Hindawi Publishing Corporation
Advances in Difference Equations
Volume 2010, Article ID 540365, 18 pages
doi:10.1155/2010/540365

Research Article

The Existence and Exponential Stability for Random Impulsive Integrodifferential Equations of Neutral Type

Huabin Chen, Xiaozhi Zhang, and Yang Zhao

Department of Mathematics, Nanchang University, Nanchang, Jiangxi 330031, China

Correspondence should be addressed to Huabin Chen, chb_00721@126.com

Received 24 March 2010; Revised 9 July 2010; Accepted 28 July 2010

Academic Editor: Claudio Cuevas

By applying the Banach fixed point theorem and using an inequality technique, we investigate a kind of random impulsive integrodifferential equations of neutral type. Some sufficient conditions, which can guarantee the existence, uniqueness, and exponential stability in mean square for such systems, are obtained. Compared with the previous works, our method is new and our results can generalize and improve some existing ones. Finally, an illustrative example is given to show the effectiveness of the proposed results.

1. Introduction

Since impulsive differential systems have been highly recognized and applied in a wide spectrum of fields such as mathematical modeling of physical systems, technology, population and biology, etc., some qualitative properties of the impulsive differential equations have been investigated by many researchers in recent years, and a lot of valuable results have been obtained (see, e.g., [1–10] and references therein). For the general theory of impulsive differential systems, the readers can refer to [11, 12]. For an impulsive differential equations, if its impulsive effects are random variable, their solutions are stochastic processes. It is different from the deterministic impulsive differential equations and stochastic differential equations. Thus, the random impulsive differential equations are more realistic than deterministic impulsive systems. The investigation for the random impulsive differential equations is a new area of research. Recently, the p-moment boundedness, exponential stability and almost sure stability of random impulsive differential systems were studied by using the Lyapunov functional method in [13–15], respectively. In [16] Wu and Duan have investigated the oscillation, stability and boundedness in mean square of second-order random impulsive differential systems; Wu et al. in [17] studied the existence and uniqueness of the solutions to random impulsive differential equations, and in [18] Zhao and Zhang discussed the exponential stability of random impulsive integro-differential equations by employing the comparison theorem. Very recently, the existence, uniqueness and stability results of random impulsive semilinear differential equations, the existence and uniqueness for neutral functional differential equations with random impulses are discussed by using the Banach fixed point theorem in [19, 20], respectively.

It is well known that the nonlinear impulsive delay differential equations of neutral type arises widely in scientific fields, such as control theory, bioscience, physics, etc. This class of equations play an important role in modeling phenomena of the real world. So it is valuable to discuss the properties of the solutions of these equations. For example, Xu et al. in [21], have considered the exponential stability of nonlinear impulsive neutral differential equations with delays by establishing singular impulsive delay differential inequality and transforming the n-dimensional impulsive neutral delay differential equation into a $2n$-dimensional singular impulsive delay differential equations; and the results about the global exponential stability for neutral-type impulsive neural networks are obtained by using the linear matrix inequality (LMI) in [9, 10], respectively.

However, most of these studies are in connection with deterministic impulses and finite delay. And, to the best of author's knowledge, there is no paper which investigates the existence, uniqueness and exponential stability in mean square of random impulsive integrodifferential equation of neutral type. One of the main reason is that the methods to discuss the exponential stability of deterministic impulsive differential equations of neutral type and the exponential stability for random differential equations can not be directly adapted to the case of random impulsive differential equations of neutral type, especially, random impulsive integrodifferential equations of neutral type. That is, the methods proposed in [15, 16] are ineffective for the exponential stability in mean square for such systems. Although the exponential stability of nonlinear impulsive neutral integrodifferential equations can be derived in [22], the method used in [22] is only suitable for the deterministic impulses. Besides, the methods introduced to deal with the exponential stability of random impulsive integrodifferential equations in [18] and study the exponential stability in mean square of random impulsive differential equations in [19], can not be applied to deal with our problem since the neutral item arises. So, the technique and the method dealt with the exponential stability in mean square of random impulsive integrodifferential equations of neutral type are in need of being developed and explored. Thus, with these aims, we will make the first attempt to study such problems to close this gap in this paper.

The format of this work is organized as follows. In Section 2, some necessary definitions, notations and lemmas used in this paper will be introduced. In Section 3, The existence and uniqueness of random impulsive integrodifferential equations of neutral type are obtained by using the Banach fixed point theorem. Some sufficient conditions about the exponential stability in mean square for the solution of such systems are given in Section 4. Finally, an illustrative example is provided to show the obtained results.

2. Preliminaries

Let $|\cdot|$ denote the Euclidean norm in R^n. If A is a vector or a matrix, its transpose is denoted by A^T; and if A is a matrix, its Frobenius norm is also represented by $|\cdot| = \sqrt{\mathrm{trace}(A^T A)}$. Assumed that Ω is a nonempty set and τ_k is a random variable defined from Ω to $D_k \triangleq (0, d_k)$ for all $k = 1, 2, \ldots$, where $0 < d_k \leq +\infty$. Moreover, assumed that τ_i and τ_j are independent with each other as $i \neq j$ for $i, j = 1, 2, \ldots$.

Let $\mathrm{BC}(X,Y)$ be the space of bounded and continuous mappings from the topological space X into Y, and $\mathrm{BC}^1(X,Y)$ be the space of bounded and continuously differentiable mappings from the topological space X into Y. In particular, Let $\mathrm{BC} \triangleq \mathrm{BC}((-\infty,0],R^n)$ and $\mathrm{BC}^1 \triangleq \mathrm{BC}^1((-\infty,0],R^n)$. $\mathrm{PC}(J,R^n) = \{\phi : J \to R^n \mid \phi(s)$ is bounded and almost surely continuous for all but at most countable points $s \in J$ and at these points $s \in J$, $\phi(s^+)$ and $\phi(s^-)$ exist, $\phi(s) = \phi(s^+)\}$, where $J \subset R$ is an interval, $\phi(s^+)$ and $\phi(s^-)$ denote the right-hand and left-hand limits of the function $\phi(s)$, respectively. Especially, let $\mathrm{PC} \triangleq \mathrm{PC}((-\infty,0],R^n)$. $\mathrm{PC}^1(J,R^n) = \{\phi : J \to R^n \mid \phi(s)$ is bounded and almost surely continuously differentiable for all but at most countable points $s \in J$ and at these points $s \in J$, $\phi(s^+)$ and $\phi(s^-)$, $\phi(s) \triangleq \phi(s^+)$, $\phi'(s) \triangleq \phi'(s^+)\}$, where $\phi'(s)$ denote the derivative of $\phi(s)$. Especially, let $\mathrm{PC}^1 \triangleq \mathrm{PC}^1((-\infty,0],R^n)$.

