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## Research Article

# **A Note on Some Strongly Sequence Spaces**

## Ekrem Savaş<sup>1</sup> and Adem Kılıçman<sup>2</sup>

<sup>1</sup> Istanbul Commerce University, 34840 Istanbul, Turkey

Correspondence should be addressed to Adem Kılıçman, akilicman@putra.upm.edu.my

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We introduce and study new sequence spaces which arise from the notions of generalized de la Vallée-Poussin means, invariant means, and modulus functions.

#### 1. Introduction

Let w be the set of all real or complex sequences and let  $l_{\infty}$ , c, and  $c_0$  be the Banach spaces of bounded, convergent, and null sequences  $x = (x_k)$ , respectively, with the usual norm  $||x|| = \sup_n |x_n|$ .

A sequence  $x = (x_k) \in l_{\infty}$  is said to be almost convergent if its Banach limit coincides. Let  $\hat{c}$  denote the space of all almost convergent sequences. Lorentz [1] proved that

$$\widehat{c} = \left\{ x \in l_{\infty} : \lim_{m} t_{mn}(x) \text{ exist uniformly in } n \right\}, \tag{1.1}$$

where

as

$$t_{mn}(x) = \frac{x_n + x_{n+1} + \dots + x_{n+m}}{m+1}.$$
 (1.2)

The space  $[\hat{c}]$  of strongly almost convergent sequences was introduced by Maddox [2]

$$[\hat{c}] = \left\{ x \in l_{\infty} : \lim_{m} t_{mn}(|x - \ell e|) \text{ exist uniformly in } n \text{ for some } \ell \in \mathbb{C} \right\}, \tag{1.3}$$

where e = (1, 1, ...).

<sup>&</sup>lt;sup>2</sup> Department of Mathematics and Institute for Mathematical Research, University of Putra Malaysia, Serdang, Selangor 43400, Malaysia

Let  $\sigma$  be a one-to-one mapping from the set of positive integers into itself such that  $\sigma^m(n) = \sigma^{m-1}(\sigma(n))$ ,  $m = 1, 2, 3, \ldots$ , where  $\sigma^m(n)$  denotes the mth iterate of the mapping  $\sigma$  in n, see [3]. A continuous linear functional  $\varphi$  on  $l_\infty$  is said to be an invariant mean or a  $\sigma$ -mean, if and only if,

- (i)  $\varphi(x) \ge 0$ , when the sequence  $x = (x_n)$  is such that  $x_n \ge 0$  for all n,
- (ii)  $\varphi(e) = 1$ , where e = (1, 1, ...),
- (iii)  $\varphi(x_{\sigma(n)}) = \varphi(x)$ , for all  $x \in l_{\infty}$ .

For a certain kind of mapping  $\sigma$ , every invariant mean  $\varphi$  extends the functional limit on the space c, in the sense that  $\varphi(x) = \lim x$  for all  $x \in c$ . Consequently,  $c \in V_{\sigma}$ , where  $V_{\sigma}$  is the set of bounded sequences with equal  $\sigma$ -means. Schaefer [3] proved that

$$V_{\sigma} = \left\{ x \in l_{\infty} : \lim_{k} t_{km}(x) = L \text{ uniformly in } m \text{ for some } L = \sigma - \lim x \right\}, \tag{1.4}$$

where

$$t_{km}(x) = \frac{x_m + x_{\sigma(m)} + \dots + x_{\sigma^k(m)}}{k+1}, \qquad t_{-1,m} = 0.$$
 (1.5)

Thus we say that a bounded sequence  $x = (x_k)$  is  $\sigma$ -convergent, if and only if,  $x \in V_{\sigma}$  such that  $\sigma^k(n) \neq n$  for all  $n \geq 0$ ,  $k \geq 1$ . Note that similarly as the concept of almost convergence leads naturally to the concept of strong almost convergence, the  $\sigma$ -convergence leads naturally to the concept of strong  $\sigma$ -convergence.

