Research Article

# How to Implement Return Policies in a Two-Echelon Supply Chain? 

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#### Abstract

We integrate a retailer's return policy and a supplier's buyback policy within a modeling framework. In this setting, consumers decide whether to buy and then whether to return the product, the retailer sets the retail price, quantity, and refund price, and the supplier chooses the wholesale price and buyback price. Both the demand uncertainty and consumers' valuation uncertainty are considered; consumers realize their valuations only after purchase. We discuss four scenarios for each party in the supply chain that may offer or not offer return policy. We characterize each party's optimal decisions for all scenarios and we show that the supplier's best choice is to provide buyback policy and the retailer's optimal response is to set refund price to be the same as supplier's buyback price.


## 1. Introduction

There has been a growing trend towards the consumer returns in recent years. A large portion of customer returns may be nondefective because the consumer does not like the product as much as anticipated, or the returned product does not fit the customer's need or expectation. Sciarrotta [1] reports that the nondefective returns rate was very high. Lawton [2] points out that only about $5 \%$ of customer returns were truly defective. Accepting customer returns is a common practice in the retail industry, where it has the objective of keeping customers' loyalty and maintaining customer satisfaction. In practice, most retailers implement various customer returns policies, such as full refund, partial refund, or store credit. "Customer returns policy" refers here to the money back policy offered by a retailer to her customers. It reduces the customer's risk of having to keep an unwanted product.

Suppliers whose products are subject to uncertain demand face a problem of inducing retailers to stock those products. A supplier may attempt to compensate his retailers by
accepting returns of unsold goods for full or partial refunds of their purchase price. The practice of returns policy has been reported widely in both research literature and business, see Bose and Anand [3]. The main objective of the return policy is to mitigate the risk of overstocking, caused by uncertain demand that retailers face. Buyback policies are commonly used in many industries, especially for products with short life cycles such as books, CDs, holiday gifts, and computers.

Although the approaches to dealing with customer returns and the supplier's buyback policies have been well studied, a very few research integrates customer returns policy with a buyback policy. In reality, supply chain is a network of suppliers, retailers, and customers. Some electronic firms (e.g., HP, Lenovo, Nokia) procure modules from their suppliers and sell various fashion electronic products to their customers. To keep the customer's loyalty, the firms offer return policies to their customers; on the other hand, the upstream supplier might also offer return policies to encourage the firms to order more modules to improve the service level for their customers. So consideration of return policies should not be limited within only two levels.

We consider a two-echelon supply chain with a single supplier who supplies a product to a retailer, and the retailer sells the product to the uncertain market. They both offer a return policy to their adjacent downstream firm. The consumers return unsatisfactory product back to the retailer, then the retailer will return unsold items, together with the consumer returns, back to the supplier. In order to differentiate supplier-retailer refund policy from retailer-customer return policy, we use the term "return policy" to refer to the retailercustomer agreement, and "buyback policy" to refer specifically to arrangements between the supplier and his retailers. Most research on buyback agreements only considers unsold inventory resulting from demand uncertainty being returned to the supplier. We also consider a buyback agreement that includes customer returns.

In this paper, we develop an integrated model that investigates the supplier's buyback policy and the retailer's return policy. Based on firms' operational policies, we study both the demand uncertainty and the customers' valuation uncertainty. Specially, we model the above decisions as a Stackelberg game.

Stage 1. The supplier, acting as the leader, offers the retailer a take-it-or-leave-it contract, specifying the wholesale price and buyback price for the returned items.

Stage 2. Given the supplier's decisions, the retailer chooses the order quantity, retail price, and refund price for the customers' returns under both demand uncertainty and valuation uncertainty.

Stage 3. Market demand uncertainty is realized. Based on the retailer's decisions, consumers make two sequential decisions. Initially, facing uncertainty in their own valuations, they decide whether to purchase the product or not. If they buy the product, after realizing their own valuation, they go on to decide whether to keep or return the product.

Stage 4. Unsatisfied consumers return products back to retailer for a refund, the supplier buys back all leftovers from the retailer. These leftover units include units that were unsold and units that were sold but returned.

The purpose of this paper is to shed light on the supplier's and retailer's optimal policy in the presence of customer valuation uncertainty and demand uncertainty. Considering supplier and retailer may offer or not offer return policy, we examine four combinations.

For each scenario, we derive each party's optimal decisions. Finally, we figure out that it is optimal for the supplier to present buyback policy and the retailer's ideal response refund price should be as same as supplier's buyback price.

In summary, this paper contributes to the literature by: (1) constructing a game model of two-echelon return policies that involves both demand uncertainty and valuation uncertainty; (2) analyzing the equilibrium solution of the model; (3) providing new insights for the return policies in a two echelon supply chain.

The remainder of this paper is organized as follows. Section 2 provides literature review. Section 3 introduces the basic model. In Section 4, we discuss both parties' optimal solutions for all scenarios. We conduct performance comparison under uniform distribution in Section 5. Finally, we provide concluding remarks and offer some directions for future work in Section 6.

## 2. Literature Review

The model setting we consider in this paper is a combination of three distinctive features: (1) a buyback policy offered by the upstream supplier to the downstream retailer; (2) a return policy offered by the retailer to the end consumer; (3) consumers are uncertain over the valuation. In the following, we provide a brief review of papers that relate to these model features.

A buyback contract is an arrangement where an upstream supplier agrees to provide a retailer credit for unsold product (see [4, 5]). Pasternack [6] first considers buyback policy for a seasonal product with stochastic demand under the newsvendor framework, with underage and overage costs as a vehicle to share the risks resulting from demand uncertainty. Most research on buyback agreements only considers unsold inventory resulting from demand uncertainty being returned to the supplier [6-8]. We consider a buyback agreement that also includes customer returns to retailers as that in Chen and Bell [9].

There is a stream of literatures on consumer returns. Some papers investigate how to prevent inappropriate returns from consumers with no intention of keeping their purchase (Hess et al. [10-12]). Most of the literatures focus on full refunds, including Marvel and Peck [13], Xiao et al. [14], Chen and Bell [9, 15, 16]. Su [17] discusses partial refund and shows that it is optimal for a retailer to offer the partial refund policy. We do not restrict our analysis on full refund policy.

Fit risk is an important component of purchase uncertainty. For example, when purchasing clothes, buyers are not completely sure whether the new clothes fit into their daily life and the rest of their wardrobe; children may not like the musical instrument that their parents bought for them [18]. Dana and Spier [19], Xie and Shugan [20] consider firms may wish to offer advance purchase discounts to compensate them for bearing risk when consumers face valuation uncertainty. Alexandrov and Lariviere [21] study the value of offering reservations to consumers who face valuation uncertainty. Su [17], Xiao et al. [14] study the role of consumer returns policy as a risk-sharing mechanism when valuations of consumers are uncertain. Along the line, we characterize the consumers' purchase and return behavior depend on the retailer's decision variables (such as retail price and refund).

