorem 3.3 f is intuitionistic fuzzy gpr-open mapping.

Theorem 3.5: *If $f: (X, \Im) \to (Y, \sigma)$ and $g: (Y, \sigma) \to (Z, \mu)$ be two intuitionistic fuzzy mappings and $gof : (X,\Im) \to (Z,\mu)$ is intuitionistic fuzzy gpr-open. If $g :(Y, \sigma) \to (Z, \mu)$ is intuitionistic fuzzy gpr-irresolute then $f : (X, \Im) \to (Y, \sigma)$ is intuitionistic fuzzy gpr-open mapping.*

Proof: Let H be an intuitionistic fuzzy open set of intuitionistic fuzzy topological space $(X,\Im)$. Then $(go\,f)$ (H) is intuitionistic fuzzy gpr-open set of Z because gof is intuitionistic fuzzy gpr-open mapping. Now since g: $(Y, \sigma) \to (Z, \mu)$ is intuitionistic fuzzy gpr-irresolute and $(go\,f)$ (H) is intuitionistic fuzzy gpr-open set of Z therefore $g^{-1}(gof(\mathrm{H})) = f(\mathrm{H})$ is intuitionistic fuzzy gpr-open set in intuitionistic fuzzy topological space Y. Hence f is intuitionistic fuzzy gpr-open mapping.

Theorem 3.6: *If f: (X, ℑ) →(Y, σ) is intuitionistic fuzzy open and g: (Y, σ) → (Z, μ) is intuitionistic fuzzy gpr-open mappings then gof : (X,ℑ) → (Z, μ) is intuitionistic fuzzy gpr-open.*

Proof: Let H be an intuitionistic fuzzy open set of intuitionistic fuzzy topological space (X,ℑ). Then *f* (H) is intuitionistic fuzzy-open set of Y because *f* is intuitionistic fuzzy open mapping. Now since *g* : (Y, σ) →(Z, μ) is intuitionistic fuzzy gpr–open, *g(f*(H)) = (*go f*) (H) is intuitionistic fuzzy gpr-open set of Z. Hence *gof* is intuitionistic fuzzy gpr-open mapping.

Theorem 3.7: *If f: (X, ℑ)→(Y, σ) is intuitionistic fuzzy rg-open and g:(Y, σ) → (Z, μ) is intuitionistic fuzzy gpr-open mappings such that Y is intuitionistic fuzzy* $rT_{1/2}$*- space then gof : (X,ℑ) →(Z,μ) is intuitionistic fuzzy gpr-open.*

Proof: Let H be an intuitionistic fuzzy open set of intuitionistic fuzzy topological space (X,ℑ). Then *f* (H) is intuitionistic fuzzy rg-open set of Y because *f* is intuitionistic fuzzy rg-open mapping. Now since Y is intuitionistic fuzzy $rT_{1/2}$- space, *f* (H) is intuitionistic fuzzy -open set of Y. Therefore *g(f*(H)) = (*go f*) (H) is intuitionistic fuzzy gpr-open set of Z because *g* : (Y, σ) →(Z, μ) is intuitionistic fuzzy gpr –open. Hence *gof* is intuitionistic fuzzy gpr-open mapping.

Theorem 3.8: *If f: (X, ℑ)→(Y, σ) is intuitionistic fuzzy g-open and g :(Y, σ) → (Z, μ) is intuitionistic fuzzy gpr-open mappings such that Y is intuitionistic fuzzy* $T_{1/2}$*- space then gof : (X,ℑ) →(Z,μ) is intuitionistic fuzzy gpr-open.*

Proof: Let H be an intuitionistic fuzzy open set of intuitionistic fuzzy topological space(X, ℑ). Then *f* (H) is intuitionistic fuzzy g-open set of Y because *f* is intuitionistic fuzzy g-open mapping. Now since Y is intuitionistic fuzzy $T_{1/2}$- space, *f* (H) is intuitionistic fuzzy-open set of Y. Therefore *g(f*(H)) = (*go f*) (H) is intuitionistic fuzzy gpr-open set of Z because *g* : (Y, σ) →(Z, μ) is intuitionistic fuzzy gpr–open. Hence *gof* is intuitionistic fuzzy gpr-open mapping.

