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# On Fuzzy Pre-Baire Spaces

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**Abstract**

*In this paper the concepts of fuzzy pre-Baireness in fuzzy topological spaces are introduced. For this purpose we define fuzzy pre-nowhere dense, fuzzy pre first category and fuzzy pre-second category sets. Several Characterizations of fuzzy pre-Baire spaces are also studied. Several examples are given to illustrate the concepts introduced in this paper.*

**Keywords:** *Fuzzy pre-open set, Fuzzy pre-nowhere dense set, Fuzzy pre-first category, Fuzzy pre-second category and Fuzzy pre-Baire spaces.*

## 1 Introduction

The fuzzy concept has invaded almost all branches of Mathematics ever since the introduction of fuzzy set by L.A. Zadeh [15]. The theory of fuzzy topological spaces was introduced and developed by C.L. Chang [5]. Since then much attention has been paid to generalize the basic concepts of General Topology in fuzzy setting and thus a modern theory of Fuzzy Topology has been developed. In recent years, fuzzy topology has been found to be very useful in solving many practical problems. It has been shown that the fuzzy Kahler manifolds which are based on a topology play the important role in $\in^{\infty}$theory [9].

The pre-open sets were introduced by A.S. Mashhour., M.E. Abd El-Monsef and S.N. El-Deeb [8] in 1982 and the above concept was introduced and studied in fuzzy setting by A.S. Bin Shahna [4]. The concepts of Baire spaces have been studied extensively in classical topology in [6], [7], [10] and [16]. The concept of Baire space in fuzzy setting was introduced and studied by the authors in [12]. The aim of this paper is to introduce the concepts of fuzzy pre-Baireness in fuzzy topological spaces. For this purpose in section 3, we introduce fuzzy pre-nowhere dense sets and we study some of their properties. Also we discuss the relationship between fuzzy pre-nowhere dense sets and fuzzy nowhere dense sets. Using the fuzzy pre-nowhere dense sets, in section 4 we introduce fuzzy pre-Baire spaces and study several characterizations of fuzzy pre-Baire spaces. In section 5, the inter–relationships between fuzzy pre-Baire spaces and fuzzy Baire spaces are also investigated. Several examples are given to illustrate the concepts introduced in this paper.

## 2 Preliminaries

Now we introduce some basic notions and results that are used in the sequel. In this work by a fuzzy topological space we shall mean a non-empty set X together with a fuzzy topology T (in the sense of Chang) and denote it by (X, T). The interior, closure and the complement of a fuzzy set λ will be denoted by int(λ), cl(λ) and $1-\lambda$ respectively.

**Definition2.1:** *Let(X,T) be any fuzzy topological space and λ be any fuzzy set in (X, T). We define* $cl(\lambda) = \wedge\{\mu / \lambda \leq \mu, 1-\mu \in T\}$ *and* $int(\lambda) = \vee\{\mu / \mu \leq \lambda, \mu \in T\}$.

*For any fuzzy set in a fuzzy topological space (X, T), it is easy to see that* $1-cl(\lambda) = int(1-\lambda)$ *and* $1-int(\lambda) = cl(1-\lambda)$*[1].*

**Definition 2.2[4]:** *A fuzzy set λ in a fuzzy topological space X is called fuzzy pre-open if* $\lambda \leq int\, cl(\lambda)$ *and fuzzy pre-closed if* $cl\, int(\lambda) \leq \lambda$.

**Definition 2.3[11]:** *Let (X, T) be any fuzzy topological space and λ be any fuzzy set in(X, T).We define the fuzzy pre-interior and the fuzzy pre-closure of λ as follows:*

*(1)* $pcl(\lambda) = \wedge\{\mu / \lambda \leq \mu, \mu$ *is fuzzy pre-closed set of X}*

*(2)* $pint(\lambda) = \vee\{\mu / \mu \leq \lambda, \mu$ *is fuzzy pre-open set of X }.*

**Definition 2.4[2]:** *A fuzzy set λ in a fuzzy topological space(X, T) is called a fuzzy pre-closed set if* $\lambda = pcl(\lambda)$ *and fuzzy pre-open set if* $\lambda = pint(\lambda)$.

**Lemma 2.1[11]:** *Let $\lambda$ be a fuzzy set of a fuzzy topological space (X, T).Then,*

*(1) $1 - pcl(\lambda) = pint(1 - \lambda)$;*
*(2) $1 - pint(\lambda) = pcl(1 - \lambda)$.*

**Definition 2.5[13]:** *A fuzzy set $\lambda$ in a fuzzy topological space (X, T) is called fuzzy dense if there exists no fuzzy closed set $\mu$ in (X, T) such that $\lambda < \mu < 1$.*

**Definition 2.6[13]:** *A fuzzy set $\lambda$ in a fuzzy topological space (X, T) is called fuzzy nowhere dense if there exists no non-zero fuzzy open set $\mu$ in (X, T) such that $\mu < cl(\lambda)$. That is, $int\, cl(\lambda)=0$.*

**Definition 2.7[13]:** *A fuzzy set $\lambda$ in a fuzzy topological space (X, T) is called fuzzy first category if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where $\lambda_i$'s are fuzzy nowhere dense sets in (X, T). Any other fuzzy set in (X, T) is said to be of second category.*

**Definition 2.8[12]:** *Let $\lambda$ be a fuzzy first category set in (X, T). Then $1 - \lambda$ is called a fuzzy residual set in (X, T).*

**Definition 2.9[13]:** *A fuzzy topological space (X, T) is called fuzzy first category if $1 = \vee_{i=1}^{\infty}(\lambda_i)$, where $\lambda_i$'s are fuzzy nowhere dense sets in (X, T). A topological space which is not of fuzzy first category, is said to be of fuzzy second category.*

**Theorem 2.1[3]:** *Let $\lambda$ be a fuzzy set of a fuzzy topological space (X, T).Then,*

*(1) $pcl(\lambda) \geq \lambda \vee cl\,int(\lambda)$;*
*(2) $pint(\lambda) \leq \lambda \wedge int\, cl(\lambda)$.*

**Theorem 2.2[3]:** *Let $\lambda$ be a fuzzy set of a fuzzy topological space (X, T). Then the following properties hold:*

*(1) $int(pcl(\lambda)) \leq int\, cl(\lambda)$;*
*(2) $int(pcl(\lambda)) \geq int(cl\,int(\lambda))$.*

**Theorem 2.3[3]:** *Let $\lambda$ be a fuzzy set of a fuzzy topological space (X, T). Then*

$$int(\lambda) \leq pint(\lambda) \leq \lambda \leq pcl(\lambda) \leq cl(\lambda).$$

**Lemma 2.2[2]:** *Let $\lambda$ and $\mu$ be fuzzy sets of a fuzzy topological space (X, T). Then,*

*(1)* $pcl(\lambda \vee \mu) \geq pcl(\lambda) \vee pcl(\mu)$;
*(2)* $pcl(\lambda \wedge \mu) \leq pcl(\lambda) \wedge pcl(\mu)$.

