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Research Article

On the Recursive Sequence $x_{n+1} = A + (x_{n-1}^p / x_n^q)$

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In this paper we study the boundedness, the persistence, the attractivity and the stability of the positive solutions of the nonlinear difference equation $x_{n+1} = \alpha + (x_{n-1}^p / x_n^q)$, $n = 0, 1, \ldots$, where $\alpha, p, q \in (0, \infty)$ and $x_{-1}, x_0 \in (0, \infty)$. Moreover we investigate the existence of a prime two periodic solution of the above equation and we find solutions which converge to this periodic solution.

1. Introduction

Difference equations have been applied in several mathematical models in biology, economics, genetics, population dynamics, and so forth. For this reason, there exists an increasing interest in studying difference equations (see [1–28] and the references cited therein).

The investigation of positive solutions of the following equation

$$x_n = A + \frac{x_{n-k}^p}{x_{n-m}^q}, \quad n = 0, 1, \ldots, \tag{1.1}$$

where $A, p, q \in [0, \infty)$ and $k, m \in N$, $k \neq m$, was proposed by Stević at numerous conferences. For some results in the area see, for example, [3–5, 8, 11, 12, 19, 22, 24, 25, 28].

In [22] the author studied the boundedness, the global attractivity, the oscillatory behavior, and the periodicity of the positive solutions of the equation

$$x_{n+1} = a + \frac{x_{n-1}^p}{x_n^p}, \quad n = 0, 1, \ldots, \tag{1.2}$$

where $a,\ p$ are positive constants, and the initial conditions x_{-1}, x_0 are positive numbers (see also [5] for more results on this equation).

In [11] the authors obtained boundedness, persistence, global attractivity, and periodicity results for the positive solutions of the difference equation

$$x_{n+1} = a + \frac{x_{n-1}}{x_n^p}, \quad n = 0, 1, \ldots, \tag{1.3}$$

where $a,\ p$ are positive constants and the initial conditions x_{-1}, x_0 are positive numbers.

Motivating by the above papers, we study now the boundedness, the persistence, the existence of unbounded solutions, the attractivity, the stability of the positive solutions, and the two-period solutions of the difference equation

$$x_{n+1} = A + \frac{x_{n-1}^p}{x_n^q}, \quad n = 0, 1, \ldots, \tag{1.4}$$

where $A, p,$ and q are positive constants and the initial values x_{-1}, x_0 are positive real numbers.

Finally equations, closely related to (1.4), are considered in [1–11, 14, 16–23, 26, 27], and the references cited therein.

2. Boundedness and Persistence

The following result is essentially proved in [22]. Hence, we omit its proof.

Proposition 2.1. *If*

$$0 < p < 1, \tag{2.1}$$

then every positive solution of (1.4) *is bounded and persists.*

In the next proposition we obtain sufficient conditions for the existence of unbounded solutions of (1.4).

Proposition 2.2. *If*

$$p > 1 \tag{2.2}$$

then there exist unbounded solutions of (1.4).

Proof. Let x_n be a solution of (1.4) with initial values x_{-1}, x_0 such that

$$x_{-1} > \max\left\{(A+1)^{p/q}, (A+1)^{q/(p-1)}\right\}, \quad x_0 < A + 1. \tag{2.3}$$

Then from (1.4), (2.2), and (2.3) we have

$$\begin{aligned} x_1 &= A + \frac{x_{-1}^p}{x_0^q} > A + \frac{x_{-1}^p}{(A+1)^q} - x_{-1} + x_{-1} \\ &= A + x_{-1}\left(\frac{x_{-1}^{p-1}}{(A+1)^q} - 1\right) + x_{-1} > A + x_{-1}, \\ x_2 &= A + \frac{x_0^p}{x_1^q} < A + \frac{(A+1)^p}{x_{-1}^q} < A + 1. \end{aligned} \tag{2.4}$$

Moreover from (1.4), and (2.3) we have

$$x_1 = A + \frac{x_{-1}^p}{x_0^q} > A + \frac{(A+1)^{qp/(p-1)}}{(A+1)^q} = A + (A+1)^{q/(p-1)} > (A+1)^{q/(p-1)}. \tag{2.5}$$