4 Intuitionistic Fuzzy GPR-Closed Mappings

Definition 4.1: *A mapping f : (X, ℑ) →(Y, σ) is intuitionistic fuzzy gpr-closed if image of every intuitionistic fuzzy closed set of X is intuitionistic fuzzy gpr-closed set in Y.*

Remark 4.1: *Every intuitionistic fuzzy closed mapping is intuitionistic fuzzy gpr-closed but converse may not be true. For,*

Example 4.1: *Let X = {a, b}, Y = {x, y} and the intuitionistic fuzzy set U and V are defined as follows:*

U= {< a, 0.3, 0.6 >, <b, 0.4, 0.6 >}

V = {< x, 0.5. 0.4 >, < y ,0.6,0.3>}

Then $\Im = \{\tilde{0}, U, \tilde{1}\}$ and $\sigma = \{\tilde{0}, V, \tilde{1}\}$ be intuitionistic fuzzy topologies on X and Y respectively. Then the mapping $f : (X, \Im) \to (Y, \sigma)$ defined by *f*(a) = x and *f*(b) = y is intuitionistic fuzzy gpr-closed but it is not intuitionistic fuzzy closed.

Remark 4.2: *Every intuitionistic fuzzy w-closed mapping is intuitionistic fuzzy gpr-closed but converse may not be true. For,*

Example 4.2: *Let X = {a, b}, Y = {x, y} and the intuitionistic fuzzy set U and V are defined as follows:*

U= {< a, 0.7, 0.2 >, <b, 0.8, 0.1 >}
V = {< x, 0.7. 0.2 >, < y ,0.6,0.3>}

Then $\Im = \{\tilde{0}, U, \tilde{1}\}$ and $\sigma = \{\tilde{0}, V, \tilde{1}\}$ be intuitionistic fuzzy topologies on X and Y respectively. Then the mapping $f : (X, \Im) \to (Y, \sigma)$ defined by *f*(a) = x and *f*(b) = y is intuitionistic fuzzy gpr-closed but it is not intuitionistic fuzzy w-closed.

Remark 4.3: *Every intuitionistic fuzzy rw-closed mapping is intuitionistic fuzzy gpr-closed but converse may not be true. For,*

Example 4.3: *Let X = {a, b, c, d, e} Y = {p, q, r, s, t} and intuitionistic fuzzy sets O, U, V, W defined as follows:*

O = { < a,0.9,0.1> ,<b,0.8,0.1>,<c,0,1>,<d,0,1> <e,0,1>}
U = { < a,0,1,> ,<b,0,1>,<c,0.8,0.1>,<d,0.7,0.2> <e,0,1>}
V = { < a,0.9,0.1> ,<b,0.8,0.1>,<c,0.8,0.1>, <d,0.7,0.2>,<e,0,1>}
W = {<p, 0.9, 0.1 > ,<q ,0, 1 >, <r ,0, 1>, <s, 0, 1>, <t ,0,1>}

Let $\Im = \{\tilde{0}, O, U, V, \tilde{1}\}$ and $\sigma = \{\tilde{0}, W, \tilde{1}\}$ be an intuitionistic fuzzy topology on X and Y respectively. Then the mapping $f : (X, \Im) \to (Y, \sigma)$ defined by *f*(a) = p, *f*(b) = q, *f*(c) = r, *f*(d)= s, *f*(e)= t is intuitionistic fuzzy gpr-closed but it is not intuitionistic fuzzy rw-closed.

