Research Article

Dynamics of a Stage Structure Pest Control Model with Impulsive Effects at Different Fixed Time

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Many existing pest control models, which control pests by releasing natural enemies, neglect the effect that natural enemies may get killed. From this point of view, we formulate a pest control model with stage structure for the pest with constant maturation time delay (through-stage time delay) and periodic releasing natural enemies and natural enemies killed at different fixed time and perform a systematic mathematical and ecological study. By using the comparison theorem and analysis method, we obtain the conditions for the global attractivity of the pest-eradication periodic solution and permanence of the system. We also present a pest management strategy in which the pest population is kept under the economic threshold level (ETL) when the pest population is uniformly permanent. We show that maturation time delay, impulsive releasing, and killing natural enemies can bring great effects on the dynamics of the system. Numerical simulations confirm our theoretical results.

1. Introduction

The warfare between man and pests has been on for thousands of years, and pest outbreaks often cause serious ecological and economic problems. With the development of society, human beings have come up with numerous methods to control pests, for instance, biological, cultural, physical, mechanical, and chemical tools in a way that minimize economics. Among those methods, releasing natural enemies is an effective and basic one, and it cannot pollute the environment. Morever, it is relatively simple to implement.

The pest control models by releasing natural enemies have been studied by many researchers [1–3]. All of them have invariably assumed that the releasing enemies can all have an effect on the models, whereas it is often the case that the natural enemies may get killed. For example, releasing frogs may prevent locusts. But in many places in China, especially in

some southern areas, it is customary to eat frogs, and in markets the price of a frog is 15 yuan; so the number of the released frogs is reduced in a period as a result of their getting killed by people. According to the above biological background, we formulate a pest control model with periodic impulsive releasing natural enemies and natural enemies killed at different fixed time. We assume that the pest has a stage structure with constant maturation time delay (through-stage time delay). Many stage structure models with time delay were extensively studied (see [4–9]). In practice, from the principle of ecosystem balance, we need only to control the pest population under the economic threshold level (ETL) and not to eradicate natural enemy totally and hopes pest population and natural enemy population can coexist when the pests do not bring about immense economic losses. Then the questions that arise here are the following: how do the impulse period and time delay affect the extinction of the pest and permanence of the system? How many natural enemies should we release to control pests? How to keep the pests under ETL?

For these purposes, in Section 2 we suggest a delay impulsive differential equation to model the process of impulsive releasing natural enemies and natural enemies killed at different fixed time and introduce lemmas which will be used in this paper. Impulsive differential equations are found in almost every domain of applied science and have been studied in many investigations [10–17], and it can describe population dynamic models, since many life phenomena and human exploitation are almost impulsive in the natural world, and impulsive delay differential equations are almost analyzed in theory (see [18–20]). Time delay and impulse are introduced into pest control models with stage structure, which greatly enrich biologic background, but the system becomes nonautonomous and quite complicated, which causes great difficulties for us to study the model. In Sections 3 and 4, the conditions for the global attractivity of the pest-eradication periodic solution and permanence of the system are obtained. We give a brief conclusion of our results in the last section. Numerical simulations are presented to illustrate our theoretical results.

2. Model Formulation and Auxiliary Lemmas

A model of single pest population growth incorporating stage structure as a reasonable generalization of the logistic model was derived as follows in [21]:

$$
\begin{aligned}
\frac{dx_j(t)}{dt} &= \alpha x(t) - dx_j(t) - \alpha e^{-d\tau} x(t-\tau), \\
\frac{dx(t)}{dt} &= \alpha e^{-d\tau} x(t-\tau) - \beta x^2(t),
\end{aligned}
\tag{2.1}
$$

where $x_j(t)$ and $x(t)$ represent the immature and mature pest population densities, respectively. This model assumes an average age to maturity which appears as a constant time delay reflecting a delayed birth of immature and a reduced survival of immature to their maturity. $\alpha > 0$ is the coefficient of birth rate of the mature population in this environment; τ, called maturation time delay, is the time to maturity; $d > 0$ is the coefficient of death rate of the immature population; $\beta > 0$ is the mature pest death and overcrowding rate; the term $\alpha e^{-d\tau} x(t-\tau)$ represents the immature pests which are born at the time $t-\tau$ (i.e., $\alpha x(t-\tau)$) and survive at the time t (with the immature pest death rate d) and therefore, represents the transformation of immatures to matures.

