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Research Article

On Certain Sufficiency Criteria for p-Valent Meromorphic Spiralike Functions

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We consider some subclasses of meromorphic multivalent functions and obtain certain simple sufficiency criteria for the functions belonging to these classes. We also study the mapping properties of these classes under an integral operator.

1. Introduction

Let $\sum(p,n)$ denote the class of functions $f(z)$ of the form

$$f(z) = z^{-p} + \sum_{k=n}^{\infty} a_k z^{k-p+1} \quad (p \in \mathbb{N}), \tag{1.1}$$

which are analytic and p-valent in the punctured unit disk $\mathbb{U} = \{z : 0 < |z| < 1\}$. Also let $\sum_{\lambda}^{*}(p,n,\alpha)$ and $\sum_{c}^{\lambda}(p,n,\alpha)$ denote the subclasses of $\sum(p,n)$ consisting of all functions $f(z)$ which are defined, respectively, by

$$\begin{gathered}\operatorname{Re}\left(-e^{i\lambda}\frac{zf'(z)}{f(z)}\right) > \alpha\cos\lambda \quad (z \in \mathbb{U}),\\ \operatorname{Re}\left(-e^{i\lambda}\frac{(zf'(z))'}{f'(z)}\right) > \alpha\cos\lambda \quad (z \in \mathbb{U}).\end{gathered} \tag{1.2}$$

We note that for $\lambda = 0$ and $n = 1$, the above classes reduce to the well-known subclasses of $\sum(p)$ consisting of meromorphic multivalent functions which are, respectively, starlike and convex of order α $(0 \leq \alpha < p)$. For the detail on the subject of meromorphic spiral-like functions and related topics, we refer the work of Liu and Srivastava [1], Goyal and Prajapat [2], Raina and Srivastava [3], Xu and Yang [4], and Spacek [5] and Robertson [6].

Analogous to the subclass of $\sum(1,1)$ for meromorphic univalent functions studied by Wang et al. [7] and Nehari and Netanyahu [8], we define a subclass $\sum_N^\lambda(p,n,\alpha)$ of $\sum(p,n)$ consisting of functions $f(z)$ satisfying

$$-\operatorname{Re} e^{i\lambda}\left(\frac{(zf'(z))'}{f'(z)}\right) < \alpha\cos\lambda \quad (\alpha > p, z \in \mathbb{U}). \tag{1.3}$$

For more details of the above classes see also [9, 10].

Motivated from the work of Frasin [11], we introduce the following integral operator of multivalent meromorphic functions $\sum(p)$

$$H_{m,p}(z) = \frac{1}{z^{p+1}}\int_0^z \prod_{j=1}^m \left(u^p f_j(u)\right)^{\alpha_j} du. \tag{1.4}$$

For $p = 1$, (1.4) reduces to the integral operator introduced and studied by Mohammed and Darus [12, 13]. Similar integral operators for different classes of analytic, univalent, and multivalent functons in the open unit disk are studied by various authors, see [14–19].

In this paper, first, we find sufficient conditions for the classes $\sum_\lambda^*(p,n,\alpha)$ and $\sum_c^\lambda(p,n,\alpha)$ and then study some mapping properties of the integral operator given by (1.4).

We will assume throughout our discussion, unless otherwise stated, that λ is real with $|\lambda| < \pi/2$, $0 \leq \alpha < p$, $p, n \in \mathbb{N}$, $\alpha_j > 0$ for $j \in \{1,\ldots m\}$.

To obtain our main results, we need the following Lemmas.

Lemma 1.1 (see [20]). *If $q(z) \in \sum(1,n)$ with $n \geq 1$ and satisfies the condition*

$$\left|z^2 q'(z) + 1\right| < \frac{n}{\sqrt{n^2+1}} \quad (z \in \mathbb{U}), \tag{1.5}$$

then

$$q(z) \in \sum_0^*(1,n,0). \tag{1.6}$$

Lemma 1.2 (see [21]). *Let Ω be a set in the complex plane $\mathbb{C}$ and suppose that Ψ is a mapping from $\mathbb{C}^2 \times \mathbb{U}$ to $\mathbb{C}$ which satisfies $\Psi(ix,y,z) \notin \Omega$ for $z \in \mathbb{U}$, and for all real x, y such that $y \leq (-n/2)(1+x^2)$. If $h(z) = 1 + c_n z^n + \cdots$ is analytic in $\mathbb{U}$ and $\Psi(h(z), zh'(z), z) \in \Omega$ for all $z \in \mathbb{U}$, then* $\operatorname{Re} h(z) > 0$.

