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Research Article

On the Existence of Solutions for Dynamic Boundary Value Problems under Barrier Strips Condition

Hua Luo[1] and Yulian An[2]

[1] *School of Mathematics and Quantitative Economics, Dongbei University of Finance and Economics, Dalian 116025, China*
[2] *Department of Mathematics, Shanghai Institute of Technology, Shanghai 200235, China*

Correspondence should be addressed to Hua Luo, luohuanwnu@gmail.com

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By defining a new terminology, scatter degree, as the supremum of graininess functional value, this paper studies the existence of solutions for a nonlinear two-point dynamic boundary value problem on time scales. We do not need any growth restrictions on nonlinear term of dynamic equation besides a barrier strips condition. The main tool in this paper is the induction principle on time scales.

1. Introduction

Calculus on time scales, which unify continuous and discrete analysis, is now still an active area of research. We refer the reader to [1–5] and the references therein for introduction on this theory. In recent years, there has been much attention focused on the existence and multiplicity of solutions or positive solutions for dynamic boundary value problems on time scales. See [6–17] for some of them. Under various growth restrictions on nonlinear term of dynamic equation, many authors have obtained many excellent results for the above problem by using Topological degree theory, fixed-point theorems on cone, bifurcation theory, and so on.

In 2004, Ma and Luo [18] firstly obtained the existence of solutions for the dynamic boundary value problems on time scales

$$\begin{gathered} x^{\Delta\Delta}(t) = f\left(t, x(t), x^{\Delta}(t)\right), \quad t \in [0,1]_{\mathbb{T}}, \\ x(0) = 0, \qquad x^{\Delta}(\sigma(1)) = 0 \end{gathered} \tag{1.1}$$

under a barrier strips condition. A barrier strip P is defined as follows. There are pairs (two or four) of suitable constants such that nonlinear term $f(t,u,p)$ does not change its sign on sets of the form $[0,1]_{\mathbb{T}} \times [-L,L] \times P$, where L is a nonnegative constant, and P is a closed interval bounded by some pairs of constants, mentioned above.

The idea in [18] was from Kelevedjiev [19], in which discussions were for boundary value problems of ordinary differential equation. This paper studies the existence of solutions for the nonlinear two-point dynamic boundary value problem on time scales

$$\begin{gathered} x^{\Delta\Delta}(t) = f\left(t, x^{\sigma}(t), x^{\Delta}(t)\right), \quad t \in \left[a, \rho^2(b)\right]_{\mathbb{T}}, \\ x^{\Delta}(a) = 0, \qquad x(b) = 0, \end{gathered} \tag{1.2}$$

where $\mathbb{T}$ is a bounded time scale with $a = \inf \mathbb{T}$, $b = \sup \mathbb{T}$, and $a < \rho^2(b)$. We obtain the existence of at least one solution to problem (1.2) without any growth restrictions on f but an existence assumption of barrier strips. Our proof is based upon the well-known Leray-Schauder principle and the induction principle on time scales.

The time scale-related notations adopted in this paper can be found, if not explained specifically, in almost all literature related to time scales. Here, in order to make this paper read easily, we recall some necessary definitions here.

A time scale $\mathbb{T}$ is a nonempty closed subset of $\mathbb{R}$; assume that $\mathbb{T}$ has the topology that it inherits from the standard topology on $\mathbb{R}$. Define the forward and backward jump operators $\sigma, \rho : \ \mathbb{T} \to \mathbb{T}$ by

$$\sigma(t) = \inf\{\tau > t \mid \tau \in \mathbb{T}\}, \qquad \rho(t) = \sup\{\tau < t \mid \tau \in \mathbb{T}\}. \tag{1.3}$$

In this definition we put $\inf \emptyset = \sup \mathbb{T}$, $\sup \emptyset = \inf \mathbb{T}$. Set $\sigma^2(t) = \sigma(\sigma(t))$, $\rho^2(t) = \rho(\rho(t))$. The sets $\mathbb{T}^k$ and $\mathbb{T}_k$ which are derived from the time scale $\mathbb{T}$ are as follows:

$$\begin{aligned} \mathbb{T}^k &:= \left\{t \in \mathbb{T} : t \text{ is not maximal or } \rho(t) = t\right\}, \\ \mathbb{T}_k &:= \{t \in \mathbb{T} : t \text{ is not minimal or } \sigma(t) = t\}. \end{aligned} \tag{1.4}$$

Denote interval I on $\mathbb{T}$ by $I_{\mathbb{T}} = I \cap \mathbb{T}$.

