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Research Article

Profitability Analysis of Price-Taking Strategy in Disequilibrium

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Conventional economic assumption that more sophistication in decision making is better than less is challenged with a profitability analysis conducted with an oligopolistic model consisting of a naive firm and a group of sophisticated firms. While the naive firm is assumed to adopt a simple Cobweb strategy by equating its marginal cost of current production to the last period's price, the sophisticated firms can take either individually or collusively any conventional sophisticated strategy such as Cournot and Stackelberg strategies. Contrary to the economic intuition, it is not the sophisticated firms but the naive firm who triumphs in equilibrium as well as during the dynamical transitionary periods, no matter what strategies the sophisticated firms may take. Moreover, when the economy turns cyclic or chaotic, a combination of the Cobweb strategy with a cautious adjustment strategy could also bring relative higher average profits for the naive firm than its rivals.

1. Introduction

Oligopoly is considered to be one of the most important types of markets. Since the classic work by Cournot in 1838, research interests in oligopoly were almost all concentrated on competitions between firms which were homogeneous in the strategy, either in output or price, although some generalizations have been made to the cases involving product differentiation, randomness in demand, and the mixture of price and quantity strategies. Following the conventional economic intuitions that greater sophistication in decision making is better than less and more information is beneficial, if output is the only choice variable, "price-taking strategy" (Cobweb strategy), in which a firm ignores its output impact to the market and sets its output through equating the estimated market price to the marginal cost, has long been regarded as an inferior strategy comparing to the more sophisticated strategies such as the Cournot or Stackelberg strategy, or to a collusion in terms of economic efficiency. Such beliefs are challenged in Huang [1, 2], where the Cobweb strategy, despite being simple and naive, has been shown to be the most effective and efficient among all possible alternative strategies in the sense that it always results in a profit higher than or equal to its rivals in equilibrium, no matter what strategies the latter may take (including acting as "relative profit maximizer"). The current research takes a step further by showing that a naive firm that adopts the Cobweb strategy could make higher profit than its rivals during the dynamic transition process. Moreover, a combination of Cobweb strategy with a cautious adjustment strategy could bring higher relative profits for the naive firm than its rivals if the oligopolistic market turns cyclic or chaotic.

The paper is organized as follows. In Section 2, a heterogeneous oligopoly model is set up. Section 3 generalizes the results of Huang [2] and shows that a naive firm (i.e., the firm taking a Cobweb strategy) always outperforms its sophisticated rivals in equilibrium. Section 4 then examines the relative profitability of a naive firm in transitionary dynamics. Section 5 modifies the original model by incorporating a cautious adjustment strategy in the naive firm's production decision. Section 6 examines the relative profitability of a naive firm with the cautious adjustment strategy in a chaotic environment. It is shown, both in theory and by simulations, that through limiting the output growth rate to a certain level, a naive firm does not only stabilize the economy, but also makes a higher average profit than its sophisticated rivals. Concluding remarks and issues for further studies are offered in Section 7.

2. A heterogeneous oligopoly model

Consider an oligopoly market, in which $N = n + m$ firms produce a homogeneous product with quantity q_t^i, $i = 1,2,\dots,n+m$, at period t. The inverse market demand for the product is given by $p_t = D(q_t^d)$, where $D' \le 0$. The conventional assumption that $q_t^d = \sum_{i=1}^N q_t^i$, that is, *the actual market price adjusts to the demand so as to clear the market at every period*, applies.

All firms are assumed to have an identical technology and hence an identical cost function $C(q)$.

The firms can be classified into two categories: *the naiver* and *the sophisticated*. The first n firms are the naiver, who are either deficient in market information or less strategic in market competition. They make their production decision based on a simple Cobweb strategy, that is, acting as *price-takers* with naive price expectations: $\hat{p}_t^i = p_{t-1}$ and planning their production based on $p_{t-1} = MC_t^i$, for $i = 1,2,\dots,n$. By taking into account the fact that all naiver have the same cost function, they should all produce an identical output, x_t, with

$$C'(x_t) = p_{t-1}, \tag{2.1}$$

which defines implicitly an identical reaction function R_x for all naiver.

In contrast, the other m firms are the sophisticated. They are assumed to command complete market information, such as the current and historical market demand, the market share, and/or the possible reaction functions of other firms. They are capable of forming accurate market expectations based on available information as well as taking into account the reactions of the others. Let $\hat{p}_t^j$ and y_t^j be the price expectation and the output by the j's sophisticated firm, $j = 1,2,\ldots,m$, respectively. Then the expected profit of the sophisticated is given by

$$\hat{\pi}_j^y = \hat{p}_t^j y_t^j - C(y_t^j), \quad j = 1,2,\ldots,m. \tag{2.2}$$

Without loss of generality, it is assumed that the conventional strategies such as Cour-not or Stackelberg leader/follower are adopted to maximize the expected profit, which gives rise to the following implicitly defined optimal reaction function r_j:

$$\hat{p}_t^j + y_t^j \frac{d\hat{p}_t^j}{dy_t^j} = C'(y_t^j), \tag{2.3}$$

where $\hat{p}_t^j$ and $d\hat{p}_t^j/dy_t^j$ depend on the current and historical data $\{p_{t-s-1}, x_{t-s}, y_{t-s}\}_{s=0}^t$, as well as their knowledge about the other firms.

Equations (2.1) and (2.3) together form a discrete dynamical process:

$$\begin{aligned} x_t &= R_x(p_{t-1}) = r_x(x_{t-1}, \{y_{t-1}^j\}_{j=1,2,\ldots,m}), \\ y_t^j &= R_j(x_t, x_{t-1}, x_{t-2}, \ldots; \{y_{t-1}^j\}_{j=1,2,\ldots,m}, \{y_{t-2}^j\}_{j=1,2,\ldots,m}, \ldots) \\ &= r_j(x_{t-1}, x_{t-2}, \ldots; \{y_{t-1}^j\}_{j=1,2,\ldots,m}, \{y_{t-2}^j\}_{j=1,2,\ldots,m}, \ldots), \quad j = 1,2,\ldots,m. \end{aligned} \tag{2.4}$$

For the convenience of reference, we will call the above model a *general heterogeneous oligopoly model* (a GHO model).

We will concentrate on a dynamical process that is economically meaningful in the following sense.

Definition 2.1. An output bundle $(x_t, \{y_t\})$ for the *general heterogeneous oligopoly model* is said to be *economically meaningful* if the following inequalities are met:

(i) $x_t > 0$ and $y_t^j > 0$, $j = 1,2,\ldots,m$, that is, positive outputs for all firms;

(ii) $0 < p_t = D(x_t + \sum_{j=1}^m y_t^j) < \infty$, that is, positive and limited price.

To illustrate the main points of the study with a deep understanding of the role of the naiver in the very oligopolistic game, a model that is analytically manipulable is needed. The following *linear heterogeneous oligopoly model* (an LHO model) will serve our purpose. In particular, to exemplify the role of the naiver, we will concentrate on the case of $n = 1$, that is, there is only one naiver in the market. (However, all results revealed in this study apply to arbitrary n and the case of differentiated costs.) We also assume that all the sophisticated firms form a collusion and produce at an identical quantity y_t. The market demand is assumed to be linear so that its inverse demand function is given by (For the convenience of graphical illustration and comparison, the current demand function is adopted instead of the conventional setting in Huang [2] as $p_t = 1 - x_t - my_t$.)

$$p_t = D(x_t + my_t) = m + 1 - x_t - my_t, \tag{2.5}$$

whereas the marginal cost is linear so that the cost function adopts the form of

$$C(q) = \frac{\sigma q^2}{2}. \tag{2.6}$$

It follows from (2.1) that the naiver's reaction function is:

$$x_t = R_x(p_{t-1}) = \frac{p_{t-1}}{\sigma}, \tag{2.7}$$

or, alternatively,

$$x_t = r_x(x_{t-1}, y_{t-1}) = \frac{m+1-x_{t-1}-my_{t-1}}{\sigma}. \tag{2.8}$$

The collusion formed by the sophisticated is assumed to take the Cournot strategy with rational expectation (or exact knowledge of the naiver's current output), (Discussion on more complicated strategies, such as relative profit maximization, can be found in Huang [2].) whose reaction function is thus derived from the first-order profit maximization condition

$$D(x_t + my_t) + \frac{dD(x_t + my_t)}{dy_t} y_t = C'(y_t), \tag{2.9}$$

which is simplified to

$$y_t = R_y(x_t) \doteq \frac{m+1-x_t}{2m+\sigma}, \tag{2.10}$$

that is,

$$\begin{aligned} y_t &= r_y(x_{t-1}, y_{t-1}) = R_y(r_x(x_{t-1}, y_{t-1})) \\ &= \frac{(m+1)(\sigma-1)+x_{t-1}+my_{t-1}}{\sigma(2m+\sigma)}. \end{aligned} \tag{2.11}$$

3. Profitability in equilibrium

Before analyzing the dynamical characteristics of the discrete process (2.4) (as well as (2.7) and (2.10)) under the different possible strategic specifications for the sophisticated firms, relative profitability in equilibrium for the two types of oligopolistic firms needs to be addressed.

Assume that the dynamic process (2.4) reaches an economically meaningful equilibrium at $(\bar{x}, \bar{y}^1, \bar{y}^2, \ldots, \bar{y}^m)$, then which type of firms will make more profit, the sophisticated or the naiver?

The following counter-intuitive result revealed in Huang [2] for the duopoly is generalized into the following.