2. Some Properties of the Classes $\sum_{\lambda}^{*}(p,n,\alpha)$ and $\sum_{c}^{\lambda}(p,n,\alpha)$

Theorem 2.1. *If $f(z) \in \sum(p,n)$ satisfies*

$$\left|\left(z^p f(z)\right)^{e^{i\lambda}/(p-\alpha)\cos\lambda}\left\{e^{i\lambda}\frac{zf'(z)}{f(z)}+\alpha\cos\lambda+ip\sin\lambda\right\}+(p-\alpha)\cos\lambda\right| < \frac{n}{\sqrt{n^2+1}}(p-\alpha)\cos\lambda \quad (z\in\mathbb{U}), \tag{2.1}$$

then $f(z) \in \sum_{\lambda}^{*}(p,n,\alpha)$.

Proof. Let us set a function $h(z)$ by

$$h(z)=\frac{1}{z}\left(z^p f(z)\right)^{e^{i\lambda}/(p-\alpha)\cos\lambda}=\frac{1}{z}+\frac{e^{i\lambda}a_n}{(p-\alpha)\cos\lambda}z^n+\cdots \tag{2.2}$$

for $f(z)\in\sum(p,n)$. Then clearly (2.2) shows that $h(z)\in\sum(1,n)$.

Differentiating (2.2) logarithmically, we have

$$\frac{h'(z)}{h(z)}=\frac{e^{i\lambda}}{(p-\alpha)\cos\lambda}\left[\frac{f'(z)}{f(z)}+\frac{p}{z}\right]-\frac{1}{z} \tag{2.3}$$

which gives

$$\left|z^2h'(z)+1\right| = \left|\left(z^p f(z)\right)^{e^{i\lambda}/(p-\alpha)\cos\lambda}\frac{1}{(p-\alpha)\cos\lambda}\left\{e^{i\lambda}\frac{zf'(z)}{f(z)}+\alpha\cos\lambda+ip\sin\lambda\right\}+1\right|. \tag{2.4}$$

Thus using (2.1), we have

$$\left|z^2h'(z)+1\right| \le \frac{n}{\sqrt{n^2+1}} \quad (z\in\mathbb{U}). \tag{2.5}$$

Hence, using Lemma 1.1, we have $h(z)\ \sum_0^{*}(1,n,0)$.

From (2.3), we can write

$$\frac{zh'(z)}{h(z)}=\frac{1}{(p-\alpha)\cos\lambda}\left[e^{i\lambda}\frac{zf'(z)}{f(z)}+\left(\alpha\cos\lambda+ip\sin\lambda\right)\right]. \tag{2.6}$$

Since $h(z)\in\sum_0^{*}(1,n,0)$, it implies that $\mathrm{Re}(-zh'(z)/h(z))>0$. Therefore, we get

$$\frac{1}{(p-\alpha)\cos\lambda}\left[\mathrm{Re}\left(-e^{i\lambda}\frac{zf'(z)}{f(z)}\right)-\alpha\cos\lambda\right]=\mathrm{Re}\left(-\frac{zh'(z)}{h(z)}\right)>0 \tag{2.7}$$

or

$$\mathrm{Re}\left(-e^{i\lambda}\frac{zf'(z)}{f(z)}\right) > \alpha\cos\lambda, \tag{2.8}$$

and this implies that $f(z) \in \sum_{\lambda}^{*}(p,n,\alpha)$.

If we take $\lambda = 0$, we obtain the following result. □

Corollary 2.2. *If $f(z) \in \sum(p,n)$ satisfies*

$$\left|\left(z^p f(z)\right)^{1/(p-\alpha)}\left\{e^{i\lambda}\frac{zf'(z)}{f(z)} + \alpha\right\} + (p-\alpha)\right| < \frac{1}{\sqrt{2}}(p-\alpha) \quad (z \in \mathbb{U}), \tag{2.9}$$

then $f(z) \in \sum^{*}(p,n,\alpha)$.

