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Research Article

On New Inequalities via Riemann-Liouville Fractional Integration

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We extend the Montgomery identities for the Riemann-Liouville fractional integrals. We also use these Montgomery identities to establish some new integral inequalities. Finally, we develop some integral inequalities for the fractional integral using differentiable convex functions.

1. Introduction

The inequality of Ostrowski [1] gives us an estimate for the deviation of the values of a smooth function from its mean value. More precisely, if $f : [a,b] \to \mathbb{R}$ is a differentiable function with bounded derivative, then

$$\left| f(x) - \frac{1}{b-a}\int_a^b f(t)dt \right| \le \left[\frac{1}{4} + \frac{(x-(a+b)/2)^2}{(b-a)^2} \right](b-a)\left\| f' \right\|_\infty, \tag{1.1}$$

for every $x \in [a,b]$. Moreover, the constant $1/4$ is the best possible.

For some generalizations of this classic fact see ([2], (pages 468–484)) by Mitrinović et al. A simple proof of this fact can be done by using the following identity [2].

If $f : [a,b] \to \mathbb{R}$ is differentiable on $[a,b]$ with the first derivative f' integrable on $[a,b]$, then Montgomery identity holds

$$f(x) = \frac{1}{b-a}\int_a^b f(t)dt + \int_a^b P_1(x,t) f'(t)dt, \tag{1.2}$$

where $P_1(x,t)$ is the Peano kernel defined by

$$P_1(x,t) := \begin{cases} \dfrac{t-a}{b-a}, & a \leq t < x, \\ \dfrac{t-b}{b-a}, & x \leq t \leq b. \end{cases} \tag{1.3}$$

Recently, several generalizations of the Ostrowski integral inequality are considered by many authors; for instance, covering the following concepts: functions of bounded variation, Lipschitzian, monotonic, absolutely continuous, and n-times differentiable mappings with error estimates with some special means together with some numerical quadrature rules. For recent results and generalizations concerning Ostrowski's inequality, we refer the reader to the recent papers [3–10].

In this paper, we extend the Montgomery identities for the Riemann-Liouville fractional integrals. We also use these Montgomery identities to establish some new integral inequalities of Ostrowski's type. Finally, we develop some integral inequalities for the fractional integral using differentiable convex functions. Later, we develop some integral inequalities for the fractional integral using differentiable convex functions. From our results, the weighted and the classical Ostrowski's inequalities can be deduced as some special cases.

2. Fractional Calculus

Firstly, we give some necessary definitions and mathematical preliminaries of fractional calculus theory which are used further in this paper. For more details, one can consult [11, 12].

Definition 2.1. The Riemann-Liouville fractional integral operator of order $\alpha \geq 0$ with $a \geq 0$ is defined as

$$\begin{gathered} J_a^\alpha f(x) = \frac{1}{\Gamma(\alpha)} \int_a^x (x-t)^{\alpha-1} f(t)dt, \\ J_a^0 f(x) = f(x). \end{gathered} \tag{2.1}$$

Recently, many authors have studied a number of inequalities by using the Riemann-Liouville fractional integrals, see [13–16] and the references cited therein.

3. Main Results

In order to prove some of our results, by using a different method of proof, we give the following identities, which are proved in [13]. Later, we will generalize the Montgomery identities in the next theorem.

Lemma 3.1. *Let $f : I \subset \mathbb{R} \to \mathbb{R}$ be a differentiable function on I° with $a, b \in I (a < b)$ and $f' \in L_1[a,b]$, then*

$$f(x) = \frac{\Gamma(\alpha)}{b-a}(b-x)^{1-\alpha} J_a^\alpha f(b) - J_a^{\alpha-1}\left(P_2(x,b)f(b)\right) + J_a^\alpha\left(P_2(x,b)f'(b)\right), \quad \alpha \geq 1, \tag{3.1}$$

where $P_2(x,t)$ is the fractional Peano kernel defined by

$$P_2(x,t) := \begin{cases} \dfrac{t-a}{b-a}(b-x)^{1-\alpha}\Gamma(\alpha), & a \le t < x, \\ \dfrac{t-b}{b-a}(b-x)^{1-\alpha}\Gamma(\alpha), & x \le t \le b. \end{cases} \tag{3.2}$$

