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Research Article

Robust Stability Analysis and Synthesis for Switched Discrete-Time Systems with Time Delay

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The problems of robust stability analysis and synthesis for a class of uncertain switched time-delay systems with polytopic type uncertainties are addressed. Based on the constructive use of an appropriate switched Lyapunov function, sufficient linear matrix inequalities (LMIs) conditions are investigated to make such systems a uniform quadratic stability with an L_2-gain smaller than a given constant level. System synthesis is to design switched feedback schemes, whether based on state, output measurements, or by using dynamic output feedback, to guarantee that the corresponding closed-loop system satisfies the LMIs conditions. Two numerical examples are provided that demonstrate the efficiency of this approach.

1. Introduction

The stability analysis and synthesis problem of switched systems is one of the fundamental and challenging research topics, and various approaches has been obtained so far. For arbitrary switching law, a common Lyapunov function gives stability [1–3]. Liberzon and Hespanha have studied the global uniform asymptotic stability problem from the viewpoint of Lie algebra [4, 5]. On the other hand, Branicky [6], Johansen and Rantzer [7], respectively, have proposed the multiple Lyapunov function method for analysis and synthesis of switched systems with prescribed switching law. Furthermore, average dwell time technique [8–12] is an effective tool of choosing such switching law. In the recent literatures [13–16], stability analysis and synthesis of discrete time switched system were studied. A survey of switched systems problems have been proposed by Liberzon and Morse [5].

On the other hand, in many industrial hybrid systems such as power systems [17] and network control systems [13], time delay often occurs in the transition of the discrete states and the interior running of each subsystem. Therefore, more recently, much research attention has been devoted to the study of switched systems with time delay. In [18], L_2-gain properties under arbitrary switching for a class of switched symmetric delay systems were studied. In [19], sufficient conditions of asymptotical stability were established for switched linear delay systems. Based on switched Lyapunov function approach, H_∞ filtering problem of discrete-time switched systems with state delay was developed. In these papers mentioned above, only stable analysis is considered and state feedback results are proposed. In addition, output measurement and dynamic output feedback that are important synthesis methods for switched systems without delay are also expected to be effective for switched delay systems. However, no such results have been available up to now.

This paper studies the robust stability analysis and synthesis problems for uncertain switched delay systems with polytopic type uncertainties. Compared with the existing results on switched delay systems, the results of this paper have two features. Firstly, we give design of a novel switched Lyapunov function while the existing works commonly aim at the design of the common Lyapunov function. Secondly, dynamic output feedback is achieved while the existing literature usually addresses the state feedback control.

The rest of the paper is organized as follows. Section 2 briefly presents the uncertain switched time delay systems with polytopic type uncertainties and robust stability analysis with L_2-gain, based on the switched Lyapunov function. Section 3 presents the robust synthesis with the switched state feedback or output feedback design schemes, while Section 4 reports the results for switched dynamic output feedback. Finally, the main conclusions are summarized in Section 5.

Notation

The notation used in this paper is fairly standard. The superscript T stands for the matrix transposition, the notation $\|\cdot\|$ refers to the Euclidea vector norm, R^n denotes the n dimensional Euclidean space. In addition, in the symmetric block matrices or long matrix expressions, we use $*$ as an ellipsis for the terms that are introduced by symmetry, and $\operatorname{diag}\{\cdots\}$ stands for a block-diagonal matrix. A symmetric matrix $P > 0$ (≥ 0) means that P is a positive (semipositive) definite matrix.

2. Problem Formulation

Given a class of linear discrete time switched systems with time delay

$$x(k+1) = A_\sigma x(k) + A_{d\sigma} x(k-d) + B_\sigma u(k) + \Gamma_\sigma w(k), \tag{2.1}$$

$$z(k) = C_\sigma x(k) + C_{d\sigma} x(k-d) + D_\sigma u(k) + \Phi_\sigma w(k), \tag{2.2}$$

$$y(k) = L_\sigma x(k), \quad \sigma \in \{1, \ldots, N\}. \tag{2.3}$$

$x(k) \in R^n$, $u(k) \in R^m$, $w(k) \in R^q$, $y(k) \in R^p$, $z(k) \in R^r$ are the system state vector, control input, exogenous disturbance, measured output, and controlled output, respectively, and d presents the time delay. The particular mode σ at any given time instant may be a selective procedure characterized by a switching rule of the following form:

$$\sigma(k+1) = \delta(\sigma(k), x(k)). \tag{2.4}$$

The function $\delta(\cdot)$ is usually defined using a partition of the continuous state space. Let S denote the set of all selective rules. Therefore, the linear discrete time switched systems under consideration are composed of N subsystems, each of which is activated at particular switching instant.

For a switching mode $j \in \{1,\ldots,N\}$, the associated matrices $A_j,\ldots,\Phi_j$ contain uncertainties represented by a real convex-bounded polytopic model of the type

$$\begin{bmatrix} A_j & A_{dj} & B_j & \Gamma_j & L_j \\ C_j & C_{dj} & D_j & \Phi_j & 0 \end{bmatrix} \triangleq \left\{ \sum_{p=1}^{M_j} \lambda_{ip} \begin{bmatrix} A_{jp} & A_{djp} & B_{jp} & \Gamma_{jp} & L_{jp} \\ C_{jp} & C_{djp} & D_{jp} & \Phi_{jp} & 0 \end{bmatrix} \right\}, \tag{2.5}$$

where $\lambda_j = (\lambda_{j1}, \lambda_{j2}, \ldots, \lambda_{jM_j}) \in \Lambda_j$ belongs to the unit simplex of M_j vertices

$$\Lambda_j \triangleq \left\{ \lambda_j : \sum_{p=1}^{M_j} \lambda_{jp} = 1, \quad \lambda_{jp} \geq 0 \right\}, \tag{2.6}$$

where $A_{jp},\ldots, \Phi_{jp},\ p = 1,\ldots,M_j$ are known as real constant matrices of appropriate dimensions.

Distinct from (2.1)–(2.3) is the free switched system

$$x(k+1) = A_\sigma x(k) + A_{d\sigma} x(k-d) + \Gamma_\sigma w(k), \tag{2.7}$$

$$z(k) = C_\sigma x(k) + C_{d\sigma} x(k-d) + \Phi_\sigma w(k), \tag{2.8}$$

$\sigma \in \{1,\ldots,N\}$.