For $\phi \in \mathrm{PC}^1$, we introduce the following norm:

$$\|\phi\|_\infty = \max\left\{ \sup_{-\infty<\theta\leq 0} |\phi(\theta)|, \sup_{-\infty<\theta\leq 0} |\phi'(\theta)| \right\}. \tag{2.1}$$

In this paper, we consider the following random impulsive integrodifferential equations of neutral type:

$$x'(t) = Ax(t) + Dx'(t-r) + f_1(t, x(t-h(t))) + \int_{-\infty}^{0} f_2(\theta, x(t+\theta))d\theta, \quad t \neq \xi_k,\ t \geq 0, \tag{2.2}$$

$$x(\xi_k) = b_k(\tau_k)x(\xi_k^-), \quad k = 1,2,\ldots, \tag{2.3}$$

$$x_{t_0} = \varphi \in \mathrm{PC}^1, \tag{2.4}$$

where $A,\ D$ are two matrices of dimension $n \times n$; $f_1 : [0,+\infty) \times R^n \to R^n$ and $f_2 : (-\infty,0] \times R^n \to R^n$ are two appropriate functions; $b_k : D_k \to R^{n\times n}$ is a matrix valued functions for each $k = 1,2,\ldots$; assume that $t_0 \in [0,+\infty)$ is an arbitrary real number, $\xi_0 = t_0$ and $\xi_k = \xi_{k-1} + \tau_k$ for $k = 1,2,\ldots$; obviously, $t_0 = \xi_0 < \xi_1 < \xi_2 < \cdots < \xi_k < \cdots$; $x(\xi_k^-) = \lim_{t\to\xi_k-0} x(t)$; $h : [0,+\infty) \to [0,\rho]$ $(\rho > 0)$ is a bounded and continuous function and $\tau = \max\{r,\rho\}$ $(r > 0)$. $x_t : x_t(s) = x(t+s)$ for all $s \in (-\infty,0]$. Let us denote by $\{B_t,\ t \geq 0\}$ the simple counting process generated by $\{\xi_n\}$, that is, $\{B_t \geq n\} = \{\xi_n \leq t\}$, and present $\Im_t$ the σ-algebra generated by $\{B_t,\ t \geq 0\}$. Then, $(\Omega, \{\Im_t\}, P)$ is a probability space.

Firstly, define the space $\mathfrak{B}$ consisting of $\mathrm{PC}^1((-\infty,T],R^n)$ $(T > t_0)$-valued stochastic process $\varphi : (-\infty,T] \to R^n$ with the norm

$$\|\varphi\|^2 = E \sup_{-\infty<\theta\leq T} |\varphi(\theta)|^2. \tag{2.5}$$

It is easily shown that the space $(\mathfrak{B}, \|\cdot\|)$ is a completed space.

Definition 2.1. A function $x \in \mathfrak{B}$ is said to be a solution of (2.2)–(2.4) if x satisfies (2.2) and conditions (2.3) and (2.4).

Definition 2.2. The fundamental solution matrix $\{\Phi(t) = \exp(At),\ t \geq 0\}$ of the equation $x'(t) = Ax(t)$ is said to be exponentially stable if there exist two positive numbers $M \geq 1$ and $a > 0$ such that $|\Phi(t)| \leq Me^{-at}$, for all $t \geq 0$.

Definition 2.3. The solution of system (2.2) with conditions (2.3) and (2.4) is said to be exponentially stable in mean square, if there exist two positive constants $C_1 > 0$ and $\lambda > 0$ such that

$$E|x(t)|^2 \leq C_1 e^{-\lambda t}, \quad t \geq 0. \tag{2.6}$$

Lemma 2.4 (see [23]). *For any two real positive numbers $a, b > 0$, then*

$$(a+b)^2 \leq \nu^{-1} a^2 + (1-\nu)^{-1} b^2, \tag{2.7}$$

where $\nu \in (0,1)$.

Lemma 2.5 (see [23]). *Let u, φ, and χ be three real continuous functions defined on $[a,b]$ and $\chi(t) \geq 0$, for $t \in [a,b]$, and assumed that on $[a,b]$, one has the inequality*

$$u(t) \leq \varphi(t) + \int_a^t \chi(s) u(s) ds. \tag{2.8}$$

If φ is differentiable, then

$$u(t) \leq \varphi(a) \exp\left(\int_a^t \chi(s) ds\right) + \int_a^t \exp\left(\int_s^t \chi(r) dr\right) \varphi'(s) ds, \tag{2.9}$$

for all $t \in [a,b]$.

In order to obtain our main results, we need the following hypotheses.

(H_1) The function f_1 satisfies the Lipschitz condition: there exists a positive constant $L_1 > 0$ such that

$$\left|f_1(t,x) - f_1(t,y)\right| \leq L_1 |x-y|, \tag{2.10}$$

for $x, y \in R^n$, $t \in [0,T]$, and $f_1(t,0) = 0$.

(H_2) The function f_2 satisfies the following condition: there also exist a positive constant $L_2 > 0$ and a function $k : (-\infty, 0] \to [0, +\infty)$ with two important properties, $\int_{-\infty}^0 k(t) dt = 1$ and $\int_{-\infty}^0 k(t) e^{-lt} dt < +\infty$ $(l > 0)$, such that

$$\left|f_2(t,x) - f_2(t,y)\right| \leq L_2 k(t) |x-y|, \tag{2.11}$$

for $x, y \in R^n$, $t \in [0,T]$, and $f_2(t,0) = 0$.

(H_3) $E(\max_{i,k}\{\prod_{j=i}^k |b_j(\tau_j)|^2\})$ is uniformly bounded. That is, there exists a positive constant $L > 0$ such that

$$E\left(\max_{i,k}\left\{\prod_{j=i}^{k} \left|b_j(\tau_j)\right|^2\right\}\right) \leq L, \tag{2.12}$$

for all $\tau_j \in D_j$ and $j = 1, 2, \ldots$.

(H_4) $\kappa \triangleq \sqrt{\max\{L, 1\}} |D| \in (0,1)$.

3. Existence and Uniqueness

In this section, to make this paper self-contained, we study the existence and uniqueness for the solution to system (2.2) with conditions (2.3) and (2.4) by using the Picard iterative method under conditions (H_1)–(H_4). In order to prove our main results, we firstly need the following auxiliary result.

Lemma 3.1. *Let $f_1 : [0,+\infty)\times R^n \to R^n$ and $f_2 : (-\infty,0]\times R^n \to R^n$ be two continuous functions. Then, x is the unique solution of the random impulsive integrodifferential equations of neutral type:*

$$\begin{gathered} x'(t) = Ax(t) + Dx'(t-r) + f_1(t, x(t-h(t))) + \int_{-\infty}^{0} f_2(\theta, x(t+\theta))d\theta, \quad t \neq \xi_k,\ t \geq 0, \\ x(\xi_k) = b_k(\tau_k)x(\xi_k^-), \quad k = 1,2,\ldots, \\ x_{t_0} = \varphi \in PC^1, \end{gathered} \tag{3.1}$$

if and only if x is a solution of impulsive integrodifferential equations:

(i) $x_{t_0}(\theta) = \varphi(\theta)$, $\theta \in (-\infty, 0]$,

(ii)

$$\begin{aligned} x(t) = \sum_{k=0}^{+\infty}\Bigg[&\prod_{i=1}^{k} b_i(\tau_i)\Phi(t-t_0)x_0 + \sum_{i=1}^{k}\prod_{j=i}^{k} b_j(\tau_j)\int_{\xi_{i-1}}^{\xi_i}\Phi(t-s)Ddx(s-r) \\ &+ \int_{\xi_k}^{t}\Phi(t-s)Ddx(s-r) + \sum_{i=1}^{k}\prod_{j=i}^{k} b_j(\tau_j) \\ &\times \int_{\xi_{i-1}}^{\xi_i}\Phi(t-s)f_1(s, x(s-h(s)))ds \\ &+ \int_{\xi_k}^{t}\Phi(t-s)f_1(s, x(s-h(s)))ds + \sum_{i=1}^{k}\prod_{j=i}^{k} b_j(\tau_j)\int_{\xi_{i-1}}^{\xi_i}\Phi(t-s) \\ &\times \int_{-\infty}^{0} f_2(\theta, x(s+\theta))d\theta ds \\ &+ \int_{\xi_k}^{t}\Phi(t-s)\int_{-\infty}^{0} f_2(\theta, x(s+\theta))d\theta ds\Bigg] I_{[\xi_k,\xi_{k+1})}(t), \end{aligned} \tag{3.2}$$

for all $t \in [t_0, T]$, where $\prod_{j=m}^{n}(\cdot) = 1$ as $m > n$, $\prod_{j=i}^{k} b_j(\tau_j) = b_k(\tau_k)b_{k-1}(\tau_{k-1})\cdots b_i(\tau_i)$, and $I_{\Omega'}(\cdot)$ denotes the index function, that is,

$$I_{\Omega'}(t) = \begin{cases} 1, & \text{if } t \in \Omega', \\ 0, & \text{if } t \notin \Omega'. \end{cases} \tag{3.3}$$

Proof. The approach of the proof is very similar to those in [17, 19, 20]. Here, we omit it. □

Theorem 3.2. *Provided that conditions* (H_1)*–*(H_4) *hold, then the system* (2.2) *with the conditions* (2.3) *and* (2.4) *has a unique solution on* $\mathfrak{B}$.

Proof. Define the iterative sequence $\{x^n(t)\}$ $(t \in (-\infty, T],\ n = 0, 1, 2, \ldots)$ as follows:

$$
\begin{aligned}
x^0(t) &= \sum_{k=0}^{+\infty}\left[\prod_{i=1}^{k} b_i(\tau_i)\Phi(t-t_0)x_0\right] I_{[\xi_k,\xi_{k+1})}(t), \quad t \in [t_0, T],\\
x^n(t) &= \sum_{k=0}^{+\infty}\Bigg[\prod_{i=1}^{k} b_i(\tau_i)\Phi(t-t_0)x_0 + \sum_{i=1}^{k}\prod_{j=i}^{k} b_j(\tau_j)\int_{\xi_{i-1}}^{\xi_i}\Phi(t-s)Ddx^n(s-r)\\
&\quad + \sum_{i=1}^{k}\prod_{j=i}^{k} b_j(\tau_j)\int_{\xi_{i-1}}^{\xi_i}\Phi(t-s)f_1\Big(s, x^{n-1}(s-h(s))\Big)ds\\
&\quad + \int_{\xi_k}^{t}\Phi(t-s)f_1\Big(s, x^{n-1}(s-h(s))\Big)ds\\
&\quad + \sum_{i=1}^{k}\prod_{j=i}^{k} b_j(\tau_j)\int_{\xi_{i-1}}^{\xi_i}\Phi(t-s)\int_{-\infty}^{0} f_2\Big(\theta, x^{n-1}(s+\theta)\Big)d\theta ds\\
&\quad + \int_{\xi_k}^{t}\Phi(t-s)Ddx^n(s-r) + \int_{\xi_k}^{t}\Phi(t-s)\int_{-\infty}^{0} f_2\Big(\theta, x^{n-1}(s+\theta)\Big)d\theta ds\Bigg]\\
&\quad \times I_{[\xi_k,\xi_{k+1})}(t), \quad t \in [t_0, T],\ n = 1, 2, \ldots,\\
x_{t_0}^n(\theta) &= \varphi(\theta), \quad \theta \in (-\infty, 0],\ n = 0, 1, 2, \ldots.
\end{aligned}
\tag{3.4}
$$

Thus, due to Lemma 2.4, it follows that

$$
\begin{aligned}
&\left|x^{n+1}(t) - x^n(t)\right|^2\\
&\quad = \Bigg|\sum_{k=0}^{+\infty}\Bigg[\prod_{i=1}^{k}\sum_{j=i}^{k} b_j(\tau_j)\int_{\xi_{i-1}}^{\xi_i}\Phi(t-s)Dd\Big[x^n(s-r) - x^{n-1}(s-r)\Big]\\
&\qquad + \sum_{i=1}^{k}\prod_{j=i}^{k} b_j(\tau_j)\int_{\xi_{i-1}}^{\xi_i}\Phi(t-s)\Big[f_1(s, x^n(s-h(s))) - f_1\Big(s, x^{n-1}(s-h(s))\Big)\Big]ds\\
&\qquad + \sum_{i=1}^{k}\prod_{j=i}^{k} b_j(\tau_j)\int_{\xi_{i-1}}^{\xi_i}\Phi(t-s)\int_{-\infty}^{0}\Big[f_2(\theta, x^n(s+\theta)) - f_2\Big(\theta, x^{n-1}(s+\theta)\Big)\Big]d\theta ds\\
&\qquad + \int_{\xi_k}^{t}\Phi(t-s)Dd\Big[x^n(s-r) - x^{n-1}(s-r)\Big]
\end{aligned}
$$

$$
\begin{aligned}
&+\int_{\xi_k}^{t}\Phi(t-s)\Big[f_1(s,x^n(s-h(s)))-f_1\Big(s,x^{n-1}(s-h(s))\Big)\Big]ds\\
&+\int_{\xi_k}^{t}\Phi(t-s)\int_{-\infty}^{0}[f_2(\theta,x^n(s+\theta))-f_2(\theta,x^{n-1}(s+\theta))]d\theta ds\Bigg]I_{[\xi_k,\xi_{k+1})}(t)\Bigg|^2\\
\le{}&\frac{1}{\kappa}\max\left\{\max_{i,k}\left\{\prod_{j=i}^{k}|b_j(\tau_j)|^2\right\},1\right\}|D|^2|x^{n+1}(t-r)-x^n(t-r)|^2\\
&+\frac{3}{1-\kappa}\max\left\{\max\prod_{j=i}^{k}\left|b_j(\tau_j)\right|^2,1\right\}\\
&\times|D|^2|A|^2\left[\int_{t_0}^{t}\Phi(t-s)\left|x^{n+1}(s-r)-x^n(s-r)\right|^2ds\right]^2\\
&+\frac{3}{1-\kappa}\max\left\{\max\left\{\prod_{j=i}^{k}\left|b_j(\tau_j)\right|^2\right\},1\right\}\\
&\times L_1^2\left[\int_{t_0}^{t}\Phi(t-s)\left|x^n(s-h(s))-x^{n-1}(s-h(s))\right|^2ds\right]^2\\
&+\frac{3}{1-\kappa}\max\left\{\max\left\{\prod_{j=i}^{k}\left|b_j(\tau_j)\right|^2\right\},1\right\}\\
&\times L_2^2\left[\int_{t_0}^{t}\Phi(t-s)\int_{-\infty}^{0}\left[x^n(s+\theta)-x^{n-1}(s+\theta)\right]d\theta ds|^2ds\right]^2\\
\le{}&\frac{1}{\kappa}\max\left\{\max_{i,k}\left\{\prod_{j=i}^{k}|b_j(\tau_j)|^2\right\},1\right\}|D|^2\sup_{-\infty<s\le t}|x^{n+1}(s)-x^n(s)|^2\\
&+\frac{3}{a(1-\kappa)}\max\left\{\max\left\{\prod_{j=i}^{k}\left|b_j(\tau_j)\right|^2\right\},1\right\}\\
&\times|D|^2|A|^2M^2\int_{t_0}^{t}\sup_{-\infty<u\le s}|x^{n+1}(u)-x^n(u)|^2ds\\
&+\frac{3}{a(1-\kappa)}\max\left\{\max\left\{\prod_{j=i}^{k}\left|b_j(\tau_j)\right|^2\right\},1\right\}\\
&\times M^2\Big(L_1^2+L_2^2\Big)\int_{t_0}^{t}\sup_{-\infty<\theta\le s}\left|x^{n+1}(\theta)-x^n(\theta)\right|^2ds.
\end{aligned}
\tag{3.5}
$$