Definition 1.1. If $f : \mathbb{T} \to \mathbb{R}$ is a function and $t \in \mathbb{T}^k$, then the delta derivative of f at the point t is defined to be the number $f^{\Delta}(t)$ (provided it exists) with the property that, for each $\varepsilon > 0$, there is a neighborhood U of t such that

$$\left| f(\sigma(t)) - f(s) - f^{\Delta}(t)(\sigma(t) - s) \right| \leqslant \varepsilon |\sigma(t) - s| \tag{1.5}$$

for all $s \in U$. The function f is called Δ-differentiable on $\mathbb{T}^k$ if $f^{\Delta}(t)$ exists for all $t \in \mathbb{T}^k$.

Definition 1.2. If $F^{\Delta} = f$ holds on $\mathbb{T}^k$, then we define the Cauchy Δ-integral by

$$\int_s^t f(\tau)\Delta\tau = F(t) - F(s), \quad s, t \in \mathbb{T}^k. \tag{1.6}$$

Lemma 1.3 (see [2, Theorem 1.16 (SUF)]). *If f is Δ-differentiable at $t \in \mathbb{T}^k$, then*

$$f(\sigma(t)) = f(t) + (\sigma(t) - t) f^{\Delta}(t). \tag{1.7}$$

Lemma 1.4 (see [18, Lemma 3.2]). *Suppose that $f : [a,b]_{\mathbb{T}} \to \mathbb{R}$ is Δ-differentiable on $[a,b]_{\mathbb{T}}^k$, then*

(i) *f is nondecreasing on $[a,b]_{\mathbb{T}}$ if and only if $f^{\Delta}(t) \geq 0$, $t \in [a,b]_{\mathbb{T}}^k$,*

(ii) *f is nonincreasing on $[a,b]_{\mathbb{T}}$ if and only if $f^{\Delta}(t) \leq 0$, $t \in [a,b]_{\mathbb{T}}^k$.*

Lemma 1.5 (see [4, Theorem 1.4]). *Let $\mathbb{T}$ be a time scale with $\tau \in \mathbb{T}$. Then the induction principle holds.*

Assume that, for a family of statements $A(t)$, $t \in [\tau, +\infty)_{\mathbb{T}}$, the following conditions are satisfied.

(1) *$A(\tau)$ holds true.*

(2) *For each $t \in [\tau, +\infty)_{\mathbb{T}}$ with $\sigma(t) > t$, one has $A(t) \Rightarrow A(\sigma(t))$.*

(3) *For each $t \in [\tau, +\infty)_{\mathbb{T}}$ with $\sigma(t) = t$, there is a neighborhood U of t such that $A(t) \Rightarrow A(s)$ for all $s \in U$, $s > t$.*

(4) *For each $t \in (\tau, +\infty)_{\mathbb{T}}$ with $\rho(t) = t$, one has $A(s)$ for all $s \in [\tau, t)_{\mathbb{T}} \Rightarrow A(t)$.*

Then $A(t)$ is true for all $t \in [\tau, +\infty)_{\mathbb{T}}$.

Remark 1.6. For $t \in (-\infty, \tau]_{\mathbb{T}}$, we replace $\sigma(t)$ with $\rho(t)$ and $\rho(t)$ with $\sigma(t)$, substitute $<$ for $>$, then the dual version of the above induction principle is also true.

