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Research Article

An Order on Subsets of Cone Metric Spaces and Fixed Points of Set-Valued Contractions

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In this paper at first we introduce a new order on the subsets of cone metric spaces then, using this definition, we simplify the proof of fixed point theorems for contractive set-valued maps, omit the assumption of normality, and obtain some generalization of results.

1. Introduction and Preliminary

Cone metric spaces were introduced by Huang and Zhang [1]. They replaced the set of real numbers by an ordered Banach space and obtained some fixed point theorems for mapping satisfying different contractions [1]. The study of fixed point theorems in such spaces followed by some other mathematicians, see [2–8]. Recently Wardowski [9] was introduced the concept of set-valued contractions in cone metric spaces and established some end point and fixed point theorems for such contractions. In this paper at first we will introduce a new order on the subsets of cone metric spaces then, using this definition, we simplify the proof of fixed point theorems for contractive set-valued maps, omit the assumption of normality, and obtain some generalization of results.

Let E be a real Banach space. A nonempty convex closed subset $P \subset E$ is called a cone in E if it satisfies.

(i) P is closed, nonempty, and $P \neq \{0\}$,

(ii) $a, b \in \mathbb{R}$, $a, b \geq 0$, and $x, y \in P$ imply that $ax + by \in P$,

(iii) $x \in P$ and $-x \in P$ imply that $x = 0$.

The space E can be partially ordered by the cone $P \subset E$; that is, $x \leq y$ if and only if $y - x \in P$. Also we write $x \ll y$ if $y - x \in P^o$, where P^o denotes the interior of P.

A cone P is called normal if there exists a constant $K > 0$ such that $0 \leq x \leq y$ implies $\|x\| \leq K\|y\|$.

In the following we always suppose that E is a real Banach space, P is a cone in E, and $\leq$ is the partial ordering with respect to P.

Definition 1.1 (see [1]). Let X be a nonempty set. Assume that the mapping $d : X \times X \to E$ satisfies

(i) $0 \leq d(x, y)$ for all $x, y \in X$ and $d(x, y) = 0$ iff $x = y$

(ii) $d(x, y) = d(y, x)$ for all $x, y \in X$

(iii) $d(x, y) \leq d(x, z) + d(z, y)$ for all $x, y, z \in X$.

Then d is called a cone metric on X, and (X, d) is called a cone metric space.

In the following we have some necessary definitions.

(1) Let (M, d) be a cone metric space. A set $A \subseteq M$ is called *closed* if for any sequence $\{x_n\} \subseteq A$ convergent to x, we have $x \in A$.

(2) A set $A \subseteq M$ is called *sequentially compact* if for any sequence $\{x_n\} \subseteq A$, there exists a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ is convergent to an element of A.

(3) Denote $N(M)$ a collection of all nonempty subsets of M, $C(M)$ a collection of all nonempty closed subsets of M and $K(M)$ a collection of all nonempty sequentially compact subsets of M.

(4) An element $x \in M$ is said to be an *endpoint* of a set-valued map $T : M \to N(M)$, if $Tx = \{x\}$. We denote a set of all endpoints of T by $\mathrm{End}(T)$.

(5) An element $x \in M$ is said to be a *fixed point* of a set-valued map $T : M \to N(M)$, if $x \in Tx$. Denote $\mathrm{Fix}(T) = \{x \in M \mid x \in Tx\}$.

(6) A map $f : M \to \mathbb{R}$ is called *lower semi-continuous,* if for any sequence $\{x_n\}$ in M and $x \in M$, such that $x_n \to x$ as $n \to \infty$, we have $f(x) \leq \liminf_{n\to\infty} f(x_n)$.

(7) A map $f : M \to E$ is called *have lower semi-continuous property,* and denoted by *lsc property* if for any sequence $\{x_n\}$ in M and $x \in M$, such that $x_n \to x$ as $n \to \infty$, then there exists $N \in \mathbb{N}$ that $f(x) \leq f(x_n)$ for all $n \geq N$.

(8) P called *minihedral* cone if $\sup\{x, y\}$ exists for all $x, y \in E$, and *strongly minihedral* if every subset of E which is bounded from above has a supremum [10]. Let (M, d) a cone metric space, cone P is strongly minihedral and hence, every subset of P has infimum, so for $A \in C(M)$, we define $d(x, A) = \inf_{y\in A} d(x, y)$.

Example 1.2. Let $E := \mathbb{R}^n$ with $P := \{(x_1, x_2, \dots, x_n) : x_i \geq 0$ for all $i = 1, 2, \dots, n\}$. The cone P is normal, minihedral and strongly minihedral with $P^o \neq \emptyset$.

Example 1.3. Let $D \subseteq \mathbb{R}^n$ be a compact set, $E := C(D)$, and $P := \{f \in E : f(x) \geq 0$ for all $x \in D\}$. The cone P is normal and minihedral but is not strongly minihedral and $P^o \neq \emptyset$.

Example 1.4. Let (X, S, μ) be a finite measure space, S countably generated, $E := L^p(X)$, $(1 < p < \infty)$, and $P := \{f \in E : f(x) \geq 0 \mu$ a.e. on $X\}$. The cone P is normal, minihedral and strongly minihedral with $P^o = \emptyset$.