From condition (H_3), we have

$$E\left(\sup_{-\infty<s\le t}\left|x^{n+1}(s)-x^{n}(s)\right|^2\right)\le\frac{3M^2\left|D\right|^2\left|A\right|^2\max\{1,L\}}{a(1-\kappa)^2}\int_{t_0}^{t}E\left(\sup_{-\infty<\theta\le s}\left|x^{n+1}(\theta)-x^{n}(\theta)\right|^2\right)ds$$
$$+\frac{3M^2\left(L_1^2+L_2^2\right)\max\{1,L\}}{a(1-\kappa)^2}\int_{t_0}^{t}E\left(\sup_{-\infty<\theta\le s}\left|x^{n}(\theta)-x^{n-1}(\theta)\right|^2\right)ds. \tag{3.6}$$

In view of Lemma 2.5, it yields that

$$E\left(\sup_{-\infty<s\le t}\left|x^{n+1}(s)-x^{n}(s)\right|^2\right)\le\Lambda_1\int_{t_0}^{t}E\left(\sup_{-\infty<\theta\le s}\left|x^{n}(\theta)-x^{n-1}(\theta)\right|^2\right)ds, \tag{3.7}$$

where $\Lambda_1 = 3M^2|D|^2|A|^2\max\{1,L\}/a(1-\kappa)^2\exp[(3M^2(L_1^2+L_2^2)\max\{1,L\}/a(1-\kappa)^2)(T-t_0)]$. Furthermore,

$$\begin{aligned}E&\left(\sup_{-\infty<s\le t}\left|x^{1}(s)-x^{0}(s)\right|^2\right)\\&\le\frac{4\kappa^2M^2E\|\varphi\|_\infty^2}{(1-\kappa)^2}+\frac{4\max\{L,1\}M^2\left(L_1^2+L_2^2\right)}{(1-\kappa)^2a}\int_{t_0}^{t}E\left(\sup_{-\infty<u\le s}\left|x^{0}(u)\right|^2\right)ds\\&+\frac{4\max\{L,1\}|D|^2|A|^2M^2}{(1-\kappa)^2a}\int_{t_0}^{t}E\sup_{-\infty<u\le s}\left|x^{1}(u)\right|^2ds.\end{aligned} \tag{3.8}$$

By (3.4), we can obtain that

$$\begin{aligned}E\left(\sup_{-\infty<s\le t}\left|x^{1}(s)\right|^2\right)\le{}&\frac{5LM^2E\|\varphi\|_\infty^2+5\max\{L,1\}M^2|D|^2E\|\varphi\|_\infty^2}{(1-\kappa)^2}\\&+\frac{5\max\{L,1\}M^2|D|^2|A|^2}{(1-\kappa)^2a}\int_{t_0}^{t}E\sup_{-\infty<u\le s}\left|x^{1}(u)\right|^2ds\\&+\frac{5\max\{L,1\}M^2\left(L_1^2+L_2^2\right)}{(1-\kappa)^2a}\int_{t_0}^{t}E\sup_{-\infty<u\le s}\left|x^{0}(u)\right|^2ds,\end{aligned} \tag{3.9}$$

$$\begin{aligned}E\left(\sup_{-\infty<s\le t}\left|x^{0}(s)\right|^2\right)&\le E\left(\sup_{-\infty<\theta\le 0}\left|\varphi(\theta)\right|^2\right)+E\left(\sup_{0\le s\le t}\left|x^{0}(s)\right|^2\right)\\&\le\left(1+LM^2\right)\|\phi\|_\infty\\&\triangleq\Lambda_2.\end{aligned} \tag{3.10}$$

From the Gronwall inequality, (3.9) implies that

$$E\left(\sup_{-\infty<t\le T}\left|x^1(t)\right|^2\right)\le \Lambda_3\exp[\Lambda_4(T-t_0)], \tag{3.11}$$

where $\Lambda_3 = (5LM^2E\|\varphi\|_\infty^2 + 5\max\{L,1\}M^2|D|^2E\|\varphi\|_\infty^2/(1-\kappa)^2) + (5\max\{L,1\}M^2(L_1^2 + L_2^2)\Lambda_2(T-t_0)/(1-\kappa)^2a)$ and $\Lambda_4 = 5\max\{L,1\}M^2|D|^2|A|^2/(1-\kappa)^2a$.

From (3.8) and (3.11), we have

$$E\left(\sup_{-\infty<s\le t}\left|x^1(s)-x^0(s)\right|^2\right)\le \Lambda_5, \tag{3.12}$$

for all $t\in[0,T]$, where

$$\begin{aligned}\Lambda_5 = {}&\frac{4\kappa^2M^2E\|\varphi\|_\infty^2}{(1-\kappa)^2}+\frac{4\max\{L,1\}M^2\left(L_1^2+L_2\right)}{(1-\kappa)^2a}\Lambda_2(T-t_0)\\ &+\frac{4\max\{L,1\}|D|^2|A|^2M^2}{(1-\kappa)^2a}\Lambda_3\exp[\Lambda_4(T-t_0)](T-t_0).\end{aligned} \tag{3.13}$$

From (3.4), it follows that

$$\begin{aligned}&\left|x^2(t)-x^1(t)\right|^2\\ &\quad\le\frac{1}{\kappa}\max\left\{\max_{i,k}\left\{\prod_{j=i}^{k}\left|b_j(\tau_j)\right|^2\right\},1\right\}|D|^2\sup_{-\infty<s\le t}\left|x^2(s)-x^1(s)\right|^2\\ &\qquad+\frac{3}{a(1-\kappa)}\max\left\{\max\left\{\prod_{j=i}^{k}\left|b_j(\tau_j)\right|^2\right\},1\right\}|D|^2|A|^2M^2\int_{t_0}^{t}\sup_{-\infty<u\le s}\left|x^1(u)-x^0(u)\right|^2ds\\ &\qquad+\frac{3}{a(1-\kappa)}\max\left\{\max\left\{\prod_{j=i}^{k}\left|b_j(\tau_j)\right|^2\right\},1\right\}M^2\left(L_1^2+L_2^2\right)\int_{t_0}^{t}\sup_{-\infty<\theta\le s}\left|x^1(\theta)-x^0(\theta)\right|^2ds.\end{aligned} \tag{3.14}$$

By virtue of condition (H_3) and Lemma 2.5,

$$E\left(\sup_{-\infty<s\le t}\left|x^2(t)-x^1(t)\right|^2\right)\le\Lambda_1\Lambda_5(t-t_0). \tag{3.15}$$

Now, for all $n\ge 0$ and $t\in[0,T]$, we claim that

$$E\left(\sup_{-\infty<s\le t}\left|x^{n+1}(s)-x^n(s)\right|^2\right)\le\Lambda_5\frac{[\Lambda_1(t-t_0)]^n}{n!}. \tag{3.16}$$