A sequence  $x = (x_k)$  is said to be strongly  $\sigma$ -convergent (see, Mursaleen [4]), if there exists a number  $\ell$  such that

$$\frac{1}{k} \sum_{i=1}^{k} \left| x_{\sigma^{i}(m)} - \ell \right| \longrightarrow 0, \tag{1.6}$$

as  $k \to \infty$  uniformly in m. We write  $[V_\sigma]$  to denote the set of all strong  $\sigma$ -convergent sequences and when (1.6) holds, we write  $[V_\sigma] - \lim x = \ell$ . Taking  $\sigma(m) = m + 1$ , we obtain  $[V_\sigma] = [\widehat{c}]$ . Then the strong  $\sigma$ -convergence generalizes the concept of strong almost convergence. We also note that

$$[V_{\sigma}] \subset V_{\sigma} \subset l_{\infty}. \tag{1.7}$$

It is also well known that the concept of paranorm is closely related to linear metric spaces. In fact, it is a generalization of absolute value. Let X be a linear space. A function  $p: X \to \mathbb{R}$  is called a paranorm, if

- (P:1)  $p(0) \ge 0$ ,
- (P:2)  $p(x) \ge 0$ , for all  $x \in X$ ,
- (P:3) p(-x) = p(x), for all  $x \in X$ ,

- (P:4)  $p(x + y) \le p(x) + p(y)$ , for all  $x, y \in X$  (triangle inequality),
- (P:5) if  $(\lambda_n)$  is a sequence of scalars, with  $\lambda_n \to \lambda$   $(n \to \infty)$ , and  $(x_n)$  is a sequence of vectors with  $p(x_n x) \to 0$   $(n \to \infty)$ , then  $p(\lambda_n x_n \lambda x) \to 0$   $(n \to \infty)$  (continuity of multiplication by scalars).

A complete linear metric space is said to be a Fréchet space. A Fréchet sequence space X is said to be an FK space, if its metric is stronger than the metric of w on X, that is, convergence in the sequence space X implies coordinatewise convergence (the letters F and K stand for Fréchet and Koordinate, the German word for coordinate).

Note that, by Ruckle in [5], a modulus function f is a function from  $[0, \infty)$  to  $[0, \infty)$  such that

- (i) f(x) = 0, if and only if, x = 0,
- (ii)  $f(x + y) \le f(x) + f(y)$ , for all  $x, y \ge 0$ ,
- (iii) f increasing,
- (iv) *f* is continuous from the right at zero.

Since  $|f(x) - f(y)| \le f(|x - y|)$ , it follows from condition (*iv*) that f is continuous on  $[0, \infty)$ . Furthermore, from condition (ii), we have  $f(nx) \le nf(x)$  for all  $n \in \mathbb{N}$ , and thus

$$f(x) = f\left(nx\frac{1}{n}\right) \le nf\left(\frac{x}{n}\right),\tag{1.8}$$

hence

$$\frac{1}{n}f(x) \le f\left(\frac{x}{n}\right), \quad \forall n \in \mathbb{N}. \tag{1.9}$$

In [5], Ruckle used the idea of a modulus function f in order to construct a class of FK spaces

$$L(f) = \left\{ x = (x_k) : \sum_{k=1}^{\infty} f(|x_k|) < \infty \right\}.$$
 (1.10)

From the definition, we can easily see that the space L(f) is closely related to the space  $l_1$ , if we consider f(x) = x for all real numbers  $x \ge 0$ . Several authors study these types of spaces. For example, Maddox introduced and examined some properties of the sequence spaces  $w_0(f)$ , w(f) and  $w_{\infty}(f)$ , defined by using a modulus f, which generalized the well-known spaces  $w_0$ , w and  $w_{\infty}$  of strongly summable sequences, see [6]. Similarly, Savaş in [7] generalized the concept of strong almost convergence by using a modulus f and examined some further properties of the corresponding new sequence spaces.