**Remarks:** From the above Lemma we can easily establish the following properties:

(1) $pint(\lambda \vee \mu) \geq pint(\lambda) \vee pint(\mu)$;
(2) $pint(\lambda \wedge \mu) \leq pint(\lambda) \wedge pint(\mu)$.

# 3 Fuzzy Pre-Nowhere Dense Sets

**Definition 3.1:** *Let (X, T) be a fuzzy topological space. A fuzzy set λ in (X, T) is called a fuzzy pre-nowhere dense set if there exists no non–zero fuzzy pre-open set μ in (X,T) such that* $\mu < pcl(\lambda)$. *That is,* $pint\ pcl(\lambda) = 0$.

**Example 3.1:** Let X = {a, b, c}, The fuzzy sets λ, μ,υ,$\alpha$, $\beta$and $\gamma$ are defined on X as follows:

$\lambda : X \rightarrow [0, 1]$ is defined as $\lambda(a) = 1$; $\lambda(b) = 0.2$; $\lambda(c) = 0.7$.
$\mu : X \rightarrow [0, 1]$ is defined as $\mu(a) = 0.3$; $\mu(b) = 1$; $\mu(c) = 0.2$.
$\upsilon : X \rightarrow [0, 1]$ is defined as $\upsilon(a) = 0.7$; $\upsilon(b) = 0.4$; $\upsilon(c) = 1$.
$\alpha : X \rightarrow [0, 1]$ is defined as $\alpha(a) = 0.7$; $\alpha(b) = 0.4$; $\alpha(c) = 0.9$.
$\beta : X \rightarrow [0, 1]$ is defined as $\beta(a) = 0.3$; $\beta(b) = 0.9$; $\beta(c) = 0.4$.
$\delta : X \rightarrow [0, 1]$ is defined as $\delta(a) = 0.8$; $\delta(b) = 0.4$; $\delta(c) = 0.3$.

Then, $T = \{0, \lambda, \mu, \upsilon, (\lambda \vee \mu), (\lambda \vee \upsilon), (\mu \vee \upsilon), (\lambda \wedge \mu), (\lambda \wedge \upsilon), (\mu \wedge \upsilon), [\lambda \vee (\mu \wedge \upsilon)], [\mu \vee (\lambda \wedge \upsilon)], [\upsilon \wedge (\lambda \vee \mu)], 1\}$ is a fuzzy topology on X. The non-zero fuzzy pre-open sets in (X, T) are

$\lambda, \mu, \upsilon, (\lambda \vee \mu), (\lambda \vee \upsilon), (\mu \vee \upsilon), (\lambda \wedge \mu), (\lambda \wedge \upsilon), (\mu \wedge \upsilon)$,

$[\lambda \vee (\mu \wedge \upsilon)], [\mu \vee (\lambda \wedge \upsilon)], [\upsilon \wedge (\lambda \vee \mu)], \alpha, \beta, \delta, (\alpha \vee \beta), (\alpha \vee \delta), (\beta \vee \delta), (\alpha \wedge \beta)$,
$(\alpha \wedge \delta), \alpha \wedge [\beta \vee \delta], 1$.

Now the fuzzy sets $1 - \lambda, 1 - \mu, 1 - \upsilon, 1 - (\lambda \vee \mu), 1 - (\lambda \vee \upsilon), 1 - (\mu \vee \upsilon), 1 - [\lambda \vee (\mu \wedge \upsilon)], 1 - [\mu \vee (\lambda \wedge \upsilon)], 1 - \alpha, 1 - \beta, 1 - \delta, 1 - (\alpha \vee \beta), 1 - (\alpha \vee \delta), 1 - (\beta \vee \delta), 1 - (\alpha \vee \beta \vee \delta)$ are fuzzy pre-nowhere dense sets in (X, T).

**Remarks:**

(1) If λ and μ are fuzzy pre-nowhere dense sets in a fuzzy topological space (X, T), then $(\lambda \vee \mu)$ need not be a fuzzy pre-nowhere dense set in (X, T). For, in example 3.1, $1-\alpha, 1-\beta$ are fuzzy pre-nowhere dense sets in (X, T). But$(1-\alpha) \vee (1-\beta) = 1-(\alpha \wedge \beta)$ is not a fuzzy pre-nowhere dense set in (X, T).

(2) The complement of a fuzzy pre-nowhere dense set in a fuzzy topological space (X, T), need not be a fuzzy pre-nowhere dense set. For, in example3.1, $(1-\alpha)$ , is a fuzzy pre-nowhere dense set in (X, T), whereas$\alpha = 1-(1-\alpha)$ , is not a fuzzy pre-nowhere dense set in (X, T).

**Proposition 3.1:** *If $\lambda$ is a fuzzy nowhere dense set in a fuzzy topological space (X,T), then pint ($\lambda$) = 0.*

**Proof:** Let λ be a fuzzy nowhere dense set in (X, T). Then, we have $int\ cl(\lambda) = 0$. By theorem 2.1, we have $pint(\lambda) \leq \lambda \wedge int\ cl(\lambda)$. Then, $pint(\lambda) \leq \lambda \wedge 0 = 0$. That is, $pint(\lambda) = 0$.

**Proposition 3.2:** *If $\lambda$ is a fuzzy nowhere dense set in a fuzzy topological space (X,T), then int pcl($\lambda$)= 0.*

**Proof:** Let λ be a fuzzy nowhere dense set in (X, T). Then, we have $int\ cl(\lambda) = 0$. By theorem 2.2, we have $int(pcl(\lambda)) \leq int\ cl(\lambda)$. Then, $int(pcl(\lambda)) \leq 0$. That is, $int(pcl(\lambda)) = 0$.

**Proposition 3.3:** *If the fuzzy sets $\lambda$ and $\mu$ are fuzzy pre-nowhere dense sets in a fuzzy topological space (X, T), then ($\lambda \wedge \mu$) is a fuzzy pre-nowhere dense set in (X,T).*

**Proof:** Let the fuzzy sets λ and μ be fuzzy pre-nowhere dense sets in (X, T).

Now $pint\big(pcl(\lambda \wedge \mu)\big) \leq pint[pcl(\lambda) \wedge pcl(\mu)] \leq pint\big(pcl(\lambda)\big) \wedge pint\big(pcl(\mu)\big) \leq 0 \wedge 0$ [Since $pint\ pcl(\lambda) = 0$ and $pint\ pcl(\mu) = 0$]. That is, $pint\ pcl(\lambda \wedge \mu) = 0$. Hence, (λ∧μ) is a fuzzy pre-nowhere dense set in (X,T).