We will show (3.16) by mathematical induction. From (3.12), it is easily seen that (3.16) holds as $n=0$. Under the inductive assumption that (3.16) holds for some $n \geq 1$. We will prove that (3.16) still holds when $n+1$. Notice that

$$\begin{aligned}
\left|x^{n+2}(t)-x^{n+1}(t)\right|^2 \leq{}& \frac{1}{\kappa}\max\left\{\max_{i,k}\left\{\prod_{j=i}^{k}\left|b_j(\tau_j)\right|^2\right\},1\right\}|D|^2\sup_{-\infty<s\leq t}\left|x^{n+2}(s)-x^{n+1}(s)\right|^2 \\
&+\frac{3}{a(1-\kappa)}\max\left\{\max\left\{\prod_{j=i}^{k}\left|b_j(\tau_j)\right|^2\right\},1\right\}|D|^2|A|^2M^2 \\
&\times\int_{t_0}^{t}\sup_{-\infty<\theta\leq s}\left|x^{n+2}(\theta)-x^{n+1}(\theta)\right|^2 ds \\
&+\frac{3}{a(1-\kappa)}\max\left\{\max\left\{\prod_{j=i}^{k}\left|b_j(\tau_j)\right|^2\right\},1\right\}M^2\left(L_1^2+L_2^2\right) \\
&\times\int_{t_0}^{t}\sup_{-\infty<\theta\leq s}\left|x^{n+2}(\theta)-x^{n+1}(\theta)\right|^2 ds.
\end{aligned} \tag{3.17}$$

From condition (H_3), we have

$$\begin{aligned}
&E\left(\sup_{-\infty<s\leq t}\left|x^{n+2}(s)-x^{n+1}(s)\right|^2\right) \\
&\quad\leq \frac{3M^2|D|^2|A|^2\max\{1,L\}}{a(1-\kappa)^2}\int_{t_0}^{t}E\left(\sup_{-\infty<\theta\leq s}\left|x^{n+2}(\theta)-x^{n+1}(\theta)\right|^2\right)ds \\
&\qquad+\frac{3M^2\left(L_1^2+L_2^2\right)\max\{1,L\}}{a(1-\kappa)^2}\int_{t_0}^{t}E\left(\sup_{-\infty<\theta\leq s}\left|x^{n+1}(\theta)-x^{n}(\theta)\right|^2\right)ds.
\end{aligned} \tag{3.18}$$

In view of Lemma 2.5 and (3.16), it yields that

$$\begin{aligned}
E\left(\sup_{-\infty<s\leq t}\left|x^{n+2}(s)-x^{n+1}(s)\right|^2\right) &\leq \Lambda_1\int_{t_0}^{t}E\left(\sup_{-\infty<\theta\leq s}\left|x^{n+1}(\theta)-x^{n}(\theta)\right|^2\right)ds \\
&\leq \frac{\Lambda_1\Lambda_5}{n!}\int_{t_0}^{t}\left[\Lambda_1(s-t_0)\right]^n ds \\
&\leq \Lambda_5\frac{\left[\Lambda_1(t-t_0)\right]^{n+1}}{(n+1)!}, \quad t\in[t_0,T].
\end{aligned} \tag{3.19}$$

That is, (3.16) holds for $n+1$. Hence, by induction, (3.16) holds for all $n \geq 0$.

For any $m > n \geq 1$, it follows that

$$\begin{aligned}
\|x^m - x^n\| &= \left[E\left(\sup_{-\infty<t\leq T} |x^m(t) - x^n(t)|^2\right)\right]^{1/2} \\
&\leq \sum_{k=n}^{+\infty}\left[E\left(\sup_{-\infty<t\leq T} \left|x^{k+1}(t) - x^k(t)\right|^2\right)\right]^{1/2} \\
&\leq \sum_{k=n}^{+\infty}\left[\Lambda_5 \frac{[\Lambda_1(T-t_0)]^k}{(k)!}\right]^{1/2} \longrightarrow 0,
\end{aligned} \tag{3.20}$$

as $n \to +\infty$. Thus, $\{x^n(t)\}_{n\geq 0}(t \in (-\infty, T])$ is a Cauchy sequence in Banach space $\mathfrak{B}$. Denote the limit by $x \in \mathfrak{B}$. Now, letting $n \to +\infty$ in both sides of (3.4), we obtain the existence for the solution of system (2.2) with conditions (2.3) and (2.4).

Uniqueness. Let $x, y \in \mathfrak{B}$ be two solutions of system (2.2) with conditions (2.3) and (2.4). By the same ways as above, we can yield that

$$\begin{aligned}
E\left(\sup_{-\infty<t\leq T} \left|x(s) - y(s)\right|^2\right) &= E\left(\sup_{t_0\leq t\leq T} \left|x(t) - y(t)\right|^2\right) \\
&\leq \Lambda_1 \int_{t_0}^{T} E\left(\sup_{-\infty<s\leq t} \left|x(s) - y(s)\right|^2\right) dt.
\end{aligned} \tag{3.21}$$

Applying the Gronwall inequality into (3.21), it follows that

$$E\left(\sup_{-\infty<t\leq T} \left|x(t) - y(t)\right|^2\right) = 0. \tag{3.22}$$

That is, $\|x - y\| = 0$. So, the uniqueness is also proved. The proof of this theorem is completed. □

4. Exponential Stability

In this section, the exponential stability in mean square for system (2.2) with initial conditions (2.3) and (2.4) is shown by using an integral inequality.

Theorem 4.1. *Supposed that the conditions of Theorem* 3.2 *holds, then the solution of the system* (2.2) *with conditions* (2.3) *and* (2.4) *is exponential stable in mean square if the inequality*

$$\frac{4M^2 \max\{1, L\}\left(|D|^2|A|^2 + L_1^2 + L_2^2\right)}{(1-\kappa)^2 a} < a \leq l \tag{4.1}$$

holds.

