

## Research Article

# Stability Analysis of Stochastic Reaction-Diffusion Cohen-Grossberg Neural Networks with Time-Varying Delays

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This paper is concerned with  $p$ th moment exponential stability of stochastic reaction-diffusion Cohen-Grossberg neural networks with time-varying delays. With the help of Lyapunov method, stochastic analysis, and inequality techniques, a set of new sufficient conditions on  $p$ th moment exponential stability for the considered system is presented. The proposed results generalized and improved some earlier publications.

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## 1. Introduction

Since the seminal work for Cohen-Grossberg neural networks by Cohen and Grossberg [1], theoretical understanding of neural network dynamics has advanced greatly. The model can be described by a system of ordinary differential equations

$$\dot{x}_i(t) = -\alpha_i(x_i(t)) \left[ \beta_i(x_i(t)) - \sum_{j=1}^n a_{ij} g_j(x_j(t)) \right], \quad (1.1)$$

where  $t \geq 0$ ,  $n \geq 2$ ;  $n$  corresponds to the number of units in a neural network;  $x_i(t)$  denotes the potential (or voltage) of cell  $i$  at time  $t$ ;  $f_j(\cdot)$  denotes a nonlinear output function between cell  $i$  and  $j$ ;  $\alpha_i(\cdot) > 0$  represents an amplification function;  $\beta_i(\cdot)$  represents an appropriately behaved function; the  $n \times n$  connection matrix  $A = (a_{ij})_{n \times n}$  denotes the strengths of connectivity

between cells, and if the output from neuron  $j$  excites (resp., inhibits) neuron  $i$ , then  $a_{ij} \geq 0$  (resp.,  $a_{ij} \leq 0$ ).

During hardware implementation, time delays do exist due to finite switching speed of the amplifiers and communication time and, thus, delays should be incorporated into the model equations of the network. For model (1.1), Ye et al. [2] introduced delays by considering the following delay differential equations:

$$\dot{x}_i(t) = -\alpha_i(x_i(t)) \left[ \beta_i(x_i(t)) - \sum_{j=1}^n a_{ij} f_j(x_j(t - \tau_j)) \right]. \quad (1.2)$$

Some other more detailed justifications for introducing delays into model equations of neural networks can be found in [3–13] and references therein. It is seen that (1.2) is quite general and it includes several well-known neural networks models as its special cases such as Hopfield neural networks, cellular neural networks, and bidirectional association memory neural networks (see, e.g., [14–18]).

In addition to the delay effects, stochastic effects constitute another source of disturbances or uncertainties in real systems [19]. A lot of dynamical systems have variable structures subject to stochastic abrupt changes, which may result from abrupt phenomena such as stochastic failures and repairs of the components, changes in the interconnections of subsystems, and sudden environment changes. In the recent years, the stability investigation of stochastic Neural Networks is interesting to many investigators, and a large number of stability criteria of these systems have been reported [20–30]. The stochastic model can be described by a system of stochastic differential equations

$$\begin{aligned} dx_i(t) = & -\alpha_i(x_i(t)) \left[ \beta_i(x_i(t)) - \sum_{j=1}^n a_{ij} g_j(x_j(t)) - \sum_{j=1}^n b_{ij} g_j(x_j(t - \tau_j(t))) \right] dt \\ & + \sum_{j=1}^n \sigma_{ij}(t, x_i(t), x_j(t - \tau_j(t))) d\omega_j(t). \end{aligned} \quad (1.3)$$

However, besides delay and stochastic effects, diffusion effect cannot be avoided in the neural networks when electrons are moving in asymmetric electromagnetic fields [31], so we must consider the activations vary in space as well as in time. In [32–36], authors have considered the stability of reaction-diffusion neural networks with constant or time-varying delays, which are expressed by partial differential equations. To the best of our knowledge, few authors have considered the problem of  $p$ th moment stability for stochastic Cohen-Grossberg neural networks with both time-varying delays and reaction-diffusion terms. Motivated by the above discussions, in this paper, we consider the stochastic reaction-diffusion Cohen-Grossberg neural networks with time-varying delays described by

the following stochastic partial differential equations:

$$\begin{aligned}
dy_i(x, t) = & \sum_{k=1}^m \frac{\partial}{\partial x_k} \left( D_{ik} \frac{\partial y_i}{\partial x_k} \right) dt - \alpha_i(y_i(x, t)) \\
& \times \left[ \beta_i(y_i(x, t)) - \sum_{j=1}^n a_{ij}(t) g_j(y_j(x, t)) - \sum_{j=1}^n b_{ij}(t) g_j(y_j(x, t - \tau_j(t))) \right] dt \quad (1.4) \\
& + \sum_{j=1}^n \sigma_{ij}(t, y_i(x, t), y_j(x, t - \tau_j(t))) d\omega_j(t),
\end{aligned}$$

where

$$\begin{aligned}
\frac{\partial y_i}{\partial n} & := \left( \frac{\partial y_i}{\partial x_1}, \dots, \frac{\partial y_i}{\partial x_m} \right)^T = 0, \quad t \geq 0, \quad i = 1, \dots, n, \quad x \in \Omega, \quad (1.5) \\
y_i = y_i(x, s) & = \phi_i(x, s), \quad -\infty < s \leq 0, \quad x \in \Omega.
\end{aligned}$$

In the above model,  $n \geq 2$  corresponds to the number of units in the neural network,  $x = (x_1, \dots, x_m)^T$  is space variable, and  $y_i(x, t)$  denotes the state variable of cell  $i$  at time  $t$  in space variable  $x$ ; smooth function  $D_{ik} = D_{ik}(x, y, t) \geq 0$  is a diffusion operator;  $\Omega$  is a compact set with smooth boundary  $\partial\Omega$  and the measure  $\text{mes } \Omega > 0$  in  $R^m$ ,

$$\frac{\partial y_i}{\partial n} \Big|_{\partial\Omega} = 0, \quad (1.6)$$

and  $\phi_i(x, s)$  are the boundary value and initial value, respectively;  $a_{ij}(t)$  and  $b_{ij}(t)$  denote the strengths of connectivity between cell  $i$  and  $j$  at time  $t$ , respectively;  $\tau_j(t)$  is time delay and satisfies  $0 \leq \tau_j(t) \leq \tau$ ;  $\sigma = (\sigma_{ij}(t, y_i(x, t), y_j(x, t - \tau_j(t))))_{n \times n}$  is the diffusion coefficient matrix, and  $\omega(t) = (\omega_1(t), \dots, \omega_n(t))^T$  is an  $n$ -dimensional Brownian motion defined on a complete probability space  $(\Omega, \mathcal{F}, \mathbf{P})$  with a natural filtration  $\{\mathcal{F}_t\}_{t \geq 0}$  by standard Brownian motion  $\{w(s) : 0 \leq s \leq t\}$ . As a standing hypothesis, we assume that  $g_j(\cdot)$  and  $\sigma(t, \cdot, \cdot)$  satisfy the Lipschitz condition and the linear growth condition and that (1.4) has a solution on  $t \geq 0$  for the initial conditions.

