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Research Article

Large Solutions of Quasilinear Elliptic System of Competitive Type: Existence and Asymptotic Behavior

Lin Wei[1] and Zuodong Yang[1, 2]

[1] *Institute of Mathematics, School of Mathematics Science, Nanjing Normal University, Jiangsu, Nanjing 210046, China*
[2] *College of Zhongbei, Nanjing Normal University, Jiangsu, Nanjing 210046, China*

Correspondence should be addressed to Zuodong Yang, zdyang_jin@263.net

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We study the existence and asymptotic behavior of positive solutions for a class of quasilinear elliptic systems in a smooth boundary via the upper and lower solutions and the localization method. The main results of the present paper are new and extend some previous results in the literature.

1. Introduction

This paper is concerned with the study of positive boundary blow-up solutions to a quasilinear elliptic system of competitive type:

$$\begin{gathered} \Delta_p u = a(x)u^a v^b \quad \text{in } \Omega, \\ \Delta_p v = b(x)u^c v^e \quad \text{in } \Omega, \\ u = v = +\infty \quad \text{on } \partial\Omega, \end{gathered} \tag{1.1}$$

where Ω is a bounded C^2 domain of $\mathbf{R}^{\mathbf{N}}$ and Δ_p stands for the p-Laplacian operator defined by $\Delta_p u = \operatorname{div}(|\nabla u|^{p-2}\nabla u), p > 1$. The exponents a, b, c, e verify $a, e > p-1, b, c > 0, (a-p+1)(e-p+1) > bc$. There exists $C(x), D(x) \in C(\overline{\Omega}, R^+), \gamma(x), \eta(x) \in C(\overline{\Omega}, R^+)$ such that

$$\lim_{x \to x_0} \frac{a(x)}{C(x_0)d(x)^{\gamma(x_0)}} = 1, \qquad \lim_{x \to x_0} \frac{b(x)}{D(x_0)d(x)^{\eta(x_0)}} = 1, \tag{1.2}$$

where $x_0 \in \partial\Omega, d(x) = \operatorname{dist}(x, \partial\Omega)$.

We must emphasize that the weight functions $a(x), b(x)$ are allowed decaying to zero on Ω with arbitrary rate, depending upon the particular point of $\partial\Omega$. The boundary condition is to be understood $u(x) \to \infty, v(x) \to \infty$ as $d(x) \to 0^+$. Problems like (1.1) are usually known in the literature as boundary blow-up problems, and their solutions are also named large solutions or boundary blow-up solutions.

The problem of the previous form is mathematical models occuring in studies of the p-Laplace system, generalized reaction-diffusion theory, non-Newtonian fluid theory [1, 2], non-Newtonian filtration [3], and the turbulent flow of a gas in porous medium. In the non-Newtonian fluid theory, the quantity p is a characteristic of the medium. Media with $p > 2$ are called dilatant fluids and those with $p < 2$ are called pseudoplastics. If $p = 2$, they are Newtonian fluids. When $p \neq 2$, the problem becomes more complicated since certain nice properties inherent to the case $p = 2$ seem to be lost or at least difficult to verify. The main differences between $p = 2$ and $p \neq 2$ can be founded in [4, 5].

When $p = 2$, system (1.1) becomes

$$\begin{aligned} &\Delta u = a(x)u^a v^b \quad \text{in } \Omega, \\ &\Delta v = b(x)u^c v^e \quad \text{in } \Omega, \\ &u = v = +\infty \quad \text{on } \partial\Omega, \end{aligned} \tag{1.3}$$

for which the existence, uniqueness, and asymptotic behavior of large solutions have been investigated extensively. We list here, for example, [6–12].

This is a huge amount of literature dealing with single equation with infinite boundary conditions (see, e.g., [13–34]). This problem with more general nonlinearies and weight-function has been discussed by many authors recently [35–39].

Problem (1.1) is considered in special case. When $p = 2$, in [40], problem (1.1) was analyzed with $a(x) = 1, b(x) = 1$. In the same paper, some existence, uniqueness, and boundary behavior of solutions were obtained under the assumptions

$$a(x) \sim C_1 d(x)^{k_1}, \qquad b(x) \sim C_2 d(x)^{k_2} \tag{1.4}$$

as $d(x) \to 0^+$ for some positive constants C_1, C_2 and real numbers $k_1, k_2 > -2$. This problem was later studied in [41] with general form, where

$$\begin{aligned} &C_1 d(x)^{\gamma_1} \le a(x) \le C_2 d(x)^{\gamma_1}, \\ &C_1' d(x)^{\gamma_1} \le b(x) \le C_2' d(x)^{\gamma_1}, \end{aligned} \tag{1.5}$$

for $x \in \Omega$, $\gamma_1, \gamma_2 \in \mathbf{R^N}, C_1, C_2, C_1', C_2'$ are positive constants. The author also obtained uniqueness results.

In [42], Yang extended the quasilinear elliptic system to

$$\begin{aligned} &\Delta_p u = u^{m_1} v^{n_1} \quad \text{in } \Omega, \\ &\Delta_q v = u^{m_2} v^{n_2} \quad \text{in } \Omega, \\ &u = v = +\infty \quad \text{on } \partial\Omega, \end{aligned} \tag{1.6}$$

where $m_1 > p-1, n_2 > q-1, m_2, n_1 > 0$, and $\Omega \subseteq \mathbf{R}^\mathbf{N}$ is a smooth bounded domain, subject to three different types of Dirichlet boundary conditions: $u = \lambda, v = \mu$ or $u = v = +\infty$ or $u = +\infty, v = \mu$ on $\partial\Omega$, where $\lambda, \mu > 0$. Under several hypotheses on the parameters m_1, n_1, m_2, n_2, the author showed the existence of positive solutions and further provided the asymptotic behavior of the solutions near $\partial\Omega$.