Theorem 2.3. *If $f(z) \in \sum(p,n)$ satisfies*

$$\begin{aligned}&\left|\left(\frac{z^{p+1}f'(z)}{-p}\right)^{e^{i\lambda}/(p-\alpha)\cos\lambda}\left\{e^{i\lambda}\left(\frac{zf''(z)}{f'(z)}+1\right)+\alpha\cos\lambda+ip\sin\lambda\right\}+(p-\alpha)\cos\lambda\right|\\&\quad<\frac{(n+1)(p-\alpha)\cos\lambda}{\sqrt{(n+1)^2+1}} \quad (z\in\mathbb{U}),\end{aligned} \tag{2.10}$$

then $f(z) \in \sum_{c}^{\lambda}(p,n,\alpha)$.

Proof. Let us set

$$h(z) = -\int_0^z \frac{1}{t^2}\left(\frac{-t^{p+1}f'(t)}{p}\right)^{e^{i\lambda}/(p-\alpha)\cos\lambda} dt = \frac{1}{z} + \frac{n-p+1}{np}\frac{e^{i\lambda}a_n}{(p-\alpha)\cos\lambda}z^n + \cdots. \tag{2.11}$$

Also let

$$g(z) = -zh'(z) = \frac{1}{z}\left(\frac{-z^{p+1}f'(z)}{p}\right)^{e^{i\lambda}/(p-\alpha)\cos\lambda} = \frac{1}{z} + \frac{p-n-1}{p}\frac{e^{i\lambda}a_n}{(p-\alpha)\cos\lambda}z^n + \cdots. \tag{2.12}$$

Then clearly $h(z)$ and $g(z) \in \sum(1,n)$. Now

$$g(z) = \frac{1}{z}\left(\frac{-z^{p+1}f'(z)}{p}\right)^{e^{i\lambda}/(p-\alpha)\cos\lambda}. \tag{2.13}$$

Differentiating logarithmically and then simple computation gives us

$$\begin{aligned}
&\left|z^2 g'(z)+1\right| \\
&\quad = \left|\left(\frac{z^{p+1} f'(z)}{-p}\right)^{e^{i\lambda}/(p-\alpha)\cos\lambda} \frac{1}{(p-\alpha)\cos\lambda}\left\{e^{i\lambda}\left(\frac{zf''(z)}{f'(z)}+1\right)+\alpha\cos\lambda+ip\sin\lambda\right\}+1\right| \\
&\quad < \frac{n}{\sqrt{n^2+1}}.
\end{aligned} \tag{2.14}$$

Therefore, by using Lemma 1.1, we have

$$g(z) = -zh'(z) \in \sum_{0}^{*}(1,n,0) \tag{2.15}$$

which implies that $h(z) \in \sum_c^0(1,n,0)$. Since

$$1+\frac{zh''(z)}{h'(z)} = \frac{e^{i\lambda}}{(p-\alpha)\cos\lambda}\left\{\frac{zf''(z)}{f'(z)}+(p+1)\right\}-1, \tag{2.16}$$

therefore

$$\begin{aligned}
\operatorname{Re}\left(1+\frac{zh''(z)}{h'(z)}\right) &= \frac{1}{(p-\alpha)\cos\lambda}\operatorname{Re}\left\{e^{i\lambda}\left(1+\frac{zf''(z)}{f'(z)}\right)+pe^{i\lambda}-(p-\alpha)\cos\lambda\right\} \\
&= \frac{1}{(p-\alpha)\cos\lambda}\left\{\operatorname{Re} e^{i\lambda}\left(1+\frac{zf''(z)}{f'(z)}\right)+\alpha\cos\lambda\right\}.
\end{aligned} \tag{2.17}$$

Since $h(z) \in \sum_c^0(1,n,0)$, so

$$\frac{-1}{(p-\alpha)\cos\lambda}\left\{\operatorname{Re} e^{i\lambda}\left(1+\frac{zf''(z)}{f'(z)}\right)+\alpha\cos\lambda\right\} > 0, \tag{2.18}$$

or

$$-\operatorname{Re} e^{i\lambda}\left(1+\frac{zf''(z)}{f'(z)}\right) > \alpha\cos\lambda. \tag{2.19}$$