The remainder of this paper is organized as follows. In Section 2, the basic notations and assumptions are introduced. In Section 3, criteria are proposed to determine  $p$ th moment exponential stability for the stochastic Cohen-Grossberg neural networks with time-varying delays and reaction-diffusion term. An illustrative example is given to illustrate the effectiveness of the obtained results in Section 4. We also conclude this paper in Section 5.

## 2. Preliminaries

For any  $y(x, t) = (y_1(x, t), \dots, y_n(x, t))^T \in R^n$ , we define

$$\begin{aligned} \|y_i(x, t)\|_p &= \left[ \int_{\Omega} |y_i(x, t)|^p dx \right]^{1/p}, \\ \|y(x, t)\| &= \left[ \sum_{j=1}^n \|y_j(x, t)\|_p^p \right]^{1/p}. \end{aligned} \quad (2.1)$$

As usual, we will also assume that the following conditions are satisfied.

(H<sub>1</sub>) There exist positive constants  $\underline{\alpha}_i, \bar{\alpha}_i$ , such that

$$\underline{\alpha}_i \leq a_i(y_i(x, t)) \leq \bar{\alpha}_i. \quad (2.2)$$

(H<sub>2</sub>) For each  $i \in \{1, \dots, n\}$ , there exists positive constant  $G_i$ , such that

$$|g_i(u) - g_i(v)| \leq G_i |u - v|, \quad \forall u, v \in R. \quad (2.3)$$

(H<sub>3</sub>) There exist positive functions  $\gamma_j(t)$ , such that

$$y_j(x, t) \beta_j(y_j(x, t)) \geq \gamma_j(t) y_j^2(x, t). \quad (2.4)$$

(H'<sub>3</sub>) There exists positive constant  $\gamma_j$ , such that

$$y_j(x, t) \beta_j(y_j(x, t)) \geq \gamma_j y_j^2(x, t). \quad (2.5)$$

(H<sub>4</sub>) There are nonnegative functions  $c_{ij}^0(t), c_{ij}^1(t)$ , for all  $t, u, v \in R$ , such that

$$\sigma_{ij}^2(t, u, v) \leq c_{ij}^0(t) u^2 + c_{ij}^1(t) v^2. \quad (2.6)$$

(H'<sub>4</sub>) There are nonnegative constants  $c_{ij}^0, c_{ij}^1$ , for all  $t, u, v \in R$ , such that

$$\sigma_{ij}^2(t, u, v) \leq c_{ij}^0 u^2 + c_{ij}^1 v^2. \quad (2.7)$$

(H<sub>5</sub>) The following inequality holds:

$$\int_{\Omega} D_{ik} \left( \frac{\partial y_i}{\partial x_k} \right)_{k=1}^m \cdot \nabla y_i^{p-1} dx > 0. \quad (2.8)$$

*Definition 2.1.* The trivial solution of (1.4) is said to be  $p$ th moment exponentially stable if there is a pair of positive constants  $\lambda$  and  $G$  such that

$$E\|y(\phi, t)\|^p < GE\|\phi\|^p e^{-\lambda(t-t_0)}, \quad \text{on } t \geq t_0 \tag{2.9}$$

for any  $\phi$ , where  $\lambda$  also called as convergence rate. When  $p = 2$ , it is usually said to be exponentially stable in mean square.

*Definition 2.2.* Let  $h : R \rightarrow R$  be a continuous function,  $d^+h$ ; the upper right Dini-derivative of  $h$  is defined as

$$d^+h(t) = \limsup_{\delta \rightarrow 0^+} \frac{h(t + \delta) - h(t)}{\delta}. \tag{2.10}$$

The following lemmas are important in our approach.

**Lemma 2.3** (Hardy inequality [4]). *Assume there exist constants  $a_k \geq 0$ ,  $p_k > 0$  ( $k = 1, \dots, m + 1$ ), then the following inequality holds:*

$$\left(\prod_{k=1}^{m+1} a_k^{p_k}\right)^{1/S_{m+1}} \leq \left(\sum_{k=1}^{m+1} p_k a_k^r\right)^{1/r} S_{m+1}^{-1/r}, \tag{2.11}$$

where  $r > 0$  and  $S_{m+1} = \sum_{k=1}^{m+1} p_k$ . In (2.11), if one lets  $p_{m+1} = 1$ ,  $r = S_{m+1} = \sum_{k=1}^m p_k + 1$ , one will get

$$\left(\prod_{k=1}^m a_k^{p_k}\right) a_{m+1} \leq \frac{1}{r} \left(\sum_{k=1}^m p_k a_k^r\right) + \frac{1}{r} a_{m+1}^r, \tag{2.12}$$

if one lets  $p_{m+1} = 2$ ,  $r = S_{m+1} = \sum_{k=1}^m p_k + 2$ , one will get

$$\left(\prod_{k=1}^m a_k^{p_k}\right) a_{m+1} \leq \frac{1}{r} \left(\sum_{k=1}^m p_k a_k^r\right) + \frac{2}{r} a_{m+1}^r. \tag{2.13}$$

**Lemma 2.4** (generalized Halanay inequality [37]). *For two positive-valued functions  $a(t)$  and  $b(t)$  defined on  $[t_0, \infty)$ , assume there exists a constant number  $0 \leq \mu < 1$  satisfying  $0 < a_0 \leq a(t)$ ,  $0 < b(t) \leq \mu a(t)$  hold for all  $t \geq t_0$ ;  $y(t)$  is nonnegative continuous function on  $[t_0 - \tau, \infty)$  and satisfies the following inequality:*

$$d^+y(t) \leq -a(t)y(t) + b(t)\bar{y}(t) \quad \text{for } t \geq t_0, \tag{2.14}$$

where  $\bar{y}(t) = \sup_{t-\tau \leq s \leq t} y(s)$ ;  $\tau \geq 0$  is constant. Then one has

$$y(t) \leq \bar{y}(t_0) e^{-\lambda^*(t-t_0)}, \tag{2.15}$$

where  $\lambda^* > 0$  is defined as

$$\lambda^* = \inf_{t \geq t_0} \left\{ \lambda(t) : \lambda = a(t) - b(t)e^{\lambda\tau} \right\}. \quad (2.16)$$