When $p \neq 2$, in [43], problem (1.1) was analyzed with $a(x) = 1, b(x) = 1$ under assumption (1.4). The author obtained the existence, uniqueness, and behavior of solutions to problem (1.1).

Very recently, Huang et al. [12] obtained existence, uniqueness, and asymptotic behavior of problem (1.1) when $p = 2$, and $a(x), b(x)$ satisfy condition (1.2). Motivated by the results of the papers [12, 40, 41, 43], we consider the quasilinear elliptic system (1.1). We modify the method developed by Huang et al. [12] and extend the results to a quasilinear elliptic system (1.1) under condition (1.2).

Throughout of this paper, set

$$
\begin{gathered}
C_1 = \min_{x\in\overline{\Omega}} C(x), \qquad C_2 = \max_{x\in\overline{\Omega}} C(x), \qquad D_1 = \min_{x\in\overline{\Omega}} D(x), \qquad D_2 = \max_{x\in\overline{\Omega}} D(x), \\
\gamma_1 = \max_{x\in\overline{\Omega}} \gamma(x), \qquad \gamma_2 = \min_{x\in\overline{\Omega}} \gamma(x), \qquad \eta_1 = \max_{x\in\overline{\Omega}} \eta(x), \qquad \eta_2 = \min_{x\in\overline{\Omega}} \eta(x), \\
\alpha(x,y) = \frac{(p+x)(e-p+1)-(p+y)b}{(a-p+1)(e-p+1)-bc}, \qquad \beta(x,y) = \frac{(p+y)(a-p+1)-(p+x)c}{(a-p+1)(e-p+1)-bc}, \\
E(x,y) = \left(\frac{\left((p-1)\alpha^{p-1}(\alpha+1)\right)^{e-p+1} x^b}{\left((p-1)\beta^{p-1}(\beta+1)\right)^{b} y^{e-p+1}}\right)^{1/((a-p+1)(e-p+1)-bc)}, \\
F(x,y) = \left(\frac{\left((p-1)\beta^{p-1}(\beta+1)\right)^{a-p+1} y^c}{\left((p-1)\alpha^{p-1}(\alpha+1)\right)^{c} x^{a-p+1}}\right)^{1/((a-p+1)(e-p+1)-bc)},
\end{gathered}
\tag{1.7}
$$

n_{x_0} stands for the outward unit normal at $x_0 \in \partial\Omega$.

The paper is organized as follows. In Section 2 we consider some preliminaries which will be used in proof of Theorem 1.1. In Section 3 we will give the proof of the main theorem.

By modifications of the arguments in the proof of Theorem 1.1 in [12], we obtain the following main results.

Theorem 1.1. *Assume that Ω is a bounded C^2 domain of $\mathbf{R}^\mathbf{N}$, $a(x), b(x) \in C^\theta(\Omega)$ for some $\theta \in (0,1)$, $a(x), b(x) > 0$ in Ω and verify* (1.2), $(a-p+1)(e-p+1) > bc, a, e > p-1, b, c > 0, \gamma(x), \eta(x) \in C(\overline{\Omega}, R^+)$ *and satisfy*

$$
\frac{b}{e-p+1} < \frac{p+\gamma(x_0)}{p+\eta(x_0)} < \frac{a-p+1}{c} \quad \text{for } x_0 \in \partial\Omega. \tag{1.8}
$$

Then problem (1.1) *has a solution* (u, v) *if and only if*

$$\frac{b}{e-p+1} < \frac{p+\gamma_1}{p+\eta_2}, \qquad \frac{p+\gamma_2}{p+\eta_1} < \frac{a-p+1}{c}. \tag{1.9}$$

And one has

$$\lim_{x \to x_0} \frac{u(x)}{d(x)^{-\alpha(\gamma(x_0),\eta(x_0))} E(D(x_0), C(x_0))} = 1, \tag{1.10}$$

$$\lim_{x \to x_0} \frac{v(x)}{d(x)^{-\beta(\gamma(x_0),\eta(x_0))} F(D(x_0), C(x_0))} = 1. \tag{1.11}$$

2. Preliminaries

In this section, we will introduce some propositions.

Definition 2.1. $(\underline{u}, \underline{v})$ is a subsolution of

$$\begin{cases} \Delta_p u = a(x) u^a v^b & \text{in } \Omega, \\ \Delta_p v = b(x) u^c v^e & \text{in } \Omega, \end{cases} \quad \text{provided} \quad \begin{cases} \Delta_p \underline{u} \geq a(x) \underline{u}^a \underline{v}^b & \text{in } \Omega, \\ \Delta_p \underline{v} \leq b(x) \underline{u}^c \underline{v}^e & \text{in } \Omega. \end{cases} \tag{2.1}$$

A supersolution $(\overline{u}, \overline{v})$ is defined by reversing the inequalities.

Proposition 2.2. *Assume that* $(\underline{u}, \underline{v})$ *is a subsolution and* $(\overline{u}, \overline{v})$ *is a supersolution of problem* (1.1), *with* $\underline{u} = \underline{v} = \overline{u} = \overline{v} = +\infty$ *on* $\partial\Omega$. *Then problem* (1.1) *has at least a solution* (u, v) *with* $\underline{u} \leq u \leq \overline{u}, \underline{v} \geq v \geq \overline{v}$ *in* Ω. *In particular* $u = v = +\infty$ *on* $\partial\Omega$.