Theorem 3.1. *When an economically meaningful equilibrium is reached for the heterogeneous oligopoly model, if the cost function C is strictly convex, the naive firms perform not worse or even better than each and every sophisticated rival in terms of the profit, regardless of the types of strategies the sophisticated firms may take.*

Proof. When the equilibrium $(\overline{x},\overline{y}^1,\overline{y}^2,\ldots,\overline{y}^m)$ is arrived, the market price is fixed at an equilibrium level

$$\overline{p}=D\left(n\overline{x}+\sum_{j=1}^{m}\overline{y}^j\right). \tag{3.1}$$

By the assumption, the marginal cost of the naiver must be equal to the price level, that is, $C'(\overline{x})=\overline{p}$.

Now, compare the profit difference between the naiver (they all have, the same profit) and any of the sophisticated firms, say, firm k, whose output is $\overline{y}^k$, then we have

$$\begin{aligned}\overline{\pi}^x-\overline{\pi}^k&=\overline{p}(\overline{x}-\overline{y}^k)-(C(\overline{x})-C(\overline{y}^k))\\&=C'(\overline{x})(\overline{x}-\overline{y}^k)-(C(\overline{x})-C(\overline{y}^k)).\end{aligned} \tag{3.2}$$

It follows from the assumption of $C''(\cdot)>0$ that $C'(\overline{x})(\overline{x}-\overline{y}^k)-(C(\overline{x})-C(\overline{y}^k))\geq 0$, or equivalently,

$$\overline{\pi}^x\geq\overline{\pi}^k, \tag{3.3}$$

where the equality holds if and only if $\overline{x}=\overline{y}^k$.

Since the sophisticated firm k's production strategy is not explicitly specified in our proof, inequality (3.3) thus leads to the conclusion immediately. □

Remark 3.2. Theorem 3.1 simply states that, unless a sophisticated firm produces the same quantity as the naiver does in equilibrium, then the equilibrium profit made by the sophisticated firms will definitely be less than the one gained by the naiver, which holds true even if all sophisticated firms form a collusion, as assumed in the LHO model.

It needs to be emphasized that the conclusion in Theorem 3.1 is valid only for economically meaningful equilibriums since an equilibrium existed in theory may not be economically meaningful in economic sense. The existence of such economically meaningful equilibrium, its uniqueness, and its stability depend on its rival's strategies, the market demand, and the production technology. However, when there are more than one equilibria, the conclusion holds for all economically meaningful ones.

Table 3.1 summarizes the equilibrium outcomes for the linear model. It is immediately verified that $\overline{\Pi}^x/\overline{\Pi}^y>1$ for all σ and m.

To serve for the benchmarking purpose, two extreme situations are also included in Table 3.1, one with all $m+1$ firms adopting the Cobweb strategy and the other with all $m+1$ firms forming a collusion. (These two cases will be denoted with subscripts "w" and "u," respectively.)

In the former case, a sophisticated firm has an identical output as the naiver ($r_y\equiv r_x$), and the equilibrium is known as a *Walrasian equilibrium* (competitive equilibrium). For the latter case, all firms act together as a monopoly and maximize the total profit; the first-order condition is given by

$$D((1+m)y_t)+(m+1)y_tD'((1+m)y_t)=C'(y_t). \tag{3.4}$$

Table 3.1. Equilibrium outcomes.

	All price-taker	1 price-taker m colluded	All colluded
Reaction of the naiver	$r_x(x_{t-1}, y_{t-1})$	$r_x(x_{t-1}, y_{t-1})$	—
Reaction of the sophisticated	$r_x(x_{t-1}, y_{t-1})$	$r_y(x_{t-1}, y_{t-1})$	—
Equilibrium price	$\underline{p}_w = \dfrac{\sigma(m+1)}{1+m+\sigma}$	$\overline{P} = \dfrac{\sigma(m+1)(m+\sigma)}{2\sigma m+\sigma^2+m+\sigma}$	$\overline{p}_u = \dfrac{(m+1)(m+\sigma+1)}{2m+2+\sigma}$
Equilibrium outputs	$\overline{x}_w = \overline{y}_w = \overline{q}_w$ $\left(\overline{q}_w = \dfrac{m+1}{1+m+\sigma}\right)$	$\overline{X} = \dfrac{(m+1)(m+\sigma)}{m(2\sigma+1)+\sigma^2+\sigma}$ $\overline{Y} = \dfrac{\sigma(m+1)}{2\sigma m+\sigma^2+m+\sigma}$	$\overline{x}_u = \overline{y}_u = \overline{q}_u$ $\left(\overline{q}_u = \dfrac{m+1}{2m+2+\sigma}\right)$
Equilibrium profits	$\overline{\pi}_w^x = \overline{\pi}_w^y = \overline{\pi}_w$ $\left(\overline{\pi}_w = \dfrac{\sigma(m+1)^2}{2(1+m+\sigma)^2}\right)$	$\overline{\Pi}^x = \dfrac{1}{2}\dfrac{\sigma(m+1)^2(m+\sigma)^2}{(2\sigma m+\sigma^2+m+\sigma)^2}$ $\overline{\Pi}^y = \dfrac{1}{2}\dfrac{(2m+\sigma)(m+1)^2\sigma^2}{(2\sigma m+\sigma^2+m+\sigma)^2}$	$\overline{\pi}_u^x = \overline{\pi}_u^y = \overline{\pi}_u$ $\left(\overline{\pi}_u = \dfrac{1}{2}\dfrac{(m+1)^2}{2+2m+\sigma}\right)$
Profits ratio	$\dfrac{\overline{\Pi}^x}{\overline{\Pi}^y} = \dfrac{(m+\sigma)^2}{\sigma(2m+\sigma)} \geq 1$		

An equilibrium is reached immediately without dynamical transitions:

$$\overline{x}_u = \overline{y}_u = \frac{m+1}{2m+2+\sigma}. \tag{3.5}$$

If all firms, the sophisticated firms and the naive firms, adopt the Cournot strategy or collude together, will any firm have an incentive to "downgrade" to a price-taker? The conventional answer would be a straightforward "No," should all firms maximize their own profits instead of relative profits. It is believed that a higher relative profit enjoyed by a betrayer (a price-taker) is achieved by hurting the others (those remain in the collusion) more than themselves. The price paid by the betrayer from the collusive and/or oligopolistic commitment is the reduction in its own profit as well. Regretfully, such reasoning does not hold in general when the number of firms in an oligopolistic market is larger. A firm may prefer to take a Cobweb strategy (behaving as a "price-taker") not just for the relative profit compared to the rest but also for the sake of increasing its own profit. There exist situations in which an individual firm can achieve the dual goal of maximizing the absolute profit and relative profit simultaneously by changing from the sophisticated strategy to the Cobweb strategy. In the terminology of game theory, the "Cobweb strategy" can be a dominant strategy for a firm, regardless what other firms do. Such a discovery improves our understanding of the beauty of simple strategy in dealing with a complex and changeable environment.

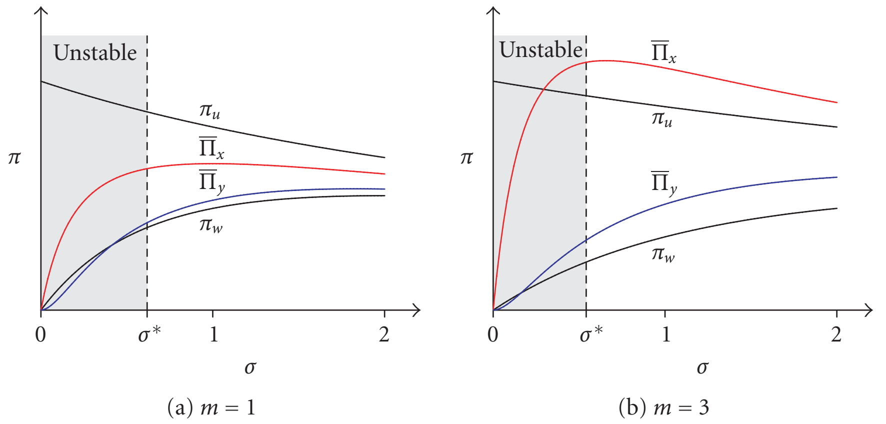


Figure 3.1. Critical π values.

Take the LHO model as an illustration and compare π_u to $(\overline{\Pi}^x, \overline{\Pi}^y)$ given in Table 3.1. While $\overline{\Pi}^y < \pi_u$ for all m and σ, we have

$$\overline{\Pi}^x - \pi_u = \frac{1}{2}(m+1)^2 \frac{\sigma m^2(2m+\sigma) - (\sigma+m)(2m\sigma+\sigma+m)}{\left(2m\sigma+\sigma^2+m+\sigma\right)^2(2+2m+\sigma)}. \tag{3.6}$$

Therefore, we have $\overline{\pi}_c^x > \pi_u$ when $\sigma > \hat{\sigma}$, where

$$\hat{\sigma} = \frac{m}{(m-1)^2-2}\left(m - m^2 + 1 + m\sqrt{m(m-2)}\right). \tag{3.7}$$

That is, there exist situations in which any firm has the incentive to betray the collusion, not only for the relative profitability but also for the instantaneous payoff.