**Proposition 3.4:** *If $\lambda$ is a fuzzy pre-nowhere dense set and $\mu$ is a fuzzy set in a fuzzy topological space (X, T), then ($\lambda \wedge \mu$) is a fuzzy pre-nowhere dense set in (X,T).*

**Proof:** Let λ be a fuzzy pre-nowhere dense set in (X, T). Then, $pint\ pcl(\lambda) = 0$. Now $pint\big(pcl(\lambda \wedge \mu)\big) \leq pint\big(pcl(\lambda)\big) \wedge pint\big(pcl(\mu)\big) \leq 0 \wedge pint\big(pcl(\mu)\big) = 0$. That is, $pint\big(pcl(\lambda \wedge \mu)\big) = 0$. Hence $(\lambda \wedge \mu)$ is a fuzzy pre-nowhere dense set in (X, T).

**Remarks:** A fuzzy nowhere dense set in a fuzzy topological space (X, T) need not be a fuzzy pre-nowhere dense set in (X, T). For, consider the following example:

**Example 3.2:** Let X ={a,b,c}. Consider the fuzzy sets λ, μ, υ, $\alpha$, $\beta$ and δ defined on X as follows:

$\lambda : X \to [0,1]$ is defined as $\lambda(a) = 0.5$; $\lambda(b) = 0.6$; $\lambda(c) = 0.7$.
$\mu : X \to [0,1]$ is defined as $\mu(a) = 0.8$; $\mu(b) = 0.4$; $\mu(c) = 0.2$.
$\upsilon : X \to [0,1]$ is defined as $\upsilon(a) = 0.7$; $\upsilon(b) = 0.5$; $\upsilon(c) = 0.8$.
$\alpha : X \to [0,1]$ is defined as $\alpha(a) = 0.3$; $\alpha(b) = 0.6$; $\alpha(c) = 0.4$.
$\beta : X \to [0,1]$ is defined as $\beta(a) = 0.6$; $\beta(b) = 0.2$; $\beta(c) = 0.7$.

$\delta : X \to [0, 1]$ is defined as $\delta(a) = 0.8;\ \delta(b) = 0.3;\ \delta(c) = 0.6.$

Then, $T = \{0, \lambda, \mu, \upsilon, [\lambda \vee \mu],\ \lambda \vee \upsilon], [\mu \vee \upsilon], [\lambda \wedge \mu],\ [\lambda \wedge \upsilon], [\mu \wedge \upsilon], \lambda \vee [\mu \wedge \upsilon],$

$\mu \vee [\lambda \wedge \upsilon], \upsilon \wedge [\lambda \vee \mu], [\lambda \vee \mu \vee \upsilon], 1\}$ is clearly a fuzzy topology on X.

The non-zero fuzzy pre-open sets in (X, T) are

$\lambda, \mu, \upsilon, [\lambda \vee \mu],\quad [\lambda \vee \upsilon], [\mu \vee \upsilon], [\lambda \wedge \mu], [\lambda \wedge \upsilon],$

$[\mu \wedge \upsilon], \lambda \vee [\mu \wedge \upsilon], \mu \vee [\lambda \wedge \upsilon], \upsilon \wedge [\lambda \vee \mu], [\lambda \vee \mu \vee \upsilon], \alpha, \beta, \delta, [\alpha \vee \beta], [\alpha \vee \delta], [\beta \vee \delta], [\alpha \wedge \beta],$

$[\alpha \wedge \delta], [\beta \wedge \delta], \alpha \vee [\beta \wedge \delta], \beta \vee [\alpha \wedge \delta], \delta \wedge [\alpha \vee \beta], 1.$

Now the fuzzy sets
$1-\mu$, $1-\upsilon$, $1-[\lambda \vee \mu]$, $1-[\lambda \vee \upsilon]$, $1-[\mu \vee \upsilon]$, $1-(\lambda \vee [\mu \wedge \upsilon])$,$1-(\mu \vee [\lambda \wedge \upsilon])$,

$1-(\upsilon \wedge [\lambda \vee \mu])$, $1-[\lambda \vee \mu \vee \upsilon]$, $1-\beta$,$1-\delta$,$1-[\alpha \vee \beta]$, $1-[\alpha \vee \delta]$, $1-[\beta \vee \delta]$, $1-(\beta \vee [\alpha \wedge \delta])$

are fuzzy pre- nowhere dense sets in (X,T). The fuzzyset$1-[\mu \wedge \upsilon]$ in(X, T)is a fuzzy nowhere dense set whereas$1-[\mu \wedge \upsilon]$ is not a fuzzy pre- nowhere dense set in (X,T),since $pint\left(pcl\left(1-[\mu \wedge \upsilon]\right)\right) = \alpha \neq 0$.

If a fuzzy nowhere dense set in a fuzzy topological space (X, T) is a fuzzy pre-closed set, then it will be a fuzzy pre-nowhere dense set in (X, T).

**Proposition 3.5:** If *a fuzzy nowhere dense set λ in a fuzzy topological space (X, T) is a fuzzy pre-closed set, then λ is a fuzzy pre-nowhere dense set in (X, T).*

**Proof:** Let λ be a fuzzy nowhere dense set in (X, T). Then, we have$int\ cl(\lambda) = 0$. Then by proposition 3.1, we have $pint(\lambda) = 0$. Since λ is a fuzzy pre-closed set in (X, T), $pcl(\lambda) = \lambda$ [4]. Then $pint(pcl(\lambda)) = pint(\lambda) = 0$. Hence, λ is a fuzzypre-nowhere dense set in (X, T).

**Remarks:** A fuzzy pre-nowhere dense set in a fuzzy topological space (X, T) need not be a fuzzy nowhere dense set in (X, T). For, consider the following example:

**Example 3.3:** Let X $=\{$a, b, c$\}$. Consider the fuzzy sets λ, μ and$\alpha$ defined on X as follows:

$\lambda : X \to [0, 1]$ is defined as $\lambda(a) = 0.3;\ \lambda(b) = 0.2;\ \lambda(c) = 0.7.$
$\mu : X \to [0, 1]$ is defined as $\mu(a) = 0.8;\ \mu(b) = 0.8;\ \mu(c) = 0.4.$
$\alpha : X \to [0, 1]$ is defined as $\alpha(a) = 0.8;\ \alpha(b) = 0.7;\ \alpha(c) = 0.6.$

Then, $T = \{0, \lambda, \mu, \lambda \vee \mu, \lambda \wedge \mu, 1\}$ is clearly a fuzzy topology on X. The non-zero fuzzy pre-open sets in (X,T) are $\lambda, \mu, \lambda \vee \mu, \lambda \wedge \mu, \alpha, \lambda \vee \alpha, \mu \vee [\lambda \wedge \alpha], \mu \wedge [\lambda \vee \alpha], 1$.

Now $pint\left(pcl(1-\alpha)\right) = pint(1-\alpha) = 0$, implies that $1-\alpha$ is a fuzzy pre-nowhere dense set in (X, T). But $int\left(cl(1-\alpha)\right) = int(1-[\lambda \wedge \mu]) = \lambda \wedge \mu \neq 0$, implies that $1-\alpha$ is not a fuzzy nowhere dense set in (X,T).