### 3. Main Results

**Theorem 3.1.** Under assumptions  $(H_1), (H_2), (H_3), (H_4), (H_5)$ , if there exist a positive diagonal matrix  $Q = \text{diag}(q_1, \dots, q_n)$ , constants  $\nu_k > 0$  ( $k = 1, \dots, k_1$ ),  $\mu_k > 0$  ( $k = 1, \dots, k_2$ ),  $\rho_k > 0$  ( $k = 1, \dots, k_3$ ),  $\xi_{ij}, \xi_{ij}^*, \eta_{ij}, \eta_{ij}^*, m_{ij}, m_{ij}^*, p_{ij}, p_{ij}^*, q_{ij}, q_{ij}^* \in \mathbb{R}$ ,  $0 < \lambda_2$ , and  $0 \leq \mu < 1$ , such that

$$0 < \lambda_2 \leq \lambda_2(t) \leq \mu \lambda_1(t), \quad \text{holds for all } t \geq t_0, \quad (3.1)$$

where

$$\begin{aligned} \lambda_1(t) = \min_{1 \leq i \leq n} \left\{ p \underline{\alpha}_i \gamma_i(t) - \sum_{j=1}^n \overline{\alpha}_i \sum_{k=1}^{k_1} \nu_k |a_{ij}(t)|^{p \xi_{ij} / \nu_k} G_j^{p \eta_{ij} / \nu_k} \right. \\ \left. - \frac{1}{q_i} \sum_{j=1}^n q_j \overline{\alpha}_j |a_{ji}(t)|^{p \xi_{ji}^*} G_i^{p \eta_{ji}^*} - \sum_{j=1}^n \overline{\alpha}_i \sum_{k=1}^{k_2} \mu_k |b_{ij}(t)|^{p p_{ij} / \mu_k} G_j^{p q_{ij} / \mu_k} \right. \\ \left. - \frac{p(p-1)}{2} \sum_{j=1}^n c_{ij}^0(t) - \frac{p-1}{2} \sum_{j=1}^n \sum_{k=1}^{k_3} \rho_k |c_{ij}^1(t)|^{p m_{ij} / \rho_k} \right\}, \quad (3.2) \\ \lambda_2(t) = \max_{1 \leq i \leq n} \left[ \sum_{j=1}^n \frac{q_j}{q_i} \overline{\alpha}_j |b_{ji}(t)|^{p p_{ji}^*} G_i^{p q_{ji}^*} + (p-1) \sum_{j=1}^n \frac{q_j}{q_i} (c_{ji}^1(t))^{p m_{ji}^* / 2} \right] \end{aligned}$$

$p = \sum_{j=1}^{k_1} \nu_k + 1 = \sum_{j=1}^{k_2} \mu_k + 1 = \sum_{j=1}^{k_3} \rho_k + 2$ ,  $k_1 \xi_{ij} + \xi_{ij}^* = 1$ ,  $k_1 \eta_{ij} + \eta_{ij}^* = 1$ ,  $k_2 p_{ij} + p_{ij}^* = 1$ ,  $k_2 q_{ij} + q_{ij}^* = 1$ ,  $k_3 m_{ij} + m_{ij}^* = 1$ ,  $k_3 m_{ij} + m_{ij}^* = 1$ , then for all  $\xi \in L_{\mathcal{F}_0}^p([-\tau, 0], \mathbb{R}^n)$ , the trivial solution of system (1.4) is  $p$ th moment exponentially stable, where  $p \geq 2$  is a constant.

*Proof.* Consider the following Lyapunov function:

$$V(t, y(t)) = \int_{\Omega} \sum_{i=1}^n q_i |y_i(x, t)|^p dx. \quad (3.3)$$

Applying Itô formula to  $V(t, y(t))$ , for  $\delta > 0$ , we can get

$$\begin{aligned}
& V(t + \delta, y(t + \delta)) - V(t, y(t)) \\
&= \int_t^{t+\delta} V_t(s, y(s)) ds \\
&+ \int_t^{t+\delta} V_y(s, y(s)) \left\{ \sum_{k=1}^m \frac{\partial}{\partial x_k} \left( D_{ik} \frac{\partial y_i}{\partial x_k} \right) - \alpha_i(y_i(x, s)) \left[ \beta_i(y_i(x, s)) - \sum_{j=1}^n a_{ij}(s) g_j(y_j(x, s)) \right. \right. \\
&\qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \left. \left. - \sum_{j=1}^n b_{ij}(s) g_j(y_j(x, s - \tau_j(s))) \right] \right\} ds \\
&+ \int_t^{t+\delta} V_y(s, y(s)) \sum_{j=1}^n \sigma_{ij}(s, y_i(x, s), y_j(x, s - \tau_j(s))) d\omega_j(s) + \int_t^{t+\delta} \frac{1}{2} \text{trace}(\sigma^T V_{yy} \sigma) ds \\
&\leq \int_t^{t+\delta} \int_{\Omega} p \sum_{i=1}^n q_i |y_i^{p-2}(x, s)| y_i(x, s) \\
&\quad \times \left\{ \sum_{k=1}^m \frac{\partial}{\partial x_k} \left( D_{ik} \frac{\partial y_i}{\partial x_k} \right) - \alpha_i(y_i(x, s)) \left[ \beta_i(y_i(x, s)) - \sum_{j=1}^n a_{ij}(s) g_j(y_j(x, s)) \right. \right. \\
&\qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \left. \left. - \sum_{j=1}^n b_{ij}(s) g_j(y_j(x, s - \tau_j(s))) \right] \right\} dx ds \\
&+ \int_t^{t+\delta} V_y(s, y(s)) \sum_{j=1}^n \sigma_{ij}(s, y_i(x, s), y_j(x, s - \tau_j(s))) d\omega_j(s) + \int_t^{t+\delta} \frac{1}{2} \text{trace}(\sigma^T V_{yy} \sigma) ds \\
&\leq \int_t^{t+\delta} \int_{\Omega} p \left\{ \sum_{i=1}^n q_i |y_i^{p-1}(x, s)| \left| \sum_{k=1}^m \frac{\partial}{\partial x_k} \left( D_{ik} \frac{\partial y_i}{\partial x_k} \right) - \sum_{i=1}^n q_i \alpha_i \gamma_i(s) |y_i^p(x, s)| \right. \right. \\
&\quad \left. \left. + \sum_{i=1}^n \sum_{j=1}^n q_i \bar{\alpha}_i G_j |a_{ij}(s) y_i^{p-1}(x, s) y_j(x, s)| \right. \right. \\
&\quad \left. \left. + \sum_{i=1}^n \sum_{j=1}^n q_i \bar{\alpha}_i G_j |b_{ij}(s) y_i^{p-1}(x, s) y_j(x, s - \tau_j(s))| \right\} dx ds \\
&+ p \int_t^{t+\delta} \int_{\Omega} \sum_{i=1}^n \sum_{j=1}^n q_i |y_i^{p-1}(x, s)| |\sigma_{ij}(s, y_i(x, s), y_j(x, s - \tau_j(s)))| dx d\omega_j(s) \\
&+ \int_t^{t+\delta} \int_{\Omega} \frac{p(p-1)}{2} q_i \sum_{i=1}^n \sum_{j=1}^n [c_{ij}^0(s) |y_i^p(x, s)| + c_{ij}^1(s) |y_i^{p-2}(x, s) y_j^2(x, s - \tau_j(s))|] dx ds.
\end{aligned} \tag{3.4}$$