The basic model that we consider in this paper is the one based on the ideas that the natural enemies may be killed when they are released to control pest population; then we formulate the following two-stage pest control model with stage structure for pest and pulse releasing of natural enemies killed at different fixed time:

$$
\begin{aligned}
\frac{dx_j(t)}{dt} &= \alpha x(t) - dx_j(t) - \alpha e^{-d\tau} x(t - \tau), \\
\frac{dx(t)}{dt} &= \alpha e^{-d\tau} x(t - \tau) - d_1 x^2(t) - \beta x(t)y(t), \quad t \neq nT, t \neq (n + k - 1)T, \\
\frac{dy(t)}{dt} &= \lambda \beta x(t)y(t) - d_2 y(t)
\end{aligned}
$$

$$
\begin{aligned}
x_j(t^+) &= x_j(t), \\
x(t^+) &= x(t), \qquad t = (n + k - 1)T, \\
y(t^+) &= (1 - p)y(t),
\end{aligned}
\tag{2.2}
$$

$$
\begin{aligned}
x_j(t^+) &= x_j(t), \\
x(t^+) &= x(t), \qquad t = nT, \\
y(t^+) &= y(t) + \mu,
\end{aligned}
$$

where $x_j(t)$, $x(t)$, and $y(t)$ represent the density of the immature pest, mature pest, and natural enemy population at time t, respectively; $n \in Z_+$, and $Z_+ = \{1, 2, \ldots\}$; $d_2 > 0$ is the natural enemy death's rate; $\mu > 0$ is the natural enemy releasing amount at every impulsive period nT; $0 \leq p \leq 1$ is the killed rate of natural enemies at every impulsive period $(n+k-1)T$. The meanings of other parameters are the same as those of Model (2.1).

The first equation in system (2.2) may be rewritten as follows:

$$
x_j(t) = \int_{t-\tau}^{t} \alpha e^{-d(t-s)} x(s)\, ds,
\tag{2.3}
$$

and $x_j(0) = \int_{-\tau}^{0} \alpha e^{-ds} x(s)\, ds$; that is $x_j(t)$ can be linear expression by $x(t)$, and $x_j(t)$ does not appear in the second and third equations in system (2.2), therefore in the rest of this paper, we will study the subsystem of (2.2) as follows:

$$
\begin{aligned}
\frac{dx(t)}{dt} &= \alpha e^{-d\tau} x(t - \tau) - d_1 x^2(t) - \beta x(t)y(t), \\
\frac{dy(t)}{dt} &= \lambda \beta x(t)y(t) - d_2 y(t),
\end{aligned}
\quad t \neq nT, t \neq (n + k - 1)T,
$$

$$
\begin{aligned}
x(t^+) &= x(t), \\
y(t^+) &= (1 - p)y(t),
\end{aligned}
\quad t = (n + k - 1)T,
\tag{2.4}
$$

$$
\begin{aligned}
x(t^+) &= x(t), \\
y(t^+) &= y(t) + \mu,
\end{aligned}
\quad t = nT.
$$

The initial conditions for (2.4) are

$$(\varphi_1(s), \varphi_2(s)) \in C_+ = C\left([-\tau, 0], R_+^2\right), \quad \varphi_i(0) > 0, \ i = 1, 2. \tag{2.5}$$

From the biological point of view, we only consider system (2.4) in the biological meaning region: $D = \{x(t), y(t) \mid x(t) \geq 0, y(t) \geq 0\}$.

Lemma 2.1 (see [22]). *Consider the following equation:*

$$\frac{dx(t)}{dt} = ax(t - \tau) - bx(t) - cx^2(t), \tag{2.6}$$

where a, b, c, τ are all positive constants, $x(t) > 0$, for $-\tau \leq t \leq 0$; one has the following.

(1) *If $a < b$, then $\lim_{t \to +\infty} x(t) = 0$.*

(2) *If $a > b$, then $\lim_{t \to +\infty} x(t) = (a - b)/c$.*

Lemma 2.2. *For each solution of system (2.2) with t being large enough, one has $x_j(t), x(t), y(t) \geq L$, where $L = (\lambda\alpha + d_3)^2/4\lambda d_3 d_1 + \mu e^{d_3 T}/(e^{d_3 T} - 1)$ and $d_3 = \min\{d_2, d\}$.*

Proof. Let $V(t) = \lambda x_j(t) + \lambda x(t) + y(t)$, $d_3 = \min\{d_2, d\}$. If $t \neq nT$ and $t \neq (n + k - 1)T$, we have

$$\frac{dV(t)}{dt} \leq \lambda(\alpha + d)x(t) - \lambda d_1 x^2(t) - d_3 V \leq \frac{\lambda(\alpha + d_3)^2}{4d_1} - d_3 V, \tag{2.7}$$

If $t = (n + k - 1)T$, $V((n + k - 1)T^+) \leq V((n + k - 1)T)$; if $t = nT$, $V(nT^+) \leq V(nT^+) + \mu$. We have

$$\begin{aligned}
V(t) &\leq V(0)e^{-d_3 t} + \int_0^t \frac{(\lambda\alpha + d_3)^2}{4\lambda d_3 d_1}e^{-d_3(t-s)}ds + \sum_{0 < nT < t} \mu e^{-d_3(t-nT)} \\
&\longrightarrow \frac{\lambda(\alpha + d_3)^2}{4d_3 d_1} + \frac{\mu e^{d_3 T}}{e^{d_3 T} - 1} = L, \quad \text{as } t \longrightarrow +\infty.
\end{aligned} \tag{2.8}$$

