

Research Article

On Certain Sufficiency Criteria for p -Valent Meromorphic Spirallike Functions

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Received 10 June 2012; Accepted 6 August 2012

Academic Editor: Allan Peterson

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We consider some subclasses of meromorphic multivalent functions and obtain certain simple sufficiency criteria for the functions belonging to these classes. We also study the mapping properties of these classes under an integral operator.

1. Introduction

Let $\Sigma(p, n)$ denote the class of functions $f(z)$ of the form

$$f(z) = z^{-p} + \sum_{k=n}^{\infty} a_k z^{k-p+1} \quad (p \in \mathbb{N}), \quad (1.1)$$

which are analytic and p -valent in the punctured unit disk $\mathbb{U} = \{z : 0 < |z| < 1\}$. Also let $\Sigma_{\lambda}^*(p, n, \alpha)$ and $\Sigma_c^*(p, n, \alpha)$ denote the subclasses of $\Sigma(p, n)$ consisting of all functions $f(z)$ which are defined, respectively, by

$$\begin{aligned} \operatorname{Re} \left(-e^{i\lambda} \frac{zf'(z)}{f(z)} \right) &> \alpha \cos \lambda \quad (z \in \mathbb{U}), \\ \operatorname{Re} \left(-e^{i\lambda} \frac{(zf'(z))'}{f'(z)} \right) &> \alpha \cos \lambda \quad (z \in \mathbb{U}). \end{aligned} \quad (1.2)$$

We note that for $\lambda = 0$ and $n = 1$, the above classes reduce to the well-known subclasses of $\Sigma(p)$ consisting of meromorphic multivalent functions which are, respectively, starlike

and convex of order α ($0 \leq \alpha < p$). For the detail on the subject of meromorphic spiral-like functions and related topics, we refer the work of Liu and Srivastava [1], Goyal and Prajapat [2], Raina and Srivastava [3], Xu and Yang [4], and Spacek [5] and Robertson [6].

Analogous to the subclass of $\sum(1, 1)$ for meromorphic univalent functions studied by Wang et al. [7] and Nehari and Netanyahu [8], we define a subclass $\sum_N^\lambda(p, n, \alpha)$ of $\sum(p, n)$ consisting of functions $f(z)$ satisfying

$$-\operatorname{Re} e^{i\lambda} \left(\frac{(zf'(z))'}{f'(z)} \right) < \alpha \cos \lambda \quad (\alpha > p, z \in \mathbb{U}). \quad (1.3)$$

For more details of the above classes see also [9, 10].

Motivated from the work of Frasin [11], we introduce the following integral operator of multivalent meromorphic functions $\sum(p)$

$$H_{m,p}(z) = \frac{1}{z^{p+1}} \int_0^z \prod_{j=1}^m (u^p f_j(u))^{\alpha_j} du. \quad (1.4)$$

For $p = 1$, (1.4) reduces to the integral operator introduced and studied by Mohammed and Darus [12, 13]. Similar integral operators for different classes of analytic, univalent, and multivalent functions in the open unit disk are studied by various authors, see [14–19].

In this paper, first, we find sufficient conditions for the classes $\sum_\lambda^*(p, n, \alpha)$ and $\sum_c^\lambda(p, n, \alpha)$ and then study some mapping properties of the integral operator given by (1.4).

We will assume throughout our discussion, unless otherwise stated, that λ is real with $|\lambda| < \pi/2$, $0 \leq \alpha < p$, $p, n \in \mathbb{N}$, $\alpha_j > 0$ for $j \in \{1, \dots, m\}$.

To obtain our main results, we need the following Lemmas.

Lemma 1.1 (see [20]). *If $q(z) \in \sum(1, n)$ with $n \geq 1$ and satisfies the condition*

$$\left| z^2 q'(z) + 1 \right| < \frac{n}{\sqrt{n^2 + 1}} \quad (z \in \mathbb{U}), \quad (1.5)$$

then

$$q(z) \in \sum_0^*(1, n, 0). \quad (1.6)$$

Lemma 1.2 (see [21]). *Let Ω be a set in the complex plane \mathbb{C} and suppose that Ψ is a mapping from $\mathbb{C}^2 \times \mathbb{U}$ to \mathbb{C} which satisfies $\Psi(ix, y, z) \notin \Omega$ for $z \in \mathbb{U}$, and for all real x, y such that $y \leq (-n/2)(1 + x^2)$. If $h(z) = 1 + c_n z^n + \dots$ is analytic in \mathbb{U} and $\Psi(h(z), zh'(z), z) \in \Omega$ for all $z \in \mathbb{U}$, then $\operatorname{Re} h(z) > 0$.*