By $C^2([a,b])$, we mean the Banach space of second-order continuous Δ-differentiable functions $x : [a,b]_{\mathbb{T}} \to \mathbb{R}$ equipped with the norm

$$\|x\| = \max\left\{ |x|_0, \left|x^{\Delta}\right|_0, \left|x^{\Delta\Delta}\right|_0 \right\}, \tag{1.8}$$

where $|x|_0 = \max_{t \in [a,b]_{\mathbb{T}}} |x(t)|$, $|x^{\Delta}|_0 = \max_{t \in [a,\rho(b)]_{\mathbb{T}}} |x^{\Delta}(t)|$, $|x^{\Delta\Delta}|_0 = \max_{t \in [a,\rho^2(b)]_{\mathbb{T}}} |x^{\Delta\Delta}(t)|$. According to the well-known Leray-Schauder degree theory, we can get the following theorem.

Lemma 1.7. *Suppose that f is continuous, and there is a constant $C > 0$, independent of $\lambda \in (0,1)$, such that $\|x\| < C$ for each solution $x(t)$ to the boundary value problem*

$$\begin{gathered} x^{\Delta\Delta}(t) = \lambda f\left(t, x^{\sigma}(t), x^{\Delta}(t)\right), \quad t \in \left[a, \rho^2(b)\right]_{\mathbb{T}}, \\ x^{\Delta}(a) = 0, \qquad x(b) = 0. \end{gathered} \tag{1.9}$$

Then the boundary value problem (1.2) *has at least one solution in $C^2([a,b])$.*

Proof. The proof is the same as [18, Theorem 4.1]. □

2. Existence Theorem

To state our main result, we introduce the definition of *scatter degree*.

Definition 2.1. For a time scale $\mathbb{T}$, define the right direction scatter degree (RSD) and the left direction scatter degree (LSD) on $\mathbb{T}$ by

$$r(\mathbb{T}) = \sup\left\{\sigma(t) - t : t \in \mathbb{T}^k\right\},$$
$$l(\mathbb{T}) = \sup\left\{t - \rho(t) : t \in \mathbb{T}_k\right\}, \tag{2.1}$$

respectively. If $r(\mathbb{T}) = l(\mathbb{T})$, then we call $r(\mathbb{T})$ (or $l(\mathbb{T})$) the scatter degree on $\mathbb{T}$.

Remark 2.2. (1) If $\mathbb{T} = \mathbb{R}$, then $r(\mathbb{T}) = l(\mathbb{T}) = 0$. If $\mathbb{T} = h\mathbb{Z} := \{hk : k \in \mathbb{Z},\ h > 0\}$, then $r(\mathbb{T}) = l(\mathbb{T}) = h$. If $\mathbb{T} = q^{\mathbb{N}} := \{q^k : k \in \mathbb{N}\}$ and $q > 1$, then $r(\mathbb{T}) = l(\mathbb{T}) = +\infty$. (2) If $\mathbb{T}$ is bounded, then both $r(\mathbb{T})$ and $l(\mathbb{T})$ are finite numbers.

Theorem 2.3. *Let $f : [a, \rho(b)]_{\mathbb{T}} \times \mathbb{R}^2 \to \mathbb{R}$ be continuous. Suppose that there are constants L_i, $i = 1, 2, 3, 4$, with $L_2 > L_1 \geq 0$, $L_3 < L_4 \leq 0$ satisfying*

(*H1*) $L_2 > L_1 + Mr(\mathbb{T})$, $L_3 < L_4 - Mr(\mathbb{T})$,

(*H2*) $f(t, u, p) \leq 0$ *for* $(t, u, p) \in [a, \rho(b)]_{\mathbb{T}} \times [-L_2(b-a), -L_3(b-a)] \times [L_1, L_2]$, $f(t, u, p) \geq 0$ *for* $(t, u, p) \in [a, \rho(b)]_{\mathbb{T}} \times [-L_2(b-a), -L_3(b-a)] \times [L_3, L_4]$,

where

$$M = \sup\left\{\left|f(t, u, p)\right| : (t, u, p) \in \left[a, \rho(b)\right]_{\mathbb{T}} \times [-L_2(b-a), -L_3(b-a)] \times [L_3, L_2]\right\}. \tag{2.2}$$

Then problem (1.2) *has at least one solution in* $C^2([a, b])$.

Remark 2.4. Theorem 2.3 extends [19, Theorem 3.2] even in the special case $\mathbb{T} = \mathbb{R}$. Moreover, our method to prove Theorem 2.3 is different from that of [19].