So $V(t)$ is uniformly ultimately bounded. Hence, by the definition of $V(t)$, we obtain that each positive solution of system (2.2) is uniformly ultimately bounded. The proof is complete. $\square$

Lemma 2.3 (see [23]). *Consider the following system:*

$$\begin{aligned}
\frac{dy(t)}{dt} &= -d_2 y(t), \quad t \neq nT, \ t \neq (n + k - 1)T, \\
y(t^+) &= (1 - p)y(t), \quad t = (n + k - 1)T, \\
y(t^+) &= y(t) + \mu, \quad t = nT,
\end{aligned} \tag{2.9}$$

where d_2, p, μ, k are all positive constants; then, system (2.9) *has a globally asymptotic stable positive periodic solution:*

$$y^*(t) = \begin{cases} \dfrac{\mu e^{-d_2[t-(n-1)T]}}{1-(1-p)e^{-d_2T}}, & (n-1)T < t \le (n+k-1)T, \\ \dfrac{\mu(1-p)e^{-d_2[t-(n-1)T]}}{1-(1-p)e^{-d_2T}}, & (n+k-1)T < t \le nT. \end{cases} \tag{2.10}$$

Lemma 2.4 (see [24]). *Let* $V : R_+ \times R^3 \to R_+$ *and* $V \in V_0$. *Assume that*

$$\begin{aligned} D^+V(t,x) &\le g(t,V(t,x)), \quad t \ne nT, \\ V(t,x(t^+)) &\le \psi_n(V(t,x(t))), \quad t = nT, \end{aligned} \tag{2.11}$$

where $g : R_+ \times R_+ \to R$ *is continuous in* $(nT,(n+1)T] \times R_+$ *and for* $v \in R_+, n \in Z_+$, $\lim_{(t,y) \to (nT^+,v)} g(t,y) = g(nT^+,v)$ *exits, and* $\psi_n : R_+ \to R_+$ *is nondecreasing. Let* $r(t)$ *be the maximal solution of the scalar impulsive differential equation:*

$$\begin{aligned} \frac{du(t)}{dt} &= g(t,u), \quad t \ne nT, \\ u(t^+) &= \psi_n(u(t)), \quad t = nT \\ u(0^+) &= u_0, \end{aligned} \tag{2.12}$$

exiting on $[0,\infty)$. *Then* $V(0^+,x_0) \le u_0$ *implies that* $V(t,x(t)) \le r(t)$, $t \ge 0$, *where* $x(t)$ *is any solution of* (2.11).

3. Global Attractivity of the "Pest-Eradication" Periodic Solution

Define $\zeta_1 = [1-(1-p)e^{-d_2T}]\alpha e^{-d\tau}/\beta\mu(1-p)e^{-d_2T}$.

Theorem 3.1. *If* $\zeta_1 < 1$ *holds, the population-extinction periodic solution* $(0, y^*(t))$ *of* (2.4) *is globally attractive.*

Proof. From the second equation in system (2.4), we have $dy(t)/dt \ge -d_2 y(t)$; then consider the following system:

$$\begin{aligned} \frac{dz_1(t)}{dt} &= -d_2 z_1(t), \quad t \ne nT, \ t \ne (n+k-1)T, \\ z_1(t^+) &= (1-p)z_1(t), \quad t = (n+k-1)T, \\ z_1(t^+) &= z_1(t) + \mu, \quad t = nT. \end{aligned} \tag{3.1}$$

According to Lemma 2.3, the globally asymptotic stable positive periodic solution of system (3.1) is

$$
z_1^*(t) = \begin{cases}
\dfrac{\mu e^{-d_2[t-(n-1)T]}}{1-(1-p)e^{-d_2 T}}, & (n-1)T < t \le (n+k-1)T, \\[4mm]
\dfrac{\mu(1-p)e^{-d_2[t-(n-1)T]}}{1-(1-p)e^{-d_2 T}}, & (n+k-1)T < t \le nT.
\end{cases}
\tag{3.2}
$$

According to Lemma 2.4, for any $\epsilon_1 > 0$, there exits a n_1 such that

$$
y(t) > z_1^*(t) - \epsilon_1 > \frac{\mu(1-p)e^{-d_2 T}}{1-(1-p)e^{-d_2 T}} - \epsilon_1 =: \eta, \quad nT < t \le (n+1)T, \ n > n_1.
\tag{3.3}
$$

From the first equation of system (2.4), we can get

$$
\frac{dx(t)}{dt} < \alpha e^{-d\tau} x(t-\tau) - d_1 x^2(t) - \beta x(t)\eta.
\tag{3.4}
$$