We have the following definitions.

Definition 2.1. Switched system (2.7)-(2.8) is Uniform Quadratic Stable (UQS), if there exist a Lyapunov functional $V(x,k) > 0$, a constant $\varepsilon > 0$, such that for all admissible uncertainties satisfying (2.5)-(2.6) and arbitrary switching rule $\sigma(\cdot)$ activating subsystem $j \in \{1,\ldots,N\}$ at instant $k+1$ and subsystem $i \in \{1,\ldots,N\}$ at instant k, the Lyapunov functional difference $\Delta V(x(k),k)$ satisfies

$$\Delta V(x(k),k) \triangleq V(x(k+1),k+1) - V(x(k),k) \leq -\varepsilon x^T(k)x(k),\ \forall x(k) \neq 0. \tag{2.9}$$

Definition 2.2. Given a scalar $\gamma \geq 0$, the L_2-gain Υ of switched system (2.7)-(2.8) over S is

$$\Upsilon \triangleq \inf\left\{ \gamma \geq 0 : \|z(k)\|_2 < \gamma^2 \|w(k)\|_2,\ \forall \sigma \in S,\ \forall \lambda_j \in \Lambda_j \right\}. \tag{2.10}$$

Definition 2.3. Switched system (2.7)-(2.8) is UQS with an L_2-gain $\Upsilon < \gamma$, if for all switching signal $\sigma \in S$ and for all admissible uncertainties satisfying (2.5)-(2.6), it is UQS and

$$\forall w(k) \neq 0, \quad \|z(k)\|_2 < \gamma^2 \|w(k)\|_2. \tag{2.11}$$

We consider the following switching quadratic Lyapunov function:

$$V_\sigma(x(k), k) \triangleq V_{1\sigma}(x(k), k) + V_2(x(k), k), \tag{2.12}$$

where

$$V_{1\sigma}(x(k), k) = x^T(k) P_\sigma x(k), \quad V_2(x(k), k) = \sum_{l=1}^{d} x^T(k-l) Q_\sigma x(k-l), \tag{2.13}$$

$P_\sigma = P_\sigma^T > 0$, $Q_\sigma = Q_\sigma^T > 0$.

Theorem 2.4. *The following statements are equivalent.*

(1) *There exist a switching Lyapunov function of the type* (2.12) *with $\sigma \in S$ and a scalar $\gamma > 0$ such that switched system* (2.7)-(2.8) *with polytopic representation* (2.5)-(2.6) *is UQS with L_2-gain $\Upsilon < \gamma$.*

(2) *There exist matrices $P_i = P_i^T > 0$, $Q_i = Q_i^T > 0$, $X_j = X_j^T > 0$, and a scalar $\gamma > 0$ satisfying the LMIs*

$$\begin{bmatrix} -P_i + Q_i & 0 & 0 & A_{ip}^T & C_{ip}^T \\ * & -Q_i & 0 & A_{dip}^T & C_{dip}^T \\ * & * & -\gamma^2 I & \Gamma_{ip}^T & \Phi_{ip}^T \\ * & * & * & -X_j & 0 \\ * & * & * & * & -I \end{bmatrix} < 0, \tag{2.14}$$

$(i, j) \in \{1, \ldots, N\} \times \{1, \ldots, N\}$, $p \in \{1, \ldots, M_i\}$.

Proof. (1)⇒(2) Suppose that there exist a constant $\gamma > 0$ and a switching Lyapunov function of the type (2.12). Let the switching rule $\sigma(\cdot)$ activates subsystem j at instant $k+1$ and subsystem i at instant k. Thus,

$$\begin{aligned}\Delta V_\sigma(x(k),k) &= V_{\sigma(k+1)}(x(k+1),k+1) - V_{\sigma(k)}(x(k),k)\\ &= x^T(k+1)P_j x(k+1) - x^T(k)P_i x(k) + \sum_{l=1}^{d} x^T(k+1-l)Q_j x(k+1-l)\\ &\quad - \sum_{l=1}^{d} x^T(k-l)Q_i x(k-l),\end{aligned} \tag{2.15}$$

Then, it can be shown that

$$\begin{aligned}&\Delta V_\sigma(x(k),k) + z^T(k)z(k) - \gamma^2 w^T(k)w(k)\\ &\quad= x^T(k+1)P_j x(k+1) - x^T(k)P_i x(k) + x^T(k)Q_j x(k)\\ &\qquad - x^T(k-d)Q_i x(k-d) + z^T(k)z(k) - \gamma^2 w^T(k)w(k)\\ &\quad= [A_i x(k) + A_{di}x(k-d) + \Gamma_i w(k)]^T P_j [A_i x(k) + A_{di}x(k-d) + \Gamma_i w(k)]\\ &\qquad - x^T(k)P_i x(k) + x^T(k)Q_j x(k) - x^T(k-d)Q_i \times x(k-d) - \gamma^2 w^T(k)w(k)\\ &\qquad + [C_i x(k) + C_{di}x(k-d) + \Phi_i w(k)]^T [C_i x(k) + C_{di}x(k-d) + \Phi_i w(k)]\\ &\quad= \begin{bmatrix} x(k)\\ x(k-d)\\ w(k)\end{bmatrix}^T \tilde{\Theta} \begin{bmatrix} x(k)\\ x(k-d)\\ w(k)\end{bmatrix} < 0,\end{aligned} \tag{2.16}$$

where

$$\tilde{\Theta} = \begin{bmatrix} -P_i + Q_j + A_i^T P_j A_i + C_i^T C_i & A_i^T P_j A_{di} + C_i^T C_{di} & A_i^T P_j \Gamma_i + C_i^T \Phi_i\\ * & -Q_i + A_{di}^T P_j A_{di} + C_{di}^T C_{di} & A_{di}^T P_j \Gamma_i + C_{di}^T \Phi_i\\ * & * & -\gamma^2 I + \Gamma_i^T P_j \Gamma_i + \Phi_i^T \Phi_i \end{bmatrix}. \tag{2.17}$$

Since $\|x(k)\ \ x(k-d)\ \ w(k)\| \neq 0$, then $\tilde{\Theta} < 0$.