2. Some Properties of the Classes $\sum_{\lambda}^*(p, n, \alpha)$ and $\sum_c^{\lambda}(p, n, \alpha)$

Theorem 2.1. If $f(z) \in \sum(p, n)$ satisfies

$$\begin{aligned} & \left| (z^p f(z))^{e^{i\lambda}/(p-\alpha) \cos \lambda} \left\{ e^{i\lambda} \frac{zf'(z)}{f(z)} + \alpha \cos \lambda + ip \sin \lambda \right\} + (p - \alpha) \cos \lambda \right| \\ & < \frac{n}{\sqrt{n^2 + 1}} (p - \alpha) \cos \lambda \quad (z \in \mathbb{U}), \end{aligned} \quad (2.1)$$

then $f(z) \in \sum_{\lambda}^*(p, n, \alpha)$.

Proof. Let us set a function $h(z)$ by

$$h(z) = \frac{1}{z} (z^p f(z))^{e^{i\lambda}/(p-\alpha) \cos \lambda} = \frac{1}{z} + \frac{e^{i\lambda} a_n}{(p - \alpha) \cos \lambda} z^n + \dots \quad (2.2)$$

for $f(z) \in \sum(p, n)$. Then clearly (2.2) shows that $h(z) \in \sum(1, n)$.

Differentiating (2.2) logarithmically, we have

$$\frac{h'(z)}{h(z)} = \frac{e^{i\lambda}}{(p - \alpha) \cos \lambda} \left[\frac{f'(z)}{f(z)} + \frac{p}{z} \right] - \frac{1}{z} \quad (2.3)$$

which gives

$$\begin{aligned} & \left| z^2 h'(z) + 1 \right| \\ &= \left| (z^p f(z))^{e^{i\lambda}/(p-\alpha) \cos \lambda} \frac{1}{(p - \alpha) \cos \lambda} \left\{ e^{i\lambda} \frac{zf'(z)}{f(z)} + \alpha \cos \lambda + ip \sin \lambda \right\} + 1 \right|. \end{aligned} \quad (2.4)$$

Thus using (2.1), we have

$$\left| z^2 h'(z) + 1 \right| \leq \frac{n}{\sqrt{n^2 + 1}} \quad (z \in \mathbb{U}). \quad (2.5)$$

Hence, using Lemma 1.1, we have $h(z) \in \sum_0^*(1, n, 0)$.

From (2.3), we can write

$$\frac{zh'(z)}{h(z)} = \frac{1}{(p - \alpha) \cos \lambda} \left[e^{i\lambda} \frac{zf'(z)}{f(z)} + (\alpha \cos \lambda + ip \sin \lambda) \right]. \quad (2.6)$$

Since $h(z) \in \sum_0^*(1, n, 0)$, it implies that $\operatorname{Re}(-zh'(z)/h(z)) > 0$. Therefore, we get

$$\frac{1}{(p - \alpha) \cos \lambda} \left[\operatorname{Re} \left(-e^{i\lambda} \frac{zf'(z)}{f(z)} \right) - \alpha \cos \lambda \right] = \operatorname{Re} \left(-\frac{zh'(z)}{h(z)} \right) > 0 \quad (2.7)$$

or

$$\operatorname{Re} \left(-e^{i\lambda} \frac{zf'(z)}{f(z)} \right) > \alpha \cos \lambda, \quad (2.8)$$

and this implies that $f(z) \in \sum_{\lambda}^*(p, n, \alpha)$.

If we take $\lambda = 0$, we obtain the following result. \square

Corollary 2.2. If $f(z) \in \sum(p, n)$ satisfies

$$\left| (z^p f(z))^{1/(p-\alpha)} \left\{ e^{i\lambda} \frac{zf'(z)}{f(z)} + \alpha \right\} + (p - \alpha) \right| < \frac{1}{\sqrt{2}} (p - \alpha) \quad (z \in \mathbb{U}), \quad (2.9)$$

then $f(z) \in \sum^*(p, n, \alpha)$.