Then using (1.4), and (2.3)–(2.5) and arguing as above we get

$$\begin{aligned} x_3 &= A + \frac{x_1^p}{x_2^q} > A + \frac{x_1^p}{(A+1)^q} - x_1 + x_1 > A + x_1, \\ x_4 &= A + \frac{x_2^p}{x_3^q} < A + \frac{(A+1)^p}{x_{-1}^q} < A + 1. \end{aligned} \tag{2.6}$$

Therefore working inductively we can prove that for $n = 0, 1, \ldots$

$$x_{2n+1} > A + x_{2n-1}, \quad x_{2n} < A + 1 \tag{2.7}$$

which implies that

$$\lim_{n\to\infty} x_{2n+1} = \infty. \tag{2.8}$$

So x_n is unbounded. This completes the proof of the proposition. □

3. Attractivity and Stability

In the following proposition we prove the existence of a positive equilibrium.

Proposition 3.1. *If either*

$$0 < q < p < 1 \tag{3.1}$$

or

$$0 < p < q \tag{3.2}$$

holds, then (1.4) *has a unique positive equilibrium* $\overline{x}$.

Proof. A point $\overline{x} \in \mathbb{R}$ will be an equilibrium of (1.4) if and only if it satisfies the following equation

$$F(x) = x^{p-q} - x + A = 0. \tag{3.3}$$

Suppose that (3.1) is satisfied. Since (3.1) holds and

$$F'(x) = (p-q)x^{p-q-1} - 1, \tag{3.4}$$

we have that F is increasing in $[0, (p-q)^{1/(-p+q+1)}]$ and F is decreasing in $[(p-q)^{1/(-p+q+1)}, \infty)$. Moreover $F(0) = A > 0$ and

$$\lim_{x\to\infty} F(x) = -\infty. \tag{3.5}$$

So if (3.1) holds, we get that (1.4) has a unique equilibrium $\overline{x}$ in $(0,\infty)$.

Suppose now that (3.2) holds. We observe that $F(1) = A > 0$ and since from (3.2) and (3.4) $F'(x) < 0$, we have that F is decreasing in $(0,\infty)$. Thus from (3.5) we obtain that (1.4) has a unique equilibrium $\overline{x}$ in $(0,\infty)$. The proof is complete. □

In the sequel, we study the global asymptotic stability of the positive solutions of (1.4).

Proposition 3.2. *Consider* (1.4). *Suppose that either*

$$0 < p < 1 < q, \qquad A > (p+q-1)^{1/(q-p+1)} \tag{3.6}$$

or (3.1) *and*

$$0 < p + q \leq 1. \tag{3.7}$$

hold. Then the unique positive equilibrium of (1.4) *is globally asymptotically stable.*

Proof. First we prove that every positive solution of (1.4) tends to the unique positive equilibrium $\overline{x}$ of (1.4).

Assume first that (3.6) is satisfied. Let x_n be a positive solution of (1.4). From (3.6) and Proposition 2.1 we have

$$0 < l = \lim_{n\to\infty} \inf x_n, \qquad L = \lim_{n\to\infty} \sup x_n < \infty. \tag{3.8}$$

Then from (1.4) and (3.8) we get,

$$L \leq A + \frac{L^p}{l^q}, \qquad l \geq A + \frac{l^p}{L^q}, \tag{3.9}$$

and so

$$Ll^q \leq Al^q + L^p, \qquad lL^q \geq AL^q + l^p. \tag{3.10}$$

Thus,

$$AL^q l^{q-1} + l^p l^{q-1} \leq Al^q L^{q-1} + L^p L^{q-1}. \tag{3.11}$$

This implies that

$$AL^{q-1} l^{q-1}(L - l) \leq L^{p+q-1} - l^{p+q-1}. \tag{3.12}$$

Suppose for a while that $p + q - 2 \geq 0$. We shall prove that $l = L$. Suppose on the contrary that $l < L$. If we consider the function x^{p+q-1}, then there exists a $c \in (l, L)$ such that

$$\frac{L^{p+q-1} - l^{p+q-1}}{L - l} = (p + q - 1)c^{p+q-2} \leq (p + q - 1)L^{p+q-2}. \tag{3.13}$$

Then from (3.12) and (3.13) we obtain

$$AL^{q-1} l^{q-1} \leq (p + q - 1)L^{p+q-2} \tag{3.14}$$

or

$$AL^{1-p} l^{q-1} \leq p + q - 1. \tag{3.15}$$

Moreover, since from (1.4),

$$L \geq A, \qquad l \geq A, \tag{3.16}$$

from (3.6) and (3.15) we get

$$AA^{1-p} A^{q-1} = A^{q-p+1} \leq p + q - 1 \tag{3.17}$$

which contradicts to (3.6). So $l = L$ which implies that x_n tends to the unique positive equilibrium $\overline{x}$.