Only a few researchers integrate the buyback policy and consumer return policy. The supplier's buyback policy is exogenously given in Su [17], he studies possible approaches to achieve the channel coordination. In Xiao et al. [14]'s model, the retailer only makes quantity decision, the supplier finds an appropriate buyback policy to coordinate the whole supply chain. In Chen and Bell [16], the retailer implements a full refund policy to consumers and
retail price is determined exogenously. Chen and Bell [9] investigate full refund policy and customer returns are a fixed proportion of quantity sold. Different to these papers, both the buyback policy and refund policy are decisions in our model.

Overall, our model incorporates consumers' valuation uncertainty as well as demand uncertainty. We also consider consumer returns that depend on the retailer's decision variables. This work integrates the supplier's buyback contract, retailer's pricing, refund policy, and consumer's purchase and return behavior within a unified framework.

## 3. The Model

Consider a supply chain consisting of one supplier (he), one retailer (she), and end consumers. The upstream supplier initiates the process by offering a wholesale price $w$, for which he will sell to the retailer prior to the selling season, and a refund price $r^{S}$, for which he will buy items back from the retailer at the end of selling season. The supplier's production cost is $c$ per unit. In response to the offered wholesale and buyback prices, the retailer determines a quantity $Q$ for the supplier to deliver, a retail price $p$ during the season, and a refund price $r^{R}$ to the consumer at the end of the season. $r^{R}=0$ means that no return policy is provided and $r^{R}=p$ represents a full refund policy, that is, $100 \%$ money-backguarantee is offered to ensure consumer satisfaction [22,23]. We refer to return policy with $0<r^{R}<p$ as partial refund policy. Unsold goods and returned items have no scrap value to either the retailer or the supplier. To make our results more transparent, we let the other modeling elements be as simple as possible.

Similar to Che [24], we assume that each consumer purchases at most a unit product and consumers do not fully know their preferences for the products until they obtain some experiences with the products. Consumers make two sequential decisions. First, they decide whether to purchase the product or not. If they buy the product, they then decide whether to keep it or not after privately observing their own expected valuation.

As in Su [17], we assume the customer valuations $V$ are identically and independently drawn from the distribution with CDF $G(\cdot)$, PDF $g(\cdot)$. The consumers seek to maximize individual expected surplus. If a consumer chooses to buy, the consumer will keep the product if his valuation is at least as high as the refund $\left(V \geq r^{R}\right)$, otherwise, he will return this product for refund $r^{R}$. So in the first stage, should the consumer decides to buy, his expected utility will be $E \max \left(V, r^{R}\right)$, this is the customer's reservation price. In other words, when retail price $p$ exceeds $E \max \left(V, r^{R}\right)$, consumer leaves the market without making a purchase; however, when $p$ is less than or equal to $E \max \left(V, r^{R}\right)$, every consumer will buy the product. The market demand $X$ is a random factor with CDF $F(\cdot)$ and PDF $f(\cdot)$. Let $h(x)=f(x) /[1-F(x)]$ denote the failure rate function of the demand function. From the retailer's perspective, demand is

$$
D= \begin{cases}X, & \text { if } E \max \left(V, r^{R}\right) \geq p  \tag{3.1}\\ 0, & \text { otherwise }\end{cases}
$$

Then the retailer makes three decisions: retail price $p$, order quantity $Q$, and refund price $r^{R}$.

Her profit function is given as:

$$
\begin{align*}
\prod^{R}\left(p, Q, r^{R}\right)= & p E \min (D, Q)-r^{R} G\left(r^{R}\right) E \min (D, Q) \\
& +r^{S}\left[Q-E \min (D, Q)+G\left(r^{R}\right) E \min (D, Q)\right]-w Q \\
= & \underbrace{p \bar{G}\left(r^{R}\right) E \min (D, Q)}_{\text {net sale }}+\underbrace{\left(p-r^{R}+r^{S}\right) G\left(r^{R}\right) E \min (D, Q)}_{\text {returned by customer }}  \tag{3.2}\\
& +\underbrace{r^{S}(Q-E \min (D, Q))}_{\text {not sold }}-w Q .
\end{align*}
$$

The first term is the retailer's revenue from net sale (total sales minus total returns), each returned unit yields $p-r^{R}$ from the consumer and $r^{S}$ from the supplier, and each unsold item yields only the buyback value $r^{S}$. The last term is the retailer's order cost.

Similarly, we can give the supplier's profit function as:

$$
\begin{equation*}
\prod^{S}\left(w, r^{S}\right)=\underbrace{w \bar{G}\left(r^{R}\right) E \min (D, Q)}_{\text {net sale }}+\underbrace{\left(w-r^{S}\right)\left[Q-\bar{G}\left(r^{R}\right) E \min (D, Q)\right]}_{\text {returned by retailer }}-c Q \tag{3.3}
\end{equation*}
$$

In (3.3), we see that the supplier gains $w$ from each product that is sold and kept by the consumer, each returned unit yields $w-r^{S}$ from the retailer (unsold items and returned items).

## 4. Model Analysis

Considering each party in the supply chain system may offer or not offer return policy, we discuss four combinations in this section. There are neither the supplier nor the retailer offers a return policy (i.e., $r^{S}=0, r^{R}=0$ ); the retailer offers a return policy but the supplier does not (i.e., $r^{S}=0, r^{R} \geq 0$ ); the retailer offers no return policy but the supplier does (i.e., $r^{S} \geq$ $0, r^{R}=0$ ) and both the supplier and the retailer offer a return policy (i.e., $r^{S} \geq 0, r^{R} \geq 0$ ).

We introduce a dummy variable that takes the values 0 or 1 to indicate the absence or presence of return policy. We will use subscripts and superscripts to facilitate the expression of the model variables. The superscripts $R$ and $S$ will denote the retailer and the supplier. The first subscript refers to the supplier's decision and the second subscript characterizes the retailer's decision. For example, $\Pi_{0,1}^{S}$ is the supplier's profit of the setting, in which the supplier does not offer while the retailer provides a return policy, notations introduced later indicate the similar meanings.

### 4.1. Scenario 1: Supplier and Retailer Do Not Offer Return Policy

In this scenario, the returns are not accepted in both the supplier and retailer, which means $r^{S}=0$ and $r^{R}=0$. Now the retailer decides her retail price and order quantity which maximizes the expected profit:

$$
\begin{equation*}
\prod_{0,0}^{R}(p, Q)=p E \min (D, Q)-w Q \tag{4.1}
\end{equation*}
$$

Observe that when consumer returns are not accepted, the highest price consumers are willing to pay is $E V$, their expected valuation for the product. In this case, the retailer faces a standard newsvendor problem with the selling price $p_{0,0}^{*}=E V$ and stocking quantity satisfy $\bar{F}\left(Q_{0,0}^{*}\right)=w / E V$.