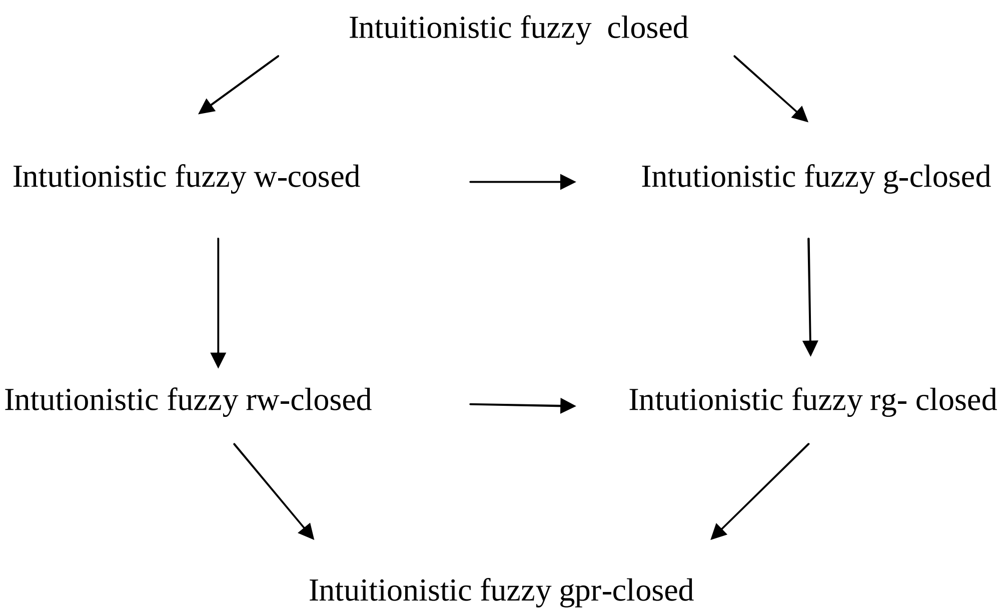
Remark 4.4: *Every intuitionistic fuzzy rg- closed mapping is intuitionistic fuzzy gpr-closed but converse may not be true. For,*

Example 4.4: *Let X = {a, b, c, d, e} Y = { p, q ,r, s, t} and intuitionistic fuzzy sets O, U, V, W defined as follows*

O = { < a,0.9,0.1> ,<b,0.8,0.1>,<c,0,1>,<d,0,1> <e,0,1>}
U = { < a,0,1,> ,<b,0,1>,<c,0.8,0.1>,<d,0.7,0.2> <e,0,1>}
V = { < a,0.9,0.1> ,<b,0.8,0.1>,<c,0.8,0.1>, <d,0.7,0.2>,<e,0,1>}
W = {<p, 0.9, 0.1 > ,<q ,0, 1 >, <r ,0, 1>, <s, 0, 1>, <t ,0,1>}

Let $\Im = \{\{\tilde{0}, O, U, V, \tilde{1}\}$ and $\sigma = \{\tilde{0}, W, \tilde{1}\}$ be an intuitionistic fuzzy topology on X and Y respectively. Then the mapping $f : (X, \Im) \to (Y, \sigma)$ defined by $f(a) = p$, $f(b) = q$, $f(c) = r$, $f(d)= s$, $f(e)= t$ is intuitionistic fuzzy gpr-closed but it is not intuitionistic fuzzy rg-closed.

Remark 4.3: *From the above discussion and known results we have the following diagram of implication:*



Theorem 4.1: *A mapping $f : (X, \Im) \to (Y, \sigma)$ is intuitionistic fuzzy gpr-closed if and only if for each intuitionistic fuzzy set S of Y and for each intuitionistic fuzzy open set U of X containing $f^{-1}(S)$ there is an intuitionistic fuzzy gpr-open set V of Y such that $S \subseteq V$ and $f^{-1}(V) \subseteq U$.*

Proof:

Necessity: Suppose that f is an intuitionistic fuzzy gpr- closed mapping. Let S be the intuitionistic fuzzy closed set of Y and U is an intuitionistic fuzzy open set of X such that $f^{-1}(S) \subseteq U$. Then $V = Y - f^{-1}(U^c)$ is intuitionistic fuzzy gpr-open set of Y such that $f^{-1}(V) \subseteq U$.