Suppose that $p + q - 2 < 0$. Then from (3.12) and arguing as above we get

$$AL^{q-1}l^{q-1} \leq (p+q-1)l^{p+q-2}. \tag{3.18}$$

Then arguing as above we can prove that x_n tends to the unique positive equilibrium $\overline{x}$.

Assume now that (3.7) holds. From (3.7) and (3.12) we obtain

$$AL^{q-1}l^{q-1}(L-l) \leq \frac{1}{L^{1-p-q}} - \frac{1}{l^{1-p-q}} = \frac{l^{1-p-q} - L^{1-p-q}}{L^{1-p-q}l^{1-p-q}} \leq 0, \tag{3.19}$$

which implies that $L = l$. So every positive solution x_n of (1.4) tends to the unique positive equilibrium $\overline{x}$ of (1.4).

It remains to prove now that the unique positive equilibrium of (1.4) is locally asymptotically stable. The linearized equation about the positive equilibrium $\overline{x}$ is the following:

$$y_{n+2} + q\overline{x}^{p-q-1}y_{n+1} - p\overline{x}^{p-q-1}y_n = 0. \tag{3.20}$$

Using [13, Theorem 1.3.4] the linear (3.20) is asymptotically stable if and only if

$$q\overline{x}^{p-q-1} < -p\overline{x}^{p-q-1} + 1 < 2. \tag{3.21}$$

First assume that (3.6) holds. Since (3.6) holds, then we obtain that

$$A > (p+q)^{(p-q)/(q+1-p)}(q+p-1). \tag{3.22}$$

From (3.6) and (3.22) we can easily prove that

$$F(c) > 0, \quad \text{where } c = (p+q)^{1/(q+1-p)}. \tag{3.23}$$

Therefore

$$\overline{x} > (p+q)^{1/(q+1-p)}, \tag{3.24}$$

which implies that (3.21) is true. So in this case the unique positive equilibrium $\overline{x}$ of (1.4) is locally asymptotically stable.

Finally suppose that (3.1) and (3.7) are satisfied. Then we can prove that (3.23) is satisfied, and so the unique positive equilibrium $\overline{x}$ of (1.4) satisfies (3.24). Therefore (3.21) hold. This implies that the unique positive equilibrium $\overline{x}$ of (1.4) is locally asymptotically stable. This completes the proof of the proposition. □

4. Study of 2-Periodic Solutions

Motivated by [5, Lemma 1], in this section we show that there is a prime two periodic solution. Moreover we find solutions of (1.4) which converge to a prime two periodic solution.

Proposition 4.1. *Consider* (1.4) *where*

$$0 < p < 1 < q. \tag{4.1}$$

Assume that there exists a sufficient small positive real number ϵ_1, such that

$$\frac{1}{(A+\epsilon_1)^{q-p}} > \epsilon_1, \tag{4.2}$$

$$(A+\epsilon_1)^{p/q}\epsilon_1^{-1/q} < A + \epsilon_1^{-p/q}(A+\epsilon_1)^{(p^2-q^2)/q}. \tag{4.3}$$

Then (1.4) *has a periodic solution of prime period two.*

Proof. Let x_n be a positive solution of (1.4). It is obvious that if

$$x_{-1} = A + \frac{x_{-1}^p}{x_0^q}, \qquad x_0 = A + \frac{x_0^p}{x_{-1}^q}, \tag{4.4}$$

then x_n is periodic of period two. Consider the system

$$x = A + \frac{x^p}{y^q}, \qquad y = A + \frac{y^p}{x^q}, \tag{4.5}$$

Then system (4.5) is equivalent to

$$y - A - \frac{y^p}{x^q} = 0, \qquad y = \frac{x^{p/q}}{(x-A)^{1/q}}, \tag{4.6}$$

and so we get the equation

$$G(x) = \frac{x^{p/q}}{(x-A)^{1/q}} - A - \frac{x^{(p^2-q^2)/q}}{(x-A)^{p/q}} = 0. \tag{4.7}$$