From the boundary condition, we get

$$\begin{aligned}
& \sum_{k=1}^m \int_{\Omega} y_i^{p-1} \frac{\partial}{\partial x_k} \left( D_{ik} \frac{\partial y_i}{\partial x_k} \right) dx \\
&= \int_{\Omega} y_i^{p-1} \nabla \cdot \left( D_{ik} \frac{\partial y_i}{\partial x_k} \right)_{k=1}^m dx \\
&= \int_{\Omega} \nabla \cdot \left( y_i^{p-1} D_{ik} \frac{\partial y_i}{\partial x_k} \right)_{k=1}^m dx - \int_{\Omega} \left( D_{ik} \frac{\partial y_i}{\partial x_k} \right)_{k=1}^m \cdot \nabla y_i^{p-1} dx \\
&= \int_{\partial\Omega} \left( \left( y_i^{p-1} D_{ik} \frac{\partial y_i}{\partial x_k} \right)_{k=1}^m \right) \cdot dS - \int_{\Omega} \left( D_{ik} \frac{\partial y_i}{\partial x_k} \right)_{k=1}^m \cdot \nabla y_i^{p-1} dx \\
&= - \int_{\Omega} \left( D_{ik} \frac{\partial y_i}{\partial x_k} \right)_{k=1}^m \cdot \nabla y_i^{p-1} dx,
\end{aligned} \tag{3.5}$$

in which,  $\nabla = (\partial/\partial x_1, \dots, \partial/\partial x_m)^T$  is the gradient operator, and

$$\left( D_{ik} \frac{\partial y_i}{\partial x_k} \right)_{k=1}^m = \left( D_{i1} \frac{\partial y_i}{\partial x_1}, \dots, D_{im} \frac{\partial y_i}{\partial x_m} \right)^T. \tag{3.6}$$

On the other hand, from Lemma 2.3, we have

$$\begin{aligned}
& pG_j \left| a_{ij}(s) y_i^{p-1}(x, s) y_j(x, s) \right| \\
&= p \prod_{k=1}^{k_1} \left( |a_{ij}(s)|^{\xi_{ij}/\nu_k} G_j^{\eta_{ij}/\nu_k} |y_i(x, s)| \right)^{\nu_k} |a_{ij}(s)|^{\xi_{ij}^*} G_j^{\eta_{ij}^*} |y_j(x, s)|
\end{aligned} \tag{3.7}$$

$$\leq \sum_{k=1}^{k_1} \nu_k |a_{ij}(s)|^{p\xi_{ij}/\nu_k} G_j^{p\eta_{ij}/\nu_k} |y_i(x, s)|^p + |a_{ij}(s)|^{p\xi_{ij}^*} G_j^{p\eta_{ij}^*} |y_j(x, s)|^p,$$

$$\begin{aligned}
& pG_j \left| b_{ij}(s) y_i^{p-1}(x, s) y_j(x, s - \tau_j(s)) \right| \\
&= p \prod_{k=1}^{k_2} \left( |b_{ij}(s)|^{p_{ij}/\mu_k} G_j^{q_{ij}/\mu_k} |y_i(x, s)| \right)^{\mu_k} |b_{ij}(s)|^{p_{ij}^*} G_j^{q_{ij}^*} |y_j(x, s - \tau_j(s))|
\end{aligned} \tag{3.8}$$

$$\leq \sum_{k=1}^{k_2} \mu_k |b_{ij}(s)|^{pp_{ij}/\mu_k} G_j^{pq_{ij}/\mu_k} |y_i(x, s)|^p + |b_{ij}(s)|^{pp_{ij}^*} G_j^{pq_{ij}^*} |y_j(x, s - \tau_j(s))|^p,$$

$$\begin{aligned}
& \frac{p(p-1)}{2} c_{ij}^1(s) \left| y_i^{p-2}(x, s) y_j^2(x, s - \tau_j(s)) \right| \\
&= \frac{p(p-1)}{2} \prod_{k=1}^{k_3} \left( |c_{ij}^1(s)|^{m_{ij}/\rho_k} |y_i(x, s)| \right)^{\rho_k} \left( |c_{ij}^1(s)|^{m_{ij}^*/2} |y_j(x, s - \tau_j(s))| \right)^2
\end{aligned} \tag{3.9}$$

$$\leq \frac{p-1}{2} \sum_{k=1}^{k_3} \rho_k |c_{ij}^1(s)|^{pm_{ij}/\rho_k} |y_i(x, s)|^p + (p-1) |c_{ij}^1(s)|^{pm_{ij}^*/2} |y_j(x, s - \tau_j(s))|^p.$$