The generalized de la Vallé-Poussin mean is defined by

$$t_n(x) = \frac{1}{\lambda_n} \sum_{k \in I_n} x_k,\tag{1.11}$$

where  $I_n = [n - \lambda_n + 1, n]$  for n = 1, 2, ... Then a sequence  $x = (x_k)$  is said to be  $(V, \lambda)$ -summable to a number L (see [8]), if  $t_n(x) \to L$  as  $n \to \infty$ , and we write

$$[V,\lambda]_0 = \left\{ x : \lim_n \frac{1}{\lambda_n} \sum_{k \in I_n} |x_k| = 0 \right\},$$

$$[V,\lambda] = \left\{ x : x - \ell e \in [V,\lambda]_0 \text{ for some } \ell \in \mathbb{C} \right\},$$

$$[V,\lambda]_\infty = \left\{ x : \sup_n \frac{1}{\lambda_n} \sum_{k \in I_n} |x_k| < \infty \right\},$$

$$(1.12)$$

for the sets of sequences that are, respectively, strongly summable to zero, strongly summable, and strongly bounded by the de la Vallé-Poussin method. In the special case where  $\lambda_n = n$ , for  $n = 1, 2, 3, \ldots$ , the sets  $[V, \lambda]_0$ ,  $[V, \lambda]$ , and  $[V, \lambda]_{\infty}$  reduce to the sets  $w_0$ , w, and  $w_{\infty}$ , which were introduced and studied by Maddox, see [6].

We also note that the sets of sequence spaces such as strongly  $\sigma$ -summable to zero, strongly  $\sigma$ -summable, and strongly  $\sigma$ -bounded with respect to the modulus function were defined by Nuray and Savaş in [9].

#### 2. Main Results

Let  $p = (p_k)$  be a sequence of real numbers such that  $p_k > 0$  for all k, and  $\sup_k p_k < \infty$ . This assumption is made throughout the rest of this paper. Then we now write

$$[V_{\sigma}, \lambda, f, p]_{0} = \left\{ x : \lim_{n} \frac{1}{\lambda_{n}} \sum_{k \in I_{n}} \left\{ f(|x_{\sigma^{k}(m)}|) \right\}^{p_{k}} = 0, \text{ uniformly in } m \right\},$$

$$[V_{\sigma}, \lambda, f, p] = \left\{ x : x - \ell e \in [V_{\sigma}, \lambda, f, p]_{0} \text{ for some } \ell \in \mathbb{C} \right\},$$

$$[V_{\sigma}, \lambda, f, p]_{\infty} = \left\{ x : \sup_{n, m} \frac{1}{\lambda_{n}} \sum_{k \in I_{n}} \left\{ f(|x_{\sigma^{k}(m)}|) \right\}^{p_{k}} < \infty \right\}.$$

$$(2.1)$$

In particular, if we take  $p_k = 1$  for all k, we have

$$[V_{\sigma}, \lambda, f]_{0} = \left\{ x : \lim_{n} \frac{1}{\lambda_{n}} \sum_{k \in I_{n}} f(|x_{\sigma^{k}(m)}|) = 0, \text{ uniformly in } m \right\},$$

$$[V_{\sigma}, \lambda, f] = \left\{ x : x - \ell e \in [V_{\sigma}, \lambda, f]_{0} \text{ for some } \ell \in \mathbb{C} \right\},$$

$$[V_{\sigma}, \lambda, f]_{\infty} = \left\{ x : \sup_{n, m} \frac{1}{\lambda_{n}} \sum_{k \in I_{n}} f(|x_{\sigma^{k}(m)}|) < \infty \right\}.$$

$$(2.2)$$

Similarly, when  $\sigma(m)=m+1$ , then  $[V_{\sigma},\lambda,f,p]_0$ ,  $[V_{\sigma},\lambda,f,p]$  and  $[V_{\sigma},\lambda,f,p]_{\infty}$  are reduced to