If a fuzzy pre-nowhere dense set in a fuzzy topological space (X, T) is a fuzzy closed set, then it will be a fuzzy nowhere dense set in (X, T).

**Proposition 3.6:** *If a fuzzy pre-nowhere dense set λ in a fuzzy topological space (X,T) is a fuzzy closed set, then λ is a fuzzy nowhere dense set in (X,T).*

**Proof:** Let λ be a fuzzy pre-nowhere dense set in (X, T). Then, we have

$pint\ pcl(\lambda) = 0$. Now $\lambda \leq pcl(\lambda)$, implies that $pint(\lambda) \leq pint(pcl(\lambda))$.

Then, $pint(\lambda) \leq 0$ . That is, $pint(\lambda) = 0$. Now $int(\lambda) \leq pint(\lambda)$,

Implies that $int(\lambda) = 0$. Now $cl(\lambda) = \lambda$ [Since λ is fuzzy closed in (X,T)] and $int(\lambda) = 0$, implies that $int\ cl(\lambda) = 0$. Hence, λ is a fuzzy nowhere dense set in (X,T).

**Definition 3.2[14]:** *A fuzzy set λ in a fuzzy topological space (X, T) is called fuzzy pre-dense if there exists no fuzzy pre-closed set μ in (X, T) such that $\lambda < \mu < 1$. That is, $pcl(\lambda) = 1$.*

**Proposition 3.7:** *If $\lambda$ is a fuzzy pre-dense and fuzzy pre-open set in a fuzzy topological space (X, T) and if $\mu \leq 1 - \lambda$, then $\mu$ is a fuzzy pre-nowhere dense set in (X, T).*

**Proof:** Let $\lambda$ be a fuzzy pre-dense and fuzzy pre-open set in (X, T). Then we have $pcl(\lambda) = 1$ and $pint(\lambda) = \lambda$. Now $\mu \leq 1 - \lambda$, implies that $pcl(\mu) \leq pcl(1 - \lambda)$. Then $pcl(\mu) \leq 1 - pint(\lambda) = 1 - \lambda$. Hence $pcl(\mu) \leq (1 - \lambda)$, which implies that $pint(pcl(\mu)) \leq pint(1 - \lambda) = 1 - pcl(\lambda) = 1 - 1 = 0$.

That is, $pint(pcl(\mu)) = 0$. Hence $\mu$ is a fuzzy pre-nowhere dense set in (X, T).

**Proposition 3.8:** If $\lambda$ is a fuzzy pre-nowhere dense set in a fuzzy topological space (X, T), then $1 - \lambda$ is a fuzzy pre-dense set in (X, T).

**Proof:** Let $\lambda$ be a fuzzy pre-nowhere dense set in (XT). Then, $pint\ pcl(\lambda) = 0$

Now $\lambda \leq pcl(\lambda)$ implies that $pint(\lambda) \leq pint(pcl(\lambda)) = 0$. Then $pint(\lambda) = 0$

and $pcl(1 - \lambda) = 1 - pint(\lambda) = 1 - 0 = 1$and hence $1 - \lambda$ is a fuzzy pre-dense set in (X, T).

**Proposition 3.9:** If $\lambda$ is a fuzzy pre-nowhere dense set in a fuzzy topological space (X, T), then $1 - \lambda$ is a fuzzy dense set in (X, T).

**Proof:** Let $\lambda$ be a fuzzy pre-nowhere dense set in (X, T). Then, by proposition 3.6, $1 - \lambda$ is a fuzzy pre-dense set in (X, T). Since $pcl(1 - \lambda) \leq cl(1 - \lambda)$, we have$1 \leq cl(1 - \lambda)$. That is, $cl(1 - \lambda) = 1$ . Hence $1 - \lambda$ is a fuzzy dense set in (X, T).

**Proposition 3.10:** If $\lambda$ is a fuzzy pre-nowhere dense set in a fuzzy topological space (X, T), then $pcl(\lambda)$ is also a fuzzy pre-nowhere dense set in(X, T).

**Proof:** Let $\lambda$ be a fuzzy pre-nowhere dense set in (X, T). Then, $pint\, pcl(\lambda) = 0$. Now $pcl\, pcl(\lambda) = pcl(\lambda)$. Hence $pint(pcl[pcl(\lambda)]) = pint(pcl(\lambda)) = 0$. Therefore $pcl(\lambda)$ is also a fuzzy pre-nowhere dense set in (X, T).

**Proposition 3.1:** If $\lambda$ is a fuzzy pre-nowhere dense set in a fuzzy topological space (X, T), then $1 - pcl(\lambda)$ is a fuzzy pre-dense set in (X, T).

**Proof:** Let $\lambda$ be a fuzzy pre-nowhere dense set in (X, T). Then, by proposition 3.10, $pcl(\lambda)$ is a fuzzy pre-nowhere dense set in (X, T). Also by proposition 3.8, $1 - pcl(\lambda)$ is a fuzzy pre-dense set in (X, T).

**Proposition 3.12:** Let λ be a fuzzy pre-dense set in a fuzzy topological space (X,T). If μ is any fuzzy set in (X, T), then μ is a fuzzy pre-nowhere dense set in (X, T) if and only if$(\lambda \wedge \mu)$ is a fuzzy pre-nowhere dense set in (X, T).

**Proof:** Let $\mu$ be a fuzzy pre-nowhere dense set in (X, T). Then $pint\, pcl(\mu) = 0$.

Now
$pint\big(pcl(\lambda \wedge \mu)\big) \leq pint[pcl(\lambda) \wedge pcl(\mu)] \leq pint\big(pcl(\lambda)\big) \wedge pint\big(pcl(\mu)\big) \leq pint\big(pcl(\lambda)\big) \wedge 0 = 0$.

That is, $pint\big(pcl(\lambda \wedge \mu)\big) = 0$.Hence$(\lambda \wedge \mu)$isafuzzy pre-nowhere dense set in (X,T).

Conversely, let $(\lambda \wedge \mu)$ be a fuzzy pre-nowhere dense set in (X, T). Then $pint\big(pcl(\lambda \wedge \mu)\big) = 0$.

Then,$pint\big(pcl(\lambda)\big) \wedge pint\big(pcl(\mu)\big) = 0$.

Since $\lambda$ is a fuzzy pre-dense set in (X, T), $pcl(\lambda) = 1$.

Then, $pint(1) \wedge pint(pcl(\mu)) = 0$. That is, $(1) \wedge pint(pcl(\mu)) = 0$.

Hence $pint(pcl(\mu)) = 0$, which means that $\mu$ is a fuzzy pre-nowhere dense set in (X, T).