By the Schur complement, $\widetilde{\Theta} < 0$ is equivalent to

$$\begin{bmatrix} -P_i + Q_j & 0 & 0 & A_i^T P_j & C_i^T \\ * & -Q_i & 0 & A_{di}^T P_j & C_{di}^T \\ * & * & -\gamma^2 I & \Gamma_i^T P_j & \Phi_i^T \\ * & * & * & -P_j & 0 \\ * & * & * & * & -I \end{bmatrix} < 0. \tag{2.18}$$

Applying the congruent transformation

$$\operatorname{diag}\{I, I, I, X_j, I\}, \quad X_j = P_j^{-1}, \tag{2.19}$$

we obtain

$$\begin{bmatrix} -P_i + Q_j & 0 & 0 & A_i^T & C_i^T \\ * & -Q_i & 0 & A_{di}^T & C_{di}^T \\ * & * & -\gamma^2 I & \Gamma_i^T & \Phi_i^T \\ * & * & * & -X_j & 0 \\ * & * & * & * & -I \end{bmatrix} < 0. \tag{2.20}$$

Upon using the vertex representation (2.5)-(2.6), we get (2.14) from (2.20).

(2)⇒(1) Follow by reversing the steps in the proof and applying Definitions 2.1–2.3 to the system (2.7)-(2.8) for all modes $(i, j) \in \{1, \ldots, N\} \times \{1, \ldots, N\}$ and using (2.5)-(2.6).

The proof is complete. □

Consider switched system (2.7)-(2.8) with $w(k) = 0$, a special case of Theorem 2.4 is provided.

Corollary 2.5. *The following statements are equivalent.*

(1) *There exists a switching Lyapunov function of type* (2.12) *with $\sigma \in S$ such that switched system* (2.7)-(2.8) *with $w(k) = 0$, and polytopic representation* (2.5)-(2.6) *is UQS.*

(2) *There exist matrices* $P_i = P_i^T > 0, \quad Q_i = Q_i^T > 0$, *and* $X_j = X_j^T > 0$ *satisfying the LMIs*

$$\begin{bmatrix} -P_i + Q_j & 0 & A_{ip}^T & C_{ip}^T \\ * & -Q_i & A_{dip}^T & C_{dip}^T \\ * & * & -X_j & 0 \\ * & * & * & -I \end{bmatrix} < 0, \tag{2.21}$$

$(i, j) \in \{1, \ldots, N\} \times \{1, \ldots, N\}$, $p \in \{1, \ldots, M_i\}$.

Since the exogenous disturbance $w(k) = 0$ in the switched system (2.7)-(2.8), we could let the coefficients Γ_i, Φ_i be zero matrices in the inequality (2.20). Directly deleting the third row and the third column from the matrix inequality (2.20), that is, deleting the term relevant to $w(k)$, we get the inequality (2.21).

3. Switched Control Synthesis

Extending on Section 2, we examine here the problem of switched control synthesis using either switched state feedback or output feedback design schemes.

3.1. Switched State Feedback

With reference to system (2.1)-(2.2), we consider that the arbitrary switching rule $\sigma(\cdot)$ activates subsystem i at an instant k. Our objective herein is to design a switched state feedback $u(k) = K_i x(k), \quad i \in \{1, \ldots, N\}$ such that the closed-loop system

$$\begin{aligned} x(k+1) &= (A_i + B_i K_i)x(k) + A_{di}x(k-d) + \Gamma_i w(k) \\ &= \overline{A}_i x(k) + A_{di}x(k-d) + \Gamma_i w(k), \end{aligned} \tag{3.1}$$

$$\begin{aligned} z(k) &= (C_i + D_i K_i)x(k) + C_{di}x(k-d) + \Phi_i w(k) \\ &= \overline{C}_i x(k) + C_{di}x(k-d) + \Phi_i w(k) \end{aligned} \tag{3.2}$$

is UQS with an L_2-gain $\Upsilon < \gamma$.

Theorem 3.1. *Switched system* (3.1)-(3.2) *is UQS with an* L_2*-gain* $\Upsilon < \gamma$, *if there exist matrices* $P_i = P_i^T > 0, \quad Q_i = Q_i^T > 0, \quad X_j = X_i^T > 0, \quad Z_i, T_i = X_i Q X_i$, *and a scalar* $\gamma > 0$, *such that the* *following LMIs hold:*

$$\begin{bmatrix} -X_i + T_j & 0 & 0 & X_i A_{ip}^T + Z_i^T B_{ip}^T & X_i C_{ip}^T + Z_i^T D_{ip}^T \\ * & -T_i & 0 & X_i A_{dip}^T & X_i C_{dip}^T \\ * & * & -\gamma^2 I & \Gamma_{ip}^T & \Phi_{ip}^T \\ * & * & * & -X_j & 0 \\ * & * & * & * & -I \end{bmatrix} < 0. \tag{3.3}$$

Moreover, the gain matrix is given by $K_i = Z_i X_i^{-1}$.

Proof. It follows from Theorem 2.4 that switched system (3.1)-(3.2) is UQS with an L_2-gain $\Upsilon < \gamma$, if, for $\forall (i,j) \in \{1,\ldots,N\} \times \{1,\ldots,N\}$, there exist matrices $P_i = P_i^T > 0$, $Q = Q^T > 0$, and $X_j = X_j^T > 0$ holding the following LMIs:

$$\begin{bmatrix} -P_i + Q_j & 0 & 0 & \overline{A}_i^T & \overline{C}_i^T \\ * & -Q_i & 0 & A_{di}^T & C_{di}^T \\ * & * & -\gamma^2 I & \Gamma_i^T & \Phi_i^T \\ * & * & * & -X_j & 0 \\ * & * & * & * & -I \end{bmatrix} = \begin{bmatrix} -P_i + Q_j & 0 & 0 & A_i^T + K_i^T B_i^T & C_i^T + K_i^T D_i^T \\ * & -Q_i & 0 & A_{di}^T & C_{di}^T \\ * & * & -\gamma^2 I & \Gamma_i^T & \Phi_i^T \\ * & * & * & -X_j & 0 \\ * & * & * & * & -I \end{bmatrix} < 0. \tag{3.4}$$

Applying the congruent transformation $\operatorname{diag}\{X_i, X_i, I, I, I\}$ to LMIs (3.4), and let $X_i = P_i^{-1}$, $Z_i = K_i X_i$, we obtain

$$\begin{bmatrix} -X_i + T_j & 0 & 0 & X_i A_i^T + Z_i^T B_i^T & X_i C_i^T + Z_i^T D_i^T \\ * & -T_i & 0 & X_i A_{di}^T & X_i C_{di}^T \\ * & * & -\gamma^2 I & \Gamma_i^T & \Phi_i^T \\ * & * & * & -X_j & 0 \\ * & * & * & * & -I \end{bmatrix} < 0. \tag{3.5}$$

Upon using vertex representation (2.5)-(2.6), we get (3.3) from (3.5). □

3.2. Example 1

In this example, we design a switched state feedback control for this system based on Theorem 3.1, switching occurs between two modes described by the following coefficients.