Theorem 2.3. If $f(z) \in \sum(p, n)$ satisfies

$$\begin{aligned} & \left| \left(\frac{z^{p+1} f'(z)}{-p} \right)^{e^{i\lambda}/(p-\alpha) \cos \lambda} \left\{ e^{i\lambda} \left(\frac{zf''(z)}{f'(z)} + 1 \right) + \alpha \cos \lambda + ip \sin \lambda \right\} + (p - \alpha) \cos \lambda \right| \\ & < \frac{(n+1)(p-\alpha) \cos \lambda}{\sqrt{(n+1)^2 + 1}} \quad (z \in \mathbb{U}), \end{aligned} \quad (2.10)$$

then $f(z) \in \sum_c^{\lambda}(p, n, \alpha)$.

Proof. Let us set

$$h(z) = - \int_0^z \frac{1}{t^2} \left(\frac{-t^{p+1} f'(t)}{p} \right)^{e^{i\lambda}/(p-\alpha) \cos \lambda} dt = \frac{1}{z} + \frac{n-p+1}{np} \frac{e^{i\lambda} a_n}{(p-\alpha) \cos \lambda} z^n + \dots \quad (2.11)$$

Also let

$$g(z) = -zh'(z) = \frac{1}{z} \left(\frac{-z^{p+1} f'(z)}{p} \right)^{e^{i\lambda}/(p-\alpha) \cos \lambda} = \frac{1}{z} + \frac{p-n-1}{p} \frac{e^{i\lambda} a_n}{(p-\alpha) \cos \lambda} z^n + \dots \quad (2.12)$$

Then clearly $h(z)$ and $g(z) \in \sum(1, n)$. Now

$$g(z) = \frac{1}{z} \left(\frac{-z^{p+1} f'(z)}{p} \right)^{e^{i\lambda}/(p-\alpha) \cos \lambda}. \quad (2.13)$$

Differentiating logarithmically and then simple computation gives us

$$\begin{aligned}
& \left| z^2 g'(z) + 1 \right| \\
&= \left| \left(\frac{z^{p+1} f'(z)}{-p} \right)^{e^{i\lambda}/(p-\alpha) \cos \lambda} \frac{1}{(p-\alpha) \cos \lambda} \left\{ e^{i\lambda} \left(\frac{zf''(z)}{f'(z)} + 1 \right) + \alpha \cos \lambda + ip \sin \lambda \right\} + 1 \right| \\
&< \frac{n}{\sqrt{n^2 + 1}}.
\end{aligned} \tag{2.14}$$

Therefore, by using Lemma 1.1, we have

$$g(z) = -zh'(z) \in \sum_0^*(1, n, 0) \tag{2.15}$$

which implies that $h(z) \in \sum_c^0(1, n, 0)$. Since

$$1 + \frac{zh''(z)}{h'(z)} = \frac{e^{i\lambda}}{(p-\alpha) \cos \lambda} \left\{ \frac{zf''(z)}{f'(z)} + (p+1) \right\} - 1, \tag{2.16}$$

therefore

$$\begin{aligned}
\operatorname{Re} \left(1 + \frac{zh''(z)}{h'(z)} \right) &= \frac{1}{(p-\alpha) \cos \lambda} \operatorname{Re} \left\{ e^{i\lambda} \left(1 + \frac{zf''(z)}{f'(z)} \right) + pe^{i\lambda} - (p-\alpha) \cos \lambda \right\} \\
&= \frac{1}{(p-\alpha) \cos \lambda} \left\{ \operatorname{Re} e^{i\lambda} \left(1 + \frac{zf''(z)}{f'(z)} \right) + \alpha \cos \lambda \right\}.
\end{aligned} \tag{2.17}$$

Since $h(z) \in \sum_c^0(1, n, 0)$, so

$$\frac{-1}{(p-\alpha) \cos \lambda} \left\{ \operatorname{Re} e^{i\lambda} \left(1 + \frac{zf''(z)}{f'(z)} \right) + \alpha \cos \lambda \right\} > 0, \tag{2.18}$$

or

$$-\operatorname{Re} e^{i\lambda} \left(1 + \frac{zf''(z)}{f'(z)} \right) > \alpha \cos \lambda. \tag{2.19}$$