**Definition 3.3:** *Let (X, T) be a fuzzy topological space. A fuzzy set $\lambda$ in (X, T) is called fuzzy pre-first category if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where $\lambda_i$'s are fuzzy pre-nowhere dense sets in (X,T). Any other fuzzy set in (X, T) is said to be of fuzzy pre-second category.*

**Example 3.4:** Let X = {a, b}. Consider the fuzzy sets λ, μ, $\alpha$ and $\beta$ defined on X as follows:

λ : X → [0, 1] is defined as λ(a) = 0.5; λ(b)= 0.7.
μ : X → [0, 1] is defined as μ(a) = 0.8;μ(b) = 0.9 .
$\alpha$: X → [0, 1] is defined as $\alpha$(a) = 0.6; $\alpha$(b)= 0.9 .
$\beta$ : X →[0, 1] is defined as $\beta$(a) = 0.7; $\beta$(b) = 0.8.

Then, T = {0, λ, μ, 1} is a fuzzy topology on X. Now $\lambda, \mu, \alpha, \beta, (\alpha \vee \beta), (\alpha \wedge \beta)$ and 1 are the non-zero fuzzy pre-open sets in (X, T).

Then $1-\alpha$, $1-\beta$, $1-(\alpha \vee \beta)$, $1-(\alpha \wedge \beta)$, $1-\lambda$, $1-\mu$are fuzzy pre- nowhere dense sets in (X, T) and $\left[(1-\alpha) \vee (1-\beta) \vee \{1-(\alpha \vee \beta) \vee (1-\mu)\} = 1-(\alpha \wedge \beta)\right\}$ and {1 – (α∧β)} is a fuzzy pre-first category set in(X,T).

**Definition 3.4:** Let $\lambda$ be a fuzzypre- first category set in a fuzzy topological space (X, T). Then$1-\lambda$is called afuzzy pre-residualset in (X, T).

**Proposition 3.13:** If $\lambda$ is a fuzzy pre-first category set in a fuzzy topological space (X, T), then $1-\lambda = \wedge_{i=1}^{\infty}(\mu_i)$, where $pcl(\mu_i) = 1$.

**Proof:** Let $\lambda$ be a fuzzy pre-first category set in (X, T). Then we have $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where$\lambda_i$'s are fuzzy pre-nowhere dense sets in(X, T). Now $1-\lambda = 1-[\vee_{i=1}^{\infty}(\lambda_i)] = \wedge_{i=1}^{\infty}(1-\lambda_i)$. Let$\mu_i = 1-\lambda_i$.Then $1-\lambda = \wedge_{i=1}^{\infty}(\mu_i)$. Since$\lambda_i$'s are fuzzy pre-nowhere dense sets in (X, T), by proposition 3.7, we have $(1-\lambda_i)$'s are fuzzy pre-dense sets in (X, T). Hence $pcl(\mu_i) = pcl(1-\lambda_i) = 1$. Therefore we have $1-\lambda = \wedge_{i=1}^{\infty}(\mu_i)$, where $pcl(\mu_i) = 1$.

**Definition 3.5:** A fuzzy topological space (X, T) is called a fuzzy pre-first category space if the fuzzy set $1_X$ is a fuzzy pre-firs category set in (X, T). That is, $1_X = \vee_{i=1}^{\infty}(\lambda_i)$, where$\lambda_i$'s are fuzzy pre-nowhere dense sets in (X, T). Otherwise (X, T) will be called a fuzzy pre-second category space.

**Proposition 3.14:** If $\lambda$ is a fuzzy pre-nowhere dense set in a fuzzy topological space (X, T), then $pint(\lambda) = 0$.

**Proof:** Let $\lambda$ be a fuzzy pre-nowhere dense set in (X, T). Then, $pint\, pcl(\lambda) = 0$. Now $\lambda \leq pcl(\lambda)$ implies that $pint(\lambda) \leq pint(pcl(\lambda))$. Hence $pint(\lambda) \leq 0$. That is, $pint(\lambda) = 0$.

**Proposition 3.15:** If $\lambda$ is a fuzzy pre-closed set in a fuzzy topological space (X, T) and if $pint(\lambda) = 0$, then $\lambda$ is a fuzzypre-nowhere dense set in (X, T).

**Proof:** Let λ be a fuzzy pre-closed set in (X, T). Then we have $pcl(\lambda) = \lambda$. Now $pint(pcl(\lambda)) = pint(\lambda)$ and $pint(\lambda) = 0$, implies that $pint(pcl(\lambda)) = 0$. Hence λ is a fuzzy pre-nowhere dense set in (X, T).

**Remarks:** If $\lambda$ is a fuzzy set in a fuzzy topological space (X, T) such that $pint(\lambda) = 0$, then $\lambda$ need not be a fuzzy pre-nowhere dense set in (X, T). For, consider the following example:

**Example 3.5:** Let X = {a, b}. Consider the fuzzy sets $\lambda$, $\mu$, $\alpha$ and β defined on X as follows:

$\lambda : X \to [0,1]$ is defined as $\lambda(a) = 0.5$; $\lambda(b) = 0.7$.
$\mu : X \to [0,1]$ is defined as $\mu(a) = 0.8$; $\mu(b) = 0.9$.
$\alpha : X \to [0,1]$ is defined as $\alpha(a) = 0.6$; $\alpha(b) = 0.9$.
$\beta : X \to [0,1]$ is defined as $\beta(a) = 0.7$; $\beta(b) = 0.8$.

Then, $T = \{0, \lambda, \mu, 1\}$ is a fuzzy topology on X. Now $\lambda, \mu, \alpha, \beta, (\alpha \vee \beta), (\alpha \wedge \beta)$ and 1 are the non-zero fuzzy pre-open sets in (X, T). Now the fuzzy set $\eta : X \to [0,1]$defined as $\eta(a) = 0.4$ ; $\eta(b) = 0.5$, is not a fuzzy pre-nowhere dense set in (X, T), even though $pint(\eta) = 0$.

# 4 Fuzzy Pre-Baire Spaces

Motivated by the classical concept introduced in [7] we shall now define:

**Definition 4.1:** *Let (X, T) be a fuzzy topological space. Then (X, T) is called a fuzzy pre-Baire space if* $int(\vee_{i=1}^{\infty}(\lambda_i)) = 0$, *where* $\lambda_i$*'s are fuzzy pre-nowhere dense sets in (X, T).*

**Example 4.1:** Let X = {a, b}. Consider the fuzzy sets λ, μ, α and β defined on X as follows:

$\lambda : X \to [0,1]$ is defined as $\lambda(a) = 0.5$; $\lambda(b) = 0.7$.
$\mu : X \to [0,1]$ is defined as $\mu(a) = 0.8$; $\mu(b) = 0.9$.
$\alpha : X \to [0,1]$ is defined as $\alpha(a) = 0.6$ ; $\alpha(b) = 0.9$.
$\beta : X \to [0,1]$ is defined as $\beta(a) = 0.7$; $\beta(b) = 0.8$.