Anticipating this, the upstream supplier chooses his optimal wholesale price to maximize

$$
\begin{equation*}
\prod_{0,0}^{S}(w)=(w-c) Q_{0,0}^{*} \tag{4.2}
\end{equation*}
$$

The following proposition characterizes the supplier's optimal decision.
Proposition 4.1. If $d h(x) / d x>0$, the optimal $w_{0,0}^{*}$ is determined uniquely by:

$$
\begin{equation*}
w Q_{0,0}^{*} h\left(Q_{0,0}^{*}\right)-w+c=0 \tag{4.3}
\end{equation*}
$$

All the proofs are provided in the appendix.

### 4.2. Scenario 2: Supplier Does Not Offer Return Policy, Retailer Offer Return Policy

In this case, the returns are only accepted in the retailer, which means $r^{S}=0$ and $r^{R} \geq 0$. Now the retailer decides her retail price, refund price and order quantity which maximizes the expected profit:

$$
\begin{equation*}
\prod_{0,1}^{R}\left(p, Q, r^{R}\right)=\left[p \bar{G}\left(r^{R}\right)+\left(p-r^{R}\right) G\left(r^{R}\right)\right] E \min (D, Q)-w Q \tag{4.4}
\end{equation*}
$$

Proposition 4.2. The retailer's optimal price $p_{0,1}^{*}$, quantity $Q_{0,1}^{*}$, and refund $r_{0,1}^{R *}$ are given by

$$
\begin{gather*}
p_{0,1}^{*}=E V  \tag{4.5}\\
\bar{F}\left(Q_{0,1}^{*}\right)=\frac{w}{p_{0,1}^{*}}=\frac{w}{E V^{\prime}}  \tag{4.6}\\
r_{0,1}^{R *}=0 \tag{4.7}
\end{gather*}
$$

If the supplier does not offer buyback policy, he just find optimal wholesale price to maximize his profit:

$$
\begin{equation*}
\prod_{0,1}^{S}(w)=(w-c) Q_{0,1}^{*} . \tag{4.8}
\end{equation*}
$$

Proposition 4.3. If $d h(x) / d x>0$, the optimal $w_{0,1}^{*}$ is determined uniquely by:

$$
\begin{equation*}
w Q_{0,1}^{*} h\left(Q_{0,1}^{*}\right)-w+c=0 . \tag{4.9}
\end{equation*}
$$

Remember that the retailer's optimal refund price in this scenario is zero, so when the upstream supplier does not buyback returns, the retailer's optimal response is to choose not to provide return policy to customer either. Notice the other decisions are all the same as the correspondence in scenario 1 , so scenario 2 is degenerate into the simply scenario 1 .

From the analysis of scenario 1 and scenario 2, we find that if the upstream supplier do not provide buyback policy to the retailer, then the retailer's optimal response is do not allow return behavior of the downstream customer either.

### 4.3. Scenario 3: Supplier Offers Return Policy, Retailer Does Not Offer Return Policy

In this case, the returns are only allowed in the supplier, which means $r^{S} \geq 0$ and $r^{R}=0$. Now the retailer decides retail price and order quantity to maximize her expected profit:

$$
\begin{align*}
\prod_{1,0}^{R}(p, Q) & =p E \min (D, Q)+r^{S}[Q-E \min (D, Q)]-w Q  \tag{4.10}\\
& =\left(p-r^{S}\right) E \min (D, Q)-\left(w-r^{S}\right) Q .
\end{align*}
$$

If the retailer does not provide return policy to the consumers, the highest price consumers are willing to pay is $E V$. The retailer then faces a standard newsvendor problem with the selling price $p_{1,0}^{*}$ and stocking quantity $Q_{1,0}^{*}$ satisfying:

$$
\begin{gather*}
p_{1,0}^{*}=E V  \tag{4.11}\\
\bar{F}\left(Q_{1,0}^{*}\right)=\frac{w-r^{S}}{p_{1,0}^{*}-r^{S}}=\frac{w-r^{S}}{E V-r^{S}} . \tag{4.12}
\end{gather*}
$$

Observing the retailer's optimal response will be (4.11) and (4.12), the supplier determines $\left(w, r^{S}\right)$ to maximize his profit:

$$
\begin{align*}
\prod_{1,0}^{S}\left(w, r^{S}\right) & =(w-c) Q_{1,0}^{*}-r^{S}\left[Q_{1,0}^{*}-E \min \left(D, Q_{1,0}^{*}\right)\right]  \tag{4.13}\\
& =r^{S} E \min \left(D, Q_{1,0}^{*}\right)+\left(w-c-r^{S}\right) Q_{1,0}^{*}
\end{align*}
$$

We assume $r^{S} \leq w-c$, so the supplier can gain profit from producing products. The term $Q_{1,0}^{*}$ is shown in (4.12), which is a function of $w$ and $r^{S}$. We reduce (4.13) to an optimization problem over the single variable $r^{S}$ by first solving for the optimal value of $w$ as a function of $r^{S}$, and then substituting the result back into $\Pi_{1,0}^{S}\left(w, r^{S}\right)$ to search optimal $r^{S}$.

Proposition 4.4. For any fixed $r^{S}$, if $d h(x) / d x>0$, the optimal $w_{1,0}^{*}$ is determined uniquely as a function of $r^{S}$ :

$$
\begin{equation*}
\left(w_{1,0}^{*}-r^{S}\right) Q_{1,0}^{*} h\left(Q_{1,0}^{*}\right)-\left[r^{S} \bar{F}\left(Q_{1,0}^{*}\right)+w_{1,0}^{*}-r^{S}-c\right]=0 \tag{4.14}
\end{equation*}
$$

where $Q_{1,0}^{*}$ is given by (4.12) replacing $w$ with $w_{1,0}^{*}$.
Combine (4.12) and (4.14), we can present $Q_{1,0}^{*}$ as a function of only $r^{S}$ :

$$
\begin{equation*}
\left[r^{S} Q_{1,0}^{*} h\left(Q_{1,0}^{*}\right)+E V\left(1-Q_{1,0}^{*} h\left(Q_{1,0}^{*}\right)\right)\right] \bar{F}\left(Q_{1,0}^{*}\right)=c . \tag{4.15}
\end{equation*}
$$

We can clearly describe how $Q_{1,0}^{*}$ is affected by $r^{S}$ in the following proposition.
Proposition 4.5. $Q_{1,0}^{*}$ is increasing in $r^{S}$.
Now we plug $w_{1,0}^{*}$ into (4.13) to find optimal $r^{S}$, the following proposition characterizes our results.

Proposition 4.6. $\Pi_{1,0}^{S}\left(w_{1,0}^{*}, r^{S}\right)$ is increasing in $r^{S}$ and reaches its maximum at $r_{1,0}^{S *}=w_{1,0}^{*}-c$.