Proof. By definition of $P_1(x,t)$, we have

$$\begin{aligned} \Gamma(\alpha) J_a^\alpha\big(P_1(x,b) f'(b)\big) &= \int_a^b (b-t)^{\alpha-1} P_1(x,t) f'(t)dt \\ &= \int_a^x (b-t)^{\alpha-1}\left(\frac{t-a}{b-a}\right) f'(t)dt + \int_x^b (b-t)^{\alpha-1}\left(\frac{t-b}{b-a}\right) f'(t)dt \\ &= \frac{1}{b-a}\left[\int_a^x (b-t)^{\alpha-1}(t-a)f'(t)dt - \int_x^b (b-t)^{\alpha} f'(t)dt\right] \\ &= \frac{1}{b-a}(I_1 + I_2). \end{aligned} \tag{3.3}$$

Integrating by parts, we can state

$$\begin{aligned} I_1 = {} & (b-t)^{\alpha-1}(t-a)f(t)\Big|_a^x - \int_a^x \Big[-(\alpha-1)(b-t)^{\alpha-2}(t-a) + (b-t)^{\alpha-1}\Big] f(t)dt \\ = {} & (b-x)^{\alpha-1}(x-a)f(x) + (\alpha-1)\int_a^x (b-t)^{\alpha-2}(t-a)f(t)dt - \int_a^x (b-t)^{\alpha-1} f(t)dt, \end{aligned} \tag{3.4}$$

and similarly,

$$I_2 = -(b-t)^{\alpha} f(t)\Big|_x^b - \alpha\int_x^b (b-t)^{\alpha-1} f(t)dt = (b-x)^{\alpha} f(x) - \alpha\int_x^b (b-t)^{\alpha-1} f(t)dt. \tag{3.5}$$

Adding (3.4) and (3.5), we get

$$\begin{aligned} \Gamma(\alpha) J_a^\alpha\big(P_1(x,b) f'(b)\big) = \frac{1}{b-a}\bigg\{ & (b-a)(b-x)^{\alpha-1} f(x) + (\alpha-1)\int_a^x (b-t)^{\alpha-2}(t-a)f(t)dt \\ & -\alpha\int_x^b (b-t)^{\alpha-1} f(t)dt - \int_a^x (b-t)^{\alpha-1} f(t)dt \bigg\}. \end{aligned} \tag{3.6}$$

If we add and subtract the integral $(\alpha-1)\int_x^b (b-t)^{\alpha-2}(t-b)f(t)dt$ to the right-hand side of the equation above, then we have

$$\begin{aligned}
\Gamma(\alpha)J_a^\alpha\big(P_1(x,b)f'(b)\big) &= \frac{1}{b-a}\Bigg\{(b-a)(b-x)^{\alpha-1}f(x) \\
&\qquad + (b-a)(\alpha-1)\int_a^b (b-t)^{\alpha-2}P_1(x,t)f(t)dt \\
&\qquad - \int_a^b (b-t)^{\alpha-1}f(t)dt\Bigg\} \\
&= (b-x)^{\alpha-1}f(x) + (\alpha-1)\int_a^b (b-t)^{\alpha-2}P_1(x,t)f(t)dt \\
&\quad - \frac{1}{b-a}\int_a^b (b-t)^{\alpha-1}f(t)dt \\
&= (b-x)^{\alpha-1}f(x) - \frac{\Gamma(\alpha)}{b-a}J_a^\alpha f(b) + \Gamma(\alpha)J_a^{\alpha-1}\big(P_1(x,b)f(b)\big).
\end{aligned} \tag{3.7}$$