Then, $T = \{0, \lambda, \mu, 1\}$ is a fuzzy topology on X. Now $\lambda, \mu, \alpha, \beta, (\alpha \vee \beta), (\alpha \wedge \beta)$ are the non-zero fuzzy pre-open sets in (X, T). Then$1 - \alpha, 1 - \beta, 1 - (\alpha \vee \beta)$, $1 - (\alpha \wedge \beta)$, $1 - \lambda, 1 - \mu$ are fuzzy pre- nowhere dense sets in (X, T) and $pint[(1-\alpha) \vee (1-\beta) \vee \{1 - (\alpha \vee \beta)\} \vee \{1 - (\alpha \wedge \beta)\} \vee (1-\lambda) \vee (1-\mu)] = 0$.

Hence the fuzzy topological space (X, T) is a fuzzy pre-Baire space.

**Example 4.2:** Let X = {a, b, c}. Consider the fuzzy sets α, β, υ, λ, μ and $\delta$ defined on X as follows :

$\alpha : X \to [0, 1]$ is defined as $\alpha(a) = 0.6$; $\alpha(b) = 0.4$; $\alpha(c) = 0.3$.
$\beta : X \to [0, 1]$ is defined as $\beta(a) = 0.5$; $\beta(b) = 0.7$; $\beta(c) = 0.2$.
$\upsilon : X \to [0, 1]$ is defined as $\upsilon(a) = 0.7$; $\upsilon(b) = 0.5$; $\upsilon(c) = 0.6$.
$\lambda : X \to [0, 1]$ is defined as $\lambda(a) = 0.8$; $\lambda(b) = 0.4$; $\lambda(c) = 0.6$.
$\mu : X \to [0, 1]$ is defined as $\mu(a) = 0.4$; $\mu(b) = 0.7$; $\mu(c) = 0.3$.
$\delta : X \to [0, 1]$ is defined as $\delta(a) = 0.4$; $\delta(b) = 0.6$; $\delta(c) = 0.9$.

Then, $T = \{0, \alpha, \beta, \upsilon, \alpha \vee \beta, \beta \vee \upsilon, \alpha \wedge \beta, \beta \wedge \upsilon, 1\}$ is clearly a fuzzy topology on X. The non-zero fuzzy pre-open sets in (X, T) are α, β, υ, (α∨β), (β∨υ), (α∧β), (β∧υ), λ, μ, δ, (λ∨μ), (λ∨δ), (μ∨δ), [λ∨μ∨δ], 1.

Now the fuzzy sets
$1-\alpha, 1-\beta, 1-\upsilon, 1-(\alpha \vee \beta), 1-(\beta \vee \upsilon), 1-\lambda, 1-\mu, 1-\delta, 1-(\lambda \vee \mu)$,

$1-(\lambda \vee \delta), 1-(\mu \vee \delta), 1-(\lambda \vee \mu \vee \delta)$ are fuzzy pre-nowhere dense sets in (X,T)

and $pint[(1-\alpha) \vee (1-\beta) \vee (1-\upsilon) \vee \{1-(\alpha \vee \beta)\} \vee \{1-(\beta \vee \upsilon)\} \vee (1-\lambda) \vee$

$(1-\mu) \vee (1-\delta) \vee \{1-(\lambda \vee \mu)\} \vee \{1-(\lambda \vee \delta)\} \vee \{1-(\mu \vee \delta)\} \vee \{1-(\lambda \vee \mu \vee \delta)\}] = \alpha$
$\neq 0$.

Hence the fuzzy topological space (X, T) is not a fuzzy pre-Baire space.

**Proposition 4.1:** *If $pint(\vee_{i=1}^{\infty}(\lambda_i)) = 0$, where $pint(\lambda_i) = 0$ and $\lambda_i$'s are fuzzy pre-closed sets in a fuzzy topological space (X, T), then (X, T) is a fuzzy pre-Baire space.*

**Proof:** Let $(\lambda_i)$'s be fuzzy pre-closed sets in (X, T). Since $pint(\lambda_i) = 0$, By proposition 3.14, the $\lambda_i$'s are fuzzy pre-nowhere dense sets in (X, T).

Therefore we have $pint(\vee_{i=1}^{\infty}(\lambda_i)) = 0$ where $\lambda_i$'s are fuzzy pre-nowhere dense sets in (X, T). Hence (X, T) is a fuzzy pre-Baire space.

**Proposition 4.2:** *If* $pcl(\wedge_{i=1}^{\infty}(\lambda_i)$ *=1, where* $\lambda_i$*'s are fuzzy pre-dense and fuzzy pre-open sets in a fuzzy topological space (X, T), then (X,T) is a fuzzy pre-Baire space.*

**Proof:** Now $pcl(\wedge_{i=1}^{\infty}(\lambda_i))$ =1 implies that $1 - pcl(\wedge_{i=1}^{\infty}(\lambda_i)) = 0$. Then we have $pint(1 - (\wedge_{i=1}^{\infty}(\lambda_i)) = 0$ , which implies that $pint(\vee_{i=1}^{\infty}(1 - \lambda_i)) = 0$.

Since$\lambda_i$'s are fuzzy pre-dense sets in (X, T), $pcl(\lambda_i) = 1$ and$pint(1 - \lambda_i) = 1 - pcl(\lambda_i) = 1 - 1 = 0$.

Hence we have $pint(\vee_{i=1}^{\infty}(1 - \lambda_i)) = 0$, where $pint(1 - \lambda_i) = 0$ and$(1 - \lambda_i)$'s are fuzzy pre-closed sets in (X, T). Then, by proposition 4.1, (X, T) is a fuzzy pre-Baire space.

**Proposition 4.3:** *Let (X, T) be a fuzzy topological space. Then the following are equivalent:*

*(1) (X,T) is a fuzzy pre-Baire space.*
*(2)* $pint(\lambda) = 0$ *for every fuzzy pre-first category set* $\lambda$ *in (X,T).*
*(3)* $pcl(\mu) = 1$ *for every fuzzy pre-residual set* $\mu$ *in (X,T).*

**Proof: (1) ⇒(2).** Let λ be a fuzzy pre-first category set in (X, T). Then $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$ , where $\lambda_i$'s arefuzzy pre-nowhere dense sets in (X,T). Now $pint(\lambda) = pint(\vee_{i=1}^{\infty}(\lambda_i)) = 0$ (since (X, T) is a fuzzy pre-Baire space). Therefore $pint(\lambda) = 0$.

**(2) ⇒ (3).** Let μ be a fuzzy pre-residual set in (X, T). Then 1 – μ is a fuzzy pre-first category set in (X, T). By hypothesis, $pint(1 - \mu) = 0$ which implies that $1 - pcl(\mu) = 0$. Hence we have $pcl(\mu)) = 1$.