Remark 2.5. We can find some elementary functions which satisfy the conditions in Theorem 2.3. Consider the dynamic boundary value problem

$$x^{\Delta\Delta}(t) = -\left(x^{\Delta}(t)\right)^3 + h\left(t, x^{\sigma}(t), x^{\Delta}(t)\right), \quad t \in \left[a, \rho^2(b)\right]_{\mathbb{T}},$$
$$x^{\Delta}(a) = 0, \qquad x(b) = 0, \tag{2.3}$$

where $h(t, u, p) : [a, \rho(b)]_{\mathbb{T}} \times \mathbb{R}^2 \to \mathbb{R}$ is bounded everywhere and continuous.

Suppose that $f(t, u, p) = -p^3 + h(t, u, p)$, then for $t \in [a, \rho(b)]_{\mathbb{T}}$

$$f(t, u, p) \longrightarrow -\infty, \quad \text{if } p \longrightarrow +\infty,$$
$$f(t, u, p) \longrightarrow +\infty, \quad \text{if } p \longrightarrow -\infty. \tag{2.4}$$

It implies that there exist constants L_i, $i = 1, 2, 3, 4$, satisfying (*H1*) and (*H2*) in Theorem 2.3. Thus, problem (2.3) has at least one solution in $C^2([a, b])$.

Proof of Theorem 2.3. Define $\Phi : \mathbb{R} \to \mathbb{R}$ as follows:

$$\Phi(u) = \begin{cases} -L_2(b-a), & u \leq -L_2(b-a), \\ u, & -L_2(b-a) < u < -L_3(b-a), \\ -L_3(b-a), & u \geq -L_3(b-a). \end{cases} \tag{2.5}$$

For all $\lambda \in (0,1)$, suppose that $x(t)$ is an arbitrary solution of problem

$$\begin{gathered} x^{\Delta\Delta}(t) = \lambda f\left(t, \Phi(x^{\sigma}(t)), x^{\Delta}(t)\right), \quad t \in \left[a, \rho^2(b)\right]_{\mathbb{T}}, \\ x^{\Delta}(a) = 0, \qquad x(b) = 0. \end{gathered} \tag{2.6}$$

We firstly prove that there exists $C > 0$, independent of λ and x, such that $\|x\| < C$.

We show at first that

$$L_3 < x^{\Delta}(t) < L_2, \quad t \in \left[a, \rho(b)\right]_{\mathbb{T}}. \tag{2.7}$$

Let $A(t) : L_3 < x^{\Delta}(t) < L_2,\ t \in [a, \rho(b)]_{\mathbb{T}}$. We employ the induction principle on time scales (Lemma 1.5) to show that $A(t)$ holds step by step.

(1) From the boundary condition $x^{\Delta}(a) = 0$ and the assumption of $L_3 < 0 < L_2$, $A(a)$ holds.

(2) For each $t \in [a, \rho(b))_{\mathbb{T}}$ with $\sigma(t) > t$, suppose that $A(t)$ holds, that is, $L_3 < x^{\Delta}(t) < L_2$. Note that $-L_2(b-a) \leq \Phi(x^{\sigma}(t)) \leq -L_3(b-a)$; we divide this discussion into three cases to prove that $A(\sigma(t))$ holds.

Case 1. If $L_4 < x^{\Delta}(t) < L_1$, then from Lemma 1.3, Definition 2.1, and (*H1*) there is

$$\begin{aligned} x^{\Delta}(\sigma(t)) &= x^{\Delta}(t) + x^{\Delta\Delta}(t)(\sigma(t) - t) \\ &< L_1 + Mr(\mathbb{T}) \\ &< L_2. \end{aligned} \tag{2.8}$$

Similarly, $x^{\Delta}(\sigma(t)) > L_4 - Mr(\mathbb{T}) > L_3$.