We obtain

$$G(x) = \frac{1}{(x-A)^{1/q}}\left(x^{p/q} - x^{(p^2-q^2)/q}(x-A)^{(1-p)/q}\right) - A, \tag{4.8}$$

and so from (4.1)

$$\lim_{x \to A^+} G(x) = \infty. \tag{4.9}$$

Moreover from (4.3) we can show that

$$G(A + \epsilon_1) < 0. \tag{4.10}$$

Therefore the equation $G(x) = 0$ has a solution $\overline{x} = A + \epsilon_0$, where $0 < \epsilon_0 < \epsilon_1$, in the interval $(A, A + \epsilon_1)$. We have

$$\overline{y} = \frac{\overline{x}^{p/q}}{(\overline{x} - A)^{1/q}}. \tag{4.11}$$

We consider the function

$$H(\epsilon) = (A + \epsilon)^{p-q} - \epsilon. \tag{4.12}$$

Since from (4.1) $H'(\epsilon) = (p - q)(A + \epsilon)^{p-q-1} - 1 < 0$ and we have

$$H(\epsilon_0) > H(\epsilon_1). \tag{4.13}$$

From (4.2) we have $H(\epsilon_1) > 0$, so from (4.13)

$$H(\epsilon_0) = (A + \epsilon_0)^{p-q} - \epsilon_0 > 0, \tag{4.14}$$

which implies that

$$\overline{x} = A + \epsilon_0 < \frac{(A + \epsilon_0)^{p/q}}{\epsilon_0^{1/q}} = \overline{y}. \tag{4.15}$$

Hence, if $x_{-1} = \overline{x}$, $x_0 = \overline{y}$, then the solution x_n with initial values x_{-1}, x_0 is a prime 2-periodic solution. □

In the sequel, we shall need the following lemmas.

Lemma 4.2. *Let* $\{x_n\}$ *be a solution of* (1.4). *Then the sequences* $\{x_{2n}\}$ *and* $\{x_{2n+1}\}$ *are eventually monotone.*

Proof. We define the sequence $\{u_n\}$ and the function $h(x)$ as follows:

$$u_n = x_n - A, \qquad h(x) = x + A. \tag{4.16}$$

Then from (1.4) for $n \geq 3$ we get

$$\frac{u_n}{u_{n-2}} = \frac{(u_{n-2}+A)^p}{(u_{n-4}+A)^p}\frac{(u_{n-3}+A)^q}{(u_{n-1}+A)^q} = \frac{(h(u_{n-2}))^p}{(h(u_{n-4}))^p}\frac{(h(u_{n-3}))^q}{(h(u_{n-1}))^q}. \tag{4.17}$$

Then using (4.17) and arguing as in [5, Lemma 2] (see also in [20, Theorem 2]) we can easily prove the lemma. □

Lemma 4.3. *Consider* (1.4) *where* (4.1) *and* (4.3) *hold. Let* x_n *be a solution of* (1.4) *such that either*

$$A < x_{-1} < A+\epsilon_1, \qquad x_0 > (A+\epsilon_1)^{p/q}\epsilon_1^{-1/q} \tag{4.18}$$

or

$$A < x_0 < A+\epsilon_1, \qquad x_{-1} > (A+\epsilon_1)^{p/q}\epsilon_1^{-1/q}. \tag{4.19}$$

Then if (4.18) *holds, one has*

$$A < x_{2n-1} < A+\epsilon_1, \quad x_{2n} > (A+\epsilon_1)^{p/q}\epsilon_1^{-1/q}, \quad n = 0,1,\ldots, \tag{4.20}$$

and if (4.19) *is satisfied, one has*

$$A < x_{2n} < A+\epsilon_1, \quad x_{2n-1} > (A+\epsilon_1)^{p/q}\epsilon_1^{-1/q}, \quad n = 0,1,\ldots. \tag{4.21}$$