$$\left[\widehat{V}, \lambda, f, p\right]_{0} = \left\{x : \lim_{n} \frac{1}{\lambda_{n}} \sum_{k \in I_{n}} \left\{f(|x_{k+m}|)\right\}^{p_{k}} = 0, \text{ uniformly in } m\right\},$$

$$\left[\widehat{V}, \lambda, f, p\right] = \left\{x : x - \ell e \in \left[\widehat{V}, \lambda, f, p\right]_{0} \text{ for some } \ell \in \mathbb{C}\right\},$$

$$\left[\widehat{V}, \lambda, f, p\right]_{\infty} = \left\{x : \sup_{n, m} \frac{1}{\lambda_{n}} \sum_{k \in I_{n}} \left\{f(|x_{k+m}|)\right\}^{p_{k}} < \infty\right\}, \text{ respectively.}$$
(2.3)

In particular, when  $p_k = p$  for all k, then we have the spaces

$$\left[ \widehat{V}, \lambda, f, p \right]_0 = \left[ \widehat{V}, \lambda, f \right]_0, \qquad \left[ \widehat{V}, \lambda, f, p \right] = \left[ \widehat{V}, \lambda, f \right], \qquad \left[ \widehat{V}, \lambda, f, p \right]_\infty = \left[ \widehat{V}, \lambda, f \right]_\infty, \quad (2.4)$$

which were introduced and studied by Malkowsky and Savaş in [10]. Further, when  $\lambda_n = n$ , for n = 1, 2, 3, ..., the sets  $[\hat{V}, \lambda, f]_0$  and  $[\hat{V}, \lambda, f]$  are reduced to  $[\hat{c}(f)]$  and  $[\hat{c}_0(f)]$  respectively, see [7]. Now, if we consider f(x) = x, then one can easily obtain

$$[V_{\sigma}, \lambda, p]_{0} = \left\{ x : \lim_{n} \frac{1}{\lambda_{n}} \sum_{k \in I_{n}} \left| x_{\sigma^{k}(m)} \right|^{p_{k}} \text{ uniformly in } m \right\},$$

$$[V_{\sigma}, \lambda, p] = \left\{ x : x - \ell e \in \left[ \widehat{V}_{\sigma}, \lambda, p \right]_{0} \text{ for some } \ell \in \mathbb{C} \right\},$$

$$[V_{\sigma}, \lambda, p]_{\infty} = \left\{ x : \sup_{n, m} \frac{1}{\lambda_{n}} \sum_{k \in I_{n}} \left| x_{\sigma^{k}(m)} \right|^{p_{k}} < \infty \right\}.$$

$$(2.5)$$

If  $p_k = 1$  for all k, then we can obtain the spaces  $[V_{\sigma}, \lambda]_0$ ,  $[V_{\sigma}, \lambda]$ , and  $[V_{\sigma}, \lambda]_{\infty}$ . Throughout this paper, we use the notation  $f(|x_k|)^{p_k}$  instead of  $\{f(|x_k|)\}^{p_k}$ .

If  $p \in l_{\infty}$ , then it is clear that  $[V_{\sigma}, \lambda, f, p]_0$ ,  $[V_{\sigma}, \lambda, f, p]$ , and  $[V, \sigma, \lambda, f, p]_{\infty}$  are linear spaces over the complex field  $\mathbb{C}$ .