It follows from (3.4), (3.5), (3.7), (3.8), and (3.9) that

$$\begin{aligned}
 & V(t + \delta, \mathbf{y}(t + \delta)) - V(t, \mathbf{y}(t)) \\
 & \leq \int_t^{t+\delta} \int_{\Omega} \left\{ - \sum_{i=1}^n q_i p \underline{\alpha}_i \gamma_i(s) |y_i(x, s)|^p \right. \\
 & \quad + \sum_{i=1}^n \sum_{j=1}^n q_i \bar{\alpha}_i \left[ \sum_{k=1}^{k_1} \nu_k |a_{ij}(s)|^{p \xi_{ij} / \nu_k} G_j^{p \eta_{ij} / \nu_k} |y_i(x, s)|^p + |a_{ij}(s)|^{p \xi_{ij}^*} G_j^{p \eta_{ij}^*} |y_j(x, s)|^p \right] \\
 & \quad + \sum_{i=1}^n \sum_{j=1}^n q_i \bar{\alpha}_i \left[ \sum_{k=1}^{k_2} \mu_k |b_{ij}(s)|^{p p_{ij} / \mu_k} G_j^{p q_{ij} / \mu_k} |y_i(x, s)|^p \right. \\
 & \quad \quad \left. + |b_{ij}(s)|^{p p_{ij}^*} G_j^{p q_{ij}^*} |y_j(x, s - \tau_j(s))|^p \right] \Big\} dx ds \\
 & + p \int_t^{t+\delta} \int_{\Omega} \sum_{i=1}^n \sum_{j=1}^n q_i |y_i(x, s)|^{p-1} \sigma_{ij}(s, y_i(x, s), y_j(x, s - \tau_j(s))) dx d\omega_j(s) \\
 & + \int_t^{t+\delta} \int_{\Omega} q_i \left\{ \sum_{i=1}^n \sum_{j=1}^n \frac{p(p-1)}{2} c_{ij}^0(s) |y_i(x, s)|^p \right. \\
 & \quad + \sum_{i=1}^n \sum_{j=1}^n \left[ \frac{p-1}{2} \sum_{k=1}^{k_3} \rho_k |c_{ij}^1(s)|^{p m_{ij} / \rho_k} |y_i(x, s)|^p \right. \\
 & \quad \quad \left. \left. + (p-1) |c_{ij}^1(s)|^{p m_{ij}^* / 2} |y_j(x, s - \tau_j(s))|^p \right] \right\} dx ds \\
 & \leq \int_t^{t+\delta} - \min_{1 \leq i \leq n} \left\{ p \underline{\alpha}_i \gamma_i(s) - \sum_{j=1}^n \bar{\alpha}_i \sum_{k=1}^{k_1} \nu_k |a_{ij}(s)|^{p \xi_{ij} / \nu_k} G_j^{p \eta_{ij} / \nu_k} - \sum_{j=1}^n \frac{q_j}{q_i} \bar{\alpha}_j |a_{ji}(s)|^{p \xi_{ji}^*} G_i^{p \eta_{ji}^*} \right. \\
 & \quad - \sum_{j=1}^n \bar{\alpha}_i \sum_{k=1}^{k_2} \mu_k |b_{ij}(s)|^{p p_{ij} / \mu_k} G_j^{p q_{ij} / \mu_k} - \frac{p(p-1)}{2} \sum_{j=1}^n c_{ij}^0(s) \\
 & \quad \left. - \frac{p-1}{2} \sum_{j=1}^n \sum_{k=1}^{k_3} \rho_k |c_{ij}^1(s)|^{p m_{ij} / \rho_k} \right\} V(s, \mathbf{y}(s)) ds \\
 & + p \int_t^{t+\delta} \int_{\Omega} \sum_{i=1}^n \sum_{j=1}^n q_i |y_i(x, s)|^{p-1} \sigma_{ij}(s, y_i(x, s), y_j(x, s - \tau_j(s))) dx d\omega_j(s) \\
 & + \int_t^{t+\delta} \max_{1 \leq i \leq n} \left\{ \sum_{j=1}^n \frac{q_j}{q_i} \bar{\alpha}_j |b_{ji}(s)|^{p p_{ji}^*} G_i^{p q_{ji}^*} + (p-1) \sum_{j=1}^n \frac{q_j}{q_i} (c_{ji}^1(s))^{p m_{ji}^* / 2} \right\} V(s - \tau(s), \mathbf{y}(s - \tau(s))) ds.
 \end{aligned} \tag{3.10}$$

By [38, Theorem 4.2.8], we know that

$$E \int_t^{t+\delta} \int_{\Omega} \sum_{i=1}^n \sum_{j=1}^n q_i |y_i(x, s)|^{p-1} \sigma_{ij}(s, y_i(x, s), y_j(x, s - \tau_j(s))) dx d\omega_j(s) = 0. \quad (3.11)$$

Therefore, taking expectation on both sides of (3.10), the preceding result leads directly to

$$\begin{aligned} & EV(t + \delta, y(t + \delta)) - EV(t, y(t)) \\ & \leq \int_t^{t+\delta} - \min_{1 \leq i \leq n} \left\{ p \underline{\alpha}_i \gamma_i(s) - \sum_{j=1}^n \bar{\alpha}_i \sum_{k=1}^{k_1} \nu_k |a_{ij}(s)|^{p \xi_{ij} / \nu_k} G_j^{p \eta_{ij} / \nu_k} \right. \\ & \quad - \sum_{j=1}^n \frac{q_j}{q_i} \bar{\alpha}_j |a_{ji}(s)|^{p \xi_{ji}^*} G_i^{p \eta_{ji}^*} - \sum_{j=1}^n \bar{\alpha}_i \sum_{k=1}^{k_2} \mu_k |b_{ij}(s)|^{p \rho_{ij} / \mu_k} G_j^{p q_{ij} / \mu_k} \\ & \quad \left. - \frac{p(p-1)}{2} \sum_{j=1}^n c_{ij}^0(s) - \frac{p-1}{2} \sum_{j=1}^n \sum_{k=1}^{k_3} \rho_k |c_{ij}^1(s)|^{p m_{ij} / \rho_k} \right\} EV(s, y(s)) ds \\ & + \int_t^{t+\delta} \max_{1 \leq i \leq n} \left\{ \sum_{j=1}^n \frac{q_j}{q_i} \bar{\alpha}_j |b_{ji}(s)|^{p \rho_{ji}^*} G_i^{p q_{ji}^*} + (p-1) \sum_{j=1}^n \frac{q_j}{q_i} (c_{ji}^1(s))^{p m_{ji}^* / 2} \right\} \\ & \quad \times EV(s - \tau(s), y(s - \tau(s))) ds. \end{aligned} \quad (3.12)$$

By the mean value theorem for integrals, we have

$$d^+ EV(t, y(t)) \leq -\lambda_1(t) EV(t, y(t)) + \lambda_2(t) \sup_{t-\tau \leq s \leq t} EV(s, y(s)). \quad (3.13)$$