### 4.4. Scenario 4: Supplier and Retailer Offer Return Policy

In this case, a supplier sells the product at unit price $w$ and will buyback each returned items at $r^{S}$ from the retailer. The retailer then chooses her response order quantity $Q$, retail price $p$ and refund price $r^{R}$ to the customer. We derive the overall channel solutions by backwardinduction procedure. We begin our analysis by focusing on the retailer's decisions.

For given wholesale price $w$ and buyback price $r^{S}$ offered by the supplier, and knowing the demand is characterized by (3.1), the retailer's problem is to choose the optimal order quantity $Q$, the price $p$ sell to the market, and refund price $r^{R}$ to the unsatisfied customers, to maximize her own profit which is shown in (3.2).

The optimization problem has a structure similar to that of the centralized problem in Proposition 4.2. The following proposition characterizes the decentralized decisions $\left(p_{1,1}^{*}, Q_{1,1}^{*}, r_{1,1}^{R *}\right)$.

Proposition 4.7. The retailer's optimal price $p_{1,1^{\prime}}^{*}$, quantity $Q_{1,1^{\prime}}^{*}$ and refund $r_{1,1}^{R *}$ are given by

$$
\begin{align*}
& p_{1,1}^{*}=E \max \left(V, r^{S}\right)  \tag{4.16}\\
& \bar{F}\left(Q_{1,1}^{*}\right)=\frac{w-r^{S}}{p_{1,1}^{*}-r^{S}}=\frac{w-r^{S}}{E \max \left(V, r^{S}\right)-r^{S}}  \tag{4.17}\\
& r_{1,1}^{R *}=r^{S} . \tag{4.18}
\end{align*}
$$

Knowing that the retailer chooses $\left(p_{1,1}^{*}, Q_{1,1}^{*}, r_{1,1}^{R *}\right)$ according to (4.16) to (4.18) in response to given $w$ and $r^{S}$. The supplier sets on $\left(w, r^{S}\right)$ to maximize his own profit, which is given by (3.3).

Plugging $\left(p_{1,1}^{*}, Q_{1,1}^{*}, r_{1,1}^{R *}\right)$ into the supplier's profit function, then (3.3) can be rearranged as:

$$
\begin{equation*}
\prod_{1,1}^{S}\left(w, r^{S}\right)=r^{S} \bar{G}\left(r^{S}\right) E \min \left(X, Q_{1,1}^{*}\right)+\left(w-c-r^{S}\right) Q_{1,1}^{*} \tag{4.19}
\end{equation*}
$$

The term $Q_{1,1}^{*}$ is shown in (4.17), which is a function of $w$ and $r^{S}$. We reduce (4.19) to an optimization problem over the single variable $r^{S}$ by first solving for the optimal value of $w$ as a function of $r^{S}$, and then substituting the result back into $\Pi_{1,1}^{S}\left(w, r^{S}\right)$. Go along the same line as in scenario 3, we have.

Proposition 4.8. For any fixed $r^{S}$, if $d h(x) / d x>0$, the optimal $w_{1,1}^{*}$ is determined uniquely as a function of $r^{S}$ :

$$
\begin{equation*}
\left(w_{1,1}^{*}-r^{S}\right) Q_{1,1}^{*} h\left(Q_{1,1}^{*}\right)-\left[r^{S} \bar{G}\left(r^{S}\right) \bar{F}\left(Q_{1,1}^{*}\right)+w_{1,1}^{*}-r^{S}-c\right]=0 \tag{4.20}
\end{equation*}
$$

where $Q_{1,1}^{*}$ is given by (4.17) replacing $w$ with $w_{1,1}^{*}$.
After substituting $w_{1,1}^{*}$ in (4.19), we find that the solution depends on $F$ and $G$ distribution, and it is difficult to track analytical solution of $r^{S}$ even if the demand distribution function follows a uniform distribution. If the inverse distribution functions (such as normal distribution) are not available, the analytical solution is unsolvable.

## 5. Performance Comparisons under Uniform Distribution

In Section 4, we completely characterize the optimal decisions of both the supplier and retailer for all scenarios except scenario 4. Actually, the equilibrium solutions are not in closed form, so we turn to a specific distribution to gain insights into the performance comparisons. In some papers on supply chain management with uncertainty demand, researchers ([25,26]) use a uniform distribution in theoretical problems because of the complexity of the supply chain management. This assumption is in real existence for example in the fashion clothing and apparels market, new products market, and so forth [27]. In our paper, we suppose that the random market demand $X$ follows a uniform distribution on $[0, b]$. Then, we have $f(x)=1 / b, \bar{F}(x)=(b-x) / b$, and $h(x)=1 /(b-x)$. The uniform distribution is an IFR distribution that ensures the existence of the unique optimal wholesale price.

Recall the analysis of scenario 1 in Section 4, from (4.5) to (4.7), we have

$$
\left\{\begin{array}{l}
p_{0,0}^{*}=E V,  \tag{5.1}\\
\bar{F}\left(Q_{0,0}^{*}\right)=\frac{w}{E V}, \\
w Q_{0,0}^{*} h\left(Q_{0,0}^{*}\right)-w+c=0, \\
w_{0,0}^{*}=\frac{E V+c}{2}
\end{array}\right.
$$

Substituting all the optimal decisions into the retailer's and supplier's profit functions:

$$
\begin{align*}
\prod_{0,0}^{R *} & =p_{0,0}^{*} E \min \left(D, Q_{0,0}^{*}\right)-w_{0,0}^{*} Q_{0,0}^{*} \\
& =w_{0,0}^{*}\left[\frac{E \min \left(D, Q_{0,0}^{*}\right)}{\bar{F}\left(Q_{0,0}^{*}\right)}-Q_{0,0}^{*}\right]  \tag{5.2}\\
& =w_{0,0}^{*} \frac{\left(Q_{0,0}^{*}\right)^{2}}{2\left(b-Q_{0,0}^{*}\right)} \\
& =\frac{b}{8 E V}(E V-c)^{2}, \\
\prod_{0,0}^{S *} & =\left(w_{0,0}^{*}-c\right) Q_{0,0}^{*}  \tag{5.3}\\
& =\frac{E V-c}{2} \cdot \frac{b(E V-c)}{2 E V}
\end{align*}
$$

From (4.11), (4.12), (4.14), and Proposition 4.6 in scenario 3, we have

$$
\begin{align*}
& \left\{\begin{array}{l}
p_{1,0}^{*}=E V \\
\bar{F}\left(Q_{1,0}^{*}\right)=\frac{w-r^{S}}{E V-r^{S}} \\
r_{1,0}^{S *}=w_{1,0}^{*}-c, \\
\left(w_{1,0}^{*}-r^{S}\right) Q_{1,0}^{*} h\left(Q_{1,0}^{*}\right)-\left[r^{S} \bar{F}\left(Q_{1,0}^{*}\right)+w_{1,0}^{*}-r^{S}-c\right]=0, \\
\\
\quad \Longrightarrow\left\{\begin{array}{l}
p_{1,0}^{*}=E V, \\
Q_{1,0}^{*}=b\left(1-\sqrt{\frac{c}{E V}}\right) \\
r_{1,0}^{S *}=E V-\sqrt{c \cdot E V} \\
w_{1,0}^{*}=E V+c-\sqrt{c \cdot E V}
\end{array}\right.
\end{array} .\right.
\end{align*}
$$