Then consider the following comparison equation:

$$
\frac{dz(t)}{dt} = \alpha e^{-d\tau} z(t-\tau) - d_1 z^2(t) - \beta z(t)\eta.
\tag{3.5}
$$

Since $\zeta_1 < 1$, according to Lemma 2.1, we obtain $\lim_{t \to \infty} z(t) = 0$; then by the comparison theorem in differential equations, we get $\lim_{t \to \infty} x(t) = 0$. Without loss of generality, we assume that

$$
0 < x(t) < \epsilon, \quad \forall t \ge 0.
\tag{3.6}
$$

Consider the following system:

$$
\begin{aligned}
\frac{dY(t)}{dt} &= (\lambda\beta\epsilon - d_2)Y(t), \quad t \ne nT, \ t \ne (n+k-1)T, \\
Y(t^+) &= (1-p)Y(t), \quad t = (n+k-1)T, \\
Y(t^+) &= Y(t) + \mu, \quad t = nT.
\end{aligned}
\tag{3.7}
$$

The globally asymptotic stable positive periodic solution of system (3.7) is

$$
Y^*(t) = \begin{cases}
\dfrac{\mu e^{(\lambda\beta\epsilon-d_2)[t-(n-1)T]}}{1-(1-p)e^{(\lambda\beta\epsilon-d_2)T}}, & (n-1)T < t \le (n+k-1)T, \\[4mm]
\dfrac{\mu(1-p)e^{(\lambda\beta\epsilon-d_2)[t-(n-1)T]}}{1-(1-p)e^{(\lambda\beta\epsilon-d_2)T}}, & (n+k-1)T < t \le nT.
\end{cases}
\tag{3.8}
$$

By (3.3) and Lemma 2.4, for any $\epsilon_2 > 0$, there exits a T^*, when $t > T^*$:

$$z_1^*(t) - \epsilon_2 < y(t) < Y^*(t) + \epsilon_2. \tag{3.9}$$

Let $\epsilon \to 0$, we can get

$$y^*(t) - \epsilon_2 < y(t) < y^*(t) + \epsilon_2 \tag{3.10}$$

for t being large enough, which implies $\lim_{t \to \infty} y(t) = y^*(t)$. This completes the proof. $\qquad \square$

4. Permanence

Define $\zeta_2 = [1 - (1 - p)e^{-d_2 T}]\alpha e^{-d\tau}/\beta\mu e^{-d_2 T}$.

Definition 4.1. System (2.4) is said to be permanent if there are constants $l, L > 0$ and a finite time T_0 such that for every positive solution $(x_j(t), x(t), y(t)) \in R_+^3$ with initial conditions (2.5) satisfies $l \leq x_j(t) \leq L, l \leq x(t) \leq L, l \leq y(t) \leq L$ for all $t \geq T_0$. Here T_0 may depend on the initial condition (2.5).

Theorem 4.2. *If $\zeta_2 > 1$ holds, there exists a positive constant q such that each positive solution $(x(t), y(t))$ of (2.4) satisfies $x(t) \geq q$ with t being large enough.*

Proof. The first equation of system (2.4) may be rewritten as follows:

$$\frac{dx(t)}{dt} = \left[\alpha e^{-d\tau} - d_1 x(t) - \beta y(t)\right] x(t) - \alpha e^{-d\tau} \frac{d}{dt} \int_{t-\tau}^{t} x(s)ds. \tag{4.1}$$

Define

$$V(t) = x(t) + \alpha e^{-d\tau} \int_{t-\tau}^{t} x(s)ds. \tag{4.2}$$

Calculating the derivative of $V(t)$ along the solution to (2.4) gives

$$\frac{dV(t)}{dt} = \left[\alpha e^{-d\tau} - d_1 x(t) - \beta y(t)\right] x(t). \tag{4.3}$$

Since $\zeta_2 > 1$, we can choose m_1. Let $\epsilon > 0$ be small enough such that $d_2 - \lambda\beta m_1 > 0$ and

$$\frac{\alpha e^{-d\tau}}{d_1 m_1 + \beta\sigma} - 1 > 0, \tag{4.4}$$

where $\sigma = \mu e^-(d_2 - \lambda\beta m_1)T)/(1 - (1 - p)e^{-(d_2 - \lambda\beta m_1)T} - \epsilon$. For any positive constant $t_0 > 0$, we claim that the inequality $x(t) < m_1$ cannot hold for all $t \geq t_0$. Otherwise, there is a positive

constant t_0 such that $x(t) < m_1$ for all $t \geq t_0$. From the second and the fourth equations in system (2.4), we have

$$
\begin{aligned}
\frac{dy(t)}{dt} &\leq (\lambda \beta m_1 - d_2) y(t), \quad t \neq nT, \ t \neq (n+k-1)T, \\
y(t^+) &= (1-p) y(t), \quad t = (n+k-1)T, \\
y(t^+) &= y(t) + \mu, \quad t = nT.
\end{aligned}
\tag{4.5}
$$