By Lemma 2.4, we get

$$EV(t, y(t)) \leq \sup_{t_0 - \tau \leq s \leq t_0} EV(s, y(s)) e^{-\lambda^*(t-t_0)}, \quad (3.14)$$

where  $\lambda^* > 0$  is defined as

$$\lambda^* = \inf_{t \geq t_0} \left\{ \lambda(t) : \lambda = \lambda_1(t) - \lambda_2(t) e^{\lambda \tau} \right\}. \quad (3.15)$$

That is to say

$$E\|y(\phi, t)\|^p < GE\|\phi\|^p e^{-\lambda^*(t-t_0)}, \quad \text{on } t \geq t_0, \quad (3.16)$$

where

$$G = \frac{\max_{1 \leq i \leq n} \{m_i\}}{\min_{1 \leq i \leq n} \{m_i\}}. \quad (3.17)$$

Therefore, the trivial solution of system (1.4) is  $p$ th moment exponentially stable. Furthermore, just as discussed in [39, pp. 173–180], the trivial solution of (1.4) is also almost surely exponentially stable. Theorem 3.1 also shows that the reaction-diffusion term has no influence on the stability for system (1.4).  $\square$

In Theorem 3.1, if we take  $k_1 = k_2 = k_3 = 1$ ,  $\nu_k = \mu_k = p - 1$ ,  $\rho_k = p - 2$ ,  $\xi_{ij} = \eta_{ij} = p_{ij} = q_{ij} = (p - 1)/p$ ,  $m_{ij} = \mu_k = (p - 2)/p$ ,  $\xi_{ij}^* = \eta_{ij}^* = p_{ij}^* = q_{ij}^* = 1/p$ ,  $m_{ij}^* = \mu_k = 2/p$ , we have the following result.

**Corollary 3.2.** *Under assumptions  $(H_1), (H_2), (H_3), (H_4), (H_5)$ , if there exist a positive diagonal matrix  $Q = \text{diag}(q_1, \dots, q_n)$ ,  $0 < \lambda_2$ , and  $0 \leq \mu < 1$ , such that*

$$0 < \lambda_2 \leq \lambda_2(t) \leq \mu \lambda_1(t), \quad \text{holds for all } t \geq t_0, \quad (3.18)$$

where

$$\begin{aligned} \lambda_1(t) = \min_{1 \leq i \leq n} & \left\{ p \bar{\alpha}_i \gamma_i(t) - (p - 1) \sum_{j=1}^n \bar{\alpha}_i |a_{ij}(t)| G_j \right. \\ & - \frac{1}{q_i} \sum_{j=1}^n \bar{\alpha}_j q_j |a_{ji}(t)| G_i - (p - 1) \sum_{j=1}^n \bar{\alpha}_i |b_{ij}(t)| G_j \\ & \left. - \frac{p(p - 1)}{2} \sum_{j=1}^n c_{ij}^0(t) - \frac{(p - 1)(p - 2)}{2} \sum_{j=1}^n c_{ij}^1(t) \right\}, \quad (3.19) \\ \lambda_2(t) = \max_{1 \leq i \leq n} & \left[ \sum_{j=1}^n \frac{q_j}{q_i} |b_{ji}(t)| \bar{\alpha}_j G_i + (p - 1) \sum_{j=1}^n \frac{q_j}{q_i} c_{ji}^1(t) \right], \end{aligned}$$

then for all  $\xi \in L_{\mathcal{F}_0}^p([-\tau, 0], \mathbb{R}^n)$ , the trivial solution of system (1.4) is  $p$ th moment exponentially stable, where  $p \geq 2$  is a constant.

When  $a_{ij}(t) \equiv a_{ij}$ ,  $b_{ij}(t) \equiv b_{ij}$ , model (1.4) is reduced to the following stochastic reaction-diffusion Cohen-Grossberg neural networks with time-varying delays:

$$\begin{aligned} dy_i(x, t) = & \sum_{k=1}^m \frac{\partial}{\partial x_k} \left( D_{ik} \frac{\partial y_i}{\partial x_k} \right) dt - \alpha_i(y_i(x, t)) \\ & \times \left[ \beta_i(y_i(x, t)) - \sum_{j=1}^n a_{ij} g_j(y_j(x, t)) - \sum_{j=1}^n b_{ij} g_j(y_j(x, t - \tau_j(t))) \right] dt \\ & + \sum_{j=1}^n \sigma_{ij}(t, y_i(x, t), y_j(x, t - \tau_j(t))) d\omega_j(t), \end{aligned} \quad (3.20)$$

**Corollary 3.3.** Under assumptions  $(H_1), (H_2), (H'_3), (H'_4), (H_5)$ , if there exist a positive diagonal matrix  $Q = \text{diag}(q_1, \dots, q_n)$ , constants  $\nu_k > 0$  ( $k = 1, \dots, k_1$ ),  $\mu_k > 0$  ( $k = 1, \dots, k_2$ ),  $\rho_k > 0$  ( $k = 1, \dots, k_3$ ),  $\xi_{ij}, \xi_{ij}^*, \eta_{ij}, \eta_{ij}^*, m_{ij}, m_{ij}^*, p_{ij}, p_{ij}^*, q_{ij}, q_{ij}^* \in \mathbb{R}$ , such that

$$\lambda_2 < \lambda_1, \quad (3.21)$$

where

$$\begin{aligned} \lambda_1 = \min_{1 \leq i \leq n} \left\{ p \alpha_i \gamma_i - \sum_{j=1}^n \bar{\alpha}_i \sum_{k=1}^{k_1} \nu_k |a_{ij}|^{p \xi_{ij} / \nu_k} G_j^{p \eta_{ij} / \nu_k} - \frac{1}{q_i} \sum_{j=1}^n q_j \bar{\alpha}_j |a_{ji}|^{p \xi_{ji}^*} G_i^{p \eta_{ji}^*} \right. \\ \left. - \sum_{j=1}^n \bar{\alpha}_i \sum_{k=1}^{k_2} \mu_k |b_{ij}|^{p p_{ij} / \mu_k} G_j^{p q_{ij} / \mu_k} - \frac{p(p-1)}{2} \sum_{j=1}^n c_{ij}^0 - \frac{p-1}{2} \sum_{j=1}^n \sum_{k=1}^{k_3} \rho_k |c_{ij}^1|^{p m_{ij} / \rho_k} \right\}, \end{aligned} \quad (3.22)$$

$$\lambda_2 = \max_{1 \leq i \leq n} \left[ \sum_{j=1}^n \frac{q_j}{q_i} \bar{\alpha}_j |b_{ji}|^{p p_{ji}^*} G_i^{p q_{ji}^*} + (p-1) \sum_{j=1}^n \frac{q_j}{q_i} (c_{ji}^1)^{p m_{ji}^* / 2} \right], \quad (3.23)$$

$p = \sum_{j=1}^{k_1} \nu_k + 1 = \sum_{j=1}^{k_2} \mu_k + 1 = \sum_{j=1}^{k_3} \rho_k + 2$ ,  $k_1 \xi_{ij} + \xi_{ij}^* = 1$ ,  $k_1 \eta_{ij} + \eta_{ij}^* = 1$ ,  $k_2 p_{ij} + p_{ij}^* = 1$ ,  $k_2 q_{ij} + q_{ij}^* = 1$ ,  $k_3 m_{ij} + m_{ij}^* = 1$ ,  $k_3 m_{ij} + m_{ij}^* = 1$ , then for all  $\xi \in L_{\mathcal{F}_0}^p([-\tau, 0], \mathbb{R}^n)$ , the trivial solution of system (3.20) is  $p$ th moment exponentially stable, where  $p \geq 2$  is a constant.

