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Research Article

On the Max-Type Equation $x_{n+1} = \max\{1/x_n, A_n x_{n-1}\}$ with a Period-Two Parameter

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We study the behavior of the well-defined solutions of the max type difference equation $x_{n+1} = \max\{1/x_n, A_n x_{n-1}\}$, $n = 0, 1, \ldots$, where the initial conditions are arbitrary nonzero real numbers and $\{A_n\}$ is a period-two sequence of real numbers with $A_n \in [0, \infty)$.

1. Introduction and Preliminaries

Recently, the study of max-type difference equations attracted a considerable attention. Although max-type difference equations are relatively simple in form, it is, unfortunately, extremely difficult to understand thoroughly the behavior of their solutions; see, for example [1–39] and the relevant references cited therein. Max-type difference equations stem from certain models in automatic control theory (see [1, 24]). For some papers on periodicity of difference equation, see, for example, [15, 16, 19, 22] and the relevant references cited therein.

In [9], Simsek et al. studied the behavior of the solutions of the following max-type difference equation:

$$x_{n+1} = \max\left\{x_{n-1}, \frac{1}{x_{n-1}}\right\}, \quad n = 0, 1, \ldots, \tag{1.1}$$

where the initial conditions are nonzero real numbers.

In [10], Simsek studied the behavior of the solutions of the following max-type difference equation:

$$x_{n+1} = \max\left\{x_{n-2}, \frac{1}{x_{n-2}}\right\}, \quad n = 0, 1, \ldots, \tag{1.2}$$

where the initial conditions are negative real numbers.

In [18], Elabbasy and Elsayed studied the behavior of the solutions of (1.2) where the initial conditions are nonzero real numbers.

In [20], Elsayed and Stević showed that every well-defined solution of the difference equation

$$x_{n+1} = \max\left\{\frac{A}{x_n}, x_{n-2}\right\}, \quad n = 0, 1, \ldots, \tag{1.3}$$

where $A \in \mathbb{R}$, is eventually periodic with period three.

In [21], Elsayed and Iričanin showed that every positive solution to the following third-order nonautonomous max-type difference equation:

$$x_{n+1} = \max\left\{\frac{A_n}{x_n}, x_{n-2}\right\}, \quad n = 0, 1, \ldots, \tag{1.4}$$

where A_n is a three-periodic sequence of positive numbers and is periodic with period three.

In [29], Yalçinkaya et al. studied the behavior of the solutions of the following max-type difference equation:

$$x_{n+1} = \max\left\{\frac{1}{x_n}, Ax_{n-1}\right\}, \quad n = 0, 1, \ldots, \tag{1.5}$$

where $A \in \mathbb{R}$ and initial conditions are nonzero real numbers.

In this paper, we study the behavior of the well-defined solutions of the max type difference equation

$$x_{n+1} = \max\left\{\frac{1}{x_n}, A_n x_{n-1}\right\}, \quad n = 0, 1, \ldots, \tag{1.6}$$

where the initial conditions are arbitrary nonzero real numbers and $\{A_n\}$ is a period-two sequence of real numbers with $A_n \in [0, \infty)$.

We need the following definitions and lemmas.

Definition 1.1. A sequence $\{x_n\}_{n=-k}^{\infty}$ is said to be eventually periodic with period p if there is $n_0 \in \{-k, \ldots, -1, 0, 1, \ldots\}$ such that $x_{n+p} = x_n$ for all $n \geq n_0$. If $n_0 = -k$, then we say that the sequence $\{x_n\}_{n=-k}^{\infty}$ periodic with period p.

We make two definitions regarding (1.6).

Definition 1.2. A right semicycle is a string of terms $x_l, \ldots, x_m$ with $l \geq 1$, $m \leq \infty$ such that $x_n = A_{n-1}x_{n-2}$ for all $n = l, \ldots, m$. Furthermore, if $l > 1$, $x_{l-1} = 1/x_{l-2}$, and if $m < \infty$, $x_{m+1} = 1/x_m$.