According to Lemma 2.4, there exists $T_1 \geq t_0 + \tau$ such that for $t \geq T_1$,

$$
y(t) < \frac{\mu e^{(\lambda \beta m_1 - d_2)T}}{1 - (1-p)e^{(\lambda \beta m_1 - d_2)T}} - \epsilon =: \sigma.
\tag{4.6}
$$

From (4.3) and (4.6) we have

$$
\frac{dV(t)}{dt} > \left[\alpha e^{-d\tau} - d_1 m_1 - \beta \sigma \right] x(t) = (d_1 m_1 + \beta \sigma) \left(\frac{\alpha e^{-d\tau}}{d_1 m_1 + \beta \sigma} - 1 \right) x(t), \quad t \geq T_1.
\tag{4.7}
$$

Let $m_2 = \min_{t \in [T_1, T_1 + \tau]} x(t)$, we show that $x(t) \geq m_2$ for all $t > T_1$. Otherwise, there exists a nonnegative constant T_2, such that $x(t) \geq m_2$, for $t \in [T_1, T_1 + \tau + T_2]$, $x(T_1 + \tau + T_2) = m_2$ and $x'(T_1 + \tau + T_2) < 0$.

Thus, from the first equation of system (2.4) and (4.5), we easily see that

$$
\begin{aligned}
\frac{dx(T_1 + \tau + T_2)}{dt} &= \alpha e^{-d\tau} x(T_1 + T_2) - d_1 x^2 (T_1 + \tau + T_2) - \beta x(T_1 + \tau + T_2) y(t) \\
&\geq \left[\alpha e^{-d\tau} - d_1 m_1 - \beta \sigma \right] m_2 > (d_1 m_1 + \beta \sigma) \left(\frac{\alpha e^{-d\tau}}{d_1 m_1 + \beta \sigma} - 1 \right) m_2 > 0.
\end{aligned}
\tag{4.8}
$$

This is a contradiction. So we obtain that $x(t) \geq m_2$, for all $t > T_1$. From (4.7), we have

$$
\frac{dV(t)}{dt} > (d_1 m_1 + \beta \sigma) \left(\frac{\alpha e^{-d\tau}}{d_1 m_1 + \beta \sigma} - 1 \right) m_2,
\tag{4.9}
$$

which implies $V(t) \to +\infty$ as $t \to +\infty$. This is a contradiction to $V(t) \leq L(1 + \alpha \tau e^{-d\tau})$. Therefore, for any positive constant t_0, the inequality $x(t) < m_1$ cannot hold for all $t \geq t_0$.

If $x(t) \geq m_1$ holds true for all t large enough, then our aim is obtained. Otherwise, $x(t)$ is oscillatory about m_1. Let

$$
q = \min\left\{ \frac{m_1}{2}, m_1 e^{-(d_1 + \beta)L\tau} \right\}.
\tag{4.10}
$$

In the following, we will show that $x(t) \geq q$ for t being large enough. There exist two positive constants $\bar{t}, \omega$ such that

$$x\left(\bar{t}\right) = x\left(\bar{t} + \omega\right) = m_1,$$

$$x(t) < m_1 \quad \text{for } \bar{t} < t < \bar{t} + \omega. \tag{4.11}$$

Since $x(t)$ is continuous and bounded and is not effected by impulses, we conclude that $x(t)$ is uniformly continuous. Then there exists a constant T_3, such that $x(t) > m_1/2$ for all $\bar{t} \leq t \leq \bar{t} + T_3$.

If $\omega \leq T_3$, our aim is obtained.

If $T_3 < \omega \leq \tau$, from the first equation of (2.4) we have that

$$\frac{dx(t)}{dt} \geq -(d_1 + \beta)Lx(t) \quad \text{for } \bar{t} < t \leq \bar{t} + \omega. \tag{4.12}$$

Then we have

$$x(t) \geq m_1 e^{-(d_1+\beta)L\tau} \quad \text{for } \bar{t} < t \leq \bar{t} + \omega \leq \bar{t} + \tau. \tag{4.13}$$

It is clear that $x(t) \geq q$ for $\bar{t} < t \leq \bar{t} + \omega$.