Mode 1: consider

$$
\begin{gathered}
A_{11} = \begin{bmatrix} 0.3 & 0.1 \\ -0.4 & 0.2 \end{bmatrix}, \qquad A_{12} = \begin{bmatrix} 0.2 & 0.1 \\ 0.6 & 0.3 \end{bmatrix}, \qquad B_{11} = \begin{bmatrix} 2 & 1 \\ 0.6 & 1 \end{bmatrix}, \\
B_{12} = \begin{bmatrix} 2 & 0.9 \\ 0.7 & 2 \end{bmatrix}, \qquad A_{d11} = \begin{bmatrix} 0 & 0.1 \\ 0.2 & 0 \end{bmatrix}, \qquad A_{d12} = \begin{bmatrix} 0 & -0.1 \\ -0.1 & 0 \end{bmatrix}, \\
\Gamma_{11} = \begin{bmatrix} 0.2 \\ 0.3 \end{bmatrix}, \quad \Gamma_{12} = \begin{bmatrix} 0.2 \\ 0.8 \end{bmatrix}, \qquad C_{11} = \begin{bmatrix} 0.1 & 0.3 \end{bmatrix}, \qquad C_{12} = \begin{bmatrix} 0.7 & 0.3 \end{bmatrix}, \\
D_{11} = \begin{bmatrix} 0.1 & 0.3 \end{bmatrix}, \qquad D_{12} = \begin{bmatrix} 0.9 & 0.3 \end{bmatrix}, \qquad C_{d11} = \begin{bmatrix} 0 & 2 \end{bmatrix}, \\
C_{d12} = \begin{bmatrix} 0 & -1 \end{bmatrix}, \qquad \Phi_{11} = 0.6, \qquad \Phi_{12} = 0.1.
\end{gathered}
\tag{3.6}
$$

Mode 2: consider

$$
\begin{gathered}
A_{21} = \begin{bmatrix} 0.1 & 0.2 \\ 0.3 & 0.4 \end{bmatrix}, \quad A_{22} = \begin{bmatrix} 0.2 & 0.1 \\ 0.6 & 0.3 \end{bmatrix}, \quad B_{21} = \begin{bmatrix} 1 & 4 \\ 3 & 1 \end{bmatrix}, \\
B_{22} = \begin{bmatrix} 2 & 0.9 \\ 0.7 & 2 \end{bmatrix}, \quad A_{d21} = \begin{bmatrix} 0 & 0.1 \\ 0.2 & 0 \end{bmatrix}, \quad A_{d22} = \begin{bmatrix} 0 & -0.1 \\ -0.1 & 0 \end{bmatrix}, \\
\Gamma_{21} = \begin{bmatrix} 0.1 \\ 0.5 \end{bmatrix}, \quad \Gamma_{22} = \begin{bmatrix} 0.2 \\ 0.8 \end{bmatrix}, \quad C_{21} = \begin{bmatrix} 0.6 & 0.2 \end{bmatrix}, \quad C_{22} = \begin{bmatrix} 0.9 & 0.3 \end{bmatrix}, \\
D_{21} = \begin{bmatrix} 0.8 & 0.3 \end{bmatrix}, \quad D_{22} = \begin{bmatrix} 0.9 & 0.3 \end{bmatrix}, \quad C_{d21} = \begin{bmatrix} -0.1 & 0 \end{bmatrix}, \\
C_{d22} = \begin{bmatrix} -1 & 0 \end{bmatrix}, \quad \Phi_{21} = 0.3, \quad \Phi_{22} = 0.1.
\end{gathered}
\tag{3.7}
$$

We set time delay $d = 1$, exogenous disturbance $w(k) = 0.02\sin(0.02\pi k)$, then the feasible solution of LMIs (3.3) is given by $\gamma = 2.1$, and

$$
\begin{gathered}
X_1 = \begin{bmatrix} 0.2031 & 0.0176 \\ 0.0176 & 0.1763 \end{bmatrix}, \qquad X_2 = \begin{bmatrix} 0.2056 & -0.3407 \\ -0.3407 & 1.6231 \end{bmatrix}, \\
Z_1 = \begin{bmatrix} 0.0024 & -0.0065 \\ -0.0473 & -0.0334 \end{bmatrix}, \qquad Z_2 = \begin{bmatrix} 0.0079 & -0.0993 \\ -0.0017 & -0.0844 \end{bmatrix}, \\
T_1 = \begin{bmatrix} 0.0661 & 0.0199 \\ 0.0199 & 0.1558 \end{bmatrix}, \qquad T_2 = \begin{bmatrix} 0.1276 & -0.2793 \\ -0.2793 & -0.9434 \end{bmatrix}, \\
Q_1 = \begin{bmatrix} 1.5700 & -0.0322 \\ -0.0322 & 5.0029 \end{bmatrix}, \qquad Q_2 = \begin{bmatrix} 2.8844 & 0.2320 \\ 0.2320 & 0.3284 \end{bmatrix}.
\end{gathered}
\tag{3.8}
$$

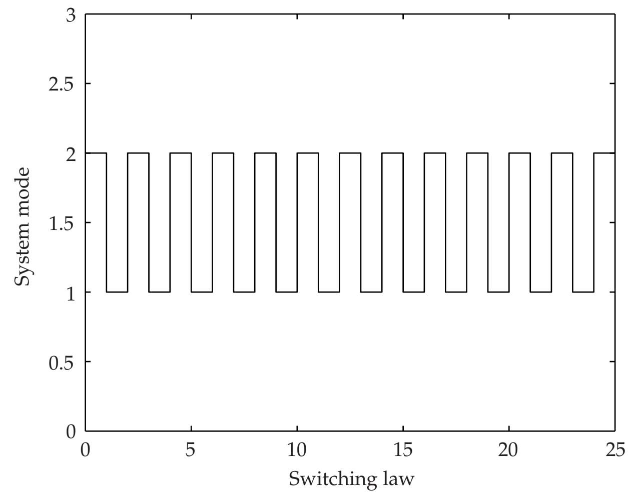

Figure 1: The switching signal of the switched systems.