Multiplying both sides by $(b-x)^{1-\alpha}$, we obtain

$$J_a^\alpha\big(P_2(x,b)f'(b)\big) = f(x) - \frac{\Gamma(\alpha)}{b-a}(b-x)^{1-\alpha}J_a^\alpha f(b) + J_a^{\alpha-1}\big(P_2(x,b)f(b)\big), \tag{3.8}$$

and so

$$f(x) = \frac{\Gamma(\alpha)}{b-a}(b-x)^{1-\alpha}J_a^\alpha f(b) - J_a^{\alpha-1}\big(P_2(x,b)f(b)\big) + J_a^\alpha\big(P_2(x,b)f'(b)\big). \tag{3.9}$$

This completes the proof. □

Now, we extend Lemma 3.1 as follows.

Theorem 3.2. *Let $f: I\subset\mathbb{R}\to\mathbb{R}$ be a differentiable function on I° with $f'\in L_1[a,b]$, then the following identity holds:*

$$\begin{aligned}
(1-2\lambda)f(x) &= \frac{\Gamma(\alpha)}{b-a}(b-x)^{1-\alpha}J_a^\alpha f(b) - \lambda\left(\frac{b-a}{b-x}\right)^{\alpha-1}f(a) \\
&\quad - J_a^{\alpha-1}\big(P_3(x,b)f(b)\big) + J_a^\alpha\big(P_3(x,b)f'(b)\big), \quad \alpha\geq 1,
\end{aligned} \tag{3.10}$$

where $P_3(x,t)$ is the fractional Peano kernel defined by

$$P_3(x,t) := \begin{cases} \dfrac{t-(1-\lambda)a-\lambda b}{b-a}(b-x)^{1-\alpha}\Gamma(\alpha), & a \le t < x, \\ \dfrac{t-(1-\lambda)b-\lambda a}{b-a}(b-x)^{1-\alpha}\Gamma(\alpha), & x \le t \le b, \end{cases} \tag{3.11}$$

for $0 \le \lambda \le 1$.

Proof. By similar way in proof of Lemma 3.1, we have

$$\begin{aligned} \Gamma(\alpha)J_a^\alpha\left(P_3(x,b)f'(b)\right) &= \int_a^b (b-t)^{\alpha-1}P_3(x,t)f'(t)dt \\ &= \frac{\Gamma(\alpha)(b-x)^{1-\alpha}}{b-a}\left[\int_a^x (b-t)^{\alpha-1}(t-(1-\lambda)a-\lambda b)f'(t)dt\right. \\ &\qquad \left. + \int_x^b (b-t)^{\alpha-1}(t-(1-\lambda)b-\lambda a)f'(t)dt\right] \\ &= \frac{\Gamma(\alpha)(b-x)^{1-\alpha}}{b-a}(J_1+J_2). \end{aligned} \tag{3.12}$$

Integrating by parts, we can state

$$\begin{aligned} J_1 = {} & (b-x)^{\alpha-1}(x-(1-\lambda)a-\lambda b)f(x) + (b-a)^\alpha f(a) \\ & + (\alpha-1)\int_a^x (b-t)^{\alpha-2}(t-(1-\lambda)a-\lambda b)f(t)dt - \int_a^x (b-t)^{\alpha-1}f(t)dt, \end{aligned} \tag{3.13}$$

and similarly,

$$\begin{aligned} J_2 = {} & -(b-x)^\alpha(x-(1-\lambda)b-\lambda a)f(x) \\ & + (\alpha-1)\int_x^b (b-t)^{\alpha-2}(t-(1-\lambda)a-\lambda b)f(t)dt - \int_x^b (b-t)^{\alpha-1}f(t)dt. \end{aligned} \tag{3.14}$$

Thus, by using J_1 and J_2 in (3.12), we get (3.10) which completes the proof. □

Remark 3.3. We note that in the special cases, if we take $\lambda = 0$ in Theorem 3.2, then we get (3.1) with the kernel $P_2(x,t)$.