For example, when $\sigma = 1$ and $m > 2$, we have

$$\overline{\Pi}^x - \pi_u = \frac{1}{2}\frac{(m+1)^2(2m(m+1)(m-2)-1)}{(3m+2)^2(3+2m)} > 0. \tag{3.8}$$

However, $\hat{\sigma}$ defined in (3.7) is valid only when $m \geq 2$. That is, *betrayal can never be beneficial either in a duopoly or in triopoly*. That may explain why it has never been revealed in the literature, since most Cournot analysis and game-theoretic researches focuss only on duopoly.

Figure 3.1 illustrates the relative profits with respect to the change of σ for the duopoly $(m = 1)$ and an oligopoly case $(m = 3)$. We see that $\pi_u > \overline{\Pi}^x$ for all σ in the duopoly, but $\overline{\Pi}^x > \pi_u$ when $\sigma > \hat{\sigma}$ for the oligopoly.

4. Profitability in transitionary dynamics

The relative profitability in an equilibrium for the naiver, though may be contrary to economic intuition, can still be justified by economic theory. In fact, at an equilibrium, the market price is fixed regardless of the output levels of all oligopolistic firms, neither the naiver nor the sophisticated firms, the equality of marginal cost to the market price is indeed the result of an optimal response for the case of diseconomies of scale. (The explicit assumption made for market structure is the strict *convexity* for the cost function so as to exclude the case of a constant marginal cost. In the latter case, the price-taking strategy becomes economically meaningless. As long as the marginal cost is not constant everywhere, the requirement for "strict convexity" can be weakened to "convexity.")

However, an equilibrium, though may be shown to exist in theory, may not necessarily converge in a dynamical adjustment process. Therefore, the scenario is unclear to us unless we further examine the relative profitability in dynamical adjustments. In this section, we will focus on the issue of relative profitability for the periods of dynamic transition, that is, from one equilibrium to the other. More complex situations such as cyclic fluctuations and chaotic fluctuations will be analyzed later.

To serve our purpose, we will concentrate our analysis on the LHO model developed in Section 2 because the multidimensional discrete process (2.8) and (2.11) can actually be simplified into a one-dimensional discrete process.

It follow from (2.7) that $x_t = p_{t-1}/\sigma$ and $y_t = R_y(x_t)$, where R_y is defined in (2.10). Therefore, the market price can be expressed as

$$\begin{aligned} p_t &= (m+1) - x_t - my_t \\ &= (m+1) - \frac{p_{t-1}}{\sigma} - mR_y\left(\frac{p_{t-1}}{\sigma}\right), \end{aligned} \tag{4.1}$$

that is,

$$p_t = f_p(p_{t-1}) \doteq \frac{(m+\sigma)(m+1)}{2m+\sigma} - \frac{m+\sigma}{\sigma(2m+\sigma)} p_{t-1}. \tag{4.2}$$

The price dynamics given by (4.2) will cyclically converge to an equilibrium $\overline{P}$ if and only if the multiplier of the steady state (the absolute value of the slope at the equilibrium $\overline{P}$) is less than unity, that is,

$$\overline{\delta} = \frac{m+\sigma}{\sigma(2m+\sigma)} < 1. \tag{4.3}$$

Since $\partial\overline{\delta}/\partial\sigma < 0$, strict inequality $\overline{\delta} < 1$ is ensured when

$$\sigma > \sigma^* = \frac{1}{2} + \frac{1}{2}\sqrt{1+4m^2} - m. \tag{4.4}$$

We have

$$\frac{m+\sigma^*}{\sigma^*(2m+\sigma^*)} < 1. \tag{4.5}$$

Figure 4.1 depicts the graph of σ^* against m. It is worth noticing that $\partial\sigma^*/\partial m < 0$ and $\lim_{m\to\infty}\sigma^* = 1/2$, that is, the larger the number of the sophisticated firms is, the smaller the value of the σ^* is, that is, the more stable the market is.

In the equilibrium $p_t = \overline{P}$, we have shown that the naiver makes higher profit than the sophisticated. Then we would expect that there exists an interval around $\overline{P}$ such that the

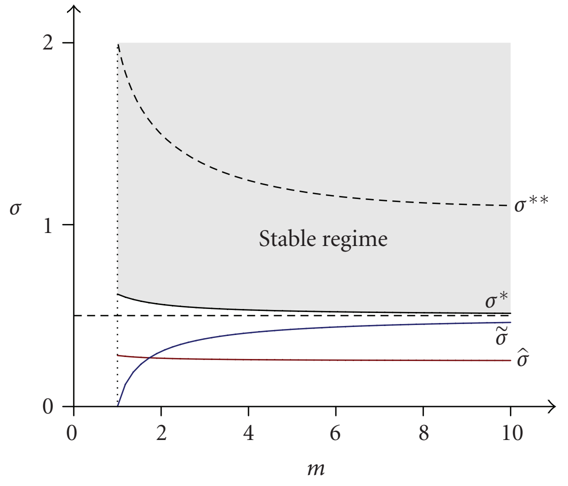


Figure 4.1. Stability parameter σ.

naiver achieves a higher relative profit than the sophisticated when the price trajectory enters into this interval. Formally, we have the following definition.

Definition 4.1. A price range Ω^p is referred to as the *naiver's profitability regime* if the naiver makes higher relative profit than the sophisticated does when the market price falls in this range.

Theorem 4.2. *For the LHO model, the following facts hold:*

(i) $x_t > y_t$ *if and only if* $p_t < P^*$;

(ii) $\pi_t^x > \pi_t^y$ *if and only if* $p_t \in \Omega^p \doteq (P_*, P^*)$ *where*

$$P_* \doteq \frac{\sigma(1+m)(m+\sigma)}{(\sigma+1)(2m+\sigma)}, \tag{4.6}$$

$$P^* \doteq \frac{(1+m)(m+\sigma)}{2m+\sigma+1}. \tag{4.7}$$

Proof. Since the sophisticated responds to the output of the naiver with R_y defined in (2.10), we have

$$x_t - y_t = x_t - R_y(x_t) = \frac{x_t(2m+\sigma+1) - m - 1}{2m+\sigma}. \tag{4.8}$$

Substituting x_t and $y_t = R_y(x_t)$ into the linear demand function (2.5) provides a relationship between x_t and the realized price p_t:

$$\begin{aligned} x_t = h^x(p_t) &\doteq \frac{(1+m)(m+\sigma) - (2m+\sigma)p_t}{m+\sigma}, \\ y_t = h^y(p_t) &= R_y(h(p_t)) = \frac{p_t}{m+\sigma}, \end{aligned} \tag{4.9}$$

so that

$$x_t - y_t = \frac{(1+m)(m+\sigma) - p_t(2m+\sigma+1)}{m+\sigma} = \frac{2m+\sigma+1}{m+\sigma}(P^* - p_t), \tag{4.10}$$

where p^* is defined in (4.7).

Therefore, $x_t > y_t$ if and only $p_t < P^*$.

We also have

$$\begin{aligned} x_t + y_t &= \frac{(1+m)(m+\sigma) - (2m+\sigma)p_t}{m+\sigma} + \frac{p_t}{m+\sigma} \\ &= \frac{1}{m+\sigma}((1+m)(m+\sigma) - p_t(2m+\sigma-1)). \end{aligned} \tag{4.11}$$

So that

$$\begin{aligned} \Delta\pi_t^{xy} &= \pi_t^x - \pi_t^y = p_t(x_t - y_t) - \frac{\sigma}{2}(x_t + y_t)(x_t - y_t) \\ &= (x_t - y_t)\left(p_t - \frac{\sigma}{2}(x_t + y_t)\right), \\ \text{that is, } \Delta\pi_t^{xy} &= \frac{(\sigma+1)(2m+\sigma)(2m+\sigma+1)}{2(m+\sigma)^2}(P^* - p_t)(p_t - P_*), \end{aligned} \tag{4.12}$$

where P_* is defined in (4.7).

It is immediately concluded that $\Delta\pi_t^{xy} > 0$ as long as $P_* < p_t < P^*$ is ensured, which completes the proof. □

It can be verified that $\overline{P} \in \Omega^p$. However, even when $\overline{P}$ is stable (the slope of f_p is steeper) and p_t runs into Ω^p in current period, it may iterate outside of Ω^p in next period. To see this, we notice that, starting with $p_t = P_*$, we have

$$p_{t+1} - P^* = f(P_*) - P^* = \frac{-m(m+\sigma)(m+1)(\sigma+2m-1)}{(2m+\sigma)^2(\sigma+1)(2m+\sigma+1)} < 0. \tag{4.13}$$

When $P_* < p_t < \overline{P}$, the increasing price (from p_t to p_{t+1}) guarantees that the price trajectory remains in the realized profitability regime. *In other words, if the naiver makes higher relative profit at a price below the equilibrium, then it should keep the same relative advantage in the next period as well.*

On the other hand, starting with $p_t = P^*$, for any $\sigma > 0$, we have

$$p_{t+1} - P_* = f_p(P^*) - P_* = \frac{(m+\sigma)(m+1)m(\sigma-1)}{\sigma(2m+\sigma)(2m+\sigma+1)(\sigma+1)}, \tag{4.14}$$

which is positive if and only if $\sigma > 1$. That implies that *if the naiver makes a higher relative profit at a price above the equilibrium, then it may lose the relative advantage in the next period, should σ be smaller than unity.*

The above reasoning leads to the following theorem.