Definition 1.3. A left semicycle is a string of terms $x_l, \ldots, x_m$ with $l \geq 1$, $m \leq \infty$ such that $x_n = 1/x_{n-1}$ for all $n = l, \ldots, m$. Furthermore, if $l > 1$, $x_{l-1} = A_{n-2}x_{n-3}$, and if $m < \infty$, $x_{m+1} = A_m x_{m-1}$.

We give the following lemmas which show us the periodic behavior of the solutions of (1.6).

Lemma 1.4. *Assume that* $\{x_n\}_{n=-1}^{\infty}$ *is a well-defined solution of* (1.6). *If* $x_{n_0} = x_{n_0+2}$ *and* $x_{n_{0+1}} = x_{n_0+3}$ *such that* $n_0 \in \mathbb{N}_0 \cup \{-1\}$, *then the solution* $\{x_n\}_{n=-1}^{\infty}$ *is eventually periodic with period two.*

Proof. We prove that

$$x_{n_0} = x_{n_0+2m}, \qquad x_{n_{0+1}} = x_{n_0+2m+1}, \tag{1.7}$$

by induction. For $m = 1$, this is, assumption. Assume that (1.7) holds for all $1 \le n \le m_0 \in \mathbb{N}$. We may assume that n_0 is odd. Then, by the inductive hypothesis, we have

$$\begin{aligned} x_{n_0+2(m_0+1)} &= \max\left\{\frac{1}{x_{n_0+2m_0+1}}, A_{n_0+2m_0+1}x_{n_0+2m_0}\right\} \\ &= \max\left\{\frac{1}{x_{n_0+1}}, A_1 x_{n_0}\right\} = x_{n_0+1} = x_{n_0}, \end{aligned} \tag{1.8}$$

from this and the inductive hypothesis, we have

$$\begin{aligned} x_{n_0+2(m_0+1)+1} &= \max\left\{\frac{1}{x_{n_0+2m_0+2}}, A_{n_0+2m_0+2}x_{n_0+2m_0+1}\right\} \\ &= \max\left\{\frac{1}{x_{n_0}}, A_0 x_{n_0+1}\right\} = x_{n_0+3} = x_{n_0+1}, \end{aligned} \tag{1.9}$$

which completes the proof (the case n_0 is even similar, so it will be omitted). □

We omit the proof of the following lemma, since it can easily be obtained by induction.

Lemma 1.5. *Assume that* $\{x_n\}_{n=-1}^{\infty}$ *is a well-defined solution of* (1.6). *If* $x_{n_0}, x_{n_{0+1}} > 0$ *such that* $n_0 \in \mathbb{N}_0 \cup \{-1\}$, *then* $x_n > 0$ *for all* $n \ge n_0$.

Lemma 1.6. *Assume that* $\{x_n\}_{n=-1}^{\infty}$ *is a well-defined solution of* (1.6) *and* $A_n \in [0, 1)$. *If this solution is eventually positive, then it is eventually periodic with period two.*

Proof. Assume that $n_0 \in \mathbb{N}_0 \cup \{-1\}$ is the smallest index such that $x_n > 0$ for all $n \ge n_0$. Then, we have

$$x_{n+1}x_n = \max\{1, A_n x_n x_{n-1}\} \quad \forall n \ge n_0 + 1. \tag{1.10}$$

Using this, we have

$$
\begin{aligned}
x_{n_0+2}x_{n_0+1} &= \max\{1, A_{n_0+1}x_{n_0+1}x_{n_0}\},\\
x_{n_0+3}x_{n_0+2} &= \max\{1, A_{n_0+2}x_{n_0+2}x_{n_0+1}\}\\
&= \max\{1, A_{n_0+2}, A_{n_0+2}A_{n_0+1}x_{n_0+1}x_{n_0}\}\\
&= \max\{1, A_{n_0+2}A_{n_0+1}x_{n_0+1}x_{n_0}\},\\
x_{n_0+4}x_{n_0+3} &= \max\{1, A_{n_0+3}A_{n_0+2}A_{n_0+1}x_{n_0+1}x_{n_0}\},\\
&\vdots
\end{aligned}
\tag{1.11}
$$

then we get

$$x_{n_0+k+1}x_{n_0+k} = \max\left\{1, x_{n_0+1}x_{n_0}\prod_{i=1}^{k} A_{n_0+i}\right\} \quad \forall k \geq 1. \tag{1.12}$$