Proof. Suppose that (4.18) is satisfied. Then from (1.4) and (4.3) we have

$$\begin{aligned} A < x_1 &= A + \frac{x_{-1}^p}{x_0^q} < A + \epsilon_1\frac{(A+\epsilon_1)^p}{(A+\epsilon_1)^p} = A+\epsilon_1, \\ x_2 &= A + \frac{x_0^p}{x_1^q} > A + (A+\epsilon_1)^{(p^2-q^2)/q}\epsilon_1^{-p/q} > (A+\epsilon_1)^{p/q}\epsilon_1^{-1/q}. \end{aligned} \tag{4.22}$$

Working inductively we can easily prove relations (4.20). Similarly if (4.19) is satisfied, we can prove that (4.21) holds. □

Proposition 4.4. *Consider* (1.4) *where* (4.1), (4.2), *and* (4.3) *hold. Suppose also that*

$$A+\epsilon_1 < 1. \tag{4.23}$$

Then every solution x_n *of* (1.4) *with initial values* x_{-1}, x_0 *which satisfy either* (4.18) *or* (4.19), *converges to a prime two periodic solution.*

Proof. Let x_n be a solution with initial values x_{-1}, x_0 which satisfy either (4.18) or (4.19). Using Proposition 2.1 and Lemma 4.2 we have that there exist

$$\lim_{n\to\infty} x_{2n+1} = L, \qquad \lim_{n\to\infty} x_{2n} = l. \tag{4.24}$$

In addition from Lemma 4.3 we have that either L or l belongs to the interval $(A, A+\epsilon_1)$. Furthermore from Proposition 3.1 we have that (1.4) has a unique equilibrium $\overline{x}$ such that $1 < \overline{x} < \infty$. Therefore from (4.23) we have that $L \neq l$. So x_n converges to a prime two-period solution. This completes the proof of the proposition. □

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References

[1] A. M. Amleh, E. A. Grove, G. Ladas, and D. A. Georgiou, "On the recursive sequence $x_{n+1} = \alpha + (x_{n-1}/x_n)$," *Journal of Mathematical Analysis and Applications*, vol. 233, no. 2, pp. 790–798, 1999.
[2] K. S. Berenhaut, J. D. Foley, and S. Stević, "The global attractivity of the rational difference equation $y_n = 1 + (y_{n-k}/y_{n-m})$," *Proceedings of the American Mathematical Society*, vol. 135, no. 4, pp. 1133–1140, 2007.
[3] K. S. Berenhaut, J. D. Foley, and S. Stević, "The global attractivity of the rational difference equation $y_n = A + (y_{n-k}/y_{n-m})^p$," *Proceedings of the American Mathematical Society*, vol. 136, no. 1, pp. 103–110, 2008.
[4] K. S. Berenhaut and S. Stević, "A note on positive non-oscillatory solutions of the difference equation $x_{n+1} = \alpha + (x_{n-k}^p/x_n^p)$," *Journal of Difference Equations and Applications*, vol. 12, no. 5, pp. 495–499, 2006.
[5] K. S. Berenhaut and S. Stević, "The behaviour of the positive solutions of the difference equation $x_{n+1} = A + (x_{n-k}^p/x_{n-1}^p)$," *Journal of Difference Equations and Applications*, vol. 12, no. 9, pp. 909–918, 2006.
[6] L. Berg, "On the asymptotics of nonlinear difference equations," *Zeitschrift für Analysis und ihre Anwendungen*, vol. 21, no. 4, pp. 1061–1074, 2002.
[7] R. DeVault, V. L. Kocic, and D. Stutson, "Global behavior of solutions of the nonlinear difference equation $x_{n+1} = p_n + (x_{n-1}/x_n)$," *Journal of Difference Equations and Applications*, vol. 11, no. 8, pp. 707–719, 2005.
[8] H. M. El-Owaidy, A. M. Ahmed, and M. S. Mousa, "On asymptotic behaviour of the difference equation $x_{n+1} = \alpha + (x_{n-k}^p/x_n^p)$," *Journal of Applied Mathematics & Computing*, vol. 12, no. 1-2, pp. 31–37, 2003.
[9] E. A. Grove and G. Ladas, *Periodicities in Nonlinear Difference Equations*, vol. 4 of *Advances in Discrete Mathematics and Applications*, Chapman & Hall/CRC, Boca Raton, Fla, USA, 2005.
[10] L. Gutnik and S. Stević, "On the behaviour of the solutions of a second-order difference equation," *Discrete Dynamics in Nature and Society*, vol. 2007, Article ID 27562, 14 pages, 2007.
[11] A. E. Hamza and A. Morsy, "On the recursive sequence $x_{n+1} = A + (x_{n-1}/x_n^k)$," *Applied Mathematics Letters*, vol. 22, no. 1, pp. 91–95, 2009.
[12] B. Iričanin and S. Stević, "On a class of third-order nonlinear difference equations," *Applied Mathematics and Computation*, vol. 213, no. 2, pp. 479–483, 2009.
[13] V. L. Kocić and G. Ladas, *Global Behavior of Nonlinear Difference Equations of Higher Order with Applications*, vol. 256 of *Mathematics and Its Applications*, Kluwer Academic Publishers, Dordrecht, The Netherlands, 1993.
[14] M. R. S. Kulenović, G. Ladas, and C. B. Overdeep, "On the dynamics of $x_{n+1} = p_n + (x_{n-1}/x_n)$ with a period-two coefficient," *Journal of Difference Equations and Applications*, vol. 10, no. 10, pp. 905–914, 2004.
[15] M. R. S. Kulenović and G. Ladas, *Dynamics of Second Order Rational Difference Equations*, Chapman & Hall/CRC, Boca Raton, Fla, USA, 2002.