Proof. From (3.2) and Lemma 2.4, we derive that

$$
\begin{aligned}
|x(t)|^2 \le {} & \Bigg[\max_{i,k}\left\{\prod_{i,k}\left|b_j(\tau_j)\right|\right\}M^2\|\varphi\|_\infty^2 e^{-at} \\
& + \max\left\{\max_{i,k}\prod_{j=i}\left|b_j(\tau_j)\right|^2, 1\right\}\sum_{k=0}^{+\infty}\left[\int_{t_0}^{t} Me^{-a(t-s)}|D|dx(s-r)\right] \\
& \times I_{[\xi_k,\xi_{k+1})}(t) + \max\left\{\max_{i,k}\left\{\prod_{j=i}^{k}\left|b_j(\tau_j)\right|^2\right\}, 1\right\} \\
& \times \sum_{k=0}^{+\infty}\left[\int_{t_0}^{t} Me^{-a(t-s)}\left|f_1(s, x(s-h(s)))\right|\mathrm{d}s\right] I_{[\xi_k,\xi_{k+1})}(t) \\
& + \max\left\{\max_{i,k}\left\{\prod_{j=i}^{k}\left|b_j(\tau_j)\right|^2\right\}, 1\right\} \\
& \times \sum_{k=0}^{+\infty}\left[\int_{t_0}^{t} Me^{-a(t-s)}\int_{-\infty}^{0}\left|f_2(\theta, x(s+\theta))\right|d\theta ds\right] I_{[\xi_k,\xi_{k+1})}(t)\Bigg]^2 \\
\le {} & \sum_{k=0}^{+\infty}\left[\frac{8\max\left\{\max_{i,k}\left\{\prod_{j=i}^{k}\left|b_j(\tau_j)\right|^2\right\}, 1\right\}M^2|D|^2\|\varphi\|_\infty^2}{1-\kappa}\right. \\
& \left. + \frac{8\max_{i,k}\left\{\prod_{j=1}^{k}\left|b_j(\tau_j)\right|^2\right\}M^2\|\varphi\|_\infty^2}{1-\kappa}\right] \\
& \times I_{[\xi_k,\xi_{k+1})}(t)e^{-a(t-t_0)} + \frac{\max\left\{\max_{i,k}\left\{\prod_{j=i}^{k}|b_j(\tau_j)|^2\right\}, 1\right\}}{\kappa}|D|^2\sum_{k=0}^{+\infty}|x(t-r)|^2 I_{[\xi_k,\xi_{k+1})}(t) \\
& + \frac{4\max\left\{\max_{i,k}\left\{\prod_{j=i}^{k}|b_j(\tau_j)|^2\right\}, 1\right\}M^2|D|^2|A|^2}{(1-\kappa)a}\sum_{k=0}^{+\infty}\left[\int_{t_0}^{t} e^{-a(t-s)}|x(s-r)|^2 ds\right] I_{[\xi_k,\xi_{k+1})}(t) \\
& + \frac{4\max\left\{\max_{i,k}\left\{\prod_{j=i}^{k}|b_j(\tau_j)|^2\right\}, 1\right\}M^2L_1^2}{(1-\kappa)a}\sum_{k=0}^{+\infty}\left[\int_{t_0}^{t} e^{-a(t-s)}|x(s-h(s))|^2 ds\right] I_{[\xi_k,\xi_{k+1})}(t) \\
& + \frac{4\max\left\{\max_{i,k}\left\{\prod_{j=i}^{k}|b_j(\tau_j)|^2\right\}, 1\right\}M^2L_2^2}{(1-\kappa)a} \\
& \times \sum_{k=0}^{+\infty}\left[\int_{t_0}^{t} e^{-a(t-s)}\int_{-\infty}^{0} k(\theta)|x(s+\theta)|^2 d\theta ds\right] I_{[\xi_k,\xi_{k+1})}(t).
\end{aligned}
\tag{4.2}
$$

Thus, it follows that

$$
\begin{aligned}
E|x(t)|^2 \le{}& \left[\frac{8\max\{L,1\}M^2\left(|D|^2+L\right)E\|\varphi\|_\infty^2}{1-\kappa}\right]e^{-a(t-t_0)} + \kappa E|x(t-r)|^2 \\
&+ \frac{4\max\{L,1\}M^2|\mathrm{D}|^2|A|^2}{(1-\kappa)a}\int_{t_0}^{t} e^{-a(t-s)}E|x(s-r)|^2 ds \\
&+ \frac{4\max\{L,1\}M^2L_1^2}{(1-\kappa)a}\int_{t_0}^{t} e^{-a(t-s)}E|x(s-h(s))|^2 ds \\
&+ \frac{4\max\{L,1\}M^2L_2^2}{(1-\kappa)a}\int_{t_0}^{t} e^{-a(t-s)}\int_{-\infty}^{0} k(\theta)E|x(s+\theta)|^2 d\theta ds \\
\le{}& \left[\frac{8\max\{L,1\}M^2|D|^2E\|\varphi\|_\infty^2}{1-\kappa} + \frac{8LM^2E\|\varphi\|_\infty^2}{1-\kappa}\right]e^{-a(t-t_0)} + \kappa \sup_{\theta\in[-\tau,0]} E|x(t+\theta)|^2 \\
&+ \frac{4\max\{L,1\}M^2\left(|D|^2|A|^2+L_1^2\right)}{(1-\kappa)a}\int_{t_0}^{t} e^{-a(t-s)}\sup_{\theta\in[-\tau,0]} E|x(s+\theta)|^2 ds \\
&+ \frac{4\max\{L,1\}M^2L_2^2}{(1-\kappa)a}\int_{t_0}^{t} e^{-a(t-s)}\int_{-\infty}^{0} k(\theta)E|x(s+\theta)|^2 d\theta ds,
\end{aligned}
\tag{4.3}
$$

and it is easily seen that there exists a positive number $M_1 > 0$ such that $E|x(t)|^2 \le M_1 e^{-a(t-t_0)}$, for all $t \in (-\infty, t_0]$.

For the convenience, setting $\lambda_1 = 8\max\{L,1\}M^2(|D|^2 + L)E\|\varphi\|_\infty^2/1-\kappa$, $\lambda_2 = 4\max\{L,1\}M^2(|D|^2|A|^2 + L_1^2)/(1-\kappa)a$, and $\lambda_3 = 4\max\{L,1\}M^2L_2^2/(1-\kappa)a$, it implies from (4.3) that

$$
E|x(t)|^2 \le
\begin{cases}
\lambda_1 e^{-a(t-t_0)} + \kappa \sup\limits_{\theta\in[-\tau,0]} E|x(t+\theta)|^2 + \lambda_2 \displaystyle\int_{t_0}^{t} e^{-a(t-s)} \sup\limits_{\theta\in[-\tau,0]} E|x(s+\theta)|^2 ds & \\
+\lambda_3 \displaystyle\int_{t_0}^{t} e^{-a(t-s)}\int_{-\infty}^{0} k(\theta)E|x(s+\theta)|^2 d\theta ds, & t \ge t_0 \\
+\lambda_1 e^{-a(t-t_0)}, & t \in (-\infty, t_0].
\end{cases}
\tag{4.4}
$$

From (4.4), letting $F(\lambda) = \kappa e^{\lambda r} + (\lambda_2/(a-\lambda))e^{\lambda\tau} + (\lambda_3/(a-\lambda))\int_{-\infty}^{0} k(\theta)e^{-\lambda\theta}d\theta - 1$, $F(0)F(a-) < 0$ holds. That is, there exists a positive constant $\mu \in (0, a)$ such that $F(\mu) = 0$.

For any $\varepsilon > 0$ and letting

$$
M_\varepsilon = \max\left\{(\lambda_1+\varepsilon)\frac{a-\mu}{\lambda_2 e^{\mu\tau} + \lambda_3\int_{-\infty}^{0} k(\theta)e^{-\mu\theta}d\theta},\ \lambda_1+\varepsilon\right\} > 0. \tag{4.5}
$$

Now, in order to show our main result, we only claim that (4.4) implies

$$E|x(t)|^2 \le M_\varepsilon e^{-\mu(t-t_0)}, \quad t \in (-\infty, +\infty). \tag{4.6}$$

It is easily seen that (4.6) holds for any $t \in (-\infty, t_0]$. Assume, for the sake of contradiction, that there exists a $t_1 > t_0$ and

$$E|x(t)|^2 < M_\varepsilon e^{-\mu(t-t_0)}, \quad t \in (-\infty, t_1), \qquad E|x(t_1)|^2 = M_\varepsilon e^{-\mu(t_1-t_0)}. \tag{4.7}$$