Observe that there exists a positive integer k such that

$$x_{n_0+1}x_{n_0}\prod_{i=1}^{k} A_{n_0+i} \leq 1. \tag{1.13}$$

From this directly follows that $\{x_n\}_{n=-1}^{\infty}$ is eventually periodic with period two. □

Lemma 1.7. *Equation* (1.6) *has no right semicycle with an infinite terms for the positive initial conditions and* $0 < A_0, A_1 < 1$.

Proof. Conversely, assume that (1.6) has a right semicycle with an infinite terms. And, let $\{a_n\}$ be periodic sequence of natural numbers with period two such that $(a_n) = (0, 1, 0, 1, \ldots)$. Without loss of generality, we denote by x_1 the first term of right semicycle with an infinite terms. There is at least $n_0 \in \mathbb{N}$. For all $n > n_0$, we can write

$$
\begin{aligned}
x_n &= \max\left\{\frac{1}{A_0^{[|n/2|]a_n}A_1^{[|n/2|]a_{n+1}}x_{-1}^{a_n}x_0^{a_{n+1}}}, A_0^{[|(n+1)/2|]a_{n+1}}A_1^{[|(n+1)/2|]a_n}x_{-1}^{a_{n+1}}x_0^{a_n}\right\}\\
&= A_0^{[|(n+1)/2|]a_{n+1}}A_1^{[|(n+1)/2|]a_n}x_{-1}x_0,
\end{aligned}
\tag{1.14}
$$

which implies

$$A_0^{[(n+1)/2]a_{n+1}+[n/2]a_n}A_1^{[(n+1)/2]a_n+[n/2]a_{n+1}}x_{-1}x_0 > 1 \quad \forall n > n_0. \tag{1.15}$$

But this is a contradiction which completes the proof. □

We omit the proof of the following lemma, since it can easily be obtained similarly.

Lemma 1.8. *Equation* (1.6) *has no right semicycle with an infinite terms for the negative initial conditions and* $A_0, A_1 > 1$.

2. Main Results

Since A_n is a two periodic, it has the form $(A_0, A_1, A_0, A_1, \ldots)$. If $A_0 = A_1 = 0$, then (1.6) becomes $x_{n+1} = 1/x_n$, from which it follows that every well-defined solution is periodic with period two. Hence, in the sequel, we will consider the case when at least one of A_0 and A_1 is not zero.

2.1. *The Case* $0 < A_0, A_1 \le 1$

Theorem 2.1. *If $0 < A_0, A_1 \le 1$ and at least one of the initial conditions is arbitrary positive real number, then every well-defined solution of* (1.6) *is eventually periodic with period two.*

Proof. Firstly, assume that $x_{-1}, x_0 > 0$. Then, we have $x_1 = \max\{1/x_0, A_0 x_{-1}\}$. There are two cases to be considered.

(a) If $A_0 x_{-1} x_0 \le 1$, then $x_1 = 1/x_0$. Hence,

$$\begin{aligned} x_2 &= \max\left\{\frac{1}{x_1}, A_1 x_0\right\} = \max\{x_0, A_1 x_0\} = x_0, \\ x_3 &= \max\left\{\frac{1}{x_2}, A_2 x_1\right\} = \max\left\{\frac{1}{x_0}, \frac{A_0}{x_0}\right\} = \frac{1}{x_0}, \\ x_4 &= \max\left\{\frac{1}{x_3}, A_3 x_2\right\} = \max\{x_0, A_1 x_0\} = x_0. \end{aligned} \tag{2.1}$$

From Lemma 1.4, the result follows.

(b) If $A_0 x_{-1} x_0 > 1$, then $x_1 = A_0 x_{-1}$. We have

$$x_2 = \max\left\{\frac{1}{x_1}, A_1 x_0\right\} = \max\left\{\frac{1}{A_0 x_{-1}}, A_1 x_0\right\}. \tag{2.2}$$

There are two subcases to be considered.