If $\omega > \tau$, by the first equation of (2.4), then we have that $x(t) \geq q$ for $\bar{t} < t \leq \bar{t} + \tau$. Thus, proceeding exactly as the proof for above claim, we can obtain the inequality $x(t) < m_1$ cannot hold for all $t > \bar{t} + \tau$, so the same arguments can be continued for for $\bar{t} + \tau < t \leq \bar{t} + \omega$, and we can get $x(t) \geq q$ for $\bar{t} + \tau < t \leq \bar{t} + \omega$. Since the interval $[\bar{t}, \bar{t} + \omega]$ is arbitrarily chosen (we only need $\bar{t}$ to be large), we get that $x(t) \geq q$ for t being large enough. In view of our arguments above, the choice of q is independent of the positive solution of (2.4) which satisfies that $x(t) \geq q$ for being sufficiently large t. This completes the proof. □

Theorem 4.3. *If $\zeta_2 > 1$, then system* (2.4) *is permanent.*

Proof. Suppose that $(x(t), y(t))$ is any solution of system (2.4) with initial condition (2.5). Let $q^* = (\mu(1-p)e^{-d_2T}/1-(1-p)e^{-d_2T})$, by (3.3) we know $y(t) \geq q^*$, and according to Theorem 4.2, there exist positive constants q, such that $x(t) \geq q$. Set

$$D = \left\{x(t), y(t) \mid q \leq x(t) \leq L, q^* \leq y(t) \leq L\right\}. \tag{4.14}$$

Then, D is a bounded compact region which has positive distance from coordinate axes. By Theorem 4.2, one obtains that every solution to system (2.4) with the initial condition (2.5) eventually enters and remains in the region D. The proof is completed.

The immature pest is hardly any harmful for the crop; so we just consider the effect of the mature pest. From the standpoint of ecological balance and saving resource, we only need to maintain the mature pest population under the economic threshold level (ETL) and not to eradicate the pest totally; then we have the following theorem. □

Theorem 4.4. *Under the condition of Theorem 4.3, if*

$$\frac{\left(d_1 E - \alpha e^{-d\tau}\right)\left(1 - (1-p)e^{-d_2 T}\right)}{\beta \mu (1-p) e^{-d_2 T}} > 1, \tag{4.15}$$

then the mature pest population is eventually under the economic threshold level E.

Proof. From (3.3) we know for any $\epsilon_1 > 0$ that there exits a n_1 such that

$$y(t) > \frac{\mu(1-p)e^{-d_2 T}}{1 - (1-p)e^{-d_2 T}} - \epsilon_1 =: \eta, \quad nT < t \le (n+1)T, \quad n > n_1. \tag{4.16}$$

From the first equation of system (2.4), we can get

$$\frac{dx(t)}{dt} < \alpha e^{-d\tau} x(t-\tau) - d_1 x^2(t) - \beta x(t)\eta. \tag{4.17}$$

Then consider the following comparison equation:

$$\frac{dz(t)}{dt} = \alpha e^{-d\tau} z(t-\tau) - d_1 z^2(t) - \beta z(t)\eta. \tag{4.18}$$

If $\varsigma_2 > 1$ holds, we can get $\alpha e^{-d\tau} > \beta \eta$; then according to Lemma(2.1), we obtain

$$\lim_{t \to \infty} z(t) = \frac{\alpha e^{-d\tau} - \beta \eta}{d_1}. \tag{4.19}$$

According to Lemma 2.4, we have $x(t) \le (\alpha e^{-d\tau} - \beta \eta)/d_1 < E$, as $t \to \infty$. The proof is completed. $\qquad\qquad\square$