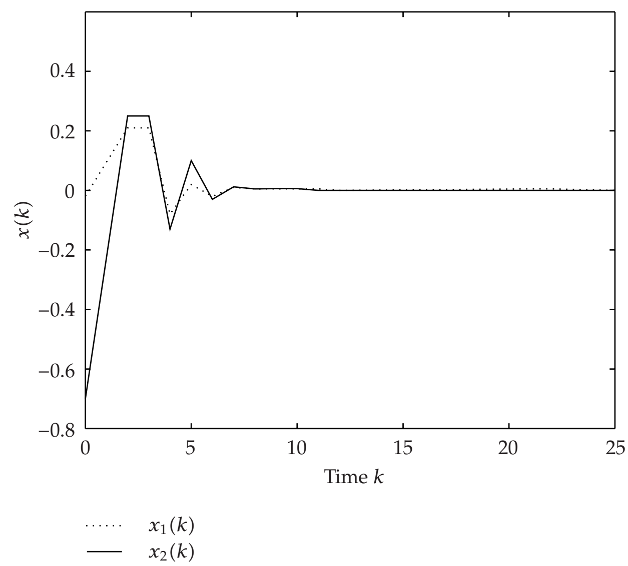

Figure 2: State response of the closed-loop system.

Since $K_i = Z_i X_i^{-1}$, the control gains are

$$K_1 = \begin{bmatrix} 0.0154 & -0.0386 \\ -0.2182 & -0.1678 \end{bmatrix}, \qquad K_2 = \begin{bmatrix} -0.0964 & -0.0814 \\ -0.1450 & -0.0825 \end{bmatrix}. \tag{3.9}$$

With the initial state $x(0) = [-0.02 \quad -0.71]^T$, and the following switching signal (Figure 1), the system state, control, and output signal are shown in Figures 2, 3, and 4, respectively.

From the simulation results, it can be clearly seen that the proposed control law guarantees the asymptotic stability of the closed-loop system.

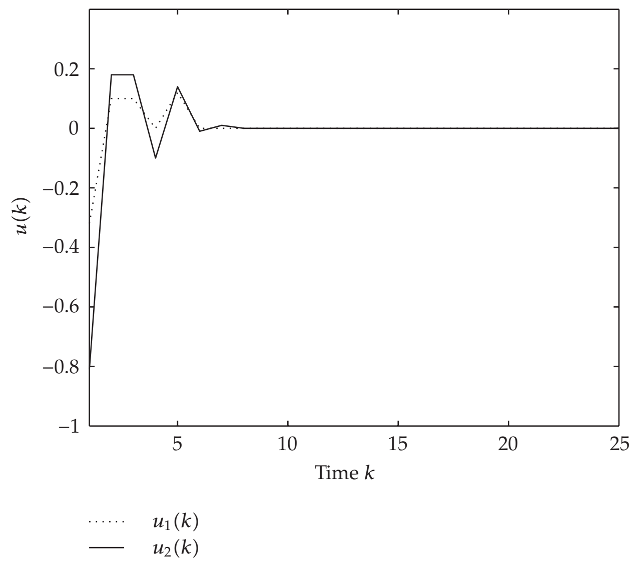

Figure 3: Control input of the switched discrete-time systems.

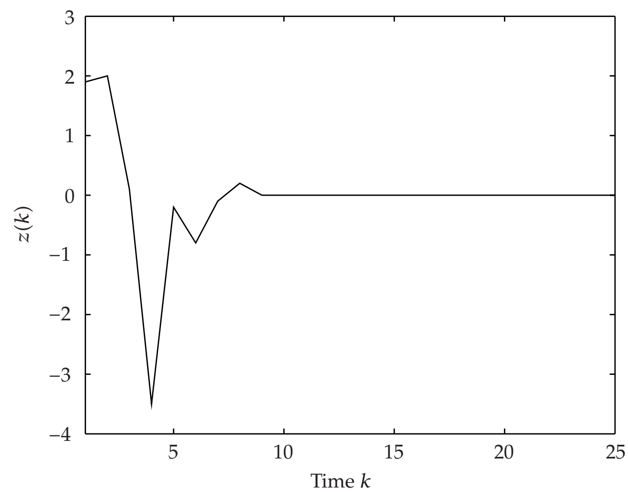

Figure 4: Controlled output signal of the closed-loop switched systems.

3.3. Switched Static Output Feedback

The objective of this subsection is to design switched output feedback $u(k) = G_i y(k)$, $i \in \{1, \ldots, N\}$, such that the closed-loop system

$$\begin{aligned} x(k+1) &= (A_i + B_i G_i L_i)x(k) + A_{di}x(k-d) + \Gamma_i w(k) = \widehat{A}_i x(k) + A_{di}x(k-d) + \Gamma_i w(k), \\ z(k) &= (C_i + D_i G_i L_i)x(k) + C_{di}x(k-d) + \Phi_i w(k) = \widehat{C}_i x(k) + C_{di}x(k-d) + \Phi_i w(k) \end{aligned} \tag{3.10}$$

is UQS with an L_2-gain $\Upsilon < \gamma$.

Before developing our main results, we give the following lemma.