It follows that $f(z) \in \sum_c^\lambda(p, n, \alpha)$. \square

Theorem 2.4. If $f(z) \in \sum(p, n)$ satisfies

$$\operatorname{Re} \left(e^{i\lambda} \frac{zf'(z)}{f(z)} \right) \left(\alpha \frac{zf''(z)}{f'(z)} - 1 \right) > \frac{M^2}{4L} + N \quad (z \in \mathbb{U}), \tag{2.20}$$

then $f(z) \in \sum_{\lambda}^*(p, n, \beta)$, where $0 \leq \alpha \leq 1$, $0 \leq \beta < p$ and

$$\begin{aligned} L &= \alpha(p - \beta) \left[\frac{n}{2} + (\beta - p) \cos 2\lambda \right] \cos \lambda, \\ M &= \alpha(\beta - p)(1 - \beta \cos \lambda) \sin 2\lambda \cos \lambda, \\ N &= \alpha \left(\beta^2 \cos^2 \lambda + \sin^2 \lambda - \frac{n}{2}(\beta - p) \right) \cos \lambda + (1 + \alpha)\beta \cos \lambda. \end{aligned} \quad (2.21)$$

Proof. Let us set

$$e^{i\lambda} \frac{zf'(z)}{f(z)} = [(\beta - p)h(z) - \beta] \cos \lambda - i \sin \lambda. \quad (2.22)$$

Then $h(z)$ is analytic in \mathbb{U} with $p(0) = 1$.

Taking logarithmic differentiation of (2.22) and then by simple computation, we obtain

$$\begin{aligned} e^{i\lambda} \frac{zf'(z)}{f(z)} \left(\alpha \frac{zf''(z)}{f'(z)} - 1 \right) &= Azh'(z) + Bh^2(z) + Ch(z) + D \\ &= \Psi(h(z), zh'(z), z) \end{aligned} \quad (2.23)$$

with

$$\begin{aligned} A &= (\beta - p)\alpha \cos \lambda, \\ B &= \alpha e^{-i\lambda} (\beta - p)^2 \cos^2 \lambda, \\ C &= (p - \beta) \left[\alpha e^{-i\lambda} \left(2\beta \cos^2 \lambda + i \sin 2\lambda \right) + (1 + \alpha)e^{-i\lambda} \cos \lambda \right], \\ D &= \alpha e^{-i\lambda} \left(\beta^2 \cos^2 \lambda - \sin^2 \lambda + i\beta \sin 2\lambda \right) + (1 + \alpha)(\beta \cos \lambda - i \sin \lambda). \end{aligned} \quad (2.24)$$

Now for all real x and y satisfying $y \leq -(n/2)(1 + x^2)$, we have

$$\Psi(ix, y, z) = Ay - Bx^2 + C(ix) + D. \quad (2.25)$$

Reputing the values of A , B , C , and D and then taking real part, we obtain

$$\begin{aligned} \operatorname{Re} \Psi(ix, y, z) &\leq -Lx^2 + Mx + N \\ &= -\left(\sqrt{Lx} - \frac{M}{2\sqrt{L}} \right)^2 + \frac{M^2}{4L} + N \\ &< \frac{M^2}{4L} + N, \end{aligned} \quad (2.26)$$

where L , M , and N are given in (2.21).

Let $\Omega = \{w : \operatorname{Re} w > (M^2/4L) + N\}$. Then $\Psi(h(z), zh'(z), z) \in \Omega$ and $\Psi(ix, y, z) \notin \Omega$, for all real x and y satisfying $y \leq -(n/2)(1 + x^2)$, $z \in \mathbb{U}$. By using Lemma 1.2, we have $\operatorname{Re} h(z) > 0$, that is $f(z) \in \sum_{\lambda}^*(p, n, \beta)$.

If we put $\lambda = 0$, we obtain the following result. \square

Corollary 2.5. *If $f(z) \in \sum(p, n)$ satisfies*

$$\operatorname{Re} \frac{zf'(z)}{f(z)} \left(\alpha \frac{zf''(z)}{f'(z)} - 1 \right) > \alpha \left(\beta^2 - \frac{n}{2}(\beta - p) \right) + (1 + \alpha)\beta \quad (z \in \mathbb{U}), \quad (2.27)$$

then $f(z) \in \sum^*(p, n, \beta)$, where $0 \leq \alpha \leq 1$, $0 \leq \beta < p$.