Case 2. If $L_1 \leq x^{\Delta}(t) < L_2$, then similar to Case 1 we have

$$\begin{aligned} x^{\Delta}(\sigma(t)) &= x^{\Delta}(t) + x^{\Delta\Delta}(t)(\sigma(t) - t) \\ &> L_4 - Mr(\mathbb{T}) \\ &> L_3. \end{aligned} \tag{2.9}$$

Suppose to the contrary that $x^{\Delta}(\sigma(t)) \geq L_2$, then

$$\lambda f\Big(t, \Phi(x^{\sigma}(t)), x^{\Delta}(t)\Big) = x^{\Delta\Delta}(t) = \frac{x^{\Delta}(\sigma(t)) - x^{\Delta}(t)}{\sigma(t) - t} > 0, \tag{2.10}$$

which contradicts $(H2)$. So $x^{\Delta}(\sigma(t)) < L_2$.

Case 3. If $L_3 < x^{\Delta}(t) \leq L_4$, similar to Case 2, then $L_3 < x^{\Delta}(\sigma(t)) < L_2$ holds.

Therefore, $A(\sigma(t))$ is true.

(3) For each $t \in [a, \rho(b))_{\mathbb{T}}$, with $\sigma(t) = t$, and $A(t)$ holds, then there is a neighborhood U of t such that $A(s)$ holds for all $s \in U,\ s > t$ by virtue of the continuity of x^{Δ}.

(4) For each $t \in (a, \rho(b)]_{\mathbb{T}}$, with $\rho(t) = t$, and $A(s)$ is true for all $s \in [a, t)_{\mathbb{T}}$, since $x^{\Delta}(t) = \lim_{s \to t, s<t} x^{\Delta}(s)$ implies that

$$L_3 \leq x^{\Delta}(t) \leq L_2, \tag{2.11}$$

we only show that $x^{\Delta}(t) \neq L_2$ and $x^{\Delta}(t) \neq L_3$.

Suppose to the contrary that $x^{\Delta}(t) = L_2$. From

$$x^{\Delta}(s) < L_2, \quad s \in [a, t)_{\mathbb{T}}, \tag{2.12}$$

$\rho(t) = t$, and the continuity of x^{Δ}, there is a neighborhood V of t such that

$$L_1 < x^{\Delta}(s) < L_2, \quad s \in [a, t)_{\mathbb{T}} \cap V. \tag{2.13}$$

So $L_1 < x^{\Delta}(s) \leq L_2,\ s \in [a, t]_{\mathbb{T}} \cap V$. Combining with $-L_2(b-a) \leq \Phi(x^{\sigma}(s)) \leq -L_3(b-a),\ s \in [a, t]_{\mathbb{T}} \cap V$, we have from $(H2)$, $x^{\Delta\Delta}(s) = \lambda f(s, \Phi(x^{\sigma}(s)),\ x^{\Delta}(s)) \leq 0,\ s \in [a, t]_{\mathbb{T}} \cap V$. So from Lemma 1.4

$$x^{\Delta}(s) \geq x^{\Delta}(t) = L_2, \quad s \in [a, t]_{\mathbb{T}} \cap V. \tag{2.14}$$

This contradiction shows that $x^{\Delta}(t) \neq L_2$. In the same way, we claim that $x^{\Delta}(t) \neq L_3$.

Hence, $A(t) : L_3 < x^{\Delta}(t) < L_2,\ t \in [a, \rho(b)]_{\mathbb{T}}$, holds. So

$$\left| x^{\Delta} \right|_0 < C_1 := \max\{-L_3, L_2\}. \tag{2.15}$$

From Definition 1.2 and Lemma 1.3, we have for $t \in [a, \rho(b)]_{\mathbb{T}}$

$$\begin{aligned} x(t) &= x\big(\rho(b)\big) - \int_t^{\rho(b)} x^{\Delta}(s)\Delta s \\ &= x(b) - x^{\Delta}\big(\rho(b)\big)\big(b - \rho(b)\big) - \int_t^{\rho(b)} x^{\Delta}(s)\Delta s. \end{aligned} \tag{2.16}$$