[16] G. Papaschinopoulos and C. J. Schinas, "On a $(k+1)$-th order difference equation with a coefficient of period $(k+1)$," *Journal of Difference Equations and Applications*, vol. 11, no. 3, pp. 215–225, 2005.

[17] G. Papaschinopoulos and C. J. Schinas, "On a nonautonomous difference equation with bounded coefficient," *Journal of Mathematical Analysis and Applications*, vol. 326, no. 1, pp. 155–164, 2007.

[18] G. Papaschinopoulos, C. J. Schinas, and G. Stefanidou, "On a difference equation with 3-periodic coefficient," *Journal of Difference Equations and Applications*, vol. 11, no. 15, pp. 1281–1287, 2005.

[19] G. Papaschinopoulos, C. J. Schinas, and G. Stefanidou, "Boundedness, periodicity and stability of the difference equation $x_{n+1} = A_n + (x_{n-1}/x_n)^p$," *International Journal of Dynamical Systems and Differential Equations*, vol. 1, no. 2, pp. 109–116, 2007.

[20] S. Stević, "On the recursive sequence $x_{n+1} = \alpha + (x_{n-1}/x_n)$. II," *Dynamics of Continuous, Discrete & Impulsive Systems. Series A*, vol. 10, no. 6, pp. 911–916, 2003.

[21] S. Stević, "A note on periodic character of a difference equation," *Journal of Difference Equations and Applications*, vol. 10, no. 10, pp. 929–932, 2004.

[22] S. Stević, "On the recursive sequence $x_{n+1} = \alpha + (x_{n-1}^p/x_n^p)$," *Journal of Applied Mathematics & Computing*, vol. 18, no. 1-2, pp. 229–234, 2005.

[23] S. Stević, "Asymptotics of some classes of higher-order difference equations," *Discrete Dynamics in Nature and Society*, vol. 2007, Article ID 56813, 20 pages, 2007.

[24] S. Stević, "On the recursive sequence $x_{n+1} = \alpha + (x_n^p/x_{n-1}^p)$," *Discrete Dynamics in Nature and Society*, vol. 2007, Article ID 34517, 9 pages, 2007.

[25] S. Stević, "On the recursive sequence $x_{n+1} = A + (x_n^p/x_{n-1}^p)$," *Discrete Dynamics in Nature and Society*, vol. 2007, Article ID 40963, 9 pages, 2007.

[26] S. Stević, "On the difference equation $x_{n+1} = \alpha + (x_{n-1}/x_n)$," *Computers & Mathematics with Applications*, vol. 56, no. 5, pp. 1159–1171, 2008.

[27] S. Stević and K. S. Berenhaut, "The behavior of positive solutions of a nonlinear second-order difference equation," *Abstract and Applied Analysis*, vol. 2008, Article ID 653243, 8 pages, 2008.

[28] S. Stević, "Boundedness character of a class of difference equations," *Nonlinear Analysis: Theory, Methods & Applications*, vol. 70, no. 2, pp. 839–848, 2009.