Lemma 3.2 (see [14]). *Given a matrix $L_i \in R^{p\times n}$, rank $[L_i] = p$, and having the singular value decomposition form*

$$L_i = U_i \left[\Lambda_{pi} \;\; 0\right] V_i^T, \tag{3.11}$$

where $U_i \in R^{p\times p}, V_i \in R^{n\times n}$ are unitary matrices and $\Lambda_{pi} \in R^{p\times p}$ is a diagonal positive matrix. Let $X_i = X_i^T > 0, X_i \in R^{n\times n}$, then there exists a matrix $0 < E_i \in R^{p\times p}$, such that

$$L_i X_i = E_i L_i \tag{3.12}$$

if and only if

$$X_i = V_i \begin{bmatrix} X_{iu} & 0 \\ * & X_{iv} \end{bmatrix} V_i^T, \quad X_{iu} \in R^{p\times p}, \quad X_{iv} \in R^{(n-p)\times(n-p)}. \tag{3.13}$$

Thus, we get $E_i^{-1} = U_i \Lambda_{pi} X_{iu}^{-1} \Lambda_{pi}^{-1} U_i^{-1}$.

Theorem 3.3. *Consider switched system* (3.1)-(3.2) *with polytopic representation* (2.5)-(2.6) *subject to the output feedback control $u(k) = G_i y(k)$, and output matrix L_i having the SVD form $L_i = U_i[\Lambda_{pi} \;\; 0]V_i^T$, $\Lambda_{pi} \in R^{p\times p}$. The resulting closed-loop system is UQS with an L_2-gain $\Upsilon < \gamma$, if there exist matrices $0 < X_{iu} = X_{iu}^T \in R^{p\times p}$, $0 < X_{iv} = X_{iv}^T \in R^{(n-p)\times(n-p)}$, $0 < X_j = X_j^T$, $0 < T_i = T_i^T$, $E_i > 0$, R_i, $L_i X_i = E_i L_i$, and a scalar $\gamma > 0$, such that the LMIs*

$$\begin{bmatrix} -X_i + T_j & 0 & 0 & X_i A_{ip}^T + L_i^T R_i^T B_{ip}^T & X_i C_{ip}^T + L_i^T R_i^T D_{ip}^T \\ * & -T_i & 0 & X_i A_{dip}^T & X_i C_{dip}^T \\ * & * & -\gamma^2 I & \Gamma_{ip}^T & \Phi_{ip}^T \\ * & * & * & -X_j & 0 \\ * & * & * & * & -I \end{bmatrix} < 0 \tag{3.14}$$

have a feasible solution. Moreover, the feedback gain is given by $G_i = R_i U_i \Lambda_{pi} X_{iu}^{-1} \Lambda_{pi}^{-1} U_i^{-1}$, $i \in \{1, \ldots, N\}$.

Proof. In view of Theorem 2.4, we have

$$\begin{bmatrix} -X_i + T_j & 0 & 0 & X_i A_i^T + X_i L_i^T G_i^T B_i^T & X_i C_i^T + X_i L_i^T G_i^T D_i^T \\ * & -T_i & 0 & X_i A_{di}^T & X_i C_{di}^T \\ * & * & -\gamma^2 I & \Gamma_i^T & \Phi_i^T \\ * & * & * & -X_j & 0 \\ * & * & * & * & -I \end{bmatrix} < 0. \quad (3.15)$$

Let $R_i = G_i E_i$, then (3.15) can be rewritten as follows:

$$\begin{bmatrix} -X_i + T_j & 0 & 0 & X_i A_i^T + L_i^T R_i^T B_i^T & X_i C_i^T + L_i^T R_i^T D_i^T \\ * & -T_i & 0 & X_i A_{di}^T & X_i C_{di}^T \\ * & * & -\gamma^2 I & \Gamma_i^T & \Phi_i^T \\ * & * & * & -X_j & 0 \\ * & * & * & * & -I \end{bmatrix} < 0. \quad (3.16)$$

Upon using vertex representation (2.5)-(2.6), we get (3.14) from (3.16). According to Lemma 3.2, we obtain that $G_i = R_i U_i \Lambda_{pi} X_{iu}^{-1} \Lambda_{pi}^{-1} U_i^{-1},\ i \in \{1, \ldots, N\}$. □

3.4. *Example 2*

Similar to Example 1, we wish to design a switched static output feedback control law based on Theorem 3.3. Using all coefficients in Example 1 that adapt to Example 2.

Mode 1: consider

$$L_{11} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}, \qquad L_{12} = \begin{bmatrix} 0.6 & 0 \\ 0 & 0.4 \end{bmatrix}. \quad (3.17)$$

Mode 2: consider

$$L_{21} = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.6 \end{bmatrix}, \qquad L_{22} = \begin{bmatrix} 0.6 & 0 \\ 0 & 0.4 \end{bmatrix}. \quad (3.18)$$

In this example, we set time delay $d = 1$, the exogenous disturbance $w(k) = 0.02\sin(0.02\pi k)$, then the feasible solution of LMIs (3.14) is given by $\gamma = 2$ and

$$\begin{aligned}
X_1 &= \begin{bmatrix} 0.1955 & 0.0254 \\ 0.0254 & 0.1803 \end{bmatrix}, & X_2 &= \begin{bmatrix} 0.1655 & -0.1082 \\ -0.1082 & 0.5388 \end{bmatrix}, \\
R_1 &= \begin{bmatrix} 0.0115 & -0.0072 \\ -0.0927 & -0.0927 \end{bmatrix}, & R_2 &= \begin{bmatrix} -0.0071 & -0.0789 \\ -0.0425 & -0.0504 \end{bmatrix}, \\
E_{11} &= \begin{bmatrix} 0.1955 & 0.0254 \\ 0.0254 & 0.1803 \end{bmatrix}, & E_{12} &= \begin{bmatrix} 0.1955 & 0.0381 \\ 0.0169 & 0.1803 \end{bmatrix}, \\
E_1 &= \begin{bmatrix} 0.1955 & -0.0343 \\ -0.0195 & 0.1803 \end{bmatrix}, & E_{21} &= \begin{bmatrix} 0.1655 & -0.0721 \\ -0.1623 & 0.5388 \end{bmatrix}, \\
E_{22} &= \begin{bmatrix} 0.1655 & -0.1623 \\ -0.0721 & 0.5388 \end{bmatrix}, & E_2 &= \begin{bmatrix} 0.1655 & -0.1352 \\ -0.0992 & 0.5388 \end{bmatrix}, \\
T_1 &= \begin{bmatrix} 0.0688 & 0.0218 \\ 0.0218 & 0.1611 \end{bmatrix}, & T_2 &= \begin{bmatrix} 0.0920 & -0.1057 \\ -0.1057 & 0.1611 \end{bmatrix}, \\
Q_1 &= \begin{bmatrix} 0.0688 & 0.0078 \\ 0.0369 & 0.1631 \end{bmatrix}, & Q_2 &= \begin{bmatrix} 0.1176 & -0.2333 \\ -0.0665 & 0.2616 \end{bmatrix}.
\end{aligned} \tag{3.19}$$