Proposition 2.3 (see [43]). *Assume that* $a(x), b(x)$ *satisfy* (1.4), *then problem* (1.1) *admits a positive solution* (u, v) *with* $u = v = +\infty$ *on* $\partial\Omega$ *if and only if* $k_1, k_2 > -p$ *and*

$$\frac{b}{e-p+1} < \frac{p+k_1}{p+k_2} < \frac{a-p+1}{c}. \tag{2.2}$$

This solution is unique and satisfies

$$\lim_{x \to x_0} \frac{u(x)}{d(x)^{-\alpha(k_1,k_2)} E(C_2, C_1)} = 1, \qquad \lim_{x \to x_0} \frac{v(x)}{d(x)^{-\beta(k_1,k_2)} F(C_2, C_1)} = 1 \tag{2.3}$$

for each $x_0 \in \partial\Omega$.

Next, we are ready to study two auxiliary problems in a ball and an annuli. To this aim, for given $0 < R_1 < R$ and $x_0 \in \mathbf{R}^\mathbf{N}$, $N \geq 1$, set

$$B_R(x_0) = \left\{ x \in \mathbf{R}^\mathbf{N} : |x - x_0| < R \right\}, \qquad A_{R_1,R}(x_0) = \left\{ x \in \mathbf{R}^\mathbf{N} : R_1 < |x - x_0| < R \right\}. \tag{2.4}$$

Proposition 2.4. *Assume* $\Omega = B_R(x_0)$, $(a-p+1)(e-p+1) > bc$, $a, e > p-1, b, c > 0$, $C(r), D(r) \in C([0,R], R^+)$ *and* $\gamma, \eta > 0$ *satisfy*

$$\frac{b}{e-p+1} < \frac{p+\gamma}{p+\eta} < \frac{a-p+1}{c}. \tag{2.5}$$

Then the following systems

$$\begin{gathered} \Delta_p \Phi = C(r)(R-r)^{\gamma}\Phi^a\Psi^b \quad \text{in } B_R(x_0), \\ \Delta_p \Psi = D(r)(R-r)^{\eta}\Phi^c\Psi^e \quad \text{in } B_R(x_0), \\ \Phi = \Psi = +\infty \quad \text{on } \partial B_R(x_0) \end{gathered} \tag{2.6}$$

possess a unique radially symmetric positive solution $(\Phi(r), \Psi(r))$ *satisfying*

$$\begin{gathered} \lim_{x \to x_0} \frac{\Phi(r)}{E(D(R), C(R))(R-r)^{-\alpha(\gamma,\eta)}} = 1, \\ \lim_{x \to x_0} \frac{\Psi(r)}{F(D(R), C(R))(R-r)^{-\beta(\gamma,\eta)}} = 1, \end{gathered} \tag{2.7}$$

where $r = |x - x_0|$.

Proof. At first, we consider the following systems

$$\begin{gathered} \left(|\Phi'|^{p-2}\Phi'\right)' + \frac{N-1}{r}\left(|\Phi'|^{p-2}\Phi'\right) = C(r)(R-r)^{\gamma}\Phi^a\Psi^b \quad \text{in } (0,R), \\ \left(|\Psi'|^{p-2}\Psi'\right)' + \frac{N-1}{r}\left(|\Psi'|^{p-2}\Psi'\right) = D(r)(R-r)^{\eta}\Phi^c\Psi^e \quad \text{in } (0,R), \\ \Phi(R) = \Psi(R) = +\infty, \qquad \Phi'(0) = \Psi'(0) = 0. \end{gathered} \tag{2.8}$$

We will show that problem (2.8) has a solution $(\Phi(r), \Psi(r))$, which provide a positive radially symmetric solution to problem (2.6). Indeed, any positive solution $(\Phi(r), \Psi(r))$ of the integral equation system

$$\begin{gathered} \Phi(r) = l + \int_0^r \left[t^{1-N}\int_0^t s^{N-1}C(s)(R-s)^{\gamma}\Phi^a\Psi^b ds\right]^{1/(p-1)} dt, \quad 0 < r < R, \\ \Psi(r) = m + \int_0^r \left[t^{1-N}\int_0^t s^{N-1}D(s)(R-s)^{\eta}\Phi^c\Psi^e ds\right]^{1/(p-1)} dt, \quad 0 < r < R, \end{gathered} \tag{2.9}$$

provides a solution of (2.8), where $\Phi(0) = l, \Psi(0) = m, \Phi(R) = +\infty, \Psi(R) = +\infty$.

Define $\Phi_0(r) = l, \Psi_0(r) = m$ for all $0 < r < R$, let $\{\Phi_k\}, \{\Psi_k\}$ be the function sequences given by

$$\Phi_k(r) = l + \int_0^r \left[t^{1-N} \int_0^t s^{N-1} C(s)(R-s)^{\gamma} \Phi_{k-1}^a \Psi_{k-1}^b ds \right]^{1/(p-1)} dt, \quad 0 < r < R,$$
$$\Psi_k(r) = m + \int_0^r \left[t^{1-N} \int_0^t s^{N-1} D(s)(R-s)^{\eta} \Phi_{k-1}^c \Psi_{k-1}^e ds \right]^{1/(p-1)} dt, \quad 0 < r < R, \tag{2.10}$$

subject to $\Phi_k(0) = l, \Psi_k(0) = m, \Phi_k(R) = \Psi_k(R) = k$.