Theorem 3.4. *Let $f : [a,b] \to \mathbb{R}$ be a differentiable on (a,b) such that $f' \in L_1[a,b]$, where $a < b$. If $|f'(x)| \leq M$ for every $x \in [a,b]$ and $\alpha \geq 1$, then the following inequality holds:*

$$\left|(1-2\lambda)f(x) - \frac{\Gamma(\alpha)}{b-a}(b-x)^{1-\alpha}J_a^{\alpha}f(b) + \lambda\left(\frac{b-a}{b-x}\right)^{\alpha-1}f(a) + J_a^{\alpha-1}\left(P_3(x,b)f(b)\right)\right|$$
$$\leq \frac{M}{\alpha(\alpha+1)}\left\{(b-a)^{\alpha}(b-x)^{1-\alpha}\left[2\lambda^{\alpha+1}+2(1-\lambda)^{\alpha+1}+\lambda(b-a)-1\right]\right.$$
$$\left.+(b-x)\left[2\alpha\frac{b-x}{b-a}-(\alpha+1)\right]\right\}. \tag{3.15}$$

Proof. From Theorem 3.2, we get

$$\left|(1-2\lambda)f(x) - \frac{\Gamma(\alpha)}{b-a}(b-x)^{1-\alpha}J_a^{\alpha}f(b) + \lambda\left(\frac{b-a}{b-x}\right)^{\alpha-1}f(a) + J_a^{\alpha-1}\left(P_3(x,b)f(b)\right)\right|$$
$$\leq \frac{1}{\Gamma(\alpha)}\int_a^b (b-t)^{\alpha-1}|P_3(x,t)|\left|f'(t)\right|dt$$
$$= \frac{(b-x)^{1-\alpha}}{b-a}\left[\int_a^x (b-t)^{\alpha-1}|t-(1-\lambda)a-\lambda b|\left|f'(t)\right|dt\right.$$
$$\left.+\int_x^b (b-t)^{\alpha-1}|t-(1-\lambda)b-\lambda a|\left|f'(t)\right|dt\right]$$
$$\leq \frac{M(b-x)^{1-\alpha}}{b-a}\left\{\int_a^x (b-t)^{\alpha-1}|t-(1-\lambda)a-\lambda b|dt + \int_x^b (b-t)^{\alpha-1}|t-(1-\lambda)b-\lambda a|dt\right\}$$
$$= \frac{M(b-x)^{1-\alpha}}{b-a}\{J_3+J_4\}. \tag{3.16}$$

By simple computation, we obtain

$$J_3 = \int_a^x (b-t)^{\alpha-1}|t-(1-\lambda)a-\lambda b|dt$$
$$= \int_a^{\lambda b+(1-\lambda)a} (b-t)^{\alpha-1}(\lambda b+(1-\lambda)a-t)dt + \int_{\lambda b+(1-\lambda)a}^x (b-t)^{\alpha-1}(t-\lambda b-(1-\lambda)a)dt$$
$$= \frac{(b-a)^{\alpha+1}}{\alpha(\alpha+1)}\left[2(1-\lambda)^{\alpha+1}+\lambda(b-a)-1\right] + \frac{(b-x)^{\alpha}}{\alpha(\alpha+1)}[\alpha(b-x)-(1-\lambda)(b-a)(\alpha+1)], \tag{3.17}$$

and similarly

$$
\begin{aligned}
J_4 &= \int_x^b (b-t)^{\alpha-1}|t-(1-\lambda)b-\lambda a|dt \\
&= \int_x^{\lambda a+(1-\lambda)b} (b-t)^{\alpha-1}(\lambda a+(1-\lambda)b-t)dt + \int_{\lambda a+(1-\lambda)b}^b (b-t)^{\alpha-1}(t-\lambda a-(1-\lambda)b)dt \\
&= \frac{2\lambda^{\alpha+1}(b-a)^{\alpha+1}}{\alpha(\alpha+1)} + \frac{(b-x)^\alpha}{\alpha(\alpha+1)}[\alpha(b-x)-\lambda(b-a)(\alpha+1)].
\end{aligned}
\tag{3.18}
$$