Theorem 2.6. *For $j \in \{1, \dots, m\}$, let $f_j(z) \in \sum(p, n)$ and satisfy (2.9). If*

$$\sum_{j=1}^m \alpha_j < \frac{p+1}{p-\beta}, \quad (2.28)$$

then $H_{m,p}(z) \in \sum_N(p, n, \zeta)$, where $\zeta > p$.

Proof. From (1.4), we obtain

$$z^{p+1} H'_{m,p}(z) + (p+1)z^p H_{m,p}(z) = \prod_{j=1}^m (z^p f_j(z))^{\alpha_j}. \quad (2.29)$$

Differentiating again logarithmically and then by simple computation, we get

$$\frac{zH''_{m,p}(z)}{H'_{m,p}(z)} + 1 + (p^2 - 1) \frac{H_{m,p}(z)}{zH'_{m,p}(z)} + 2p = \left[1 + (p+1) \frac{H_{m,p}(z)}{zH'_{m,p}(z)} \right] \left[\sum_{j=1}^m \alpha_j \left(\frac{zf'_j(z)}{f_j(z)} + p \right) - 1 \right], \quad (2.30)$$

or, equivalently we can write

$$\begin{aligned} -\left(\frac{zH''_{m,p}(z)}{H'_{m,p}(z)} + 1 \right) &= \frac{H_{m,p}(z)}{zH'_{m,p}(z)} \left[(p+1) \left(\sum_{j=1}^m \alpha_j \left(-\frac{zf'_j(z)}{f_j(z)} - p \right) + 1 \right) + (p^2 - 1) \right] \\ &\quad + \sum_{j=1}^m \alpha_j \left(-\frac{zf'_j(z)}{f_j(z)} - p \right) + (1 + 2p). \end{aligned} \quad (2.31)$$

Now taking real part on both sides, we obtain

$$\begin{aligned} -\operatorname{Re}\left(\frac{zH''_{m,p}(z)}{H'_{m,p}(z)} + 1\right) &= \operatorname{Re} \frac{H_{m,p}(z)}{zH'_{m,p}(z)} \left[(p+1) \left(\sum_{j=1}^m \alpha_j \left(-\frac{zf'_j(z)}{f_j(z)} - p \right) + 1 \right) + (p^2 - 1) \right] \\ &\quad + \sum_{j=1}^m \alpha_j \left(-\operatorname{Re} \frac{zf'_j(z)}{f_j(z)} - p \right) + (1 + 2p). \end{aligned} \tag{2.32}$$

This further implies that

$$\begin{aligned} -\operatorname{Re}\left(\frac{zH''_{m,p}(z)}{H'_{m,p}(z)} + 1\right) &\leq \left| \frac{H_{m,p}(z)}{zH'_{m,p}(z)} \left[(p+1) \left(\sum_{j=1}^m \alpha_j \left(-\frac{zf'_j(z)}{f_j(z)} - p \right) + 1 \right) + (p^2 - 1) \right] \right| \\ &\quad + \sum_{j=1}^m \alpha_j \left(-\operatorname{Re} \frac{zf'_j(z)}{f_j(z)} - p \right) + (1 + 2p). \end{aligned} \tag{2.33}$$

Let

$$\begin{aligned} \zeta &= \left| \frac{H_{m,p}(z)}{zH'_{m,p}(z)} \left[(p+1) \left(\sum_{j=1}^m \alpha_j \left(-\frac{zf'_j(z)}{f_j(z)} - p \right) + 1 \right) + (p^2 - 1) \right] \right| \\ &\quad + \left[\sum_{j=1}^m \alpha_j \left(-\operatorname{Re} \frac{zf'_j(z)}{f_j(z)} - p \right) + (1 + 2p) \right]. \end{aligned} \tag{2.34}$$

Clearly we have

$$\zeta > \left[\sum_{j=1}^m \alpha_j \left(-\operatorname{Re} \frac{zf'_j(z)}{f_j(z)} - p \right) + (1 + 2p) \right]. \tag{2.35}$$

Then by using (2.28) and Corollary 2.2, we obtain

$$\zeta > \sum_{j=1}^m \alpha_j (\beta - p) + (1 + 2p) > p. \tag{2.36}$$

Therefore $H_{m,p}(z) \in \Sigma_N(p, n, \zeta)$ with $\zeta > p$.