We remark that $\{\Phi_k\}, \{\Psi_k\}$ are nondecreasing sequences. In fact,

$$\begin{aligned}
\Phi_1(r) &= l + \left(l^a m^b\right)^{1/(p-1)} \int_0^r \left[t^{1-N} \int_0^t s^{N-1} C(s)(R-s)^{\gamma} ds \right]^{1/(p-1)} dt \\
&= l + \left(l^a m^b\right)^{1/(p-1)} A(r) \geq l = \Phi_0(r), \\
\Psi_1(r) &= m + (l^c m^e)^{1/(p-1)} \int_0^r \left[t^{1-N} \int_0^t s^{N-1} D(s)(R-s)^{\eta} ds \right]^{1/(p-1)} dt \\
&= m + (l^c m^e)^{1/(p-1)} B(r) \geq m = \Psi_0(r),
\end{aligned} \tag{2.11}$$

where

$$A(r) = \int_0^r \left[t^{1-N} \int_0^t s^{N-1} C(s)(R-s)^{\gamma} ds \right]^{1/(p-1)} dt,$$
$$B(r) = \int_0^r \left[t^{1-N} \int_0^t s^{N-1} D(s)(R-s)^{\eta} ds \right]^{1/(p-1)} dt. \tag{2.12}$$

Proceeding by the same manner, we conclude that

$$l \leq \Phi_k \leq \Phi_{k+1}, \qquad m \leq \Psi_k \leq \Psi_{k+1}. \tag{2.13}$$

We now prove that $\{\Phi_k\}, \{\Psi_k\}$ are bounded in $(0, R)$. To prove this, we consider

$$\Delta_p \Upsilon = \left(C(r)(R-r)^{\gamma} + D(r)(R-r)^{\eta} \right) \left(\Upsilon^{a+b} + \Upsilon^{c+e} \right), \tag{2.14}$$

problem (2.14) has a large radially symmetric solution $\Upsilon(r)$, and

$$\Upsilon(r) = \Upsilon(0) + \int_0^r \left[t^{1-N} \int_0^t s^{N-1} \left(C(s)(R-s)^{\gamma} + D(s)(R-s)^{\eta} \right) \left(\Upsilon^{a+b} + \Upsilon^{c+e} \right) ds \right]^{1/(p-1)} dt, \tag{2.15}$$

where $\Upsilon(0) = l + m$. It follows that

$$\begin{aligned}\Phi_1(r) &= l + \left(l^a m^b\right)^{1/(p-1)} \int_0^r \left[t^{1-N} \int_0^t s^{N-1} C(s)(R-s)^\gamma ds \right]^{1/(p-1)} dt \\ &\le \Upsilon(0) + \int_0^r \left[t^{1-N} \int_0^t s^{N-1} \left(C(s)(R-s)^\gamma + D(s)(R-s)^\eta\right)\left(\Upsilon^{a+b} + \Upsilon^{c+e}\right) ds \right]^{1/(p-1)} dt \\ &= \Upsilon(r).\end{aligned} \tag{2.16}$$

Similarly, we have $\Psi_1 \le \Upsilon(r)$.

Arguing as before, we obtain $\Phi_k \le \Upsilon(r), \Psi_k \le \Upsilon(r)$. Therefore, we show that $\{\Phi_k\}, \{\Psi_k\}$ are nondecreasing and bounded sequences in $(0, R)$, which implies that the following limit holds

$$(\Phi, \Psi) = \lim_{k\to\infty} (\Phi_k, \Psi_k), \tag{2.17}$$

we deduce that (Φ, Ψ) is a positive solution of (2.8). Then $(\Phi(x), \Psi(x)) = (\Phi(r), \Psi(r))$ is a positive radially symmetric solution to problem (2.6) and

$$\Phi(R) = \lim_{r\to R} \Phi(r) = \infty, \qquad \Psi(R) = \lim_{r\to R} \Psi(r) = \infty. \tag{2.18}$$

Secondly, it is clear that

$$C_1(R-r)^\gamma \le C(r)(R-r)^\gamma \le C_2(R-r)^\gamma, \qquad D_1(R-r)^\eta \le D(r)(R-r)^\eta \le D_2(R-r)^\eta. \tag{2.19}$$

By (2.5) and Proposition 2.3, we have

$$\begin{aligned}E(D_1, C_2) &\le \lim_{r\to R} \frac{\Phi(r)}{(R-r)^{-\alpha(\gamma,\eta)}} \le E(D_2, C_1), \\ F(D_2, C_1) &\le \lim_{r\to R} \frac{\Psi(r)}{(R-r)^{-\beta(\gamma,\eta)}} \le F(D_1, C_2).\end{aligned} \tag{2.20}$$

Denote by

$$l = \lim_{r\to R} \frac{\Phi(r)}{(R-r)^{-\alpha(\gamma,\eta)}}, \qquad k = \lim_{r\to R} \frac{\Psi(r)}{(R-r)^{-\beta(\gamma,\eta)}}. \tag{2.21}$$

By using $\gamma + p = (a-p+1)\alpha(\gamma,\eta) + b\beta(\gamma,\eta)$, $\eta + p = (e-p+1)\beta(\gamma,\eta) + c\alpha(\gamma,\eta)$ and *L'Hôpital* rule, we obtain