Since $G_i = R_i E_i^{-1}$, the control gains are

$$G_1 = \begin{bmatrix} 0.0642 & -0.0521 \\ -0.4312 & -0.4320 \end{bmatrix}, \qquad G_2 = \begin{bmatrix} -0.1537 & -0.1849 \\ -0.3685 & -0.1860 \end{bmatrix}. \tag{3.20}$$

With the initial state $x(0) = [-0.54 \quad -0.56]^T$, and the switching signal as Example 1, the system state, control, and output signal are shown as follows.

4. Switched Dynamic Output Feedback

We consider the more general case and employ at every mode $i \in \{1, \ldots, N\}$, a switched dynamic output feedback scheme of the following form:

$$\begin{aligned} \xi(k+1) &= A_{ci}\xi(k) + B_{ci}y(k), \\ u(k) &= C_{ci}\xi(k). \end{aligned} \tag{4.1}$$

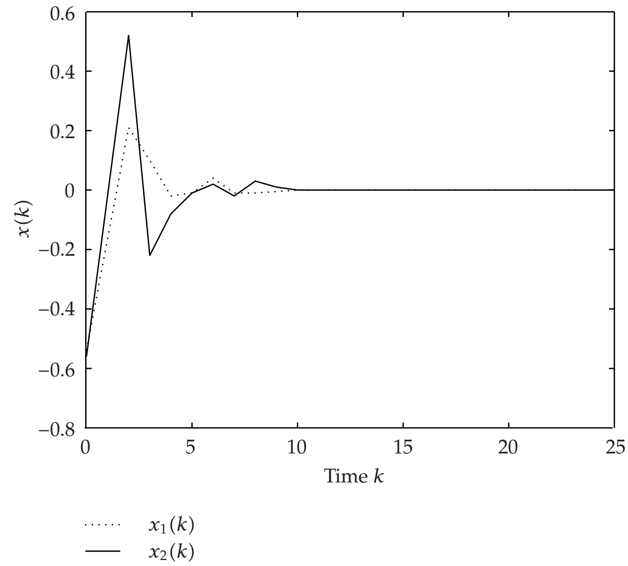

Figure 5: State response of the closed-loop switched discrete-time systems.

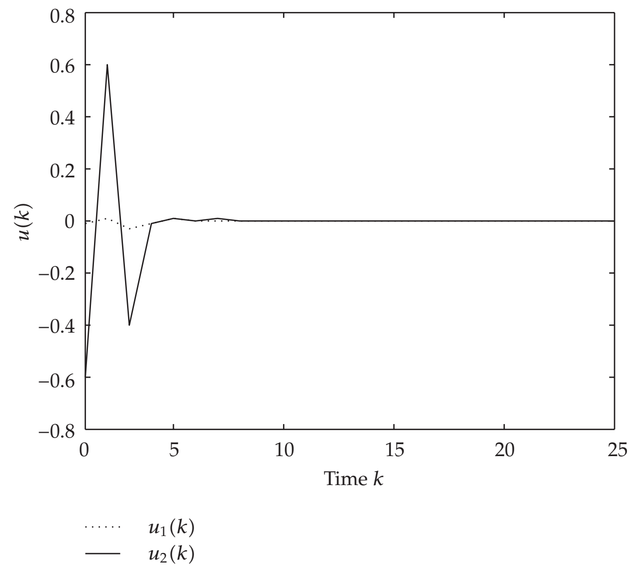

Figure 6: Control input of the switched discrete time systems.

Applying controller (4.1) to system (2.1)-(2.3) and letting $\eta^T(k) = [x^T(k) \;\; \xi^T(k)]$, we get the closed-loop system

$$\begin{aligned} \eta(k+1) &= \widetilde{A}_i\eta(k) + \widetilde{A}_{di}\eta(k-d) + \widetilde{\Gamma}_i w(k), \\ z(k) &= \widetilde{C}_i\eta(k) + \widetilde{C}_{di}\eta(k-d) + \Phi_i w(k), \end{aligned} \tag{4.2}$$

where

$$\tilde{A}_i = \begin{bmatrix} A_i & B_i C_{ci} \\ B_{ci} L_i & A_{ci} \end{bmatrix}, \qquad \tilde{A}_{di} = \begin{bmatrix} A_{di} & 0 \\ 0 & 0 \end{bmatrix}, \qquad \tilde{\Gamma}_i = \begin{bmatrix} \Gamma_i \\ 0 \end{bmatrix},$$
$$\tilde{C}_i = \begin{bmatrix} C_i & D_i C_{ci} \end{bmatrix}, \qquad \tilde{C}_{di} = \begin{bmatrix} C_{di} & 0 \end{bmatrix}. \tag{4.3}$$

Theorem 4.1. *Consider switched system* (4.2) *with polytopic representation* (2.5)-(2.6) *and output matrix L_i having the SVD form $L_i = U_i[\Lambda_{pi} \;\; 0]V_i^T$, $\Lambda_{pi} \in R^{p\times p}$, $L_i X_i = \tilde{E}_i L_i$. This system is UQS with an L_2-gain $\Upsilon < \gamma$, if there exist*

$$\tilde{P}_i = \begin{bmatrix} \tilde{P}_{si} & 0 \\ * & \tilde{P}_{ci} \end{bmatrix}, \quad \tilde{T}_i = \begin{bmatrix} \tilde{T}_{si} & 0 \\ * & \tilde{T}_{ci} \end{bmatrix} > 0, \quad \tilde{X}_i \triangleq \tilde{P}_i^{-1} = \begin{bmatrix} \tilde{X}_{si} & 0 \\ * & \tilde{X}_{ci} \end{bmatrix}, \quad 0 < \tilde{X}_{siu} = \tilde{X}_{siu}^T, \; 0 < \tilde{X}_{siv} = \tilde{X}_{siv}^T,$$
$$\tilde{Y}_j = \begin{bmatrix} \tilde{Y}_{sj} & 0 \\ * & \tilde{Y}_{cj} \end{bmatrix}, \quad 0 < \tilde{Y}_{sj} = \tilde{Y}_{sj}^T, \; 0 < \tilde{Y}_{cj} = \tilde{Y}_{cj}^T, \; \Omega_{ci}, \Pi_{ci}, \Psi_{ci}, \; (i,j) \in \{1,\ldots,N\} \times \{1,\ldots,N\}, \tag{4.4}$$