**Lemma 2.1.** Let f be any modulus. Then

$$[V_{\sigma}, \lambda, f]_{\infty} = \ell_{\infty}^{\sigma}(f) = \{x \in w : (f(|x_{\sigma^{k}(m)}|)) \in \ell_{\infty}\}.$$
(2.6)

*Proof.* Let  $x \in [V_{\sigma}, \lambda, f]_{\infty}$ . Then there is a constant M > 0 such that

$$\frac{1}{\lambda_1} f(|x_{\sigma^k(m)}|) \le \sup_{m,n} \frac{1}{\lambda_n} \sum_{k \in I_n} f(|x_{\sigma^k(m)}|) \le M, \tag{2.7}$$

for all m, and so  $(f(|x_{\sigma^k(m)}|)) \in l_{\infty}$ . Let  $x \in \ell_{\infty}^{\sigma}(f)$ . Then there is a constant M > 0 such that  $(f(|x_{\sigma^k(m)}|)) \leq M$  for all k and m, and so

$$\frac{1}{\lambda_n} \sum_{k \in I_n} f(|x_{\sigma^k(m)}|) \le M \frac{1}{\lambda_n} \sum_{k \in I_n} 1 \le M, \tag{2.8}$$

for all m and n. Thus  $x \in [V_{\sigma}, \lambda, f]_{\infty}$ . This completes the proof.

If  $x \in [V_{\sigma}, \lambda, f, p]$ , with  $(1/\lambda_n) \sum_{k \in I_n} f(|x_{\sigma^k(m)} - \ell e|)^{p_k} \to 0$  as  $n \to \infty$  uniformly in m, then we write  $x_k \to \ell[V_{\sigma}, \lambda, f, p]$ .

The following well-known inequality ([11], page 190) will be used later.

If  $0 \le p_k \le \sup p_k = H$  and  $C = \max(1, 2^{H-1})$ , then

$$|a_k + b_k|^{p_k} \le C\{|a_k|^{p_k} + |b_k|^{p_k}\},\tag{2.9}$$

for all k and  $a_k, b_k \in \mathbb{C}$ .

In the following theorem, we prove  $x_k \to \ell$  implies  $x_k \to \ell \in [V_\sigma, \lambda, f, p]$  and we also prove the uniqueness of the limit  $\ell$ . To prove the theorem, we need the following lemma.

**Lemma 2.2** (see [2]). Let  $p_k > 0$ ,  $q_k > 0$ . Then  $c_o(q) \in c_0(p)$ , if and only if,  $\lim_{k \to \infty} \inf p_k / q_k > 0$ , where  $c_0(p) = \{x : |x_k|^{p_k} \to 0 \text{ as } k \to \infty\}$ .

Note that no other relation between  $(p_k)$  and  $(q_k)$  is needed in Lemma 2.2.

**Theorem 2.3.** Let  $\lim_{k\to\infty}\inf p_k > 0$ . Then  $x_k \to \ell$  implies  $x_k \to \ell \in [V_\sigma, \lambda, f, p]$ . Let  $\lim_{k\to\infty}p_k = r > 0$ . If  $x_k \to \ell \in [V_\sigma, \lambda, f, p]$ , then  $\ell$  is unique.

*Proof.* Let  $x_k \to \ell$ . By the definition of modulus, we have  $f(|x_k - \ell|) \to 0$ . Since  $\lim_{k \to \infty} \inf p_k > 0$ , it follows from the above lemma that  $f(|x_k - \ell|)^{p_k} \to 0$  and consequently,  $x_k \to \ell \in [V_\sigma, f, p]$ .

Let  $\lim_{k\to\infty} p_k = r > 0$ . Suppose that  $x_k \to \ell_1 \in [V_\sigma, \lambda, f, p]$ ,  $x_k \to \ell_2 \in [V_\sigma, \lambda, f, p]$  and  $|\ell_1 - \ell_2|^{p_k} = a > 0$ . Now, from (2.9) and the definition of modulus, we have

$$\frac{1}{\lambda_n} \sum_{k \in I_n} f(|\ell_1 - \ell_2|)^{p_k} \leq \frac{C}{\lambda_n} \sum_{k \in I_n} f(|x_{\sigma^k(m)} - \ell_1|)^{p_k} 
+ \frac{C}{\lambda_n} \sum_{k \in I} f(|x_{\sigma^k(m)} - \ell_2|)^{p_k}.$$
(2.10)