$$
\begin{aligned}
l &= \lim_{r\to R} \frac{\Phi(0) + \int_0^r \left[t^{1-N}\int_0^t s^{N-1}C(s)(R-s)^{\gamma}\Phi^a\Psi^b ds\right]^{1/(p-1)} dt}{(R-r)^{-\alpha}} \\
&= \lim_{r\to R} \frac{\left[r^{1-N}\int_0^r t^{N-1}C(t)(R-t)^{\gamma}\Phi^a\Psi^b dt\right]^{1/(p-1)}}{\alpha(R-r)^{-\alpha-1}} \\
&= \left[\lim_{r\to R} \frac{r^{1-N}\int_0^r t^{N-1}C(t)(R-t)^{\gamma}\Phi^a\Psi^b dt}{\alpha(R-r)^{-(\alpha+1)(p-1)}}\right]^{1/(p-1)} \\
&= \left[\lim_{r\to R} \frac{(1-N)r^{-N}\int_0^r t^{N-1}C(t)(R-t)^{\gamma}\Phi^a\Psi^b dt + C(r)(R-r)^{\gamma}\Phi^a\Psi^b}{\alpha(\alpha+1)(p-1)(R-r)^{-\alpha p+\alpha-p}}\right]^{1/(p-1)} \\
&= \left[\frac{C(R)}{\alpha(\alpha+1)(p-1)}\lim_{r\to R}(R-r)^{a\alpha+b\beta}\Phi^a\Psi^b \right. \\
&\qquad \left. + \frac{1-N}{\alpha(\alpha+1)(p-1)}\lim_{r\to R}\frac{r^{-N}\int_0^r t^{N-1}C(t)(R-t)^{\gamma}\Phi^a\Psi^b dt}{(R-r)^{-\alpha p+\alpha-p}}\right]^{1/(p-1)} \\
&= \left[\frac{C(R)}{\alpha(\alpha+1)(p-1)} l^a k^b + \frac{1-N}{\alpha(\alpha+1)(p-1)}\lim_{r\to R}\frac{r^{-N}\int_0^r t^{N-1}C(t)(R-t)^{\gamma}\Phi^a\Psi^b dt}{(R-r)^{-\alpha p+\alpha-p}}\right]^{1/(p-1)}.
\end{aligned}
\tag{2.22}
$$

We note that

$$
\begin{aligned}
0 &\le \lim_{r\to R}\frac{r^{-N}\int_0^r t^{N-1}C(t)(R-t)^{\gamma}\Phi^a\Psi^b dt}{(R-r)^{-\alpha p+\alpha-p}} \\
&\le \lim_{r\to R}\frac{r^{-1}\int_0^r C(t)(R-t)^{\gamma}\Phi^a\Psi^b dt}{(R-r)^{-\alpha p+\alpha-p}} \\
&= \lim_{r\to R}\frac{C(r)(R-r)^{\gamma}\Phi^a\Psi^b dt}{R(-\alpha p+\alpha-p)(R-r)^{-\alpha p+\alpha-p+1}} \\
&= \frac{C(R)}{R(-\alpha p+\alpha-p)}\lim_{r\to R}(R-r)^{\gamma+\alpha p-\alpha+p+1}\Phi^a\Psi^b \\
&= \frac{C(R)}{R(-\alpha p+\alpha-p)} l^a k^b \lim_{r\to R}(R-r) = 0.
\end{aligned}
\tag{2.23}
$$

This implies that

$$
l^{p-1} = \frac{C(R)}{\alpha(\alpha+1)(p-1)} l^a k^b. \tag{2.24}
$$

Similarly, we obtain

$$k^{p-1} = \frac{D(R)}{\beta(\beta+1)(p-1)} l^c k^e. \tag{2.25}$$

Since

$$\begin{aligned} \frac{\alpha^{p-1}(\alpha+1)(p-1)}{C(R)} &= E(D(R),C(R))^{a-p+1} F(D(R),C(R))^b, \\ \frac{\beta^{p-1}(\beta+1)(p-1)}{D(R)} &= E(D(R),C(R))^c F(D(R),C(R))^{e-p+1}. \end{aligned} \tag{2.26}$$

If $0 < \alpha < 1, 1 < p \le 2$, then

$$E^{a-p+1}F^b = \frac{\alpha^{p-1}(\alpha+1)(p-1)}{C(R)} \ge \frac{\alpha(\alpha+1)(p-1)}{C(R)} = l^{a-p+1}k^b, \tag{2.27}$$

therefore, we get $E \ge l, F \ge k$. If $0 < \alpha < 1, p > 2$, we get $E \le l, F \le k$. So, when $0 < \alpha < 1$, we get $E = l, F = k$.

Similarly, when $\alpha \ge 1$, we also get $E = l, F = k$.

By (2.24) and (2.25), we conclude that $l = E(D(R),C(R)), k = F(D(R),C(R))$, this completes the proof. □

Proposition 2.5. *Assume* $(a-p+1)(e-p+1) > bc, a, e > p-1, b, c > 0, \gamma > 0, \eta > 0$, *and*

$$\frac{b}{e-p+1} < \frac{p+\gamma}{p+\eta} < \frac{a-p+1}{c}, \tag{2.28}$$

$D(r), C(r) \in C([R_1, R], R^+)$ *are the reflection around* $R_0 = (R_1+R)/2$ *of some functions* $\overline{C}(r), \overline{D}(r) \in C([R_0, R], R^+)$. *Then the following system*

$$\begin{aligned} \Delta_p \Phi &= C(r) d(x)^\gamma \Phi^a \Psi^b \quad \text{in } A_{R_1,R}(x_0), \\ \Delta_p \Psi &= D(r) d(x)^\eta \Phi^c \Psi^e \quad \text{in } A_{R_1,R}(x_0), \\ \Phi &= \Psi = +\infty \quad \text{on } \partial A_{R_1,R}(x_0) \end{aligned} \tag{2.29}$$

has a unique radially symmetric positive solution $(\Phi(r), \Psi(r))$ *such that*

$$\begin{aligned} \lim_{d(x)\to 0} \frac{\Phi(r)}{E(D(R),C(R)) d(x)^{-\alpha(\gamma,\eta)}} &= 1, \\ \lim_{d(x)\to 0} \frac{\Psi(r)}{F(D(R),C(R)) d(x)^{-\beta(\gamma,\eta)}} &= 1, \end{aligned} \tag{2.30}$$

where

$$d(x) = d(x, \partial A_{R_1,R}(x_0)) = \begin{cases} R - |x - x_0|, & \text{if } R_0 \le |x - x_0| \le R, \\ |x - x_0| - R_1, & \text{if } R_1 \le |x - x_0| \le R_0. \end{cases} \tag{2.31}$$

Proof. The proof is similarl to the proof of Proposition 2.4, so we omit it here. □

3. Proof of Theorem 1.1

We are now ready to prove Theorem 1.1, whose proof will be split into the following several lemmas.