**(3) ⇒(1).** Let λ be a fuzzy pre-first category set in (X, T). Then$\lambda = \vee_{i=1}^{\infty}(\lambda_i)$ , where $\lambda_i$'s are fuzzy pre-nowhere dense sets in (X, T). Since λ is a fuzzy pre-first category set in (X, T), 1 – λ is a fuzzy pre-residual set in (X, T). By hypothesis, we have $pcl(1 - \lambda) = 1$. Then $1 - pint(\lambda) = 1$, which implies that $pint(\lambda) = 0$. Hence $pint(\vee_{i=1}^{\infty}(\lambda_i)) = 0$ where $\lambda_i$'s are fuzzy pre-nowhere dense sets in (X, T). Hence(X, T) is a fuzzy pre-Baire space.

**Proposition 4.4:** *If a fuzzy topological space (X, T) is a fuzzy pre-Baire space, then (X, T) is a fuzzy pre-second category space.*

**Proof:** Let (X, T) be a fuzzy pre-Baire space. Then, $pint(\vee_{i=1}^{\infty}(\lambda_i)) = 0$ where $\lambda_i$'s are fuzzy pre-nowhere dense sets in (X, T). Then $\vee_{i=1}^{\infty}(\lambda_i) \neq 1_X$ [Otherwise, $\vee_{i=1}^{\infty}(\lambda_i) = 1_X$ implies that $pint(\vee_{i=1}^{\infty}(\lambda_i)) = pint(1_X) = 1_X$, which implies that 0 = 1, a contradiction]. Hence (X, T) is a fuzzy pre-second category space.

**Remarks:** The converse of the above proposition need not be true. A fuzzy pre-second category space need not be a fuzzy pre-Baire space.

For, in example 4.2, The fuzzy sets $1-\alpha, 1-\beta, 1-\upsilon, 1-(\alpha\vee\beta), 1-(\beta\vee\upsilon)$, $1-\lambda$, $1-\mu$, $1-\delta$, $1-(\lambda\vee\mu)$, $1-(\lambda\vee\delta)$, $1-(\mu\vee\delta)$, $1-(\lambda\vee\mu\vee\delta)$ are fuzzy pre- nowhere dense sets in (X, T) and

$$[(1-\alpha)\vee(1-\beta)\vee(1-\upsilon)\vee\{1-(\alpha\vee\beta)\}\vee\{1-(\beta\vee\upsilon)\}\vee(1-\lambda)\vee(1-\mu)\vee(1-\delta)\vee \{1-(\lambda\vee\mu)\}\vee\{1-(\lambda\vee\delta)\}\vee\{1-(\mu\vee\delta)\}\vee\{1-(\lambda\vee\mu\vee\delta)\} = \alpha \neq 1_X.$$

Hence the fuzzy topological space (X, T) is a fuzzy pre-second category space but not a fuzzy pre-Baire space.

**Proposition 4.5:** *If a fuzzy topological space (X, T) is a fuzzy pre-Baire space, then no non-zero fuzzy pre-open set in (X, T) is a fuzzy pre-first category set in (X,T).*

**Proof:** Suppose that $\lambda$ is a non-zero fuzzy pre-open set in (X, T) such that $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$ , where $\lambda_i$'s are fuzzy pre-nowhere dense sets in (X, T). Then we have $pint(\lambda) = pint(\vee_{i=1}^{\infty}(\lambda_i))$. Since $\lambda$ is a non-zero fuzzy pre-open set in (X,T) $pint(\lambda) = \lambda$. Then $pint(\vee_{i=1}^{\infty}(\lambda_i)) = \lambda \neq 0$. But this is a contradiction to (X,T) being a fuzzy pre-Baire space, in which $pint(\vee_{i=1}^{\infty}(\lambda_i)) = 0$, where $\lambda_i$'s are fuzzy pre-nowhere dense sets in (X, T). Hence we must have $\lambda \neq \vee_{i=1}^{\infty}(\lambda_i)$. Therefore no non-zero fuzzy pre-open set in (X, T) is a fuzzy pre-first category set in (X, T).

# 5 Inter–Relations between Fuzzy Baire Spaces and Fuzzy Pre-Baire Spaces

**Definition 5.1[12]:** *Let (X, T) be a fuzzy topological space. Then (X, T) is called a fuzzy Baire space if $int(\vee_{i=1}^{\infty}(\lambda_i)) = 0$, where $\lambda_i$'s are fuzzy nowhere dense sets in (X, T).*

*There are some fuzzy topological spaces which are fuzzy Baire, but not fuzzy pre-Baire. A fuzzy Baire space need not be a fuzzy pre-Baire space. For, consider the following example:*

**Example 5.1:** Let X = {a, b, c}. Consider the fuzzy sets $\lambda, \mu, \upsilon, \alpha, \beta$ and $\delta$ defined on X as follows:

$\lambda : X \to [0, 1]$ is defined as $\lambda(a) = 0.5$; $\lambda(b) = 0.6$; $\lambda(c) = 0.7$.
$\mu : X \to [0, 1]$ is defined as $\mu(a) = 0.8$; $\mu(b) = 0.4$; $\mu(c) = 0.2$.
$\upsilon : X \to [0, 1]$ is defined as $\upsilon(a) = 0.7$; $\upsilon(b) = 0.5$; $\upsilon(c) = 0.8$.
$\alpha : X \to [0, 1]$ is defined as $\alpha(a) = 0.3$; $\alpha(b) = 0.6$; $\alpha(c) = 0.4$.
$\beta : X \to [0, 1]$ is defined as $\beta(a) = 0.6$; $\beta(b) = 0.2$; $\beta(c) = 0.7$.

$\delta : X \to [0,1]$ is defined as $\delta(a) = 0.8$; $\delta(b) = 0.3$; $\delta(c) = 0.6$.

Then, $T = \{0, \lambda, \mu, \upsilon, [\lambda \vee \mu], [\lambda \vee \upsilon], [\mu \vee \upsilon], [\lambda \wedge \mu], [\lambda \wedge \upsilon], [\mu \wedge \upsilon], \lambda \vee [\mu \wedge \upsilon], \mu \vee [\lambda \wedge \upsilon], \upsilon \wedge [\lambda \vee \mu], [\lambda \vee \mu \vee \upsilon], 1\}$ is clearly a fuzzy topology on X.

The non-zero fuzzy pre-open sets in (X, T) are $\lambda,\mu,\upsilon,[\lambda\vee\mu]$, $[\lambda\vee\upsilon]$, $[\mu\vee\upsilon]$, $[\lambda\wedge\mu]$, $[\lambda\wedge\upsilon]$, $[\mu\wedge\upsilon]$, $\lambda\vee[\mu\wedge\upsilon], \mu\vee[\lambda\wedge\upsilon], \upsilon\wedge[\lambda\vee\mu], [\lambda\vee\mu\vee\upsilon], \alpha, \beta, \delta, [\alpha\vee\beta], [\alpha\vee\delta], [\beta\vee\delta], [\alpha\wedge\beta], [\alpha\wedge\delta], [\beta\wedge\delta], \alpha\vee[\beta\wedge\delta], \beta\vee[\alpha\wedge\delta], \delta\wedge[\alpha\vee\beta], 1$.