Theorem 4.3. *When $\sigma \geq 1$, if $p_{t^*} \in \Omega^p$, then $p_t \in \Omega^p$ for all $t \geq t^*$, that is, when the price wonders into the realized profitability regime, it will stay inside forever. However, if $\sigma < 1$, then $f(P_*) < P^*$ but $f_p(P^*) < P_*$.*

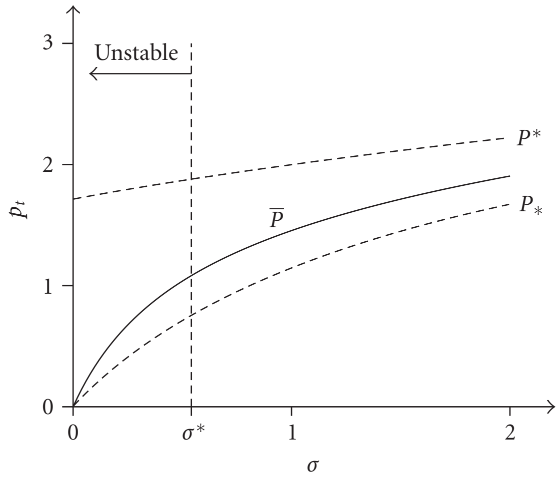


Figure 4.2. Critical price values: $m = 3$.

With the above preliminary analysis, we are ready to examine the most interesting initial situations, that is, the two extremes listed in Table 3.1.

For the first extreme, all firms adopt the Cobweb strategy and an equilibrium price is reached with

$$\underline{p}_w = \frac{\sigma(m+1)}{1+m+\sigma}. \tag{4.15}$$

Starting with this equilibrium $p_0 = \underline{p}_w < P_*$, if m sophisticated firms decide to form a collusion and adopt the Cournot strategy, at their first move, we have

$$p_1 = f_p(p_0) = \frac{(m+1)(m+\sigma)^2}{(2m+\sigma)(1+m+\sigma)} < P^*, \tag{4.16}$$

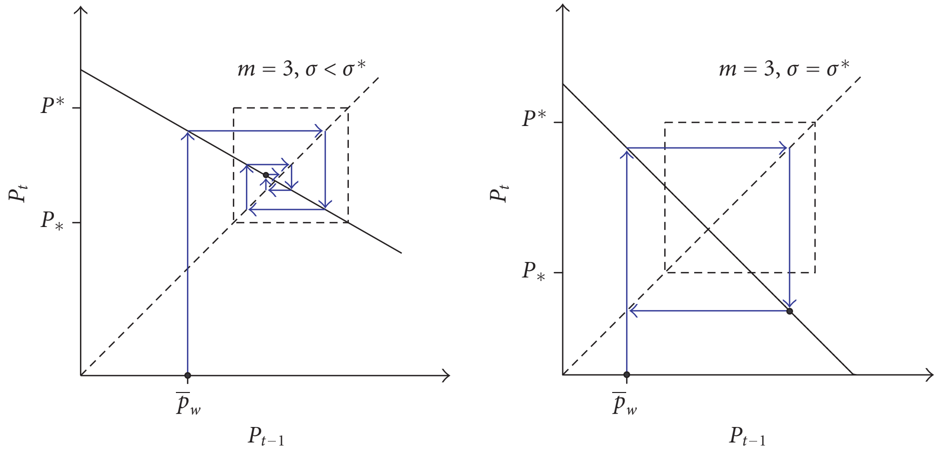
therefore, *the collusive action is harmful to them at their first move.*

But for the relative probability of the naiver for the rest moves, we need to distinguish several possibilities.

Case 1.1 ($\sigma \geq 1$). It follows from Theorem 4.3 that $p_t \in \Omega^p$ for all $t \geq 1$, that is, the naiver will maintain the relative profitability for all converging periods towards the new equilibrium price $\overline{P}$ and continue to enjoy the relative advantage forever.

Case 1.2 ($1 > \sigma > \sigma^*$). $p_2 = f_p(p_1)$ will still stay in Ω^p but $p_3 = f_p^2(p_1)$ may wonder above P^*. Due to converging characteristics of new equilibrium $\overline{P}$, it will soon reach a state that $p_t \in \Omega^p$ forever.

Case 1.3 ($\sigma = \sigma^*$). The P-dynamics ends up with a two-period cycle, $(\underline{p}_w, \overline{p}_w)$, where $\overline{p}_w = p_1 > \overline{P} > \underline{p}_w$. Now, all firms make the same profit at $\underline{p}_w$, but the naiver makes a higher relative profit at $\overline{p}_w$, and thus, *the naiver profits more than the sophisticated on average.*



(a) Stable equilibrium (b) Stable cycle

Figure 4.3. Transitionary dynamics: starting with all price-takers.

Case 1.4 ($\sigma < \sigma^*$). The new equilibrium $\overline{P}$ is unstable. The P-dynamics ends up with a divergent fluctuation. Even though the sophisticated make a loss in terms of relative profit at their first move, they may soon reverse the relative disadvantage status since the price will soon wonder away from Ω^p.

Figure 4.3 illustrates Cases 1.1 and 1.3.

As for the second extreme, all firms collude and set a monopoly price at

$$\overline{p}_u = \frac{(m+1)(m+1+\sigma)}{2(m+1)+\sigma} > \overline{P}. \tag{4.17}$$

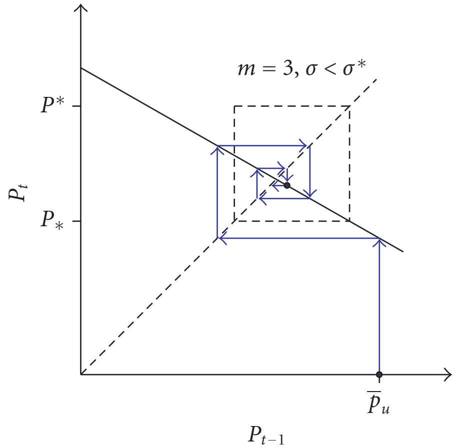
Starting with this equilibrium $p_0 = \overline{p}_u > P^*$, if one of the firms decides to betray the collusion and becomes a naiver to adopt the simple Cobweb strategy, at its first move, the market price becomes

$$p_1 = f_p(\overline{p}_u) = (m+\sigma)(m+1)\frac{2\sigma m+\sigma+\sigma^2-1-m}{\sigma(2m+\sigma)(2m+2+\sigma)}, \tag{4.18}$$

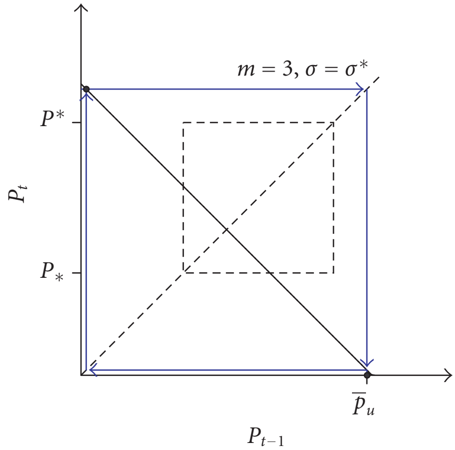
so that

$$\begin{aligned} p_1 - P_* &= (m+\sigma)(m+1)\frac{2\sigma m+\sigma+\sigma^2-1-m}{\sigma(2m+\sigma)(2m+2+\sigma)} - \frac{\sigma(1+m)(m+\sigma)}{(\sigma+1)(2m+\sigma)} \\ &= \frac{(m+\sigma)(m+1)}{m}\frac{(\sigma-\sigma^{**})}{\sigma(2m+\sigma)(2m+2+\sigma)(\sigma+1)}, \end{aligned} \tag{4.19}$$

where $\sigma^{**} \doteq 1+1/m > \sigma^*$.



(a) Stable equilibrium



(b) Stable cycle

Figure 4.4. Transitionary dynamics: starting with a full collusion.

Then there are four possibilities.

Case 2.1 ($1 < \sigma < \sigma^{**}$). We have $p_1 > P_*$, that is, the first betrayal price lies in the realized profitability regime, the betrayer (the naiver) enjoys a higher relative profit than its rivals right at the first move. According to Theorem 4.3, it will keep the same relative advantage forever.

Case 2.2 ($\sigma^* < \sigma \leq 1$). We have $p_1 < P_*$, that is, the first betrayal price is lower than the lower bound of the realized profitability regime, the betrayer (the naiver) suffers a lower relative profit than its rivals at the first move (or a few moves) but inverts the relatively losing status soon.

Case 2.3 ($\sigma = \sigma^*$). The P-dynamics process ends up with a two-period cycle $(\underline{p}_u, \overline{p}_u)$, where $\underline{p}_u = p_1 < P_*$. However, due to $\overline{p}_u > P^*$, it follows from (4.12) that $\Delta\pi_1^{xy} < 0$, the outcome is exactly contrary to Case 1.3, that is, *the colluded sophisticated firms make higher profit so that the betrayal is not awarded.*

Case 2.4 ($\sigma < \sigma^*$). The new equilibrium $\overline{P}$ is unstable. The P-dynamics ends up with a divergent fluctuation and the betrayer is not awarded at all since the naiver makes lower relative profit than the sophisticated does. Such situation continues until the market is driven into a noneconomically meaningful status.

Figure 4.4 illustrates Cases 2.2 and 2.3.

Figure 4.5 shows the numerical simulations for the case where $\sigma = 1$.