(b$_1$) If $A_0 A_1 x_{-1} x_0 \le 1$, then $x_2 = 1/(A_0 x_{-1})$. Hence,

$$\begin{aligned} x_3 &= \max\left\{\frac{1}{x_2}, A_2 x_1\right\} = \max\left\{A_0 x_{-1}, A_0^2 x_{-1}\right\} = A_0 x_{-1}, \\ x_4 &= \max\left\{\frac{1}{x_3}, A_3 x_2\right\} = \max\left\{\frac{1}{A_0 x_{-1}}, \frac{A_1}{A_0 x_{-1}}\right\} = \frac{1}{A_0 x_{-1}}. \end{aligned} \tag{2.3}$$

From Lemma 1.4, the result follows in this case.

(b$_2$) If $A_0 A_1 x_{-1} x_0 > 1$, then $x_2 = A_1 x_0$. We have

$$x_3 = \max\left\{\frac{1}{x_2}, A_2 x_1\right\} = \max\left\{\frac{1}{A_1 x_0}, A_0^2 x_{-1}\right\}. \tag{2.4}$$

There are two subcases to be considered.

(b_{21}) If $A_0^2 A_1 x_{-1} x_0 \le 1$, then $x_3 = 1/(A_1 x_0)$. We have

$$x_4 = \max\left\{\frac{1}{x_3}, A_3 x_2\right\} = \max\left\{A_1 x_0, A_1^2 x_0\right\} = A_1 x_0,$$
$$x_5 = \max\left\{\frac{1}{A_1 x_0}, A_4 x_3\right\} = \max\left\{\frac{1}{A_1 x_0}, \frac{A_0}{A_1 x_0}\right\} = \frac{1}{A_1 x_0}. \tag{2.5}$$

From Lemma 1.4, the result follows in this case.

(b_{22}) If $A_0^2 A_1 x_{-1} x_0 > 1$, then $x_3 = A_0^2 x_{-1}$. The result follows Lemma 1.7.

Secondly, assume that $x_0 < 0 < x_{-1}$, then we have

$$x_1 = \max\left\{\frac{1}{x_0}, A_0 x_{-1}\right\} = A_0 x_{-1} > 0,$$
$$x_2 = \max\left\{\frac{1}{x_1}, A_1 x_0\right\} = \frac{1}{x_1} > 0. \tag{2.6}$$

From Lemmas 1.5 and 1.6, the result follows (the case $x_{-1} < 0 < x_0$ is similar, so it will be omitted) which completes the proof.

□

Remark 2.2. If $0 < A_0, A_1 \le 1$ and $x_{-1}, x_0 < 0$, then every well-defined solution of (1.6) is not periodic.

2.2. *The Case* $A_0 = 0 < A_1 < 1$ *or* $A_1 = 0 < A_0 < 1$

Theorem 2.3. *If* $A_0 = 0 < A_1 < 1$ *or* $A_1 = 0 < A_0 < 1$, *then every well-defined solution of* (1.6) *is eventually periodic with period two.*

Proof. First assume that $A_1 = 0 < A_0 < 1$. Then, we have

$$x_1 = \max\left\{\frac{1}{x_0}, A_0 x_{-1}\right\} > 0,$$
$$x_2 = \max\left\{\frac{1}{x_1}, A_1 x_0\right\} > 0. \tag{2.7}$$

From Lemmas 1.5 and 1.6, the result follows. The case $A_0 = 0 < A_1 < 1$ is similar, so it will be omitted. □

2.3. *The Other Cases*

If at least one of A_0 and A_1 greater than one, then we have the well-defined solutions of (1.6), where the positive initial conditions are not periodic. So, there are many cases in which solutions of (1.6) are not periodic. If the solutions of (1.6) are not periodic, then general solution of (1.6) can be obtained for many subcases.

Theorem 2.4. *Assume that* $\{x_n\}_{n=-1}^{\infty}$ *is a well-defined solution of* (1.6) *for* $A_0 > 1$ *and* $A_1 = 0$.