It follows that $f(z) \in \sum_c^{\lambda}(p,n,\alpha)$. □

Theorem 2.4. *If* $f(z) \in \sum(p,n)$ *satisfies*

$$\operatorname{Re}\left(e^{i\lambda}\frac{zf'(z)}{f(z)}\right)\left(\alpha\frac{zf''(z)}{f'(z)}-1\right) > \frac{M^2}{4L}+N \quad (z \in \mathbb{U}), \tag{2.20}$$

then $f(z) \in \sum_{\lambda}^{*}(p,n,\beta)$, *where* $0 \le \alpha \le 1$, $0 \le \beta < p$ *and*

$$\begin{aligned} L &= \alpha(p-\beta)\left[\frac{n}{2} + (\beta-p)\cos 2\lambda\right]\cos\lambda, \\ M &= \alpha(\beta-p)(1-\beta\cos\lambda)\sin 2\lambda\cos\lambda, \\ N &= \alpha\left(\beta^2\cos^2\lambda + \sin^2\lambda - \frac{n}{2}(\beta-p)\right)\cos\lambda + (1+\alpha)\beta\cos\lambda. \end{aligned} \tag{2.21}$$

Proof. Let us set

$$e^{i\lambda}\frac{zf'(z)}{f(z)} = \left[(\beta-p)h(z)-\beta\right]\cos\lambda - i\sin\lambda. \tag{2.22}$$

Then $h(z)$ is analytic in $\mathbb{U}$ with $p(0)=1$.

Taking logarithmic differentiation of (2.22) and then by simple computation, we obtain

$$\begin{aligned} e^{i\lambda}\frac{zf'(z)}{f(z)}\left(\alpha\frac{zf''(z)}{f'(z)} - 1\right) &= Azh'(z) + Bh^2(z) + Ch(z) + D \\ &= \Psi(h(z), zh'(z), z) \end{aligned} \tag{2.23}$$

with

$$\begin{aligned} A &= (\beta-p)\alpha\cos\lambda, \\ B &= \alpha e^{-i\lambda}(\beta-p)^2\cos^2\lambda, \\ C &= (p-\beta)\left[\alpha e^{-i\lambda}\left(2\beta\cos^2\lambda + i\sin 2\lambda\right) + (1+\alpha)e^{-i\lambda}\cos\lambda\right], \\ D &= \alpha e^{-i\lambda}\left(\beta^2\cos^2\lambda - \sin^2\lambda + i\beta\sin 2\lambda\right) + (1+\alpha)(\beta\cos\lambda - i\sin\lambda). \end{aligned} \tag{2.24}$$

Now for all real x and y satisfying $y \le -(n/2)(1+x^2)$, we have

$$\Psi(ix, y, z) = Ay - Bx^2 + C(ix) + D. \tag{2.25}$$

Reputing the values of A, B, C, and D and then taking real part, we obtain

$$\begin{aligned} \operatorname{Re}\Psi(ix,y,z) &\le -Lx^2 + Mx + N \\ &= -\left(\sqrt{L}x - \frac{M}{2\sqrt{L}}\right)^2 + \frac{M^2}{4L} + N \\ &< \frac{M^2}{4L} + N, \end{aligned} \tag{2.26}$$

where L, M, and N are given in (2.21).

Let $\Omega = \{w : \operatorname{Re} w > (M^2/4L) + N\}$. Then $\Psi(h(z), zh'(z), z) \in \Omega$ and $\Psi(ix, y, z) \notin \Omega$, for all real x and y satisfying $y \leq -(n/2)(1+x^2)$, $z \in \mathbb{U}$. By using Lemma 1.2, we have $\operatorname{Re} h(z) > 0$, that is $f(z) \in \sum_{\lambda}^{*}(p, n, \beta)$.

If we put $\lambda = 0$, we obtain the following result. □

Corollary 2.5. *If $f(z) \in \sum(p, n)$ satisfies*

$$\operatorname{Re} \frac{zf'(z)}{f(z)}\left(\alpha \frac{zf''(z)}{f'(z)} - 1\right) > \alpha\left(\beta^2 - \frac{n}{2}(\beta - p)\right) + (1+\alpha)\beta \quad (z \in \mathbb{U}), \tag{2.27}$$

then $f(z) \sum^{*}(p, n, \beta)$, where $0 \leq \alpha \leq 1$, $0 \leq \beta < p$.

Theorem 2.6. *For $j \in \{1, \ldots m\}$, let $f_j(z) \in \sum(p, n)$ and satisfy* (2.9). *If*

$$\sum_{j=1}^{m} \alpha_j < \frac{p+1}{p-\beta}, \tag{2.28}$$

then $H_{m,p}(z) \in \sum_N(p, n, \zeta)$, where $\zeta > p$.