5. Cautious Cobweb strategy

For the LHO model, the price dynamics is relatively simple, either a convergence or a divergence or a cyclic fluctuation. Especially when $\sigma < \sigma^*$, the price diverges explosively

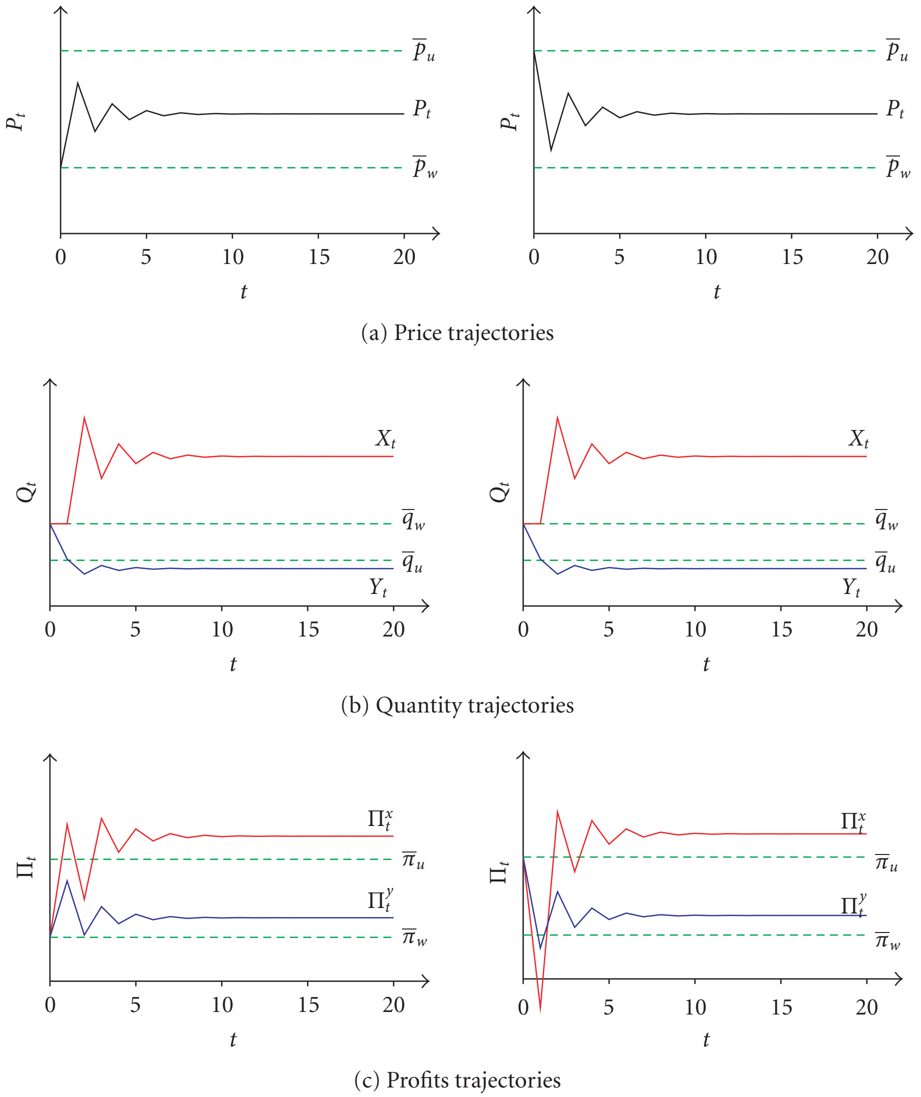


Figure 4.5. Convergency to the equilibrium, $m = 3$, $\sigma = 1$.

so as to drive the market into a noneconomically meaningful status. To force the market price to stay in an economically meaningful region, the naiver is assumed to adopt a cautious adjustment strategy. (This type of "cautious adjustment strategy" is often adopted by an economic agent who responds cautiously to the uncertain and fluctuating environment. It was first studied by Day [3] in modeling the cautious behaviors of a competitive firm in coping with the uncertain price fluctuations in a Cobweb economy and later applied by Day [4] in explaining the classical population growth behavior. Further development by Huang [1], Weddpohl [5], and Matsumoto [6] revealed the role of the cautious adjustment strategy in controlling or stabilizing an economy, and the comparative profitability for an economic agent under different dynamical environments.)

A firm is said to take the cautious adjustment strategy if it limits its output growth rate to β:

$$\frac{q_t - q_{t-1}}{q_{t-1}} \leq \beta, \tag{5.1}$$

where $\beta \geq 0$ is referred to as the *growth-rate limit*, so that

$$q_t \leq (1+\beta)q_{t-1}. \tag{5.2}$$

Therefore, if the naiver takes the Cobweb strategy and the cautious adjustment strategy simultaneously, its output is recursively determined by

$$\begin{aligned} x_t &= \min\{(1+\beta)x_{t-1}, R_x(p_{t-1})\} \\ &= \min\{(1+\beta)x_{t-1}, r_x(x_{t-1}, y_{t-1})\}. \end{aligned} \tag{5.3}$$

Substituting $y_{t-1} = R_y(x_{t-1})$ into (5.3) gives us

$$x_t = F_x(x_{t-1}) \doteq \min\{g_x(x_{t-1}), f_x(x_{t-1})\}, \tag{5.4}$$

where

$$\begin{aligned} g_x(x_{t-1}) &= (1+\beta)x_{t-1}, \\ f_x(x_{t-1}) &= \frac{(m+\sigma)}{\sigma(2m+\sigma)}(m+1-x_{t-1}). \end{aligned} \tag{5.5}$$

In this way, a two-dimensional nonlinear discrete process is reduced to a one-dimensional one.

Similarly, we can express y_t as

$$y_t = R_y(F_x(R_y^{-1}(y_{t-1}))), \tag{5.6}$$

where R_y^{-1} indicates the inverse function of R_y:

$$R_y^{-1}(y_t) = 1+m-(2m+\sigma)y_t. \tag{5.7}$$

Simple mathematical manipulation yields

$$y_t = F_y(y_{t-1}) \doteq \max\{f_y(y_{t-1}), g_y(y_{t-1})\}, \tag{5.8}$$

with

$$\begin{aligned} f_y(y_{t-1}) &= \frac{m+1}{2m+\sigma} - \frac{m+\sigma}{\sigma(2m+\sigma)}y_{t-1}, \\ g_y(y_{t-1}) &= (1+\beta)y_{t-1} - \frac{\beta(1+m)}{2m+\sigma}. \end{aligned} \tag{5.9}$$

Substituting $y_t = R_y(x_t)$ into the linear demand function (2.5) provides a relationship between x_{t-1} and the realized price p_{t-1}:

$$x_{t-1} = h^x(p_{t-1}) \doteq \frac{(1+m)(m+\sigma) - (2m+\sigma)p_{t-1}}{m+\sigma}. \tag{5.10}$$

Hence, (5.3) can be recast as

$$x_t = \min\{h(p_{t-1}), R_x(p_{t-1})\}. \tag{5.11}$$

The price dynamics (4.2) is thus modified into

$$p_t = m+1 - \min\{g_w(p_{t-1}), R_w(p_{t-1})\} - mR_y(\min\{g_w(p_{t-1}), R_w(p_{t-1})\}), \tag{5.12}$$

or, equivalently,

$$p_t = F_p(p_{t-1}) \doteq \max\{f_p(p_{t-1}), g_p(p_{t-1})\}, \tag{5.13}$$

where f_p is defined in (4.2) and

$$g_p(p_{t-1}) \doteq (1+\beta)p_t - \frac{\beta(1+m)(m+\sigma)}{2m+\sigma}. \tag{5.14}$$

The LHO model incorporating the cautious adjustment strategy is referred to as a *cautious LHO model*. For the convenience of easy reference, we will call (5.4), (5.8), and (5.13) as X-dynamics, Y-dynamics, and P-dynamics, respectively. To be consistent with the analysis in the previous section, we will limit our analysis of the cautious adjustment strategy to the P-dynamics.

We will see that, for a suitable choice of β, the price trajectory will be restricted in an economically meaningful region so that the price dynamics becomes either cyclic or chaotic. In fact, we notice that two branches of F_p intersect at

$$\hat{p} = \frac{\sigma(1+\beta)(m+\sigma)(1+m)}{\sigma(1+\beta)(2m+\sigma)+m+\sigma}. \tag{5.15}$$

As shown in Figure 5.1(a), there exists a trapping set $J_p \doteq [p_{\min}, p_{\max}]$, with

$$\begin{aligned} p_{\min} &= F_p(\hat{p}) = \frac{(1+m)(m+\sigma)(\sigma(1+\beta)(2m+\sigma)-\beta(m+\sigma))}{(2m+\sigma)(\sigma(1+\beta)(2m+\sigma)+m+\sigma)}, \\ &= F_p(p_{\min}) \\ &= \frac{(m+1)(m+\sigma)(\sigma^2(2m+\sigma)^2(1+\beta)+\beta(m+\sigma)((m+\sigma)-\sigma(2m+\sigma)))}{\sigma(2m+\sigma)^2(\sigma(1+\beta)(2m+\sigma)+m+\sigma)}, \end{aligned} \tag{5.16}$$

such that the price trajectories will be eventually trapped into it and remain inside forever. (Since we are concerned with the long-run dynamical behavior, only the trajectories inside the trapping set are meaningful. In this regard, a trapping set will be taken as the domain of relevant variable unless it is otherwise stated.)