(a) *If* $x_1 = 1/x_0$ *and* $x_0, x_{-1} > 0$ *or* $x_{-1} < 0 < x_0$, *then*

$$x_n = \left(\frac{x_0}{A_0^{[|(n-1)/2|]}}\right)^{(-1)^n}. \tag{2.8}$$

(b) *If* $x_1 = A_0 x_{-1}$ *and* $x_0, x_{-1} > 0$ *or* $x_0 < 0 < x_{-1}$, *then*

$$x_n = \left(\frac{1}{A_0^{[|(n+1)/2|]} x_{-1}}\right)^{(-1)^n}. \tag{2.9}$$

Proof. (a) It can be proved by induction. Let $x_1 = 1/x_0$ and $x_0, x_{-1} > 0$. For $n = 1$, (2.8) holds. Assume that (2.8) holds for all $1 \le m \le m_0$. We may assume that $m_0 \in \mathbb{N}$ is even (the case m_0 is odd is similar, so it will be omitted). Then, by the inductive hypothesis, we have

$$\begin{aligned} x_{m_0+1} &= \max\left\{\frac{1}{x_{m_0}}, A_{m_0} x_{m_0-1}\right\} \\ &= \max\left\{\frac{A_0^{(m_0-2)/2}}{x_0}, \frac{A_0^{m_0/2}}{x_0}\right\} = \frac{A_0^{m_0/2}}{x_0} = \left(\frac{x_0}{A_0^{[|m_0/2|]}}\right)^{(-1)^{m_0+1}}, \end{aligned} \tag{2.10}$$

which completes the proof.

(b) Also, this case can be proved similarly. □

Now, we describe the behavior of solutions of (1.6) for some other cases. We omit the proof of the following theorem, since it can easily be obtained by induction.

Theorem 2.5. *Assume that* $\{x_n\}_{n=-1}^{\infty}$ *is a well-defined solution of* (1.6).

(a) *If* $A_0 = 0, A_1 > 1$ *and* $x_0, x_{-1} > 0$ (*or* $x_{-1} < 0 < x_0$), *then*

$$x_n = \left(A_1^{[|n/2|]} x_0\right)^{(-1)^n}. \tag{2.11}$$

(b) *If* $A_0, A_1 > 1$ *and* $x_0, x_{-1} > 0$ (*or* $x_{-1} < 0 < x_0, x_1 = 1/x_0$), *then*

$$x_n = \left(A_0^{a_{n+1}} A_1^{a_n}\right)^{[|n/2|]} x_0^{(-1)^n}. \tag{2.12}$$

(c) *If* $A_0, A_1 > 1$ *and* $x_0, x_{-1} > 0$, $x_1 = A_0 x_{-1}$, $x_2 = A_1 x_0$, *then*

$$x_n = \left(A_0^{[|(n+1)/2|] a_{n+1}} A_1^{[|(n+1)/2|] a_n}\right)^{[|n/2|]} x_{-1}^{a_{n+1}} x_0^{a_n}. \tag{2.13}$$

There are many different cases. The different cases can be obtained similarly.

Theorem 2.6. *If* $A_0, A_1 > 1$ *and initial conditions are negative, then every well-defined solution of* (1.6) *is eventually periodic with period two.*

Proof. Assume that $x_0, x_{-1} < 0$. Then,

$$x_1 = \max\left\{\frac{1}{x_0}, A_0 x_{-1}\right\}. \tag{2.14}$$

There are two cases to be considered.

(a) If $x_1 = 1/x_0$, then $x_2 = x_0$, $x_3 = 1/x_0$, $x_4 = x_4 = x_0$. Then, the result follows Lemma 1.4.

(b) If $x_1 = A_0 x_{-1}$, then $x_2 = \max\{1/(A_0 x_{-1}), A_1 x_0\}$. There are two subcases.

(b$_1$) If $x_2 = 1/(A_0 x_{-1})$, then $x_3 = A_0 x_{-1}$, $x_4 = 1/(A_0 x_{-1})$. Then the result follows Lemma 1.4.

(b$_2$) If $x_2 = A_0 x_{-1}$, then there will be subcases and from Lemmas 1.4 and 1.8 which completes the proof.

□

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