Then, it, from (4.4), implies that

$$\begin{aligned}
E|x(t_1)|^2 &\le \lambda_1 e^{-a(t_1-t_0)} + \kappa M_\varepsilon e^{\mu\tau} e^{-\mu(t_1-t_0)} + \lambda_2 M_\varepsilon \int_{t_0}^{t_1} e^{-a(t_1-t_0-s)} \sup_{\theta\in[-\tau,0]} e^{-\mu(s+\theta)} ds \\
&\quad + \lambda_3 M_\varepsilon \int_{t_0}^{t_1} e^{-a(t_1-t_0-s)} \int_{-\infty}^{0} k(\theta) e^{-\mu(s+\theta)} d\theta ds \\
&\le \left[\lambda_1 - M_\varepsilon \left[\lambda_2 \frac{e^{\mu\tau}}{a-\mu} + \frac{\lambda_3}{a-\mu} \int_{-\infty}^{0} k(\theta) e^{-\mu\theta} d\theta\right]\right] e^{-a(t_1-t_0)} \\
&\quad + M_\varepsilon \left(\kappa e^{\mu r} + \frac{\lambda_2 e^{\mu\tau}}{a-\mu} + \frac{\lambda_3}{a-\mu} \int_{-\infty}^{0} k(\theta) e^{-\mu\theta} d\theta\right) e^{-\mu(t_1-t_0)}.
\end{aligned} \tag{4.8}$$

From the definitions of μ and M_ε, we have

$$\begin{aligned}
&\kappa e^{\mu r} + \frac{\lambda_2 e^{\mu\tau}}{a-\mu} + \frac{\lambda_3}{a-\mu} \int_{-\infty}^{0} k(\theta) e^{-\mu\theta} d\theta = 1, \\
&\lambda_1 - M_\varepsilon \left[\lambda_2 \frac{e^{\mu\tau}}{a-\mu} + \frac{\lambda_3}{a-\mu} \int_{-\infty}^{0} k(\theta) e^{-\mu\theta} d\theta\right] \\
&\quad \le \lambda_1 - \left[\lambda_2 \frac{e^{\mu\tau}}{a-\mu} + \frac{\lambda_3}{a-\mu} \int_{-\infty}^{0} k(\theta) e^{-\mu\theta} d\theta\right] (\varepsilon + \lambda_1) \frac{a-\mu}{\lambda_2 e^{\mu\tau} + \lambda_3 \int_{-\infty}^{0} k(\theta) e^{-\mu\theta} d\theta} \\
&\quad < 0.
\end{aligned} \tag{4.9}$$

Thus, (4.8) yields that

$$E|x(t_1)|^2 < M_\varepsilon e^{-\mu(t_1-t_0)}, \tag{4.10}$$

which contradicts (4.7), that is, (4.4) holds.

As $\varepsilon > 0$ is arbitrarily small, in view of (4.6), it follows that

$$E|x(t)|^2 \le M e^{-\mu(t-t_0)}, \quad t \ge t_0, \tag{4.11}$$

where $M = \max\{\lambda_1(a-\mu)/(\lambda_2 e^{\mu\tau} + \lambda_3 \int_{-\infty}^{0} k(\theta)e^{-\mu\theta}d\theta), \lambda_1\} > 0$.

That is,

$$E|x(t)|^2 \leq M' e^{-\alpha(t-t_0)}, \quad t \geq t_0, \tag{4.12}$$

where $\alpha \in (0, a)$ and

$$\begin{aligned} M' = \max\Biggl\{ & \frac{8\max\{L,1\}M^2\left(|D|^2 + L\right)E\|\varphi\|_\infty^2(a-\mu)}{1-\kappa} \\ & \times \left(\frac{4\max\{L,1\}M^2\left(|D|^2|A|^2 + L_1^2\right)e^{\mu\tau}}{(1-\kappa)a} + \frac{4\max\{L,1\}M^2L_2^2}{(1-\kappa)a}\int_{-\infty}^{0} k(\theta)e^{-\mu\theta}d\theta \right)^{-1}, \\ & \frac{8\max\{L,1\}M^2\left(|D|^2 + L\right)E\|\varphi\|_\infty^2(a-\mu)}{1-\kappa} \Biggr\} > 0. \end{aligned} \tag{4.13}$$

The proof is completed. □

In particular, when $D \equiv 0$, $\tau \equiv 0$, and $f_2 \equiv 0$, system (2.2) is turned into the following form:

$$\begin{aligned} x'(t) &= Ax(t) + f_1(t, x(t)), \quad t \neq \xi_k, \ t \geq 0, \\ x(\xi_k) &= b_k(\tau_k)x(\xi_k^-), \quad k = 1, 2, \ldots, \\ x_{t_0} &= x_0. \end{aligned} \tag{4.14}$$

Remark 4.2. Obviously, we can also give the existence, uniqueness, and exponential stability in mean square for the solution of system (4.14) by employing the Picard iterative method and a similar impulsive-integral inequality proposed in [24]. So, the following corollary can be given as follows.

Corollary 4.3. *Under conditions (H$_1$) and (H$_3$), the existence, uniqueness, and exponential stability in mean square for the solution of system* (4.14) *can be obtained only if the inequality*

$$\max\{1, L\}M^2L_1^2 < a^2 \tag{4.15}$$

holds.

Proof. The proofs of this corollary are very similar to those of Theorems 3.2 and 4.1. So, we omit them. □

Remark 4.4. Recently, in [19], Anguraj and Vinodkumar have derived Corollary 4.3 by using the fixed point theorem. Obviously, our results are more general than those obtained in [19]. Thus, we can generalize and improve the results in [19].

5. An Illustrative Example

Let τ_k be a random variable defined in $D_k \equiv (0, d_k)$ for all $k = 1, 2, \ldots$, where $0 < d_k \leq +\infty$. Furthermore, assume that τ_i and τ_j are independent of each other as $i \neq j$ for $i, j = 1, 2, \ldots$.

Consider the random impulsive integrodifferential equations of neutral type:

$$\begin{aligned}
\frac{dx_1(t)}{dt} &= -0.2x_1(t) + \frac{1}{2\sqrt{100}}\frac{dx_1(t-r)}{dt} - \frac{1}{2\sqrt{100}}\frac{dx_2(t-r)}{dt} + \delta_1 \cos(t)x_1(t-h(t)) \\
&\quad + \delta_3 \int_{-\infty}^{0} (-\theta)^{-(1/2)} e^{\pi^2\theta} x_1(s+\theta)ds, \\
\frac{dx_2(t)}{dt} &= -0.1x_1(t) - 0.1x_2(t) + \frac{1}{3\sqrt{100}}\frac{dx_1(t-r)}{dt} + \frac{1}{4\sqrt{100}}\frac{dx_2(t-r)}{dt} + \delta_2 \sin(t)x_2(t-h(t)) \\
&\quad + \delta_4 \int_{-\infty}^{0} (-\theta)^{-(1/2)} e^{\pi^2\theta} x_2(s+\theta)ds,
\end{aligned} \tag{5.1}$$

as $\xi_{k-1} \leq t < \xi_k$, $k = 1, 2, \ldots$, and for all $k = 1, 2, 3, \ldots$,

$$\begin{aligned}
x_1(\xi_k) &= c(k)\tau_k \cdot x_1(\xi_k^-), \\
x_2(\xi_k) &= c(k)\tau_k \cdot x_2(\xi_k^-),
\end{aligned} \tag{5.2}$$

where $\xi_0 = t_0$ and $\xi_k = \xi_{k-1} + \tau_k$ for all $k = 1, 2, \ldots$, $c(k)$ is a function of k. Denote that $c = \max_k\{c(k)\}$ and there is $\rho : 0 \leq \rho < 1$ such that $E(c\tau_k^2) \leq \rho$ for all $k = 1, 2, \ldots$. and $|\delta_i| \leq L_1 \quad (i = 1, 2)$ and $|\delta_i| \leq L_2 \quad (i = 3, 4)$. And $|D| = 0.0846$, $|A| = 0.2449$, $\kappa = 0.0846$.