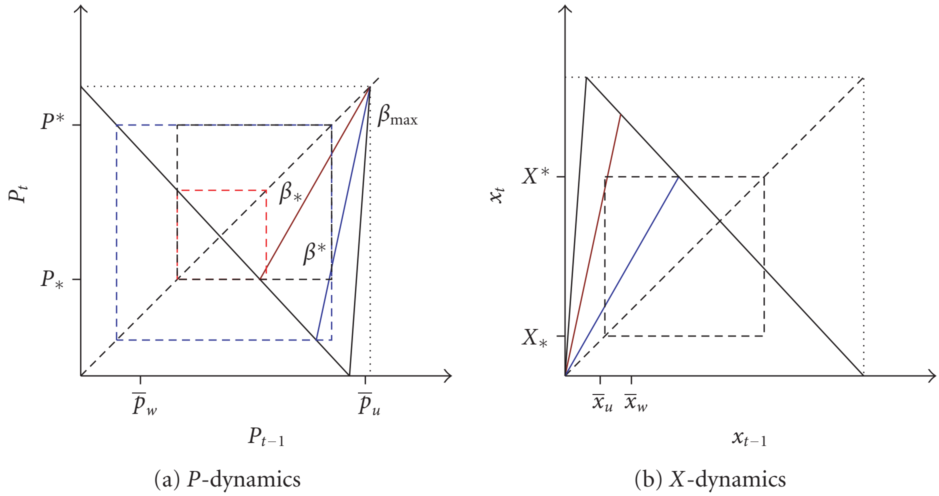


Figure 5.1. Stabilization effect of cautious strategy.

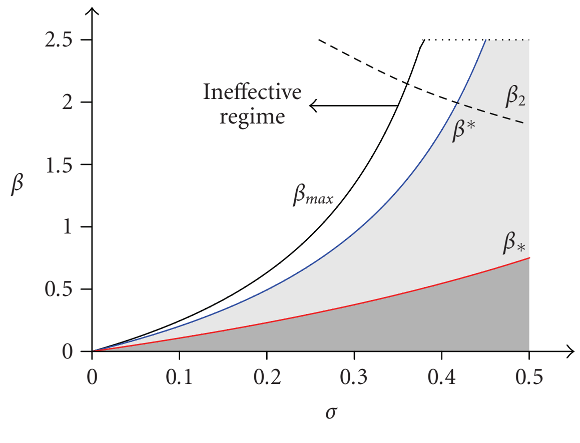


Figure 5.2. Critical β values.

An economically meaningful trapping set demands that $p_{\min} > 0$, that is,

$$\sigma(1+\beta)(2m+\sigma) > \beta(m+\sigma), \tag{5.17}$$

which demands

$$\beta < \beta_{\max} \doteq \frac{\sigma(2m+\sigma)}{(1-2\sigma)m+\sigma(1-\sigma)}. \tag{5.18}$$

There exist two steady states, one is an economically meaningful equilibrium point $\overline{P}$, the intersection of f_p with 45-degree diagonal line in the phase diagram, the other is a trivial steady state $\widetilde{P}$, the intersection of g_p with 45-degree diagonal line in the phase diagram of F_p.

While the value of $\overline{P}$, as specified in Table 5.1, is independent of the growth-rate limit β, it happens that

$$\widetilde{P} = \frac{(1+m)(m+\sigma)}{2m+\sigma} = f_p(0), \tag{5.19}$$

which is also independent of β. Economically, it corresponds to the case in which only the naiver produces all the output, while the sophisticated collusion ceases to produce. Hence, it is not economically meaningful. The fact that

$$\frac{d\beta_{\max}}{d\sigma} = \frac{2m(m+\sigma)+\sigma^2}{\left(m(1-2\sigma)+\sigma(1-\sigma)\right)^2} > 0 \tag{5.20}$$

implies *that smaller σ (more unstable) requires a relatively smaller cautious adjustment strategy to narrow down the price fluctuations.*

In general, we have $\widetilde{p} \geq p_{\max}$ and the equality holds only when $\beta = \beta_{\max}$, a case in which the trapping set J_p coincides with $[0,\widetilde{P}]$. (This is exactly what we refer to as *the full-range chaos* in the chaos theory.)

The steady states, the turning point, and the trapping set can be similarly determined for the X-dynamics F_x and the Y-dynamics F_y. The results are summarized in Table 5.1. Also provided in Table 5.1 are the $\Omega^x = [X_*, X^*]$ and $\Omega^y = [Y_*, Y^*]$, which specify the relevant relative profitability regimes (for the naiver) in terms of x_t and y_t, respectively. It can be verified that the X-dynamics, the Y-dynamics, and the P-dynamics are synchronized in the way that if $P_t \in \Omega^p$, then $x_t \in \Omega^x$ ($y_t \in \Omega^y$) and vice versa.

The mechanisms of the cautious adjustment strategy in limiting the fluctuation range of the relevant variables are illustrated in Figures 5.1(a) and 5.1(b) for the P-dynamics and the X-dynamics with three critical values of $\beta : \beta_{\max}, \beta_*$ and β^*, where β_* and β^* are two critical values defined by

$$\beta_* \doteq \frac{m\sigma}{m+\sigma-m\sigma}, \tag{5.21}$$

$$\beta^* \doteq \frac{m\sigma(2m+\sigma)^2}{(m\sigma+\sigma+m+2m^2)(m+\sigma)-m\sigma(2m+\sigma)^2}, \tag{5.22}$$

where β_* is associated with Ω^p in a way that $p_{\min}(\beta_*) = P_*$ while β^* is associated with Ω^p in a way that $p_{\max}(\beta^*) = P^*$. The meanings of β_* and β^* will be further discussed in Section 6.

6. Profitability in ergodic dynamics

We have discussed the relative profitability for the naiver both in equilibrium and in dynamical transition with an illustration of the LHO model. Now we are ready to examine the relative profitability for the naiver in an unstable and chaotic market using the cautious LHO model. We are interested in comparing the long-run average profits between the naiver and the sophisticated. A formal definition for the average is needed.

Table 5.1. Quantity dynamics.

	F_x	F_y
Relation with p_t	$h^x(p_t) = \frac{(1+m)(m+\sigma)-(2m+\sigma)p_t}{m+\sigma}$	$h^y(p_t) = \frac{p_t}{m+\sigma}$
Turning point	$\hat{x} \doteq \frac{(1+m)(m+\sigma)}{\sigma(1+\beta)(2m+\sigma)+m+\sigma}$	$\hat{y} = \frac{\sigma(1+m)(1+\beta)}{\sigma(1+\beta)(2m+\sigma)+m+\sigma}$
Lower end of the trapping set	$x_{\min} = \frac{(1+m)(m+\sigma)(\sigma(1+\beta)(2m+\sigma)-\beta(m+\sigma))}{\sigma(2m+\sigma)(\sigma(1+\beta)(2m+\sigma)+m+\sigma)}$	$y_{\min} = \frac{(1+m)(\sigma(1+\beta)(2m+\sigma)-\beta(m+\sigma))}{(2m\sigma+\sigma^2+2m\sigma\beta+\sigma^2\beta+m+\sigma)(2m+\sigma)}$
Upper end of the trapping set	$x_{\max} = \frac{(1+m)(1+\beta)(m+\sigma)}{\sigma(1+\beta)(2m+\sigma)+m+\sigma}$	$y_{\max} = \frac{(1+m)[(1+\beta)\sigma^2(2m+\sigma)^2+\beta(m+\sigma)(m(1-2\sigma)+\sigma(1-\sigma))]}{\sigma(2m+\sigma)^2(\sigma(1+\beta)(2m+\sigma)+m+\sigma)}$
Trivial fixed point	$\tilde{x} = 0$	$\tilde{y} = \frac{1+m}{2m+\sigma}$
Viable fixed point	$\overline{x} = \frac{(m+\sigma)(1+m)}{m(1+2\sigma)+\sigma(1+\sigma)}$	$\overline{y} = \frac{\sigma(1+m)}{m+\sigma+2m\sigma+\sigma^2}$
Multiplier $\overline{\delta}$	$\frac{m+\sigma}{\sigma(2m+\sigma)}$	
Upper bound of the Naiver's profitability regime	$X^* = \frac{m+1}{1+\sigma}$	$Y^* = \frac{m+1}{2m+1+\sigma}$
Lower bound of the Naiver's profitability regime	$X_* = \frac{m+1}{2m+\sigma+1}$	$Y_* = \frac{\sigma(m+1)}{(1+\sigma)(2m+\sigma)}$

Definition 6.1. For the difference in profit $\Delta\pi_t^{xy} \doteq \pi^x(P_t) - \pi^y(P_t)$, its *k-period average* for a periodic price cycle $\{P_i\}_{i=1}^k$ is defined as

$$\langle \Delta\pi^{xy} \rangle_k \doteq \frac{1}{k}\sum_{i=1}^{k} \Delta\pi_i^{xy}, \tag{6.1}$$

and its *long-run average* for a chaotic and ergodic fluctuation is given by

$$\langle \Delta\pi^{xy} \rangle \doteq \lim_{k\to\infty} \frac{1}{k}\sum_{t=1}^{k} \Delta\pi_i^{xy}. \tag{6.2}$$

Two typical numerical simulations are shown in Figures 6.1 and 6.2. We see that $\langle \Delta\pi^{xy} \rangle > 0$ for both cases.

It is noted that P_* and P^* are invariant with respect to the value of β, while $p_{\min}$ and $p_{\max}$ vary with β in an opposite direction

$$\frac{dp_{\min}}{d\beta} < 0, \qquad \frac{dp_{\max}}{d\beta} > 0, \tag{6.3}$$

we can expect that a relative profitability for the naiver can be sustained even if the market is chaotic, should the β growth-rate limit be set sufficiently small. To see this, we first note a general result observed for the GHO model when the sophisticated adopts the Cournot strategy.