Lemma 3.1. *Assume* $(a-p+1)(e-p+1) > bc, a, e > p-1, b, c > 0, C(x), D(x) \in C(\overline{\Omega}), a(x), b(x) > 0$ *in* Ω *and* (1.2) *holds,* $\gamma(x), \eta(x) > 0$ *and satisfy*

$$\frac{b}{e-p+1} < \frac{p+\gamma(x_0)}{p+\eta(x_0)} < \frac{a-p+1}{c}, \tag{3.1}$$

for each $x_0 \in \partial\Omega$, *then problem* (1.1) *has a solution* (u, v) *if*

$$\frac{b}{e-p+1} < \frac{p+\gamma_1}{p+\eta_2}, \qquad \frac{p+\gamma_2}{p+\eta_1} < \frac{a-p+1}{c}. \tag{3.2}$$

Proof. By (3.2) and Proposition 2.3, the following system

$$\begin{gathered} \Delta_p \Phi = C_1 d(x)^{\gamma_1} \Phi^a \Psi^b \quad \text{in } \Omega, \\ \Delta_p \Psi = D_2 d(x)^{\eta_2} \Phi^c \Psi^e \quad \text{in } \Omega, \\ \Phi = \Psi = +\infty \quad \text{on } \partial\Omega \end{gathered} \tag{3.3}$$

possesses a positive solution (u_1, v_1).

Next we will show that

$$(\overline{u}, \overline{v}) = \left(\left(\frac{m+n}{(m-n)^{b/(a-p+1)}} \right)^{1/(p-1)} u_1, \left(\frac{m-n}{(m+n)^{c/(e-p+1)}} \right)^{1/(p-1)} v_1 \right) \tag{3.4}$$

is a supersolution of (1.1), if m is sufficiently large and $0 < m - n < 1$, where $m, n \in R^+$ and $m > n$. In fact, by

$$\gamma_1 + p = (a-p+1)\alpha(\gamma_1, \eta_2) + b\beta(\gamma_1, \eta_2), \qquad \eta_2 + p = (e-p+1)\beta(\gamma_1, \eta_2) + c\alpha(\gamma_1, \eta_2). \tag{3.5}$$

We have $(\overline{u}, \overline{v})$ is a supersolution of (1.1) provided

$$\begin{aligned} C_1 d(x)^{\gamma_1} &\leq a(x)(m+n)^{((e-p+1)(a-p+1)-bc)/(e-p+1)(p-1)}, \\ D_2 d(x) &\geq b(x)(m-n)^{((a-p+1)(e-p+1)-bc)/(a-p+1)(p-1)}. \end{aligned} \tag{3.6}$$

Since $a(x), b(x) \in C(\overline{\Omega})$, choosing m is large enough, and $m-n>0$ is sufficiently small, we can prove that

$$(\underline{u}, \underline{v}) = \left(\left(\frac{m-n}{(m+n)^{b/(a-p+1)}} \right)^{1/(p-1)} u_2, \left(\frac{m+n}{(m-n)^{c/(e-p+1)}} \right)^{1/(p-1)} v_2 \right) \tag{3.7}$$

is a subsolution of (1.1), where (u_2, v_2) is a solution of the following problem:

$$\begin{aligned} \Delta_p \Phi &= C_2 d(x)^{\gamma_2} \Phi^a \Psi^b \quad \text{in } \Omega, \\ \Delta_p \Psi &= D_1 d(x)^{\eta_1} \Phi^c \Psi^e \quad \text{in } \Omega, \\ \Phi &= \Psi = +\infty \quad \text{on } \partial\Omega. \end{aligned} \tag{3.8}$$

Then by Proposition 2.2, problem (1.1) has a solution. □

Lemma 3.2. *Assume that problem* (1.1) *has a solution* (u, v), *then* (1.9) *holds.*

Proof. In fact, if (1.9) does not hold, it will lead to a contradiction. From Lemma 3.1, we find that if m is large enough and $m-n>0$ is sufficiently small, we have

$$u \leq \overline{u} = \left(\frac{m+n}{(m-n)^{b/(a-p+1)}} \right)^{1/(p-1)} u_1, \qquad v \leq \underline{v} = \left(\frac{m+n}{(m-n)^{c/(e-p+1)}} \right)^{1/(p-1)} v_2. \tag{3.9}$$

On the other hand, by (2.3), there exists $\varepsilon>0$ such that for $x \in \Omega_\varepsilon = \{x \in \Omega : d(x, \partial\Omega) \leq \varepsilon\}$, we get

$$u \leq \left(\frac{m+n}{(m-n)^{b/(a-p+1)}} \right)^{1/(p-1)} u_1 \leq \left(\frac{m+n}{(m-n)^{b/(a-p+1)}} \right)^{1/(p-1)} E(D_2, C_1) d(x)^{-\alpha(\gamma_1, \eta_2)}. \tag{3.10}$$

Thus, if

$$\frac{b}{e-p+1} \geq \frac{p+\gamma_1}{p+\eta_2}, \tag{3.11}$$

by the definition of $\alpha(\gamma_1, \eta_2)$, we obtain $\alpha(\gamma_1, \eta_2) \leq 0$. By (3.10), it implies that u is bounded for $x \in \Omega_\varepsilon$, which is impossible since $u(x) = +\infty$ as $d(x) = \text{dist}(x, \partial\Omega) \to 0^+$. If

$$\frac{p+\gamma_2}{p+\eta_2} \geq \frac{a-p+1}{c}, \tag{3.12}$$

it is similarly proved that v is bounded near $\partial\Omega$, which is also a contradiction. The proof of Lemma 3.2 is complete. □