Proof. From (1.4), we obtain

$$z^{p+1} H'_{m,p}(z) + (p+1) z^p H_{m,p}(z) = \prod_{j=1}^{m} \left(z^p f_j(z)\right)^{\alpha_j}. \tag{2.29}$$

Differentiating again logarithmically and then by simple computation, we get

$$\frac{zH''_{m,p}(z)}{H'_{m,p}(z)} + 1 + \left(p^2 - 1\right)\frac{H_{m,p}(z)}{zH'_{m,p}(z)} + 2p = \left[1 + (p+1)\frac{H_{m,p}(z)}{zH'_{m,p}(z)}\right]\left[\sum_{j=1}^{m} \alpha_j \left(\frac{zf'_j(z)}{f_j(z)} + p\right) - 1\right], \tag{2.30}$$

or, equivalently we can write

$$\begin{aligned} -\left(\frac{zH''_{m,p}(z)}{H'_{m,p}(z)} + 1\right) = \frac{H_{m,p}(z)}{zH'_{m,p}(z)} &\left[(p+1)\left(\sum_{j=1}^{m} \alpha_j \left(-\frac{zf'_j(z)}{f_j(z)} - p\right) + 1\right) + \left(p^2 - 1\right)\right] \\ &+ \sum_{j=1}^{m} \alpha_j \left(-\frac{zf'_j(z)}{f_j(z)} - p\right) + (1 + 2p). \end{aligned} \tag{2.31}$$

Now taking real part on both sides, we obtain

$$-\operatorname{Re}\left(\frac{zH''_{m,p}(z)}{H'_{m,p}(z)}+1\right) = \operatorname{Re}\frac{H_{m,p}(z)}{zH'_{m,p}(z)}\left[(p+1)\left(\sum_{j=1}^{m}\alpha_j\left(-\frac{zf'_j(z)}{f_j(z)}-p\right)+1\right)+\left(p^2-1\right)\right] + \sum_{j=1}^{m}\alpha_j\left(-\operatorname{Re}\frac{zf'_j(z)}{f_j(z)}-p\right)+(1+2p). \tag{2.32}$$

This further implies that

$$-\operatorname{Re}\left(\frac{zH''_{m,p}(z)}{H'_{m,p}(z)}+1\right) \leq \left|\frac{H_{m,p}(z)}{zH'_{m,p}(z)}\left[(p+1)\left(\sum_{j=1}^{m}\alpha_j\left(-\frac{zf'_j(z)}{f_j(z)}-p\right)+1\right)+\left(p^2-1\right)\right]\right| + \sum_{j=1}^{m}\alpha_j\left(-\operatorname{Re}\frac{zf'_j(z)}{f_j(z)}-p\right)+(1+2p). \tag{2.33}$$

Let

$$\zeta = \left|\frac{H_{m,p}(z)}{zH'_{m,p}(z)}\left[(p+1)\left(\sum_{j=1}^{m}\alpha_j\left(-\frac{zf'_j(z)}{f_j(z)}-p\right)+1\right)+\left(p^2-1\right)\right]\right| + \left[\sum_{j=1}^{m}\alpha_j\left(-\operatorname{Re}\frac{zf'_j(z)}{f_j(z)}-p\right)+(1+2p)\right]. \tag{2.34}$$

Clearly we have

$$\zeta > \left[\sum_{j=1}^{m}\alpha_j\left(-\operatorname{Re}\frac{zf'_j(z)}{f_j(z)}-p\right)+(1+2p)\right]. \tag{2.35}$$

Then by using (2.28) and Corollary 2.2, we obtain

$$\zeta > \sum_{j=1}^{m}\alpha_j(\beta-p)+(1+2p) > p. \tag{2.36}$$

Therefore $H_{m,p}(z) \in \sum_N(p,n,\zeta)$ with $\zeta > p$.

Making use of (2.27) and Corollary 2.5, one can prove the following result. □

Theorem 2.7. *For $j \in \{1, \dots m\}$, let $f_j(z) \in \sum(p, n)$ and satisfy* (2.27)*. If*

$$\sum_{j=1}^{m} \alpha_j < \frac{p+1}{p-\beta}, \tag{2.37}$$

then $H_{m,p}(z) \in \sum_N(p, n, \zeta)$, where $\zeta > p$.

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