Proposition 6.2. *For the GHO model, $\pi_t^y > 0$ for all t if $C(0) = 0$, that is, the sophisticated makes positive profit at each and every move when the Cournot strategy is adopted.*

Proof. Since the output y_t is determined from

$$p_t + \frac{dp_t}{dy_t} y_t = C'(y_t), \tag{6.4}$$

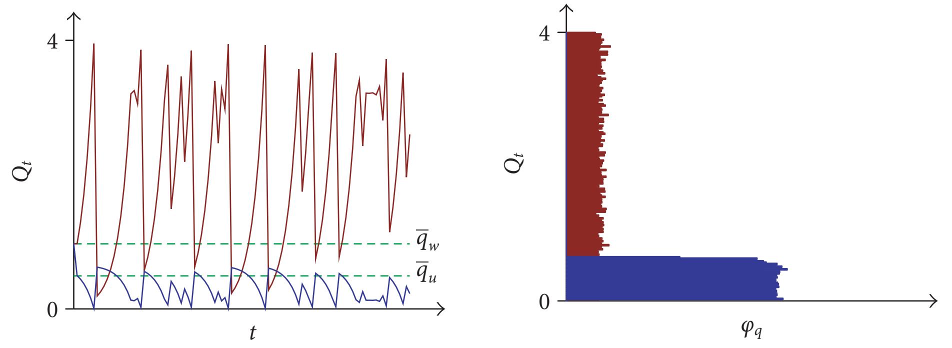
we have

$$p_t = -\frac{dp_t}{dy_t} y_t + C'(y_t) > C'(y_t). \tag{6.5}$$

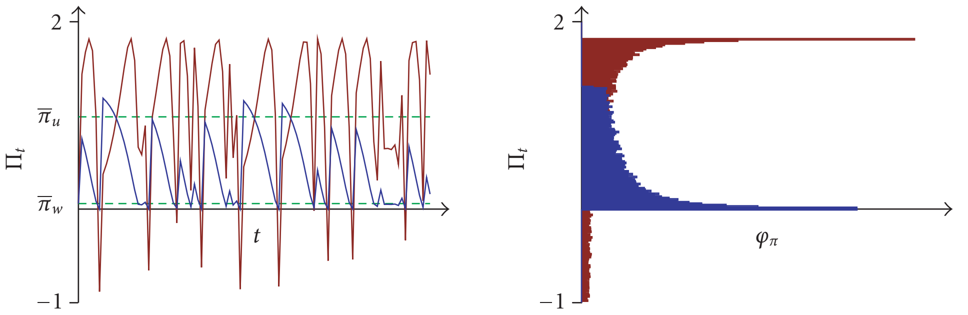
Therefore,

$$\begin{aligned} \pi_t^y &= p_t y_t - C(y_t) \geq C'(y_t) y_t - C(y_t) \\ &= y_t\left(C'(y_t) - \frac{C(y_t)}{y_t}\right) \geq 0, \end{aligned} \tag{6.6}$$

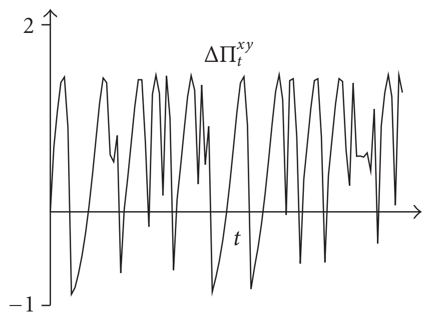
where the last inequality results from the convexity of the cost function C and $C(0) = 0$. □



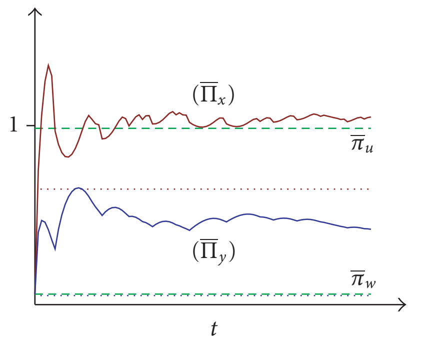
(a) Quantity trajectories and frequencies



(b) Profit trajectories and frequencies



(c) Profits difference



(d) Average profits

Figure 6.1. Chaotic fluctuations, $m = 3$, $\sigma = 1/8$, $\beta = \beta_{\max}$.

THEOREM 6.3. *For the cautious LHO model, let β_* and β^* be two critical growth-rate limits defined in (5.21) and (5.22), respectively,*

(i) *if $\beta \le \beta_*$, for all $p_t \in J_p(\beta)$, then*

$$\pi_t^x > \pi_t^y > 0; \tag{6.7}$$

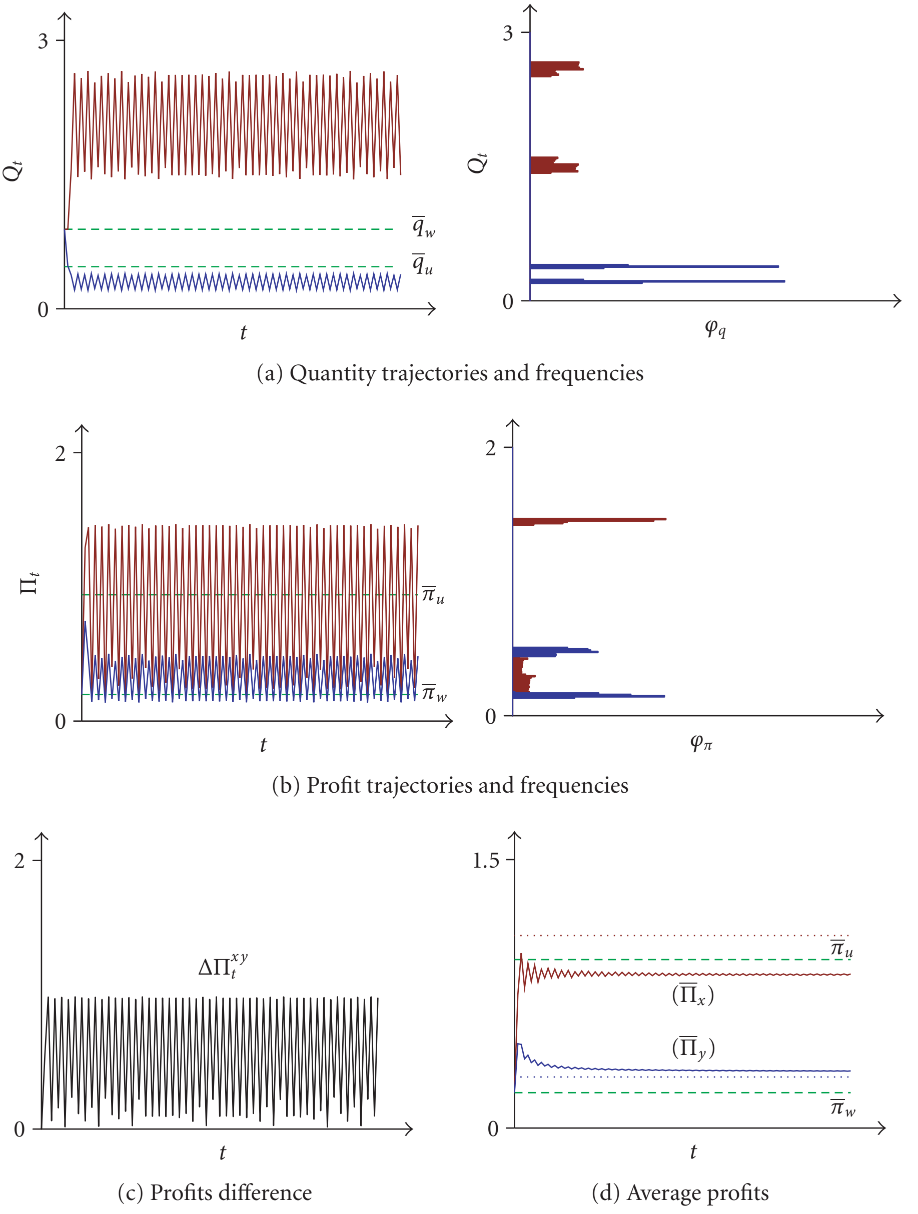


Figure 6.2. Chaotic fluctuations, $m = 3$, $\sigma = 1/2$, $\beta = \beta_*$.

(ii) *if* $\beta \le \beta^*$, *for all* $p_t \in J_p(\beta)$, *then*

$$x_t \ge \frac{1+m}{2m+\sigma+1} \ge y_t > 0. \tag{6.8}$$

Proof. (i) For a fixed m, the equilibrium $\overline{P}$ is unstable if $\sigma < \sigma^*$. It follows from Theorem 4.3 that $f(P_*) < P^*$ but $f_p(P^*) < P_*$. Therefore, as long as we can ensure that $p_{\min} > P_*$, the inequality $p_{\max} \le P^*$ will hold.

Setting $p_{\min} = P_*$ yields β_* as defined in (5.21).

(ii) It follows from Theorem 4.2 that $x_t \ge y_t$ if and only if

$$p_t \le P^* = \frac{(1+m)(m+\sigma)}{2m+\sigma+1}. \tag{6.9}$$

The condition $p_t \le P^*$ can be ensured by forcing $p_{\max} \le P^*$, which yields β^* is defined in (5.22).