Lemma 3.3. *Let* (u, v) *be a positive solution of* (1.1), *then* (1.10) *and* (1.11) *hold.*

Proof. Fix $\tau \in (0,1)$, by (1.2), there exits $\sigma \in (0,1)$ such that, if $d(x, x_0) < \sigma$,

$$a(x) \geq (1-\tau)C(x_0)d(x)^{\gamma(x_0)}, \qquad b(x) \leq (1+\tau)D(x_0)d(x)^{\eta(x_0)}, \tag{3.13}$$

where $x_0 \in \partial\Omega$. For a fixed $x_0 \in \partial\Omega$, set

$$\Sigma = \overline{B}_{\sigma/2} \bigcap \partial\Omega \tag{3.14}$$

and choose $R > 0$ small enough such that

$$K = \bigcup_{y \in \Sigma} \overline{B}_R(y - Rn_y) \subset B_\sigma(x_0) \bigcap \Omega, \tag{3.15}$$

where n_y stands for the outward unit normal at $y \in \partial\Omega$.

For $x \in B_\sigma(x_0) \cap \Omega$, we get

$$a(x) \geq (1-\tau)C(x_0)d(x)^{\gamma(x_0)}, \qquad b(x) \leq (1+\tau)D(x_0)d(x)^{\eta(x_0)}. \tag{3.16}$$

Since Ω is of C^2 bounded domain, there exit $R > 0$ and $\sigma_0 > 0$ such that

$$B_R(x_0 - (R+\sigma)n_{x_0}) \subset \Omega, \qquad B_R(x_0 - Rn_{x_0}) \bigcap \partial\Omega = \{x_0\}, \tag{3.17}$$

for each $\sigma \in (0, \sigma_0)$.

Let $(u_{B,\sigma}, v_{B,\sigma})$ be any positive radially symmetric solution to the following system:

$$\begin{gathered}
\Delta_p u = (1-\tau)C(x_0)(R - |x - x_0|)^{\gamma(x_0)} u^a v^b \quad \text{in } B_R(x_0 - (R+\sigma)n_{x_0}), \\
\Delta_p v = (1+\tau)D(x_0)(R - |x - x_0|)^{\eta(x_0)} u^c v^e \quad \text{in } B_R(x_0 - (R+\sigma)n_{x_0}), \\
u = v = +\infty \quad \text{on } \partial B_R(x_0 - (R+\sigma)n_{x_0}).
\end{gathered} \tag{3.18}$$

It is easy to see that $(\underline{u}_\sigma, \underline{v}_\sigma) = (u, v)|_{B_R(x_0 - (R+\sigma)n_{x_0})}$ is a positive smooth subsolution of (3.18), where (u, v) is a positive solution of (1.1).

Then we get

$$\underline{u}_\sigma = u|_{B_R(x_0-(R+\sigma)n_{x_0})} \le u_{B,\sigma}, \qquad \underline{v}_\sigma = v|_{B_R(x_0-(R+\sigma)n_{x_0})} \ge v_{B,\sigma}. \tag{3.19}$$

Let (u_B, v_B) be any positive solution to the following system:

$$\begin{gathered}\Delta_p u = (1-\tau)C(x_0)(R-|x-x_0|)^{\gamma(x_0)}u^a v^b \quad \text{in } B_R(x_0 - Rn_{x_0}),\\ \Delta_p v = (1+\tau)D(x_0)(R-|x-x_0|)^{\eta(x_0)}u^c v^e \quad \text{in } B_R(x_0 - Rn_{x_0}),\\ u = v = +\infty \quad \text{on } \partial B_R(x_0 - Rn_{x_0}).\end{gathered} \tag{3.20}$$

By Proposition 2.3, (u_B, v_B) satisfies

$$\lim_{r\to R}\frac{u_B}{E((1+\tau)D(x_0),(1-\tau)C(x_0))(R-r)^{-\alpha(\gamma(x_0),\eta(x_0))}} = 1, \tag{3.21}$$

$$\lim_{r\to R}\frac{v_B}{F((1+\tau)D(x_0),(1-\tau)C(x_0))(R-r)^{-\beta(\gamma(x_0),\eta(x_0))}} = 1, \tag{3.22}$$

where $r = |x - x_0|$.

Taking into account that, for $x \in B_R(x_0 - (R+\sigma)n_{x_0})$,

$$u_{B,\sigma}(x) = u_B(x + \sigma n_{x_0}), \qquad v_{B,\sigma}(x) = v_B(x + \sigma n_{x_0}), \tag{3.23}$$

by (3.19), for each $x \in B_R(x_0 - (R+\sigma)n_{x_0})$ and $\sigma \in (0, \sigma_0)$, we have

$$u(x) \le u_B(x + \sigma n_{x_0}), \qquad v(x) \ge v_B(x + \sigma n_{x_0}). \tag{3.24}$$

Let $\sigma \to 0$, we have

$$u(x) \le u_B(x), \qquad v(x) \ge v_B(x). \tag{3.25}$$

It follows immediately from (3.21), (3.22) that

$$\lim_{r\to R}\frac{u}{E(R-r)^{-\alpha}} \le \lim_{r\to R}\frac{u_B}{E(R-r)^{-\alpha}} = 1, \tag{3.26}$$

$$\lim_{r\to R}\frac{v}{F(R-r)^{-\beta}} \ge \lim_{r\to R}\frac{v_B}{F(R-r)^{-\beta}} = 1, \tag{3.27}$$

where $E = E((1+\tau)D(R),(1-\tau)C(R)), F = F((1+\tau)D(R),(1-\tau)C(R))$.