Making use of (2.27) and Corollary 2.5, one can prove the following result. \square

Theorem 2.7. For $j \in \{1, \dots, m\}$, let $f_j(z) \in \Sigma(p, n)$ and satisfy (2.27). If

$$\sum_{j=1}^m \alpha_j < \frac{p+1}{p-\beta}, \quad (2.37)$$

then $H_{m,p}(z) \in \Sigma_N(p, n, \zeta)$, where $\zeta > p$.

Acknowledgment

The author would like to thank Prof. Dr. Ihsan Ali, Vice Chancellor Abdul Wali Khan University Mardan for providing excellent research facilities and financial support.

References

- [1] J.-L. Liu and H. M. Srivastava, "A linear operator and associated families of meromorphically multivalent functions," *Journal of Mathematical Analysis and Applications*, vol. 259, no. 2, pp. 566–581, 2001.
- [2] S. P. Goyal and J. K. Prajapat, "A new class of meromorphic multivalent functions involving certain linear operator," *Tamsui Oxford Journal of Mathematical Sciences*, vol. 25, no. 2, pp. 167–176, 2009.
- [3] R. K. Raina and H. M. Srivastava, "A new class of meromorphically multivalent functions with applications to generalized hypergeometric functions," *Mathematical and Computer Modelling*, vol. 43, no. 3–4, pp. 350–356, 2006.
- [4] N. Xu and D. Yang, "On starlikeness and close-to-convexity of certain meromorphic functions," *Journal of the Korea Society of Mathematical Education B*, vol. 10, no. 1, pp. 566–581, 2003.
- [5] L. Spacek, "Prispevek k teorii funkcií prostých," *Časopis Pro Pestování Matematiky A Fysiky*, vol. 62, pp. 12–19, 1933.
- [6] M. S. Robertson, "Univalent functions $f(z)$ for which $zf'(z)$ is spirallike," *The Michigan Mathematical Journal*, vol. 16, pp. 97–101, 1969.
- [7] Z.-G. Wang, Y. Sun, and Z.-H. Zhang, "Certain classes of meromorphic multivalent functions," *Computers & Mathematics with Applications*, vol. 58, no. 7, pp. 1408–1417, 2009.
- [8] Z. Nehari and E. Netanyahu, "On the coefficients of meromorphic schlicht functions," *Proceedings of the American Mathematical Society*, vol. 8, pp. 15–23, 1957.
- [9] Z.-G. Wang, Z.-H. Liu, and R.-G. Xiang, "Some criteria for meromorphic multivalent starlike functions," *Applied Mathematics and Computation*, vol. 218, no. 3, pp. 1107–1111, 2011.
- [10] Z.-G. Wang, Z.-H. Liu, and A. Catas, "On neighborhoods and partial sums of certain meromorphic multivalent functions," *Applied Mathematics Letters*, vol. 24, no. 6, pp. 864–868, 2011.
- [11] B. A. Frasin, "New general integral operators of p -valent functions," *Journal of Inequalities in Pure and Applied Mathematics*, vol. 10, no. 4, article 109, 2009.
- [12] A. Mohammed and M. Darus, "A new integral operator for meromorphic functions," *Acta Universitatis Apulensis*, no. 24, pp. 231–238, 2010.
- [13] A. Mohammed and M. Darus, "Starlikeness properties of a new integral operator for meromorphic functions," *Journal of Applied Mathematics*, Article ID 804150, 8 pages, 2011.
- [14] M. Arif, W. Haq, and M. Ismail, "Mapping properties of generalized Robertson functions under certain integral operators," *Applied Mathematics*, vol. 3, no. 1, pp. 52–55, 2012.
- [15] K. I. Noor, M. Arif, and A. Muhammad, "Mapping properties of some classes of analytic functions under an integral operator," *Journal of Mathematical Inequalities*, vol. 4, no. 4, pp. 593–600, 2010.
- [16] N. Breaz, V. Pescar, and D. Breaz, "Univalence criteria for a new integral operator," *Mathematical and Computer Modelling*, vol. 52, no. 1–2, pp. 241–246, 2010.
- [17] B. A. Frasin, "Convexity of integral operators of p -valent functions," *Mathematical and Computer Modelling*, vol. 51, no. 5–6, pp. 601–605, 2010.
- [18] B. A. Frasin, "Some sufficient conditions for certain integral operators," *Journal of Mathematical Inequalities*, vol. 2, no. 4, pp. 527–535, 2008.
- [19] G. Saltik, E. Deniz, and E. Kadioğlu, "Two new general p -valent integral operators," *Mathematical and Computer Modelling*, vol. 52, no. 9–10, pp. 1605–1609, 2010.