Sufficiency: For the converse suppose that F is an intuitionistic fuzzy closed set of X. Then $(f(F))^c$ is an intuitionistic fuzzy set of Y and F^c is intuitionistic fuzzy open set in X such that $f^{-1}((f(F))^c) \subseteq F^c$. By hypothesis there is an intuitionistic fuzzy gpr-open set V of Y such that $(f(F))^c \subseteq V$ and $f^{-1}(V) \subseteq F^c$. Therefore $F \subseteq (f^{-1}(V))^c$. Hence $V^c \subseteq f(F) \subseteq f((f^{-1}(V))^c) \subseteq V^c$ which implies $f(F) = V^c$. Since V^c is intuitionistic fuzzy gpr-closed set of Y. Hence $f(F)$ is intuitionistic fuzzy gpr-closed in Y and thus f is intuitionistic fuzzy gpr-closed mapping.

Theorem 4.2: *If f: $(X,\Im)\rightarrow(Y, \sigma)$ is intuitionistic fuzzy R-mapping and intuitionistic gpr-closed mapping and A is an intuitionistic fuzzy gpr-closed set of X, then f (A) intuitionistic fuzzy gpr-closed.*

Proof: Let *f*(A) ⊆ O where O is an intuitionistic fuzzy regular open set of Y. Since *f* is intuitionistic fuzzy R-mapping therefore f^{-1}(O) is an intuitionistic fuzzy regular open set of X such that A ⊆ f^{-1}(O). Since A is intuitionistic fuzzy gpr-closed of X which implies that pcl(A) ⊆ (f^{-1}(O)) and hence *f*(pcl (A)) ⊆ O which implies that pcl (*f* (pcl(A)) ⊆ O therefore pcl (*f* ((A)) ⊆ O whenever *f*(A) ⊆ O where O is an intuitionistic fuzzy regular open set of Y. Hence *f*(A) is an intuitionistic fuzzy gpr-closed set of Y.

Corollary 4.1: *If f : $(X, \Im) \rightarrow(Y, \sigma)$ is intuitionistic fuzzy gpr-continuous and intuitionistic closed mapping and A is an intuitionistic fuzzy gpr-closed set of X, then f (A) intuitionistic fuzzy gpr-closed.*

Theorem 4.3: *If f: $(X, \Im)\rightarrow(Y, \sigma)$ is intuitionistic fuzzy closed and g: $(Y, \sigma) \rightarrow (Z, \mu)$ is intuitionistic fuzzy gpr-closed. Then gof : $(X, \Im) \rightarrow(Z, \mu)$ is intuitionistic fuzzy gpr-closed.*

Proof: Let H be an intuitionistic fuzzy closed set of intuitionistic fuzzy topological space $(X, \Im)$. Then *f* (H) is intuitionistic fuzzy closed set of (Y, σ) because *f* is intuitionistic fuzzy closed mapping. Now (*gof*) (H) = *g(f*(H)) is intuitionistic fuzzy gpr-closed set in intuitionistic fuzzy topological space Z because *g* is intuitionistic fuzzy gpr-closed mapping. Thus *gof* : $(X,\Im) \rightarrow(Z, \mu)$ is intuitionistic fuzzy gpr-closed mapping.