By using J_3 and J_4 in (3.16), we obtain (3.15). □

Remark 3.5. If we take $\lambda = 0$ in Theorem 3.4, then it reduces Theorem 4.1 proved by Anastassiou et al. [13]. So, our results are generalizations of the corresponding results of Anastassiou et al. [13].

Theorem 3.6. *Let $f : [a,b] \to \mathbb{R}$ be a differentiable convex function on (a,b) and $f' \in L_1[a,b]$. Then for any $x \in (a,b)$, the following inequality holds:*

$$
\begin{aligned}
&\frac{1}{\alpha(\alpha+1)}\left[\alpha\frac{(b-x)^2}{b-a}f'_+(x) - \left((b-a)^\alpha(b-x)^{1-\alpha} + \alpha\frac{(b-x)^2}{b-a} - (\alpha+1)(b-x)\right)f'_-(x)\right] \\
&\quad \le \frac{\Gamma(\alpha)}{b-a}(b-x)^{1-\alpha}J_a^\alpha f(b) - J_a^{\alpha-1}(P_2(x,b)f(b)) - f(x), \quad \alpha \ge 1.
\end{aligned}
\tag{3.19}
$$

Proof. Similarly to the proof of Lemma 3.1, we have

$$
\begin{aligned}
&f(x) - \frac{\Gamma(\alpha)}{b-a}(b-x)^{1-\alpha}J_a^\alpha f(b) + J_a^{\alpha-1}(P_2(x,b)f(b)) \\
&\quad = \frac{(b-x)^{1-\alpha}}{b-a}\left[\int_a^x (b-t)^{\alpha-1}(t-a)f'(t)dt - \int_x^b (b-t)^\alpha f'(t)dt\right].
\end{aligned}
\tag{3.20}
$$

Since f is convex, then for any $x \in (a,b)$ we have the following inequalities:

$$f'(t) \le f'_-(x) \quad \text{for a.e. } t \in [a,x], \tag{3.21}$$

$$f'(t) \ge f'_+(x) \quad \text{for a.e. } t \in [x,b]. \tag{3.22}$$

If we multiply (3.21) by $(b-t)^{\alpha-1}(t-a) \ge 0$, $t \in [a,x]$, $\alpha \ge 1$ and integrate on $[a,x]$, we get

$$
\begin{aligned}
\int_a^x (b-t)^{\alpha-1}(t-a)f'(t)dt &\le \int_a^x (b-t)^{\alpha-1}(t-a)f'_-(x)dt \\
&= \frac{1}{\alpha(\alpha+1)}\left[(b-a)^{\alpha+1} + (b-x)^\alpha[\alpha(b-x)-(\alpha+1)(b-a)]\right]f'_-(x),
\end{aligned}
\tag{3.23}
$$

and if we multiply (3.22) by $(b-t)^\alpha \geq 0,\ t \in [x,b],\ \alpha \geq 1$ and integrate on $[x,b]$, we also get

$$\int_x^b (b-t)^\alpha f'(t)dt \geq \int_x^b (b-t)^\alpha f'_+(x)dt = \frac{(b-x)^{\alpha+1}}{\alpha+1} f'_+(x). \tag{3.24}$$

Finally, if we subtract (3.24) from (3.23) and use the representation (3.20) we deduce the desired inequality (3.19). □

Corollary 3.7. *Under the assumptions Theorem 3.6 with $\alpha = 1$, one has*

$$\frac{1}{2}\left[(b-x)^2 f'_+(x) - (a-x)^2 f'_-(x)\right] \leq \int_a^b f(t)dt - (b-a)f(x). \tag{3.25}$$

The proof of Corollary 3.7 is proved by Dragomir in [6]. Hence, our results in Theorem 3.6 are generalizations of the corresponding results of Dragomir [6].