5. Discussion

In this paper, we discuss a pest control model with stage structure for the pest with constant maturation time delay (through-stage time delay) and periodic releasing natural enemies and natural enemies killed at different fixed time. From Theorems 3.1, 4.2, and 4.3, we can observe that the extinction and permanence of the population are very much dependent on T, τ, μ, p. If μ is too large, from the condition of Theorem 3.1 we know that the population will be extinct. Although in theory pests can be eliminated completely, in fact it is hard to implement. In the natural world, the immature pest does not have any effect on the crop; so we only consider the mature pest in this paper. Even though pest is extinct, the food chain between pest and natural enemy is broken, that would be another disaster. Therefore we only need to control the mature pest population under (ETL) and not to eradicate natural enemy totally and hope that pest population and natural enemy population can coexist when the pests do not bring about immense economic losses.
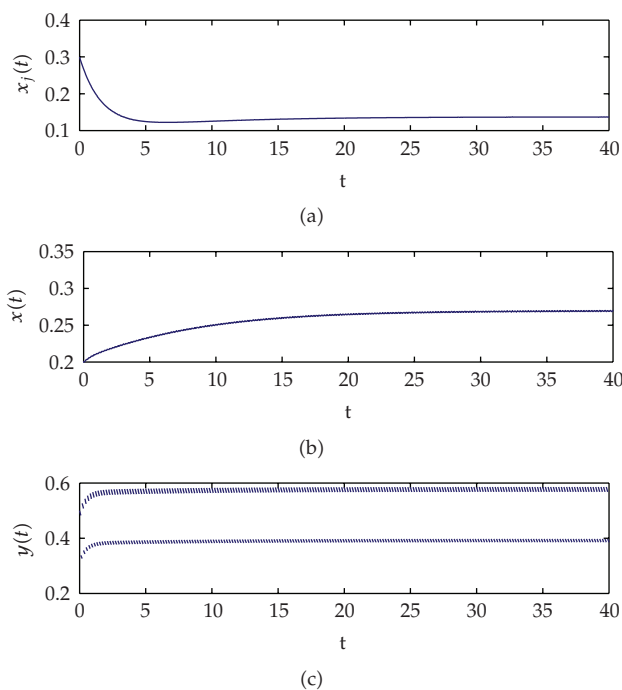
Figure 1: Time series of system (2.4) with parameters $d = 0.6$; $d_1 = 0.4$; $\alpha = 0.8$; $\tau = 0.8$; $d_2 = 0.5$; $\beta = 0.8$; $\lambda = 0.9$; $p = 0.3$; $\mu = 0.2$; $T = 0.2$.
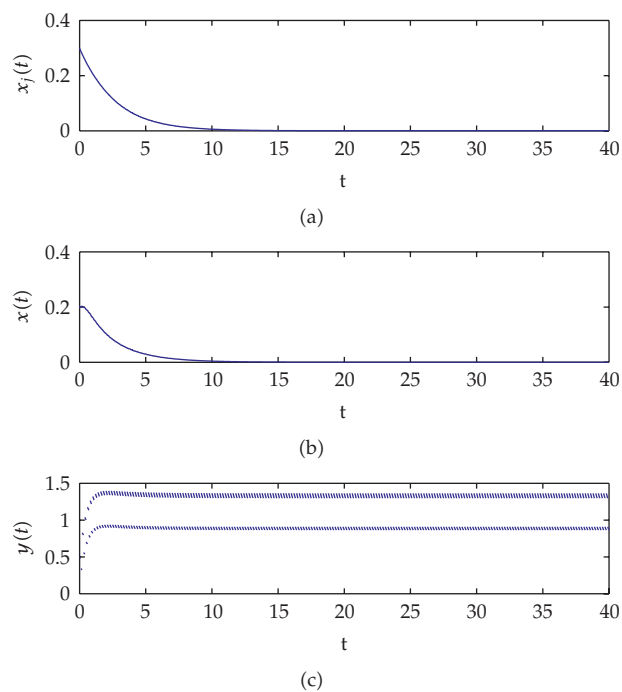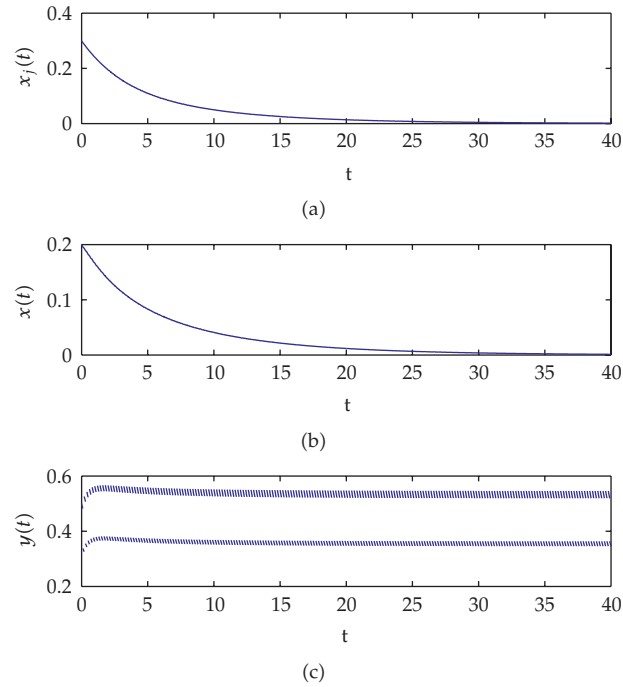

Figure 2: Time series of system (2.4) with parameters $d = 0.6$; $d_1 = 0.4$; $\alpha = 0.8$; $\tau = 0.8$; $d_2 = 0.5$; $\beta = 0.8$; $\lambda = 0.9$; $p = 0.3$; $\mu = 0.5$; $T = 0.2$.