The corresponding linear homogeneous equations:

$$\begin{pmatrix} \dfrac{dx_1(t)}{dt} \\ \dfrac{dx_2(t)}{dt} \end{pmatrix} = \begin{pmatrix} -0.2 & 0 \\ -0.1 & -0.1 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}. \tag{5.3}$$

And, the fundamental solution matrix of system (5.3) can be given by

$$\Phi(t) = \begin{pmatrix} e^{-0.2t} & 0 \\ e^{-0.2t} - e^{-0.1t} & e^{-0.1t} \end{pmatrix}. \tag{5.4}$$

Then, it is easily obtain that

$$|\Phi(t)| \leq Me^{-0.1t}, \qquad t \geq 0, \tag{5.5}$$

where $M = \sqrt{2} > 1$ and $a = 0.1 > 0$, and it is easily seen that we can derive that the functions f_1 and f_2 satisfy conditions (H_1) and (H_2) with the Lipschitz coefficients L_1 and L_2, respectively. On the other hand, hypothesis (H_3) and (H_4) are easily verified. In view of Theorems 3.2 and

4.1, the existence, uniqueness, and exponential stability in mean square of system (5.1) with (5.2) are obtained if the constants L_1 and L_2 satisfy the following inequality:

$$L_1^2 + L_2^2 < 0.02771. \tag{5.6}$$

Acknowledgment

The authors would like to thank the referee and the editor for their careful comments and valuable suggestions on this work.

References

[1] A. Anokhin, L. Berezansky, and E. Braverman, "Exponential stability of linear delay impulsive differential equations," *Journal of Mathematical Analysis and Applications*, vol. 193, no. 3, pp. 923–941, 1995.

[2] Y. Hino, S. Murakani, and T. Naito, *Functional Differential Equations with Infinite Delay*, vol. 1473 of *Lecture Notes in Mathematics*, Springer, Berlin, Germany, 1991.

[3] J. Hale and S. M. Verguyn Lunel, *Introduction to Functional Differential Equations*, vol. 99 of *Applied Mathematical Science*, Springer, New York, NY, USA, 1993.

[4] J. Henderson and A. Ouadhab, "Local and global existence and uniqueness results for second and higher order impulsive functional differential equations with infinite delay," *The Australian Journal of Mathematical Analysis and Applications*, vol. 4, pp. 1–26, 2007.

[5] A. Korzeniowski and G. S. Ladde, "Modeling hybrid network dynamics under random perturbations," *Nonlinear Analysis: Hybrid Systems*, vol. 3, no. 2, pp. 143–149, 2009.

[6] X. Liu, X. Shen, and Y. Zhang, "A comparison principle and stability for large-scale impulsive delay differential systems," *The ANZIAM Journal*, vol. 47, no. 2, pp. 203–235, 2005.

[7] X. Liu, "Stability results for impulsive differential systems with applications to population growth models," *Dynamics & Stability of Systems. Series A*, vol. 9, no. 2, pp. 163–174, 1994.

[8] X. Liu and X. Liao, "Comparison method and robust stability of large-scale dynamic systems," *Dynamics of Continuous, Discrete & Impulsive Systems. Series A*, vol. 11, no. 2-3, pp. 413–430, 2004.

[9] R. Rakkiyappan, P. Balasubramaniam, and J. Cao, "Global exponential stability results for neutral-type impulsive neural networks," *Nonlinear Analysis: Real World Applications*, vol. 11, no. 1, pp. 122–130, 2010.

[10] R. Samidurai, S. M. Anthoni, and K. Balachandran, "Global exponential stability of neutral-type impulsive neural networks with discrete and distributed delays," *Nonlinear Analysis: Hybrid Systems*, vol. 4, no. 1, pp. 103–112, 2010.

[11] V. Lakshmikantham, D. D. Baĭnov, and P. S. Simeonov, *Theory of Impulsive Differential Equations*, vol. 6 of *Series in Modern Applied Mathematics*, World Scientific, Teaneck, NJ, USA, 1989.

[12] A. M. Samoĭlenko and N. A. Perestyuk, *Impulsive Differential Equations*, vol. 14 of *World Scientific Series on Nonlinear Science. Series A: Monographs and Treatises*, World Scientific, River Edge, NJ, USA, 1995.

[13] S. Wu and X. Meng, "Boundedness of nonlinear differential systems with impulsive effect on random moments," *Acta Mathematicae Applicatae Sinica*, vol. 20, no. 1, pp. 147–154, 2004.

[14] S. Wu, X. Guo, and Y. Zhou, "p-moment stability of functional differential equations with random impulses," *Computers & Mathematics with Applications*, vol. 52, no. 12, pp. 1683–1694, 2006.

[15] S. Wu, X. Guo, and R. Zhai, "Almost sure stability of functional differential equations with random impulses," *Dynamics of Continuous, Discrete & Impulsive Systems. Series A*, vol. 15, no. 3, pp. 403–415, 2008.

[16] S. Wu and Y. Duan, "Oscillation, stability, and boundedness of second-order differential systems with random impulses," *Computers & Mathematics with Applications*, vol. 49, no. 9-10, pp. 1375–1386, 2005.

[17] S. Wu, X. Guo, and S. Lin, "Existence and uniqueness of solutions to random impulses," *Acta Mathematicae Applicatae Sinica*, vol. 22, pp. 595–600, 2004.

[18] D. Zhao and L. Zhang, "Exponential asymptotic stability of nonlinear Volterra equations with random impulses," *Applied Mathematics and Computation*, vol. 193, no. 1, pp. 18–25, 2007.

[19] A. Anguraj and A. Vinodkumar, "Existence, uniqueness and stability results of random impulsive semilinear differential systems," *Nonlinear Analysis: Hybrid Systems*, vol. 4, pp. 475–483, 2010.

[20] A. Anguraj and A. Vinodkumar, "Existence and uniqueness of neutral functional differential equations with random impulses," *International Journal of Nonlinear Science*, vol. 8, no. 4, pp. 412–418, 2009.

[21] D. Xu, Z. Yang, and Z. Yang, "Exponential stability of nonlinear impulsive neutral differential equations with delays," *Nonlinear Analysis: Theory, Methods & Applications*, vol. 67, no. 5, pp. 1426–1439, 2007.

[22] L. Xu and D. Xu, "Exponential stability of nonlinear impulsive neutral integro-differential equations," *Nonlinear Analysis: Theory, Methods & Applications*, vol. 69, no. 9, pp. 2910–2923, 2008.

[23] W. Walter, *Differential and Integral Inequalities*, Ergebnisse der Mathematik und ihrer Grenzgebiete, Band 55, Springer, New York, NY, USA, 1970.

[24] H. Chen, "Impulsive-integral inequality and exponential stability for stochastic partial differential equations with delays," *Statistics & Probability Letters*, vol. 80, no. 1, pp. 50–56, 2010.