Remark 3.8. If we take $x = (a+b)/2$ in Corollary 3.7, we get

$$0 \leq \frac{b-a}{8}\left[f'_+\left(\frac{a+b}{2}\right) - f'_-\left(\frac{a+b}{2}\right)\right] \leq \frac{1}{b-a}\int_a^b f(t)dt - f\left(\frac{a+b}{2}\right). \tag{3.26}$$

Theorem 3.9. *Let $f : [a,b] \to \mathbb{R}$ be a differentiable convex function on (a,b) and $f' \in L_1[a,b]$. Then for any $x \in [a,b]$, the following inequality holds:*

$$\begin{aligned}
&\frac{\Gamma(\alpha)}{b-a}(b-x)^{1-\alpha} J_a^\alpha f(b) - J_a^{\alpha-1}\left(P_2(x,b)f(b)\right) - f(x) \\
&\quad \leq \frac{1}{\alpha(\alpha+1)}\left[\alpha\frac{(b-x)^2}{b-a} f'_-(b)\right. \\
&\qquad\qquad \left. -\left((b-a)^\alpha (b-x)^{1-\alpha} + \alpha\frac{(b-x)^2}{b-a} - (\alpha+1)(b-x)\right)\right], \quad \alpha \geq 1.
\end{aligned} \tag{3.27}$$

Proof. Assume that $f'_+(a)$ and $f'_-(b)$ are finite. Since f is convex on $[a,b]$, then we have the following inequalities:

$$f'(t) \geq f'_+(a) \quad \text{for a.e. } t \in [a,x], \tag{3.28}$$

$$f'(t) \leq f'_-(b) \quad \text{for a.e. } t \in [x,b]. \tag{3.29}$$

If we multiply (3.28) by $(b-t)^{\alpha-1}(t-a) \geq 0$, $t \in [a,x]$, $\alpha \geq 1$ and integrate on $[a,x]$, we have

$$\begin{aligned}\int_a^x (b-t)^{\alpha-1}(t-a)f'(t)dt &\geq \int_a^x (b-t)^{\alpha-1}(t-a)f'_+(a)dt \\ &= \frac{1}{\alpha(\alpha+1)}\Big[(b-a)^{\alpha+1} + (b-x)^{\alpha}[\alpha(b-x) - (\alpha+1)(b-a)]\Big]f'_+(a),\end{aligned} \tag{3.30}$$

and if we multiply (3.29) by $(b-t)^{\alpha} \geq 0$, $t \in [x,b]$, $\alpha \geq 1$ and integrate on $[x,b]$, we also have

$$\int_x^b (b-t)^{\alpha} f'(t)dt \leq \int_x^b (b-t)^{\alpha} f'_-(b)dt = \frac{(b-x)^{\alpha+1}}{\alpha+1} f'_-(b). \tag{3.31}$$

Finally, if we subtract (3.30) from (3.31) and use the representtation (3.20) we deduce the desired inequality (3.27). □

Corollary 3.10. *Under the assumptions Theorem 3.9 with $\alpha = 1$, one*

$$\int_a^b f(t)dt - (b-a)f(x) \leq \frac{1}{2}\Big[(b-x)^2 f'_-(b) - (a-x)^2 f'_+(a)\Big]. \tag{3.32}$$

The proof of Corollary 3.10 is proved by Dragomir in [6]. So, our results in Theorem 3.9 are generalizations of the corresponding results of Dragomir [6].