Substituting P^* into (4.9) gives us inequality (6.8). □

Remark 6.4. Theorem 6.3(ii) can be alternatively proved by imposing the restriction of $y_{\max} \le x_{\min}$ where $y_{\max}$ and $x_{\min}$ are defined in Table 5.1.

Figure 5.2 depicts the monotonic relationship between σ and the critical values of β^* and β_*, that is, the smaller the value of σ is (the more unstable the equilibrium is), the smaller the values of β^* and β_* are.

When $\beta_* < \beta < \beta^*$, $x_t > (1+m)/(2m+\sigma+1) > y_t$ is guaranteed. When $\beta < \beta_*$, $\pi_t^x > \pi_t^y > 0$ is guaranteed. By the continuity of π_t^x and π_t^y as a function of β and the ergodicity of the dynamical process (5.13), we can hypothesize that there exists a $\widetilde{\beta}$ such that so long as $\beta < \widetilde{\beta}_k$, although $\Delta\pi_t^{xy} < 0$ may occur from time to time, we still have $\langle \Delta\pi^{xy} \rangle_k > 0$ for a cyclic market. Apparently, the value of $\widetilde{\beta}_k$ depends on the order of the periodic cycle.

For instance, it can be verified that there exists a unique type of period-2 cycle (P_1, P_2), where

$$\begin{aligned} P_1 &\doteq \frac{(m+1)(m+\sigma)\sigma}{(1+\beta)(m+\sigma)+\sigma(2m+\sigma)}, \\ P_2 &\doteq \frac{(m+\sigma)(m+1)\big(\beta(m+\sigma)+\sigma(2m+\sigma)\big)}{(2m+\sigma)\big((1+\beta)(m+\sigma)+\sigma(2m+\sigma)\big)}. \end{aligned} \tag{6.10}$$

It follows from (4.12) that we have

$$\begin{aligned} \langle \Delta\pi^{xy} \rangle_2 &= \frac{1}{2}\big(\Delta\pi^{xy}(P_1) + \Delta\pi^{xy}(P_2)\big) \\ &\sim \frac{1}{2}\sum_{i=1}^{2}(P^* - P_i)(P_i - P_*) \\ &\sim 2m^2(2m\sigma + m\beta + \sigma^2 + \sigma\beta) - \beta^2(\sigma^2 + 2m\sigma + 1)(m+\sigma)^2. \end{aligned} \tag{6.11}$$

So when $\beta < \beta_2$, we have $\langle \Delta\pi^{xy} \rangle_2 > 0$, where

$$\beta_2 \doteq \frac{m\Big(m + \sqrt{2\sigma(2m+\sigma)(\sigma^2+2m\sigma+1)+m^2}\Big)}{(m+\sigma)(\sigma^2+2m\sigma+1)}, \tag{6.12}$$

whose graph is depicted in Figure 5.2 in comparison with other critical β values.

Similar expectation can be obtained for the chaotic dynamics. There exists a $\widetilde{\beta}$ value such that for all $\beta < \widetilde{\beta}$, we have $\langle \Delta\pi^{xy} \rangle > 0$. There is no general estimation on the value of $\widetilde{\beta}$. Figure 6.3 depicts the numerical simulations of $\langle \pi^x \rangle$ and $\langle \pi^y \rangle$ with respect to β for several σ values. When σ is relatively large (more cyclic economy), $\widetilde{\beta}$ is smaller than β^*, as illustrated in Figure 6.3(d). On the other hand, when σ is relatively small (more divergent economy), $\widetilde{\beta}$ can be greater than its maximum possible value $\beta_{\max}$, as illustrated in Figures 6.3(a) and 6.3(b).

The following theorem provide further insights about the last observation.

Theorem 6.5. *For the cautious LHO model, when $\sigma < \widetilde{\sigma}$, where*

$$\widetilde{\sigma} \doteq \frac{1}{2}\sqrt{4m^2 - 3} + \frac{1}{2} - m, \tag{6.13}$$

then $\langle \Delta\pi^{xy} \rangle > 0$ for all $\beta < \beta_{\max}$, that is, the naiver always makes higher relative profit for all possible growth rate limit.

Proof. It follows from (4.12) that we have

$$\Delta\pi^{xy}(p) = \frac{(\sigma+1)(2m+\sigma)(2m+\sigma+1)}{2(m+\sigma)^2}(P^* - p)(p - P_*). \tag{6.14}$$

Therefore,

$$\begin{aligned} \langle \Delta\pi^{xy} \rangle &= \lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} \Delta\pi_t^{xy} = \int_{P_{\min}}^{P_{\max}} \Delta\pi^{xy}(p)\varphi(p)dp \\ &= \frac{(\sigma+1)(2m+\sigma)(2m+\sigma+1)}{2(m+\sigma)^2}\int_{P_{\min}}^{P_{\max}} (P^* - p)(p - P_*)\varphi(p)dp. \end{aligned} \tag{6.15}$$

When β is set to its maximum value $\beta_{\max}$, we have $P_{\min} = 0$, $P_{\max} = \widetilde{P} = (1+m)(m+\sigma)/(2m+\sigma)$, and $\varphi(p) = 1/\widetilde{P}$, which leads to

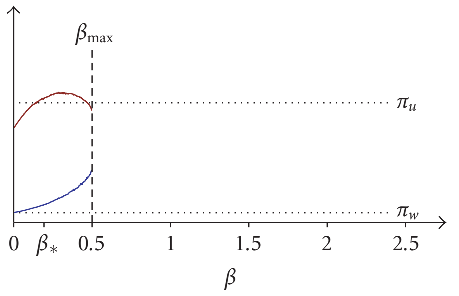
$$\langle \Delta\pi^{xy} \rangle = \frac{(\sigma+1)(2m+\sigma)(2m+\sigma+1)}{12(m+\sigma)^2}\left(3\widetilde{P}(P^* + P_*) - 2(3P^*P_* + \widetilde{P}^2)\right). \tag{6.16}$$

$\langle \Delta\pi^{xy} \rangle > 0$ if and only if $3\widetilde{P}(P^* + P_*) > 2(3P^*P_* + \widetilde{P}^2)$, or, equivalently, $\sigma < \widetilde{\sigma}$. □

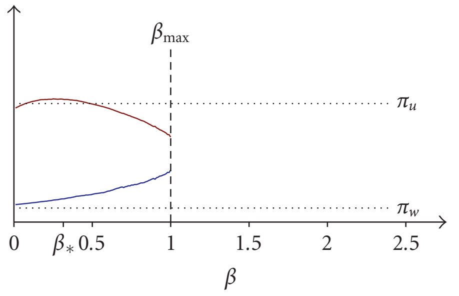
For instance, when $m = 3$, we have $\widetilde{\sigma} = 0.37228$ and $\widetilde{\beta} = \beta_{\max} = 2.3723$. For this particular set of parameters, the long-run averages of profits made by the naiver and the sophisticated are identical, which is illustrated in Figure 6.3(c). When $\sigma > \widetilde{\sigma}$, we have $\widetilde{\beta} < \beta_{\max}$, as shown in Figure 6.3(d).

7. Concluding remarks

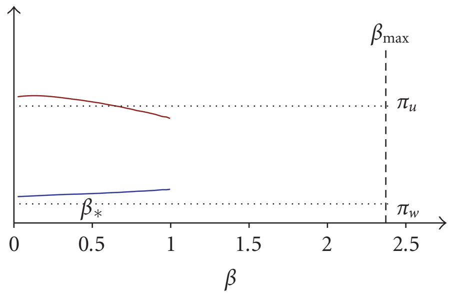
Through analyzing the relative profitability of the two types of firms in a general heterogeneous oligopoly model, we have generalized the conclusion drawn in Huang [2] that the Cobweb strategy is the most effective and efficient among all possible alternative strategies in the sense that it always results in a higher (or equal) profit than its rivals



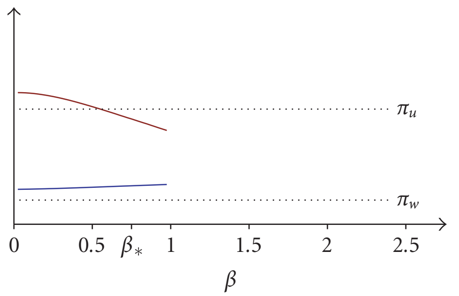
(a) $\sigma = 0.17129$



(b) $\sigma = \hat{\sigma}$



(c) $\sigma = \tilde{\sigma}$



(d) $\sigma = 1/2$

Figure 6.3. Average profits.

in equilibrium. Further analyzing a linear oligopolisitic model in a dynamical environment reveals that the naiver can still enjoy the relative profitability during most of the dynamical transitionary periods. If the economy turns into cyclic or explosive fluctuations, a combination of the Cobweb strategy with the cautious adjustment strategy could still lead to a relatively higher average profits for the naiver.

Although most dynamic analysis in the current research is conducted using a simplified linear model to derive the analytical solutions, our numerical simulations for more complex nonlinear models have confirmed that the conclusions drawn in this research have universal implications. Further studies can aim to derive some universal conditions for the relative profitability of the naiver and the sophisticated in a general setting.

Apparently, the current studies can be generalized in many ways. For instance, it is interesting to see the relative profitability of the different agents in an oligopolist economy with product differentiation. Analogous analysis can also be conducted for heterogeneous oligopsonist model.

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