Substituting all the optimal decisions into the retailer's and supplier's profit functions:

$$
\begin{align*}
\prod_{1,0}^{R *} & =\left(p_{1,0}^{*}-r_{1,0}^{S *}\right) E \min \left(D, Q_{1,0}^{*}\right)-\left(w_{1,0}^{*}-r_{1,0}^{S *}\right) Q_{1,0}^{*} \\
& =\frac{c}{\bar{F}\left(Q_{1,0}^{*}\right)} E \min \left(D, Q_{1,0}^{*}\right)-c Q_{1,0}^{*} \\
& =c\left[\frac{E \min \left(D, Q_{1,0}^{*}\right)}{\bar{F}\left(Q_{1,0}^{*}\right)}-Q_{1,0}^{*}\right]  \tag{5.5}\\
& =c \frac{\left(Q_{1,0}^{*}\right)^{2}}{2\left(b-Q_{1,0}^{*}\right)} \\
& =\frac{b}{2} \sqrt{c \cdot E V}\left(1-\sqrt{\left.\frac{c}{E V}\right)^{2},}\right. \\
\prod_{1,0}^{S *} & =r_{1,0}^{S *} E \min \left(D, Q_{1,0}^{*}\right)+\left(w_{1,0}^{*}-c-r_{1,0}^{S *}\right) Q_{1,0}^{*} \\
& =r_{1,0}^{S *} E \min \left(D, Q_{1,0}^{*}\right)  \tag{5.6}\\
& =(E V-\sqrt{c \cdot E V}) \cdot \frac{b(E V-c)}{2 E V} .
\end{align*}
$$

Make comparison between (5.2) and (5.5), (5.3) and (5.6), we can draw the following conclusion.

Proposition 5.1. $\Pi_{0,0}^{R *} \geq \Pi_{1,0}^{R *}$ and $\Pi_{0,0}^{S *} \leq \Pi_{1,0}^{S *}$.
From Proposition 5.1, we can get the conclusion that if the retailer does not intend to provide return policy to the consumer, the supplier's best choice is to offer the most generous buyback policy to the retailer. Actually, the supplier can gain more while the retailer earns less compared to that of scenario 1.

From Propositions 4.2 and 4.3 in scenario 2, we can get

$$
\left\{\begin{array} { l } 
{ p _ { 0 , 1 } ^ { * } = E V }  \tag{5.7}\\
{ \overline { F } ( Q _ { 0 , 1 } ^ { * } ) = \frac { w } { E V ^ { \prime } } , } \\
{ r _ { 0 , 1 } ^ { R * } = 0 , } \\
{ w Q _ { 0 , 1 } ^ { * } h ( Q _ { 0 , 1 } ^ { * } ) - w + c = 0 , }
\end{array} \quad \Longrightarrow \left\{\begin{array}{l}
p_{0,1}^{*}=E V \\
Q_{0,1}^{*}=\frac{b(E V-c)}{2 E V} \\
r_{0,1}^{R *}=0 \\
w_{0,1}^{*}=\frac{E V+c}{2}
\end{array}\right.\right.
$$

Substituting all the optimal decisions into the retailer's profit function (4.4) and supplier's profit function (4.8), we have:

$$
\begin{align*}
\prod_{0,1}^{R *}=\left[p_{0,1}^{*} \bar{G}\left(r_{0,1}^{R *}\right)+\right. & \left.\left(p-r_{0,1}^{R *}\right) G\left(r_{0,1}^{R *}\right)\right] E \min \left(D, Q_{0,1}^{*}\right)-w_{0,1}^{*} Q_{0,1}^{*}=\frac{b}{8 E V}(E V-c)^{2}  \tag{5.8}\\
& \prod_{0,1}^{S *}=\left(w_{0,1}^{*}-c\right) Q_{0,1}^{*}=\frac{E V-c}{2} \cdot \frac{b(E V-c)}{2 E V}
\end{align*}
$$

From Proposition 4.7 in scenario 4, we find that if the supplier presents buyback policy to the retailer, it is better for the retailer to present return policy too. Specially, the retailer's optimal return price is the same as the supplier's buyback price. The derivation of supplier's buyback price $r^{S}$ depends on the demand uncertainty distribution $F$ and valuation uncertainty distribution G. Comparing the results in scenario 2 and scenario 4 , we can derive the following proposition.

Proposition 5.2. $\Pi_{0,1}^{S *} \leq \Pi_{1,1}^{S *}$.
However, we cannot analytically make a comparison between $\Pi_{1,1}^{R *}$ and $\Pi_{1,0}^{R *}$. When we do numerical simulations, we find the result relies critically on the parameters and distribution $G$.

From the above analysis, we can draw the following conclusions: whether the retailer offers return policy to consumers or not, the upstream supplier's optimal policy is always offer buyback policy to the retailer. This is because the supplier has an incentive to enlarge the market by offering buyback policy. In addition, if the retailer does not present return policy, the supplier can profit from the most generous buyback price; if the retailer presents return policy, the supplier can gain profit from the buyback policy, the specific buyback price depends on the system parameters. For the retailer, supplier's buyback policy may hurt her profit. As she is the follower in the supply chain, her optimal response to supplier's buyback price is to implement the same return price.

We provide a guideline for the supplier offering a buyback policy for unsold inventory and customer returns as how to contract a buyback price with the retailer and also guide the retailer how to decide the return policy for the end consumers.

## 6. Conclusion

In this paper, we examine return policies in a two-echelon supply chain that comprises an upstream supplier, a downstream retailer, and end consumers. In this environment, the upstream supplier decides his wholesale price and buyback price for returned items; the downstream retailer then chooses her order quantity, retail price, and refund price for customers' returns. The end consumers face uncertainty in their valuation for products. With returns polices, the consumer can then decide whether to keep or return the product. Using this model, we put forth the following results.
(1) If the upstream supplier does not provide buyback policy, the retailer's optimal response is not to provide return policy either; otherwise, the retailer's optimal refund price would be the same as that of the buyback price.
(2) The supplier will adopt buyback policy, as he can always gain more profit than absence of buyback price. Retailer would expect the supplier not to adopt buyback policy, because she may gain a lower profit by facing the buyback policy.
(3) As supplier is the leader of this Stackelberg game, so in the two-echelon system, the supplier will present buyback policy, then the retailer will offer the same amount return policy. The analysis of the specific amount relies on all environmental parameters.