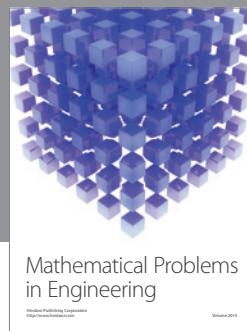
- [20] H. Al-Amiri and P. T. Mocanu, "Some simple criteria of starlikeness and convexity for meromorphic functions," *Mathematica*, vol. 37(60), no. 1-2, pp. 11–20, 1995.
- [21] S. S. Miller and P. T. Mocanu, "Differential subordinations and inequalities in the complex plane," *Journal of Differential Equations*, vol. 67, no. 2, pp. 199–211, 1987.



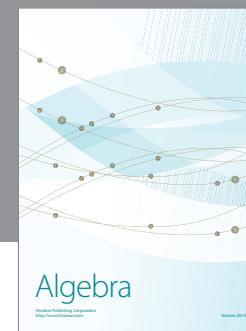
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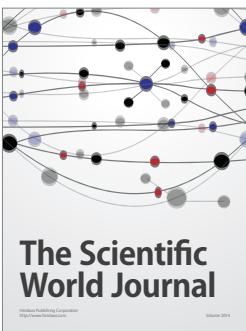
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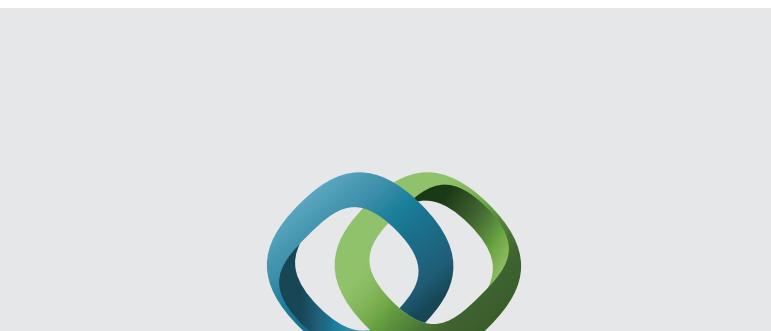
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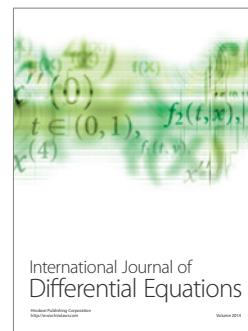


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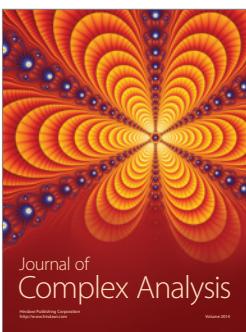
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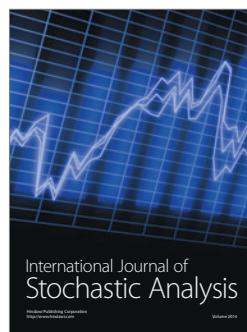
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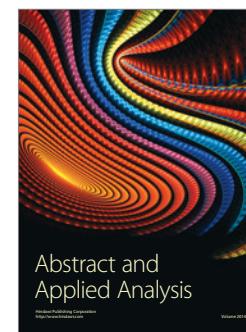
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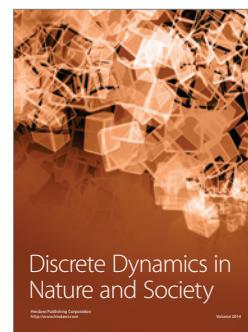
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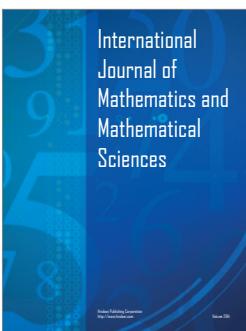
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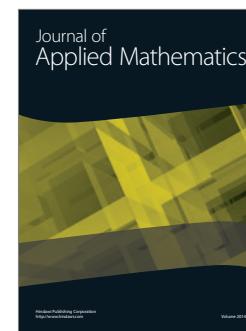
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