Theorem 4.4: *If f : $(X, \Im) \rightarrow(Y, \sigma)$ is intuitionistic fuzzy gpr–closed then gprcl (f(A)) ⊆ f (cl(A)).*

Proof: Let A is any intuitionistic fuzzy set of X and *f*: $(X, \Im) \rightarrow(Y, \sigma)$ is intuitionistic fuzzy gpr–closed which gives *f*(cl(A)) is intuitionistic fuzzy gpr-closed in Y. Now *f*(*A*) ⊆ *f* (cl(A)) which implies that gprcl (*f* (*A*)) ⊆ gprcl(*f* (cl(A))). Since *f*(cl(A)) is intuitionistic fuzzy gpr-closed in Y, gprcl(*f*(cl(A))) = *f*(cl(A)). Hence gprcl(*f*(A)) ⊆ *f*(cl(A)).

Theorem 4.5: *If gprcl(A) = rgcl(A) for every intuitionistic fuzzy set A of Y. Then following are equivalent.*

(a) *f* : $(X, \Im) \rightarrow(Y, \sigma)$ is intuitionistic fuzzy gpr –closed

(b) gprcl (*f*(A)) ⊆ *f*(cl(A))

Proof: (a) ⇒ (b) follows by theorem 4.4.

(b) $\Rightarrow$ (a) Let A be intuitionistic fuzzy closed set in X. Then A = cl(A) which implies that *f*(A) = *f*(cl(A)) $\supseteq$ gprcl(*f*(A)) by hypothesis. But we *f*(A) $\subseteq$ gprcl((*f*(A)) combing this we have *f* (A) = gprcl(*f*(A)) = rgcl(*f*(A)) (by given condition). Hence *f*(A) is intuitionistic fuzzy rg-closed, therefore intuionistic fuzzy gpr-closed. Therefore, *f* : (X, $\Im$) $\rightarrow$(Y, σ) is intuitionistic fuzzy gpr–closed mapping.

Theorem 4.6: *Let f : (X, $\Im$) $\rightarrow$(Y, σ) and g :(Y, σ) $\rightarrow$(Z, μ) are intuitionistic fuzzy gpr-closed mappings. If every intuitionistic fuzzy gpr closed set of Y is intuitionistic fuzzy regular closed then, gof : (X, $\Im$) $\rightarrow$(Z, μ) is intuitionistic fuzzy gpr-closed.*

Proof: Let H be an intuitionistic fuzzy closed set of intuitionistic fuzzy topological space (X,$\Im$). Then *f* (H) is intuitionistic fuzzy gpr- closed set of (Y, σ) because *f* is intuitionistic fuzzy gpr-closed mapping. By hypothesis *f* (H) is intuitionistic fuzzy regular-closed set of (Y, σ). Now *g*(*f*(H)) = (*g*o*f*) (H) is intuitionistic fuzzy gpr-closed set in intuitionistic fuzzy topological space Z because *g* is intuitionistic fuzzy gpr-closed mapping. Thus *g*o*f* : (X,$\Im$) $\rightarrow$(Z,μ) is intuitionistic fuzzy gpr-closed mapping.

Theorem 4.7: *If f: (X, $\Im$)$\rightarrow$(Y, σ) is intuitionistic fuzzy rg-cosed and g:(Y, σ) $\rightarrow$(Z, μ) is intuitionistic fuzzy gpr-closed mappings such that Y is intuitionistic fuzzy* $rT_{1/2-}$ *space then gof : (X, $\Im$) $\rightarrow$(Z, μ) is intuitionistic fuzzy gpr-closed.*

Proof: Let H be an intuitionistic fuzzy closed set of intuitionistic fuzzy topological space (X, $\Im$). Then *f* (H) is intuitionistic fuzzy rg-closed set of Y because *f* is intuitionistic fuzzy rg- closed mapping. Now since Y is intuitionistic fuzzy $rT_{1/2-}$ space, *f* (H) is intuitionistic fuzzy –closed set of Y. Therefore *g(f*(H)) = (*go f*) (H) is intuitionistic fuzzy gpr-cosed set of Z because *g*:(Y, σ) $\rightarrow$(Z, μ) is intuitionistic fuzzy gpr–closed. Hence *gof* is intuitionistic fuzzy gpr-closed mapping.

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