We believe this research will provide new insights for the return policies in a two echelon supply chain. This research can be enriched in several directions. In practice, many retailers permit consumer returns up to a certain time limit, to understand how the duration of returns policy take effect may be necessary. It would be interesting to investigate a related but different context, in which consumers return the product directly to the supplier rather than to the retailer. It is difficult to generate closed-form solutions for general demand distributions; this needs to be investigated in further research.

## Appendix

## Mathematical Proofs

Proof of Proposition 4.1. From (4.2), taking the first derivative with $w$, we have

$$
\begin{align*}
\frac{d \Pi_{0,0}^{S}(w)}{d w} & =\frac{\partial \Pi_{0,0}^{S}(w)}{\partial w}+\frac{\partial \Pi_{0,0}^{S}(w)}{\partial Q} \frac{d Q_{0,0}^{*}}{d w} \\
& =Q_{0,0}^{*}+(w-c) \frac{d Q_{0,0}^{*}}{d w}  \tag{A.1}\\
& =Q_{0,0}^{*}-\frac{(w-c)}{w} \frac{1}{h\left(Q_{0,0}^{*}\right)}
\end{align*}
$$

The last equation is derived by applying the implicit function rule on $\bar{F}\left(Q_{0,0}^{*}\right)=w / E V$ :

$$
\begin{equation*}
\frac{d Q_{0,0}^{*}}{d w}=-\frac{1}{f\left(Q_{0,0}^{*}\right)} \frac{1}{E V}=-\frac{1}{f\left(Q_{0,0}^{*}\right)} \frac{\bar{F}\left(Q_{0,0}^{*}\right)}{w}=-\frac{1}{w} \frac{1}{h\left(Q_{0,0}^{*}\right)} \tag{A.2}
\end{equation*}
$$

Then we have

$$
\begin{equation*}
\frac{d^{2} \Pi_{0,0}^{S}(w)}{d w^{2}}=\frac{d Q_{0,0}^{*}}{d w}-\frac{c}{w^{2}} \frac{1}{h\left(Q_{0,0}^{*}\right)}-\frac{w-c}{w} \frac{-h^{\prime}\left(Q_{0,0}^{*}\right)}{h^{2}\left(Q_{0,0}^{*}\right)} \frac{d Q_{0,0}^{*}}{d w} \tag{A.3}
\end{equation*}
$$

So if $d h(x) / d x>0, \Pi_{0,0}^{S}(w)$ is concave in $w$, which reaches its maximum at the first order condition.

Proof of Proposition 4.2. The first step in maximizing (4.4) is to find optimal $p_{0,1}^{*}, r_{0,1}^{R *}$ that maximizes the expression $\left[p \bar{G}\left(r^{R}\right)+\left(p-r^{R}\right) G\left(r^{R}\right)\right]$, subject to the constraint $p \leq E \max \left(V, r^{R}\right)$ so that consumers are willing to buy in the first place. Obviously, the optimal price is $p=E \max \left(V, r^{R}\right)$, so we have an expression in terms of only $r^{R}$ :

$$
\begin{equation*}
p \bar{G}\left(r^{R}\right)+\left(p-r^{R}\right) G\left(r^{R}\right)=p-r^{R} G\left(r^{R}\right)=E \max \left(V, r^{R}\right)-r^{R} G\left(r^{R}\right)=\int_{r^{R}}^{\infty} v g(v) d v \tag{A.4}
\end{equation*}
$$

This term is decreasing in $r^{R}$ and maximized when $r^{R}=0$. So we have $r_{0,1}^{R *}=0$ and $p_{0,1}^{*}=E V$. Then we solve the resulting newsvendor problem in $Q$ to obtain (4.6).

Proof of Proposition 4.3. The proof here is totally the same as that of Proposition 4.1.
Proof of Proposition 4.4. From (4.12), by the implicit function rule, we can derive

$$
\begin{equation*}
\frac{\partial Q_{1,0}^{*}}{\partial w}=-\frac{1}{f\left(Q_{1,0}^{*}\right)} \frac{1}{E V-r^{S}}=-\frac{1}{w-r^{S}} \frac{1}{h\left(Q_{1,0}^{*}\right)} \tag{A.5}
\end{equation*}
$$

Then taking the partial derivative of (4.13) w.r.t. $w$,

$$
\begin{align*}
\frac{\partial \Pi_{1,0}^{S}\left(w, r^{S}\right)}{\partial w} & =Q_{1,0}^{*}+\left[\left(w-r^{S}-c\right)+r^{S} \bar{F}\left(Q_{1,0}^{*}\right)\right] \frac{\partial Q_{1,0}^{*}}{\partial w} \\
& =Q_{1,0}^{*}-\left[\frac{\left(w-r^{S}-c\right)+r^{S} \bar{F}\left(Q_{1,0}^{*}\right)}{w-r^{S}}\right] \frac{1}{h\left(Q_{1,0}^{*}\right)}  \tag{A.6}\\
& =Q_{1,0}^{*}-\left[\frac{E V\left(w-r^{S}\right)-c\left(E V-r^{S}\right)}{\left(w-r^{S}\right)\left(E V-r^{S}\right)}\right] \frac{1}{h\left(Q_{1,0}^{*}\right)}
\end{align*}
$$

The last equation is obtained by substituting (4.12) into the second equation, we continue to have

$$
\begin{equation*}
\frac{\partial^{2} \Pi_{1,0}^{S}\left(w, r^{S}\right)}{\partial w^{2}}=\frac{\partial Q_{1,0}^{*}}{\partial w}-\frac{c}{\left(w-r^{S}\right)^{2}} \frac{1}{h\left(Q_{1,0}^{*}\right)}-\left[\frac{E V\left(w-r^{S}\right)-c\left(E V-r^{S}\right)}{\left(w-r^{S}\right)\left(E V-r^{S}\right)}\right] \frac{-h^{\prime}\left(Q_{1,0}^{*}\right)}{h^{2}\left(Q_{1,0}^{*}\right)} \frac{\partial Q_{1,0}^{*}}{\partial w} . \tag{A.7}
\end{equation*}
$$

Therefore, if $h^{\prime}(x)>0, \Pi_{1,0}^{S}\left(w, r^{S}\right)$ is concave in $w$, there exists a unique optimal $w_{1,0}^{*}$. Replacing $w$ with $w_{1,0}^{*}$ in (4.12) and $\partial \Pi_{1,0}^{S}\left(w, r^{S}\right) / \partial w$, setting $\partial \Pi_{1,0}^{S}\left(w, r^{S}\right) / \partial w=0$, we have (4.14).