We next have to prove the inverse inequalities. Similarly, there exits $R > R_1 > 0$ and $\sigma_0 > 0$ such that $\Omega \subset \bigcap_{0<\sigma<\sigma_0} A_{R_1,R}(x_0 + (R+\sigma)n_{x_0})$ and $A_{R_0,R}(x_0 + R_1 n_{x_0}) \bigcap \partial\Omega = \{x_0\}$.

Fix a sufficiently small τ, there exit radially symmetric functions $\overline{a} : A_{R_1,R}(x_0 + R_1 n_{x_0}) \to R^+$ and $\underline{b} : A_{R_1,R}(x_0 + R_1 n_{x_0}) \to R^+$ such that $\overline{a} \geq a, \underline{b} \leq b$ in Ω, and

$$\max_{\overline{A}_{R_1,R(x_0+R_1 n_{x_0})}} \overline{a} \leq \max_{\Omega} a + 1, \qquad \max_{\Omega} b + 1 \leq \max_{\overline{A}_{R_1,R(x_0+R_1 n_{x_0})}} \underline{b}, \tag{3.28}$$

and for each $x \in A_{R_1,R}(x_0 + R_1 n x_0)$

$$\begin{aligned}\overline{a}(x) &= a_1(|x - x_0 - R_1 n_{x_0}|)\left[d\left(x, \partial A_{R_1,R\left(x_0+R_1 n_{x_0}\right)}\right)\right]^{\gamma(x_0)}, \\ \underline{b}(x) &= b_1(|x - x_0 - R_1 n_{x_0}|)\left[d\left(x, \partial A_{R_1,R\left(x_0+R_1 n_{x_0}\right)}\right)\right]^{\eta(x_0)},\end{aligned} \tag{3.29}$$

where $a_1, b_1 \in C([R_1, R], R^+)$, satisfing

$$a_1(R_1) = C(x_0) + \tau, \qquad b_1(R_1) = D(x_0) - \tau. \tag{3.30}$$

We now consider the system

$$\begin{aligned}\Delta_p u &= \overline{a}(x) u^a v^b \quad \text{in } A_{R_1,R}(x_0 + R_1 n_{x_0}), \\ \Delta_p v &= \underline{b}(x) u^c v^e \quad \text{in } A_{R_1,R}(x_0 + R_1 n_{x_0}), \\ u &= v = +\infty \quad \text{on } \partial A_{R_1,R}(x_0 + R_1 n_{x_0}).\end{aligned} \tag{3.31}$$

By Proposition 2.5, problem (3.31) possesses a solution (u_A, v_A).

But for the system

$$\begin{aligned}\Delta_p u &= \overline{a}(x) u^a v^b \quad \text{in } A_{R_1,R}(x_0 + (R_1 + \sigma) n_{x_0}), \\ \Delta_p v &= \underline{b}(x) u^c v^e \quad \text{in } A_{R_1,R}(x_0 + (R_1 + \sigma) n_{x_0}), \\ u &= v = +\infty \quad \text{on } \partial A_{R_1,R}(x_0 + (R_1 + \sigma) n_{x_0}),\end{aligned} \tag{3.32}$$

it has a solution $(u_{A,\sigma}, v_{A,\sigma})$, and for each $x \in A_{R_1,R}(x_0 + (R_1 + \sigma) n_{x_0})$, we have

$$(u_{A,\sigma}(x), v_{A,\sigma}(x)) = (u_A(x - \sigma n_{x_0}),\ v_A(x - \sigma n_{x_0})). \tag{3.33}$$

It is also clear that $(\underline{u}_A(x), \underline{v}_A(x)) = (u_{A,\sigma}(x), v_{A,\sigma}(x))|_\Omega$ is a subsolution of problem (1.1). Thus for each $x \in A_{R_1,R}(x_0 + (R_1 + \sigma)n_{x_0})$, we get $u_A(x - \sigma n_{x_0}) \le u(x), v_A(x - \sigma n_{x_0}) \ge v(x)$. Let $\sigma \to 0$, we have $\underline{u}_A(x) \le u(x), \underline{v}_A(x) \ge v(x)$. Thus for $x \in K$, we get

$$\begin{aligned} 1 &= \lim_{|x| \to R} \frac{\underline{u}_A(x)}{E\left(\overline{a}(x), \underline{b}(x)\right)(R - |x|)^{-\alpha\left(\gamma(x_0), \eta(x_0)\right)}} \\ &\le \lim_{d(x) \to 0} \frac{u(x)}{E\left(\overline{a}(x), \underline{b}(x)\right)(R - |x|)^{-\alpha\left(\gamma(x_0), \eta(x_0)\right)}}, \end{aligned} \tag{3.34}$$

$$\begin{aligned} 1 &= \lim_{|x| \to R} \frac{\underline{v}_A(x)}{F\left(\overline{a}(x), \underline{b}(x)\right)(R - |x|)^{-\beta(\gamma(x_0), \eta(x_0))}} \\ &\ge \lim_{d(x) \to 0} \frac{v(x)}{F\left(\overline{a}(x), \underline{b}(x)\right)(R - |x|)^{-\beta(\gamma(x_0), \eta(x_0))}} \end{aligned} \tag{3.35}$$

but we have $\lim_{\tau \to 0} K = \{x_0\}$. Therefore, by (3.26), (3.27), (3.34), and (3.35), we finish (1.10) and (1.11). The proof of Lemma 3.3 is complete. From Lemma 3.1 to Lemma 3.3, we finish the proof of Theorem 1.1. □

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