(a)

(b)

(c)

Figure 3: Time series of system (2.4) with parameters $d = 0.6$; $d_1 = 0.4$; $\alpha = 0.8$; $\tau = 2$; $d_2 = 0.5$; $\beta = 0.8$; $\lambda = 0.9$; $p = 0.3$; $\mu = 0.2$; $T = 0.2$.
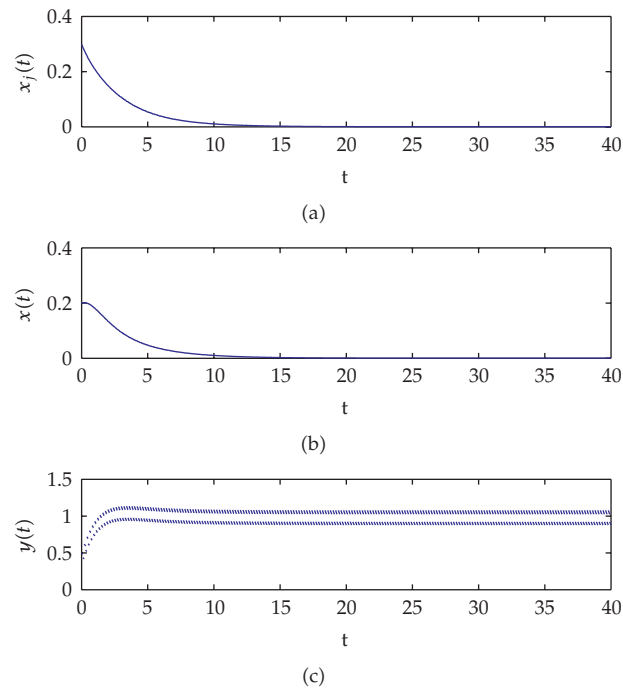

(a)

(b)

(c)

Figure 4: Time series of system (2.4) with parameters $d = 0.6$; $d_1 = 0.4$; $\alpha = 0.8$; $\tau = 0.8$; $d_2 = 0.5$; $\beta = 0.8$; $\lambda = 0.9$; $p = 0.1$; $\mu = 0.2$; $T = 0.2$.
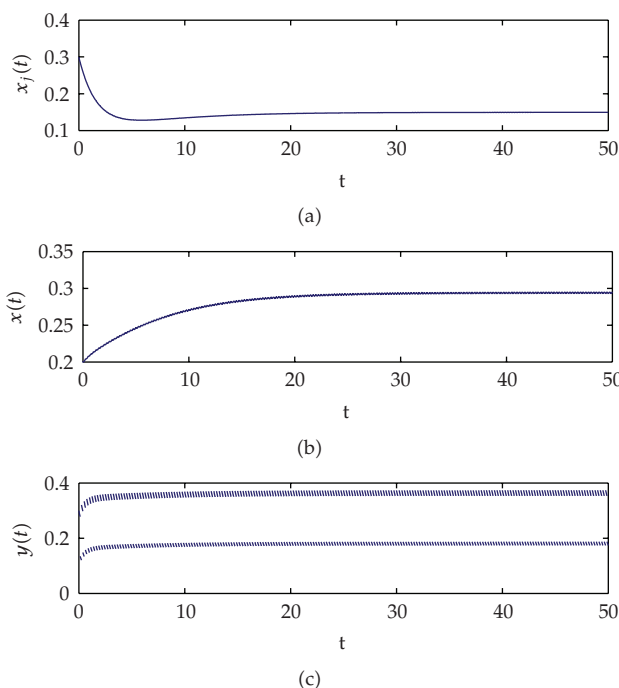
(a)

(b)

(c)

Figure 5: Time series of system (2.4) with parameters $d = 0.6$; $d_1 = 0.4$; $\alpha = 0.8$; $\tau = 0.8$; $d_2 = 0.5$; $\beta = 0.8$; $\lambda = 0.9$; $p = 0.3$; $\mu = 0.2$; $T = 0.25$.

To verify the theoretical results obtained in this paper, in the following we will give some numerical simulations and take $d = 0.6$; $d_1 = 0.4$; $\alpha = 0.8$; $\tau = 0.8$; $d_2 = 0.5$; $\beta = 0.8$; $\lambda = 0.9$; $p = 0.3$; $\mu = 0.2$; $T = 0.2$ (see Figure 1); here we can compute $\zeta_2 = 1.253 > 1$; from Theorem 4.3 we know that system (2.4) is permanent. If we increase the natural enemy input amount to $\mu = 0.5$ ($\zeta_1 = 0.716 < 1$) or increase the maturation time delay $\tau = 2$ ($\zeta_1 = 0.872 < 1$), other parameters are the same with those in Figure 1, and the pest will be extinct (see Figure 2 and Figure 3). If we decrease the killed rate of natural enemies to $p = 0.1$, then $\zeta_1 = 0.635 < 1$, the pest also will be extinct (see Figure 4). It shows that if the maturation time delay is too long, or the natural enemy releasing amount is too large, or fewer natural enemies are killed, the permanence of the system disappears and the pest population dies out. This implies that pulse releasing natural enemies, the maturation time delay and natural enemies killed bring great effects on the dynamics behaviors of the model. Suppose that the economic threshold level, E, is 0.3, then Figure 5 shows that if we choose an appropriate pulse releasing natural enemy period (with T), we can control the pest population under the ETL and not eradicate natural enemy totally. This gives us some reasonable suggestions for pest management.

Acknowledgments

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