Proof of Proposition 4.5. We define

$$
\begin{equation*}
\nabla=\left[r^{s} Q_{1,0}^{*} h\left(Q_{1,0}^{*}\right)+E V\left(1-Q_{1,0}^{*} h\left(Q_{1,0}^{*}\right)\right)\right] \bar{F}\left(Q_{1,0}^{*}\right)-c, \tag{A.8}
\end{equation*}
$$

by the implicit function rule, we have

$$
\begin{align*}
& \frac{d Q_{1,0}^{*}}{d r^{S}} \\
& =-\frac{\partial \nabla / \partial r^{S}}{\partial \nabla / \partial Q_{1,0}^{*}} \\
& =\frac{-Q_{1,0}^{*} h\left(Q_{1,0}^{*}\right) \bar{F}\left(Q_{1,0}^{*}\right)}{-f\left(Q_{1,0}^{*}\right)\left[r^{S} Q_{1,0}^{*} h\left(Q_{1,0}^{*}\right)+E V\left(1-Q_{1,0}^{*} h\left(Q_{1,0}^{*}\right)\right)\right]-\left(E V-r^{S}\right) \bar{F}\left(Q_{1,0}^{*}\right)\left[h\left(Q_{1,0}^{*}\right)+Q_{1,0}^{*} h^{\prime}\left(Q_{1,0}^{*}\right)\right]} \\
& =\frac{-Q_{1,0}^{*} f\left(Q_{1,0}^{*}\right)}{-c h\left(Q_{1,0}^{*}\right)-\left(E V-r^{S}\right) \bar{F}\left(Q_{1,0}^{*}\right)\left[h\left(Q_{1,0}^{*}\right)+Q_{1,0}^{*} h^{\prime}\left(Q_{1,0}^{*}\right)\right]}>0 . \tag{A.9}
\end{align*}
$$

Proof of Proposition 4.6. We can rewrite (4.15) as

$$
\begin{equation*}
\left[r^{S} Q_{1,0}^{*} h\left(Q_{1,0}^{*}\right)+E V\left(1-Q_{1,0}^{*} h\left(Q_{1,0}^{*}\right)\right)\right]\left(w_{1,0}^{*}-r^{S}\right)=c\left(E V-r^{S}\right), \tag{A.10}
\end{equation*}
$$

then

$$
\begin{equation*}
w_{1,0}^{*}-r^{S}-c=\frac{\left[\left(E V-r^{S}\right) Q_{1,0}^{*} h\left(Q_{1,0}^{*}\right)-r^{S}\right] c}{r^{S} Q_{1,0}^{*} h\left(Q_{1,0}^{*}\right)+E V\left(1-Q_{1,0}^{*} h\left(Q_{1,0}^{*}\right)\right)} . \tag{A.11}
\end{equation*}
$$

$$
\begin{align*}
\prod_{1,0}^{S}\left(w_{1,0}^{*}, r^{S}\right) & =r^{S} E \min \left(D, Q_{1,0}^{*}\right)+\left(w_{1,0}^{*}-c-r^{S}\right) Q_{1,0}^{*} \\
& =r^{S} E \min \left(D, Q_{1,0}^{*}\right)+\frac{\left[\left(E V-r^{S}\right) Q_{1,0}^{*} h\left(Q_{1,0}^{*}\right)-r^{S}\right] c Q_{1,0}^{*}}{r^{S} Q_{1,0}^{*} h\left(Q_{1,0}^{*}\right)+E V\left(1-Q_{1,0}^{*} h\left(Q_{1,0}^{*}\right)\right)} \tag{A.12}
\end{align*}
$$

Taking the derivative with $r^{S}$, we have

$$
\begin{aligned}
& \frac{d \Pi_{1,0}^{S}\left(w_{1,0}^{*}, r^{S}\right)}{d r^{S}}=\frac{\partial \Pi_{1,0}^{S}\left(w_{1,0}^{*}, r^{S}\right)}{\partial r^{S}}+\frac{\partial \Pi_{1,0}^{S}\left(w_{1,0}^{*}, r^{S}\right)}{\partial Q} \frac{d Q_{1,0}^{*}}{d r^{S}} \\
& =E \min \left(D, Q_{1,0}^{*}\right)+\frac{-E V c Q_{1,0}^{*}}{\left[r^{s} Q_{1,0}^{*} h\left(Q_{1,0}^{*}\right)+E V\left(1-Q_{1,0}^{*} h\left(Q_{1,0}^{*}\right)\right)\right]^{2}} \\
& +\left[r^{s} \bar{F}\left(Q_{1,0}^{*}\right)+\frac{\left[\left(E V-r^{S}\right) Q_{1,0}^{*} h\left(Q_{1,0}^{*}\right)-r^{s}\right] c}{r^{s} Q_{1,0}^{*} h\left(Q_{1,0}^{*}\right)+E V\left(1-Q_{1,0}^{*} h\left(Q_{1,0}^{*}\right)\right)}\right. \\
& +\frac{\left(E V-r^{S}\right)^{2}\left[h\left(Q_{1,0}^{*}\right)+Q_{1,0}^{*} h^{\prime}\left(Q_{1,0}^{*}\right)\right] c Q_{1,0}^{*}}{\left.\left[r^{S} Q_{1,0}^{*} h\left(Q_{1,0}^{*}\right)+E V\left(1-Q_{1,0}^{*} h\left(Q_{1,0}^{*}\right)\right)\right]^{2}\right]} \frac{d Q_{1,0}^{*}}{d r^{S}} \\
& =E \min \left(D, Q_{1,0}^{*}\right)+\frac{-E V Q_{1,0}^{*} \bar{F}^{2}\left(Q_{1,0}^{*}\right)}{c} \\
& +\left[r^{s} \bar{F}\left(Q_{1,0}^{*}\right)+\left[\left(E V-r^{S}\right) Q_{1,0}^{*} h\left(Q_{1,0}^{*}\right)-r^{S}\right] \bar{F}\left(Q_{1,0}^{*}\right)\right. \\
& \left.+\left(E V-r^{S}\right)^{2}\left[h\left(Q_{1,0}^{*}\right)+Q_{1,0}^{*} h^{\prime}\left(Q_{1,0}^{*}\right)\right] c Q_{1,0}^{*} \frac{\bar{F}^{2}\left(Q_{1,0}^{*}\right)}{c^{2}}\right] \frac{d Q_{1,0}^{*}}{d r^{S}} \\
& =E \min \left(D, Q_{1,0}^{*}\right)-\frac{E V Q_{1,0}^{*} \bar{F}^{2}\left(Q_{1,0}^{*}\right)}{c}
\end{aligned}
$$

$$
\begin{align*}
& +\left[\left(E V-r^{S}\right) Q_{1,0}^{*} h\left(Q_{1,0}^{*}\right) \bar{F}\left(Q_{1,0}^{*}\right)\right. \\
& \left.\quad+\left(E V-r^{S}\right)^{2}\left[h\left(Q_{1,0}^{*}\right)+Q_{1,0}^{*} h^{\prime}\left(Q_{1,0}^{*}\right)\right] Q_{1,0}^{*} \frac{\bar{F}^{2}\left(Q_{1,0}^{*}\right)}{c}\right] \frac{d Q_{1,0}^{*}}{d r^{S}} . \tag{A.13}
\end{align*}
$$