In Corollary 3.3, if we take  $k_1 = k_2 = k_3 = 1$ ,  $\nu_k = \mu_k = p - 1$ ,  $\rho_k = p - 2$ ,  $\xi_{ij} = \eta_{ij} = p_{ij} = q_{ij} = (p - 1)/p$ ,  $m_{ij} = \mu_k = (p - 2)/p$ ,  $\xi_{ij}^* = \eta_{ij}^* = p_{ij}^* = q_{ij}^* = 1/p$ ,  $m_{ij}^* = \mu_k = 2/p$ , we have the following result.

**Corollary 3.4.** Under assumptions  $(H_1), (H_2), (H'_3), (H'_4), (H_5)$ , if there exists a positive diagonal matrix  $Q = \text{diag}(q_1, \dots, q_n)$ , such that

$$\lambda_1 > \lambda_2, \quad (3.24)$$

where

$$\lambda_1 = \min_{1 \leq i \leq n} \left\{ p \underline{\alpha}_i \gamma_i - (p-1) \sum_{j=1}^n \bar{\alpha}_i |a_{ij}| G_j - \frac{1}{q_i} \sum_{j=1}^n \bar{\alpha}_j q_j |a_{ji}| G_i - (p-1) \sum_{j=1}^n \bar{\alpha}_i |b_{ij}| G_j - \frac{p(p-1)}{2} \sum_{j=1}^n c_{ij}^0 - \frac{(p-1)(p-2)}{2} \sum_{j=1}^n c_{ij}^1 \right\}, \quad (3.25)$$

$$\lambda_2 = \max_{1 \leq i \leq n} \left[ \frac{1}{q_i} \sum_{j=1}^n q_j |b_{ji}| \bar{\alpha}_j G_i + (p-1) \frac{1}{q_i} \sum_{j=1}^n q_j c_{ji}^1 \right],$$

then for all  $\xi \in L_{\varphi_0}^p([-\tau, 0], R^n)$ , the trivial solution of system (3.20) is  $p$ th moment exponentially stable.

*Remark 3.5.* Model (3.20) has been studied in [40] and the main results in [40, Theorem 1, Corollaries 1 and 2] are the direct results of Corollary 3.4 in our paper when we choose  $p = 2$ .

*Remark 3.6.* When  $D_{ik} = 0$  ( $i = 1, n, k = i, \dots, m$ ) system (3.20) is reduced to the stochastic Cohen-Grossberg neural networks (1.3), which has been studied in [24, 28]. Unfortunately, the assumed condition  $(A_2)$  in [28] is not correct, a defect appearing in the main result in [28] when  $p = 2k + 1, k \in Z^+, x(t) < 0$ , just from the constructed Lyapunov function, one can find that the term " $x^{p/2}(t)$ " is a blemish. The constructed Lyapunov function should be replaced with  $V(t, x(t)) = \sum_{i=1}^n q_i |x_i(t)|^p$ . Noticing that  $\partial |x_i(t)|^p / \partial x_i = p |x_i|^{p-1} \text{sgn}\{x_i\} = p |x_i|^{p-2} x_i$ , we have  $(\partial |x_i(t)|^p / \partial x_i) \beta_i(x_i(t)) = p |x_i|^{p-2} x_i \beta_i(x_i(t))$ , so the assumed condition  $(A_2)$  should be revised as  $(H'_3)$ . On the other hand, there is an error appear in (1.3), the coefficient of  $\lambda_1$ , the term " $(p-1)(p-2)$ " should be replaced with  $((p-1)(p-2))/2$ , therefore, the main results obtained in [28] are somewhat errors. Obviously, Theorem 3.1 in our paper modifies and generalizes the main results in [28] greatly. Just choosing  $p = 2$ , one can get a set of corollary easily, which also generalizes the main results in [24].

*Remark 3.7.* When  $D_{ik} = 0$  ( $i = 1, n, k = i, \dots, m$ ),  $\sigma_{ij}(t, \cdot, \cdot) = 0$ , system (1.4) is reduced to a deterministic Cohen-Grossberg neural networks with time-varying delays model, just choosing some special parameters, using Theorem 3.1, one can get a set of corollary easily, which also generalizes some corresponding results in [4].

## 4. An Illustrative Example

In this section, a numerical example is presented to illustrate the correctness of our main result.

*Example 4.1.* Consider a two-dimensional stochastic reaction-diffusion Cohen-Grossberg neural networks with time-varying delays as follows:

$$\begin{aligned}
dy(t) = & \left( \sum_{k=1}^2 \frac{\partial}{\partial x_k} \left( D_{1k} \frac{\partial y_1}{\partial x_k} \right) \right) dt - \begin{pmatrix} \alpha_1(y_1(x,t)), & 0 \\ 0 & \alpha_2(y_2(x,t)) \end{pmatrix} \\
& \times \left[ \begin{pmatrix} \beta_1(y_1(x,t)), & 0 \\ 0 & \beta_2(y_2(x,t)) \end{pmatrix} \begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix} - \begin{pmatrix} a_{11}(t), & a_{12}(t) \\ a_{21}(t), & a_{22}(t) \end{pmatrix} \begin{pmatrix} \tanh(y_1(t)) \\ \tanh(y_2(t)) \end{pmatrix} \right. \\
& \left. - \begin{pmatrix} b_{11}(t), & b_{12}(t) \\ b_{21}(t), & b_{22}(t) \end{pmatrix} \begin{pmatrix} \tanh(y_1(t - \tau_1(t))) \\ \tanh(y_2(t - \tau_2(t))) \end{pmatrix} \right] dt \\
& + \sigma(t, y(t), y(t - \tau(t))) dw(t), \quad t \geq 0,
\end{aligned} \tag{4.1}$$