and a scalar $\gamma > 0$ satisfying the systems LMIs

$$\begin{bmatrix} -\tilde{X}_{si} + \tilde{T}_{si} & 0 & 0 & 0 & 0 & \tilde{X}_{si} A_{ip}^T & L_{ip}^T \Omega_{ci}^T & \tilde{X}_{si} C_{ip}^T \\ * & -\tilde{X}_{ci} + \tilde{T}_{ci} & 0 & 0 & 0 & \Pi_{ci}^T B_{ip}^T & \Psi_{ci}^T & \Pi_{ci}^T D_{ip}^T \\ * & * & -\tilde{T}_{si} & 0 & 0 & \tilde{X}_{si} A_{dip}^T & 0 & \tilde{X}_{si} C_{dip}^T \\ * & * & * & -\tilde{T}_{ci} & 0 & 0 & 0 & 0 \\ * & * & * & * & -\gamma^2 I & \Gamma_{ip}^T & 0 & \Phi_{ip}^T \\ * & * & * & * & * & -\tilde{Y}_{sj} & 0 & 0 \\ * & * & * & * & * & * & -\tilde{Y}_{cj} & 0 \\ * & * & * & * & * & * & * & -I \end{bmatrix} < 0. \tag{4.5}$$

Moreover, the gain matrix is given by $A_{ci} = \Psi_{ci} \tilde{X}_{ci}^{-1}$, $C_{ci} = \Pi_{ci} \tilde{X}_{ci}^{-1}$, and $B_{ci} = \Omega_{ci} \tilde{E}_i^{-1} = \Omega_{ci} U_i \times \Lambda_{pi} \tilde{X}_{siu}^{-1} \Lambda_{pi}^{-1} U_i^{-1}$.

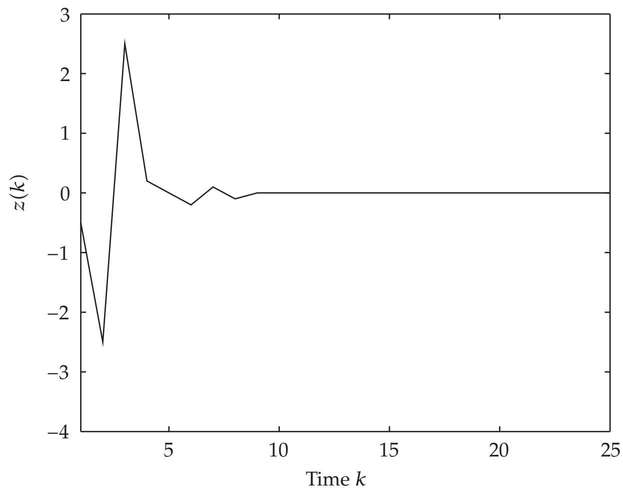

Figure 7: Output signal of the closed-loop switched discrete time systems.

Proof. In view of Theorem 2.4, we have

$$\begin{bmatrix} -\widetilde{X}_i + \widetilde{T}_i & 0 & 0 & \widetilde{X}_i \widetilde{A}_i^T & \widetilde{X}_i \widetilde{C}_i^T \\ * & -\widetilde{T}_i & 0 & \widetilde{X}_i \widetilde{A}_{di}^T & \widetilde{X}_i \widetilde{C}_{di}^T \\ * & * & -\gamma^2 I & \widetilde{\Gamma}_i^T & \Phi_i^T \\ * & * & * & -\widetilde{Y}_j & 0 \\ * & * & * & * & -I \end{bmatrix} < 0. \tag{4.6}$$

By applying (4.3)-(4.4) to (4.6), we obtain

$$\begin{bmatrix} -\widetilde{X}_{si} + \widetilde{T}_{si} & 0 & 0 & 0 & 0 & \widetilde{X}_{si} A_i^T & \widetilde{X}_{si} L_i^T B_{ci}^T & \widetilde{X}_{si} C_i^T \\ * & -\widetilde{X}_{ci} + \widetilde{T}_{ci} & 0 & 0 & 0 & \widetilde{X}_{ci} C_{ci}^T B_i^T & \widetilde{X}_{ci} A_{ci}^T & \widetilde{X}_{ci} C_{ci}^T D_i^T \\ * & * & -\widetilde{T}_{si} & 0 & 0 & \widetilde{X}_{si} A_{di}^T & 0 & \widetilde{X}_{si} C_{di}^T \\ * & * & * & -\widetilde{T}_{ci} & 0 & 0 & 0 & 0 \\ * & * & * & * & -\gamma^2 I & \Gamma_i^T & 0 & \Phi_i^T \\ * & * & * & * & * & -\widetilde{Y}_{sj} & 0 & 0 \\ * & * & * & * & * & * & -\widetilde{Y}_{cj} & 0 \\ * & * & * & * & * & * & * & -I \end{bmatrix} < 0. \tag{4.7}$$

Let $\Psi_{ci} = A_{ci}\widetilde{X}_{ci}, \Omega_{ci} = B_{ci}\widetilde{E}_i, \Pi_{ci} = C_{ci}\widetilde{X}_{ci}$, and using vertex representation (2.5)-(2.6), we get (4.5) from (4.7). □

5. Conclusions

We have examined L_2-gain analysis and control synthesis for a class of switched time-delay systems with convex-bounded parameter uncertainties in the system matrices using an appropriate switched Lyapunov functional. LMI-based feasibility conditions have been developed to ensure that the linear switched time-delay system with polytopic uncertainties is UQS with an L_2-gain. Switched feedback schemes have been designed using state output measurements and by using dynamic output feedback to guarantee that the corresponding closed-loop system enjoys the uniform quadratic stability with an L_2-gain smaller than a prescribed constant level.

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