Remark 3.11. If we take $x = (a+b)/2$ in Corollary 3.10, we get

$$0 \leq \frac{1}{b-a}\int_a^b f(t)dt - f\left(\frac{a+b}{2}\right) \leq \frac{b-a}{8}\left[f'_-(b) - f'_+(a)\right]. \tag{3.33}$$

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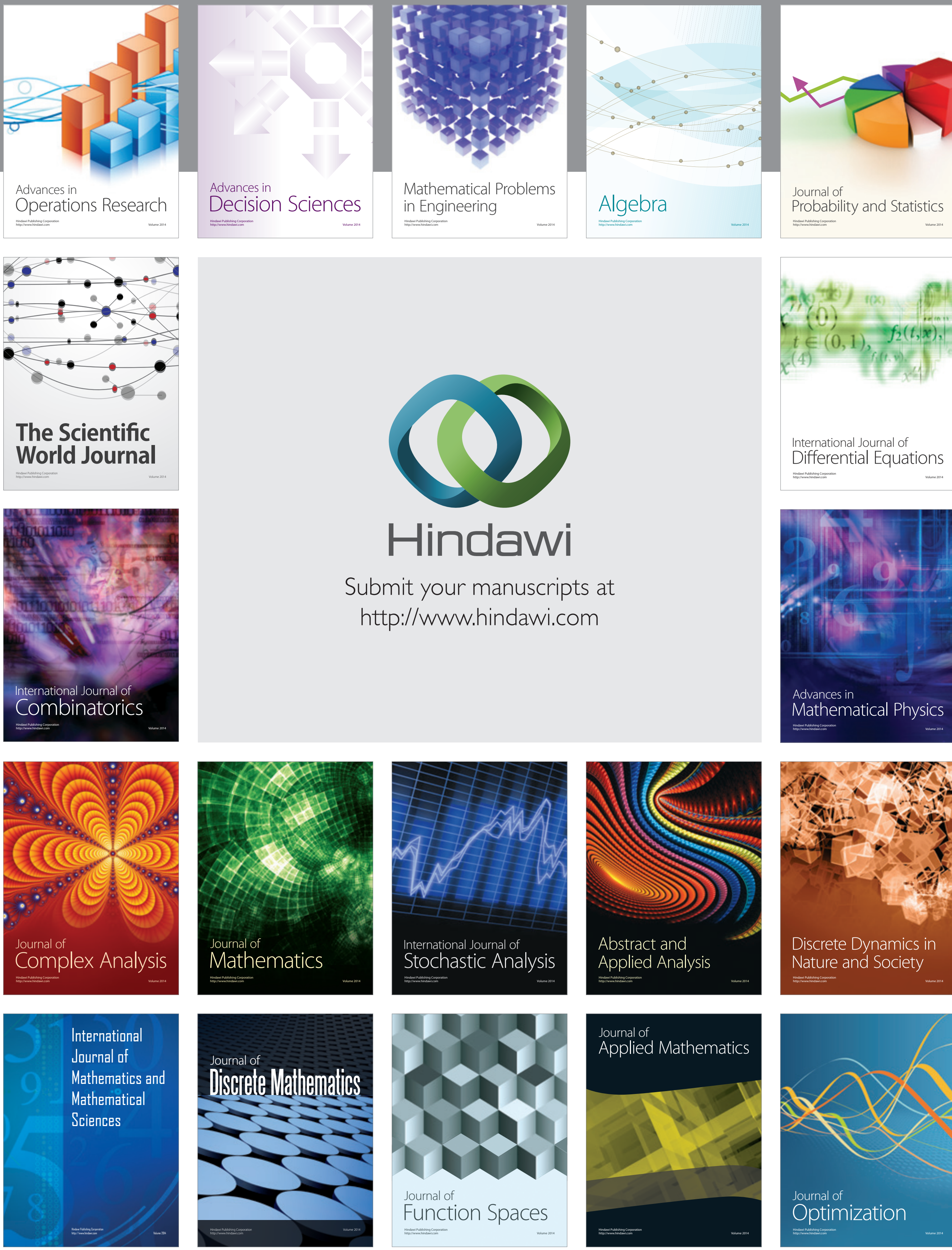

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