Hence,

$$\frac{1}{\lambda_n} \sum_{k \in I_n} f(|\ell_1 - \ell_2|)^{p_k} = 0.$$
 (2.11)

Further,  $f(|\ell_1 - \ell_2|)^{p_k} \to f(a)^r$  as  $k \to \infty$  and, therefore,

$$\lim_{n \to \infty} \frac{1}{\lambda_n} \sum_{k \in I_n} f(|\ell_1 - \ell_2|)^{p_k} = f(a)^r.$$
 (2.12)

From (2.11) and (2.12), it follows that f(a) = 0 and by the definition of modulus, we have a = 0. Hence  $\ell_1 = \ell_2$  and this completes the proof.

**Theorem 2.4.** (i) Let  $0 < \inf_k p_k \le p_k \le 1$ . Then,

$$[V_{\sigma}, \lambda, f, p] \subset [V_{\sigma}, \lambda, f]. \tag{2.13}$$

(ii) Let  $0 < p_k \le \sup_k p_k < \infty$ . Then,

$$[V_{\sigma}, \lambda, f] \subset [V_{\sigma}, \lambda, f, p]. \tag{2.14}$$

*Proof.* (i) Let  $x \in [V_{\sigma}, \lambda, f, p]$ . Since  $0 < \inf_k p_k \le 1$ , we get

$$\frac{1}{\lambda_n} \sum_{k \in I_n} \left\{ f(\left| x_{\sigma^k(m)} - \ell e \right|) \right\} \le \frac{1}{\lambda_n} \sum_{k \in I_n} \left\{ f(\left| x_{\sigma^k(m)} - \ell e \right|) \right\}^{p_k}, \tag{2.15}$$

and hence  $x \in [V_{\sigma}, \lambda, f]$ .

(ii) Let  $p \ge 1$  for each k, and  $\sup_k p_k < \infty$ . Let  $x \in [V_\sigma, \lambda, f]$ . Then, for each k,  $0 < \varepsilon < 1$ , there exists a positive integer N such that

$$\frac{1}{\lambda_n} \sum_{k \in L} \left\{ f(\left| x_{\sigma^k(m)} - \ell e \right|) \right\} \le \varepsilon < 1, \tag{2.16}$$

for all  $m \ge N$ . This implies that

$$\frac{1}{\lambda_n} \sum_{k \in I_n} \left\{ f(\left| x_{\sigma^k(m)} - \ell e \right|) \right\}^{p_k} \le \frac{1}{\lambda_n} \sum_{k \in I_n} \left\{ f(\left| x_{\sigma^k(m)} - \ell e \right|) \right\}. \tag{2.17}$$

Therefore,  $x \in [V_{\sigma}, \lambda, f, p]$ . This completes the proof.

Finally, we conclude this paper by stating the following theorem. We omit the proof, since it involves routine verification and can be obtained by using standard techniques.

**Theorem 2.5.**  $[V_{\sigma}, \lambda, f, p]_0$  and  $[V_{\sigma}, \lambda, f, p]$  are complete linear topological spaces, with paranorm g, where g is defined by

$$g(x) = \sup_{m,n} \left( \frac{1}{\lambda_n} \sum_{k \in I_n} \{ f(|x_{\sigma^k(m)}|) \}^{p_k} \right)^M,$$
 (2.18)

where  $M = \max(1, \{\sup_k p_k\})$ .

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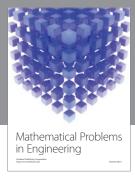
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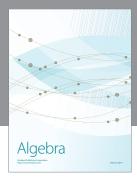
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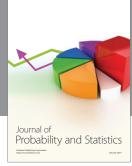
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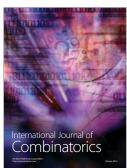








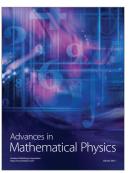


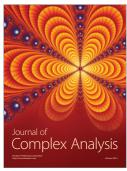




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