Now the fuzzy sets

$1-\mu$, $1-\upsilon$, $1-[\lambda\vee\mu]$, $1-[\lambda\vee\upsilon]$, $1-[\mu\vee\upsilon]$, $1-(\lambda\vee[\mu\wedge\upsilon])$, $1-\mu\vee[\lambda\wedge\upsilon])$, $1-(\upsilon\wedge[\lambda\vee\mu])$, $1-[\lambda\vee\mu\vee\upsilon]$,

$1-\beta, 1-\delta, 1-[\alpha\vee\beta], 1-[\alpha\vee\delta], 1-[\beta\vee\delta], 1-(\beta\vee[\alpha\wedge\delta]$ are fuzzy pre- nowhere dense sets in (X, T)

And $pint([1-\mu]\vee[1-\upsilon]\vee\{1-[\lambda\vee\mu]\}\vee\{1-[\lambda\vee\upsilon]\}\vee\{1-[\mu\vee\upsilon]\}\vee\{1-(\lambda\vee[\mu\wedge\upsilon])\}\vee$

$\{1-(\mu\vee[\lambda\wedge\upsilon])\}\vee\{1-(\upsilon\wedge[\lambda\vee\mu])\}\vee\{1-[\lambda\vee\mu\vee\upsilon]\}$

$\vee[1-\beta]\vee[1-\delta]\vee 1-[\alpha\vee\beta]\vee\{1-[\alpha\vee\delta]\}\vee\{1-[\beta\vee\delta]\}\vee) = \alpha \neq 0$.

Hence the fuzzy topological space (X, T) is not a fuzzy pre-Baire space. But (X,T) is a fuzzy Baire space. Since, for the fuzzy nowhere dense sets

$1-\mu$, $1-\upsilon$, $1-[\lambda\vee\mu]$, $1-[\lambda\vee\upsilon]$, $1-[\mu\vee\upsilon]$, $1-(\lambda\vee[\mu\wedge\upsilon])$, $1-(\mu\vee[\lambda\wedge\upsilon])$, $1-(\upsilon\wedge[\lambda\vee\mu]$, $1-[\lambda\vee\mu\vee\upsilon]$ in (X,T)

we have $int([1-\mu]\vee[1-\upsilon]\vee\{1-[\lambda\vee\mu]\}\vee\{1-[\lambda\vee\upsilon]\}\vee$

$\{1-[\mu\vee\upsilon]\}\vee\{1-(\lambda\vee[\mu\wedge\upsilon])\}\vee\{1-(\mu\vee[\lambda\wedge\upsilon])\}\vee\{1-(\upsilon\wedge[\lambda\vee\mu])\}\vee\{1-[\lambda\vee\mu\vee\upsilon]\}) = 0$.

Hence a fuzzy Bairespace need not be afuzzy pre-Baire space.

There are some fuzzy topological spaces which are neither fuzzy Baire nor fuzzy pre-Baire.

**Example 5.2:** Let X ={a, b, c}. Consider the fuzzy sets $\lambda$, $\mu$ and $\alpha$ defined on X as follows:

$\lambda : X \to [0,1]$ is defined as $\lambda(a) = 0.3$; $\lambda(b) = 0.2$; $\lambda(c) = 0.7$.
$\mu : X \to [0,1]$ is defined as $\mu(a) = 0.8$; $\mu(b) = 0.8$; $\mu(c) = 0.4$.

$\alpha: X \to [0,1]$ is defined as $\alpha(a) = 0.8$; $\alpha(b) = 0.7$; $\alpha(c) = 0.6$

Then, $T = \{0, \lambda, \mu, \lambda \vee \mu, \lambda \wedge \mu, 1\}$is clearly a fuzzy topology on X.

The non-zero fuzzy pre-open sets in (X, T) are$\lambda, \mu, \lambda \vee \mu, \lambda \wedge \mu, \alpha, \lambda \vee \alpha, \mu \vee [\lambda \wedge \alpha]$, $\mu \wedge [\lambda \vee \alpha], 1$.

The fuzzy nowhere dense sets in (X, T) are$1-\lambda$, $1-\mu$, $1-[\lambda\vee\mu]$, $1-[\lambda\vee\alpha]$, $1-(\mu\vee[\lambda\wedge\alpha])$ and $int\,\{(1-\lambda)\vee(1-\mu)\vee(1-[\lambda\vee\mu])\vee(1-[\lambda\vee\alpha])\vee(1-(\mu\vee[\lambda\wedge\alpha])\}$ $= \lambda\wedge\mu \neq 0$.

Hence (X, T) is no ta fuzzy Baire space. Also the fuzzy pre-nowhere dense sets in (X, T) are$1-\lambda$, $1-\mu$, $1-[\lambda\vee\mu]$, $1-\alpha$,$1-[\lambda\vee\alpha]$, $1-(\mu\vee[\lambda\wedge\alpha])$,

$1-(\mu\wedge[\lambda\vee\alpha])$ and $pint\,\{(1-\lambda)\vee(1-\mu)\vee(1-\alpha)\vee(1-[\lambda\vee\mu])\vee(1-[\lambda\vee\alpha])$

$\vee(1-(\mu\vee[\lambda\wedge\alpha])\vee(1-(\mu\wedge[\lambda\vee\alpha]))\} = \lambda\wedge\mu \neq 0$.

Hence (X, T) is not a fuzzy pre-Baire space.

**Proposition 5.1:** *If a fuzzy topological space (X, T) is a fuzzy pre-Baire space and if every fuzzy pre-nowhere dense set λ in (X, T) is a fuzzy closed set, then (X,T) is a fuzzy Baire space.*

**Proof:** Let (X, T) be a fuzzy pre-Baire space such that every fuzzy pre-nowhere dense set in (X, T) is a fuzzy closed set in (X, T). Since (X, T) is a fuzzy pre-Baire space, by proposition 4.3, $pint(\lambda) = 0$ for every fuzzy pre-first category set λ in (X, T). That is, $pint(\vee_{i=1}^{\infty}(\lambda_i)) = 0$, where $\lambda_i$'s are fuzzy pre-nowhere dense sets in (X, T). Since the fuzzy pre-nowhere dense sets$\lambda_i$'s in (X, T) are all fuzzy closed sets in (X, T), by proposition 3.6, $\lambda_i$'s are fuzzy nowhere dense sets in (X, T). Now $int(\lambda) \leq pint(\lambda)$ implies that $int(\vee_{i=1}^{\infty}(\lambda_i)) \leq pint(\vee_{i=1}^{\infty}(\lambda_i)$ and $pint(\vee_{i=1}^{\infty}(\lambda_i)) = 0$, implies that $int(\vee_{i=1}^{\infty}(\lambda_i)) = 0$. Hence we have $int(\vee_{i=1}^{\infty}(\lambda_i)) = 0$, where $\lambda_i$'s are fuzzy nowhere dense sets in (X, T). Therefore (X, T) is a fuzzy Baire space.

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