where  $\alpha_1(y_1(x,t)) = 3 + \sin(y_1(x,t))$ ,  $\alpha_2(y_2(x,t)) = 3 - \cos(y_2(x,t))$ ,  $\beta_1(y_1(x,t)) = (4 + (3t/100))y_1(x,t)$ ,  $\beta_2(y_2(x,t)) = (5 + (3t/100))y_2(x,t)$ ,  $a_{11}(t) = -1/8 - t/100$ ,  $a_{12}(t) = 1/4$ ,  $a_{21}(t) = 3/4$ ,  $a_{22}(t) = -1/4 - (1/200)t$ ,  $b_{11}(t) = -1/8 - (1/1600)t$ ,  $b_{12}(t) = 1/4$ ,  $b_{21}(t) = 1/8$ ,  $b_{22}(t) = -1/4 - (1/1600)t$ ,  $\tau_i(t)$  is a bounded positive function and  $\sigma : R_+ \times R^2 \times R^2 \rightarrow R^2 \times R^2$  satisfies

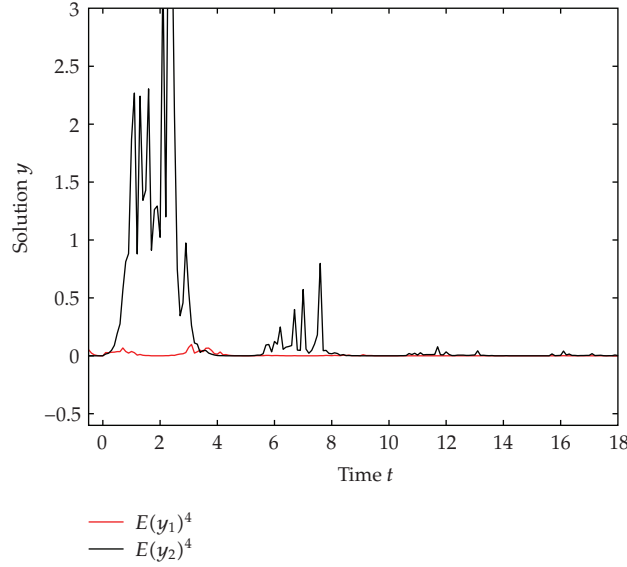
$$\text{trace} \left[ \sigma^T(t, u, v) \sigma(t, u, v) \right] \leq u_1^2 + u_2^2 + v_1^2 + v_2^2. \tag{4.2}$$

In the example, let  $p = 4$ ,  $D_{ik}$  be a positive constant, take  $c_{ij}^0 = c_{ij}^1 = q_i \equiv 1$ , by simple computation, we get

$$\int_{\Omega} D_{ik} \left( \frac{\partial y_i}{\partial x_k} \right)_{k=1}^2 \cdot \nabla y_i^{4-1} dx = \sum_{k=1}^2 \int_{\Omega} 3y_i^2 D_{ik} \left( \frac{\partial y_i}{\partial x_k} \right)^2 dx > 0. \tag{4.3}$$

According to Corollary 3.2, one can get that

$$\begin{aligned}
\lambda_1(t) = & \min_{1 \leq i \leq n} \left\{ p \underline{\alpha}_i \gamma_i(t) - (p-1) \sum_{j=1}^n \bar{\alpha}_i |a_{ij}(t)| G_j \right. \\
& - \frac{1}{q_i} \sum_{j=1}^n \bar{\alpha}_j q_j |a_{ji}(t)| G_i - (p-1) \sum_{j=1}^n \bar{\alpha}_i |b_{ij}(t)| G_j \\
& \left. - \frac{p(p-1)}{2} \sum_{j=1}^n c_{ij}^0(t) - \frac{(p-1)(p-2)}{2} \sum_{j=1}^n c_{ij}^1(t) \right\} = \frac{23}{2} + \frac{29}{400} t,
\end{aligned} \tag{4.4}$$



**Figure 1:** Numerical solution  $E(y_1^4(t)), E(y_2^4(t))$  of system (4.1).

$$\lambda_2(t) = \max_{1 \leq i \leq n} \left[ \sum_{j=1}^n \frac{q_j}{q_i} |b_{ji}(t)| \bar{\alpha}_j G_i + (p-1) \sum_{j=1}^n \frac{q_j}{q_i} c_{ji}^1(t) \right] = \frac{15}{2} + \frac{t}{400}, \tag{4.5}$$

using Corollary 3.2, system (4.1) is 4th moment exponential stable.

*Remark 4.2.* One can find that models considered in [20, 22–29] are special cases of model (1.4). To the best of our knowledge, few authors have considered the  $p$ th moment exponential stability for Stochastic reaction-diffusion Neural Networks with time-varying connection matrix and delays. It is assumed in [22, 23, 25, 26] that delays are constants, the delay functions appear in [29] are differential and their derivatives are simultaneously required to be not greater than 1, the activation functions appear in [22, 26] are bounded. Obviously, we have dropped out these basic assumptions in this paper.

It is obvious that the results in [20–30] and the references therein cannot be applicable to system (4.1) even if we remove the reaction-diffusion terms from the system for the connection matrix and delays considered in this example are time-varying. This implies that the results of this paper are essentially new. Just choose  $x \equiv$  constant these conclusions can be verified by the numerical simulations shown in Figure 1.

### 5. Conclusions

In this paper, stochastic Cohen-Grossberg neural networks with time-varying delays and reaction-diffusion have been investigated. All features of stochastic systems (especially the connection matrices and delays are time-varying) reaction-diffusion systems have been taken into account in the neural networks. Without requiring the differential and monotonicity of the activation functions and the symmetry of the connection matrices, a set of new sufficient conditions for checking  $p$ th moment exponential stability of the trivial solution

of the considered system is presented by using of Lyapunov function, stochastic analysis technique, and the generalized Halanay inequality. The proposed results generalized and improved some of the earlier published results greatly. The results obtained in this paper are independent of the magnitude of delays and diffusion effect, which implies that strong self-regulation is dominant in the networks. In addition, the methods used in this paper are also applicable to other neural networks, such as stochastic Hopfield neural networks with time-varying delays and reaction-diffusion terms and stochastic bidirectional associative memory (BAM) neural networks with time-varying delays and reaction-diffusion terms. If we remove the noise and reaction-diffusion terms from the system, the derived conditions for stability of general deterministic neural networks can be viewed as byproducts of our results.

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