The third equation is derived from (4.15), from Proposition 4.5, we know $d Q_{1,0}^{*} / d r^{S}$ is positive, so is the last term. To the end, it suffices to show that the sum of the first two terms is nonnegative. In fact, we get

$$
\begin{align*}
& \frac{d}{d Q_{1,0}^{*}}\left[E \min \left(D, Q_{1,0}^{*}\right)-\frac{E V Q_{1,0}^{*} \bar{F}^{2}\left(Q_{1,0}^{*}\right)}{c}\right] \\
& =\bar{F}\left(Q_{1,0}^{*}\right)-\frac{E V}{c}\left[\bar{F}^{2}\left(Q_{1,0}^{*}\right)-2 Q_{1,0}^{*} f\left(Q_{1,0}^{*}\right) \bar{F}\left(Q_{1,0}^{*}\right)\right] \\
& =\frac{c \bar{F}\left(Q_{1,0}^{*}\right)-E V \bar{F}^{2}\left(Q_{1,0}^{*}\right)+2 E V Q_{1,0}^{*} f\left(Q_{1,0}^{*}\right) \bar{F}\left(Q_{1,0}^{*}\right)}{c} \\
& =\frac{\bar{F}^{2}\left(Q_{1,0}^{*}\right)\left[r^{s} Q_{1,0}^{*} h\left(Q_{1,0}^{*}\right)+E V\left(1-Q_{1,0}^{*} h\left(Q_{1,0}^{*}\right)\right)\right]-E V \bar{F}^{2}\left(Q_{1,0}^{*}\right)+2 E V Q_{1,0}^{*} f\left(Q_{1,0}^{*}\right) \bar{F}\left(Q_{1,0}^{*}\right)}{c} \\
& =\frac{\left(E V+r^{s}\right) Q_{1,0}^{*} f\left(Q_{1,0}^{*}\right) \bar{F}\left(Q_{1,0}^{*}\right)}{c} \geq 0 . \tag{A.14}
\end{align*}
$$

The third equation is derived from (4.15), and $E \min \left(D, Q_{1,0}^{*}\right)-E V Q_{1,0}^{*} \bar{F}^{2}\left(Q_{1,0}^{*}\right) / c$ equals zero at $Q_{1,0}^{*}=0$, then this term turns out to be nonnegative for any nonnegative $Q_{1,0}^{*}$.

Above all, $\Pi_{1,0}^{S}\left(w_{1,0}^{*}, r^{S}\right)$ is increasing in $r^{S}$ and reaches its maximum at the upper bound $w_{1,0}^{*}-c$.

Proof of Proposition 4.7. We can rewrite (3.2) as:

$$
\begin{equation*}
\prod_{1,1}^{R}\left(p, Q, r^{R}\right)=\left[\left(p-r^{S}\right) \bar{G}\left(r^{R}\right)+\left(p-r^{R}\right) G\left(r^{R}\right)\right] E \min (D, Q)-\left(w-r^{S}\right) Q \tag{A.15}
\end{equation*}
$$

Firstly, we aim to find optimal $p_{1,1}^{*}$ and $r_{1,1}^{R *}$ that maximizes $\left[\left(p-r^{S}\right) \bar{G}\left(r^{R}\right)+\left(p-r^{R}\right) G\left(r^{R}\right)\right]$ under the constraint $p \leq E \max \left(V, r^{R}\right)$. Clearly, the retailer will set the maximum possible price $p=E \max \left(V, r^{R}\right)$, then we have

$$
\begin{align*}
\left(p-r^{S}\right) \bar{G}\left(r^{R}\right)+\left(p-r^{R}\right) G\left(r^{R}\right) & =p-r^{S} \bar{G}\left(r^{R}\right)-r^{R} G\left(r^{R}\right) \\
& =E \max \left(V, r^{R}\right)-r^{S} \bar{G}\left(r^{R}\right)-r^{R} G\left(r^{R}\right)  \tag{A.16}\\
& =\int_{r^{R}}^{\infty}\left(v-r^{S}\right) g(v) d v
\end{align*}
$$

This is maximized when $r^{R}=r^{S}$, so we have $r_{1,1}^{R *}=r^{S}$ and $p_{1,1}^{*}=E \max \left(V, r^{S}\right)$. We can easily get $Q_{1,1}^{*}$ by solving a resulting newsvendor problem.

Proof of Proposition 4.8. The analysis here is strikingly similar to that of Proposition 4.4, so we omit the details here.

Proof of Proposition 5.1. To show $\Pi_{0,0}^{R *} \geq \Pi_{1,0}^{R *}$ is equivalent to show

$$
\begin{gather*}
4 E V \sqrt{c \cdot E V}\left(1-\sqrt{\frac{c}{E V}}\right)^{2} \leq(E V-c)^{2} \\
\Longleftarrow 4(E V+c) \sqrt{c \cdot E V} \leq(E V+c)^{2}+4 c \cdot E V  \tag{A.17}\\
\Longleftarrow 0 \leq[(E V+c)-2 \sqrt{c \cdot E V}]^{2}
\end{gather*}
$$

To show $\Pi_{0,0}^{S *} \leq \Pi_{1,0}^{S *}$ is equivalent to show $E V+c \geq 2 \sqrt{c \cdot E V}$.
Proof of Proposition 5.2. From (4.16), (4.17), (4.18), and Proposition 4.8, we have

$$
\begin{gather*}
p_{1,1}^{*}=E \max \left(V, r^{S}\right) \\
\bar{F}\left(Q_{1,1}^{*}\right)=\frac{w-r^{S}}{p_{1,1}^{*}-r^{S}}=\frac{w-r^{S}}{E \max \left(V, r^{S}\right)-r^{S}}  \tag{A.18}\\
r_{1,1}^{R *}=r^{S} \\
\left(w_{1,1}^{*}-r^{S}\right) Q_{1,1}^{*} h\left(Q_{1,1}^{*}\right)-\left[\left(w_{1,1}^{*}-r^{S}-c\right)+r^{S} \bar{G}\left(r^{S}\right) \bar{F}\left(Q_{1,1}^{*}\right)\right]=0 .
\end{gather*}
$$

All of decisions are decided by $r^{S}$, and we find that it is difficult to find optimal $r^{S}$ even for $F$ that follows this uniform distribution.

If we fix $r^{S}=0$, we can easily find that the decisions in scenario 4 are the same as those in scenario 2, then $\left.\Pi_{1,1}^{S}\right|_{\left(r^{S}=0\right)}=\Pi_{0,1}^{S *}$. As $\Pi_{1,1}^{S *}$ can be achieved for an optimal $r^{S}$, so $\Pi_{1,1}^{S *} \geq\left.\Pi_{1,1}^{S}\right|_{\left(r^{S}=0\right)}$, then $\Pi_{1,1}^{S *} \geq \Pi_{1,0}^{S *}$.

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