There are, from $x(b) = 0$ and (2.7),

$$x(t) < -L_3(b - \rho(b)) - L_3(\rho(b) - t) \leq -L_3(b - a),$$
$$x(t) > -L_2(b - \rho(b)) - L_2(\rho(b) - t) \geq -L_2(b - a) \tag{2.17}$$

for $t \in [a, \rho(b)]_{\mathbb{T}}$. In addition,

$$-L_2(b - a) < x(b) = 0 < -L_3(b - a). \tag{2.18}$$

Thus,

$$-L_2(b - a) < x(t) < -L_3(b - a), \quad t \in [a, b]_{\mathbb{T}}, \tag{2.19}$$

that is,

$$|x|_0 < C_1(b - a). \tag{2.20}$$

Moreover, by the continuity of f, the equation in (2.6), (2.7) and the definition of Φ

$$\left|x^{\Delta\Delta}\right|_0 < M, \tag{2.21}$$

where M is defined in (2.2). Now let $C = \max\{C_1, C_1(b - a), M\}$. Then, from (2.15), (2.20), and (2.21),

$$\|x\| < C. \tag{2.22}$$

Note that from (2.19) we have

$$-L_2(b - a) < x^\sigma(t) < -L_3(b - a), \quad t \in \left[a, \rho(b)\right]_{\mathbb{T}}, \tag{2.23}$$

that is, $\Phi(x^\sigma(t)) = x^\sigma(t)$, $t \in [a, \rho(b)]_{\mathbb{T}}$. So x is also an arbitrary solution of problem

$$x^{\Delta\Delta}(t) = \lambda f\left(t, x^\sigma(t), x^\Delta(t)\right), \quad t \in \left[a, \rho^2(b)\right]_{\mathbb{T}},$$
$$x^\Delta(a) = 0, \qquad x(b) = 0. \tag{2.24}$$

According to (2.22) and Lemma 1.7, the dynamic boundary value problem (1.2) has at least one solution in $C^2([a, b])$. □

3. An Additional Result

Parallel to the definition of delta derivative, the notion of nabla derivative was introduced, and the main relations between the two operations were studied in [7]. Applying to the dual version of the induction principle on time scales (Remark 1.6), we can obtain the following result.

Theorem 3.1. *Let* $g : [\sigma(a), b]_{\mathbb{T}} \times \mathbb{R}^2 \to \mathbb{R}$ *be continuous. Suppose that there are constants* I_i, $i = 1, 2, 3, 4$, *with* $I_2 > I_1 \geq 0$, $I_3 < I_4 \leq 0$ *satisfying*

(*S1*) $I_2 > I_1 + Nl(\mathbb{T})$, $I_3 < I_4 - Nl(\mathbb{T})$,

(*S2*) $g(t, u, p) \geq 0$ *for* $(t, u, p) \in [\sigma(a), b]_{\mathbb{T}} \times [I_3(b-a), I_2(b-a)] \times [I_1, I_2]$, $g(t, u, p) \leq 0$ *for* $(t, u, p) \in [\sigma(a), b]_{\mathbb{T}} \times [I_3(b-a), I_2(b-a)] \times [I_3, I_4]$,

where

$$N = \sup\{|g(t, u, p)| : (t, u, p) \in [\sigma(a), b]_{\mathbb{T}} \times [I_3(b-a), I_2(b-a)] \times [I_3, I_2]\}. \tag{3.1}$$

Then dynamic boundary value problem

$$\begin{gathered} x^{\nabla\nabla}(t) = g\left(t, x^{\rho}(t), x^{\nabla}(t)\right), \quad t \in \left[\sigma^2(a), b\right]_{\mathbb{T}}, \\ x(a) = 0, \qquad x^{\nabla}(b) = 0 \end{gathered} \tag{3.2}$$

has at least one solution.

Remark 3.2. According to Theorem 3.1, the dynamic boundary value problem related to the nabla derivative

$$\begin{gathered} x^{\nabla\nabla}(t) = \left(x^{\nabla}(t)\right)^3 + k\left(t, x^{\rho}(t), x^{\nabla}(t)\right), \quad t \in \left[\sigma^2(a), b\right]_{\mathbb{T}}, \\ x(a) = 0, \qquad x^{\nabla}(b) = 0 \end{gathered} \tag{3.3}$$

has at least one solution. Here $k(t, u, p) : [\sigma(a), b]_{\mathbb{T}} \times \mathbb{R}^2 \to \mathbb{R}$ is bounded everywhere and continuous.

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