For more details about above examples, see [11].

Example 1.5. Let $E := C^2([0,1],\mathbb{R}^+)$ with norm $\|f\| = \|f\|_\infty + \|f'\|_\infty$ and $P := \{f \in E : f \geq 0\}$ that is not normal cone by [12] and not minihedral by [10].

Example 1.6. Let $E := \mathbb{R}^2$ and $P := \{(x_1, 0) : x_1 \geq 0\}$. This P is strongly minihedral but not minihedral by [10].

Throughout, we will suppose that P is strongly minihedral cone in E with nonempty interior and $\leq$ be a partial ordering with respect to P.

2. Main Results

Let (M, d) be a cone metric space and $T : M \to C(M)$. For $x, y \in M$, Let

$$\begin{aligned} D(x, Ty) &= \{d(x, z) : z \in Ty\}, \\ S(x, Ty) &= \{u \in D(x, Ty) : \|u\| = \inf\{\|v\| : v \in D(x, Ty)\}\}. \end{aligned} \tag{2.1}$$

At first we prove the closedness of $\mathrm{Fix}(T)$ without the assumption of normality.

Lemma 2.1. *Let (M, d) be a complete cone metric space and $T : M \to C(M)$. If the function $f(x) = \inf_{y \in Tx} \|d(x, y)\|$ for $x \in M$ is lower semi-continuous, then* $\mathrm{Fix}(T)$ *is closed.*

Proof. Let $x_n \in Tx_n$ and $x_n \to x$. We show that $x \in Tx$. Since

$$\begin{aligned} f(x) &\leq \liminf_{n \to \infty} f(x_n) = \liminf_{n \to \infty} \inf_{y \in Tx_n} \|d(x_n, y)\|, \\ &\leq \liminf_{n \to \infty} \|d(x_n, x_n)\| = 0, \end{aligned} \tag{2.2}$$

so $f(x) = 0$ which implies $d(y_n, x) \to 0$ for some $y_n \in Tx$. Let $c \in E$ with $c \gg 0$ then, there exists N such that for $n \geq N$, $d(y_n, x) \ll (1/2)c$. Now, for $n > m$, we have,

$$d(y_n, y_m) \leq d(y_n, x) + d(x, y_m) \ll \frac{1}{2}c + \frac{1}{2}c = c. \tag{2.3}$$

So $\{y_n\}$ is a Cauchy sequence in complete metric space, hence there exist $y^* \in M$ such that $y_n \to y^*$. Since Tx is closed, thus $y^* \in Tx$. Now by uniqueness of limit we conclude that $x = y^* \in Tx$. □

Definition 2.2. Let A and B are subsets of E, we write $A \preceq B$ if and only if there exist $x \in A$ such that for all $y \in B$, $x \leq y$. Also for $x \in E$, we write $x \preceq B$ if and only if $\{x\} \preceq B$ and similarly $A \preceq x$ if and only if $A \preceq \{x\}$.

Note that $aA + B := \{ax + y : x \in A, y \in B\}$, for every scaler $a \in \mathbb{R}^+$ and A, B subsets of E.

The following lemma is easily proved.

Lemma 2.3. *Let* $A, B, C \subseteq E$, $x, y \in E$, $a \in \mathbb{R}^+$, *and* $a \neq 0$.

(1) *If* $A \preceq B$, *and* $B \preceq C$, *then* $A \preceq C$,

(2) $A \preceq B \Leftrightarrow aA \preceq aB$,

(3) *If* $x \preceq B$, *then* $ax \preceq aB$,

(4) *If* $A \preceq y$, *then* $aA \preceq ay$,

(5) $x \leq y \Leftrightarrow \{x\} \preceq \{y\}$,

(6) *If* $A \preceq B$, *then* $A \preceq B + P$.

The order "$\preceq$" is not antisymmetric, thus this order is not partially order.

Example 2.4. Let $E := \mathbb{R}$ and $P := \mathbb{R}^+$. Put $A := [1, 3)$ and $B := [1, 4]$ so $A \preceq B, B \preceq A$ but $A \neq B$.

Theorem 2.5. *Let* (M, d) *be a complete cone metric space,* $T : M \to C(M)$, *a set-valued map and the function* $f : M \to P$ *defined by* $f(x) = d(x, Tx)$, $x \in M$ *with lsc property. If there exist real numbers* $a, b, c, e \geq 0$ *and* $q > 1$ *with* $k := aq + b + ceq < 1$ *such that for all* $x \in M$ *there exists* $y \in Tx$:

$$\begin{aligned} d(x, y) &\preceq qD(x, Tx), \\ D(y, Tx) &\preceq ed(x, y), \\ D(y, Ty) &\preceq ad(x, y) + bD(x, Tx) + cD(y, Tx), \end{aligned} \tag{2.4}$$

then $\mathrm{Fix}(T) \neq \emptyset$.

Proof. Let $x \in M$, then there exists $y \in Tx$ such that

$$\begin{aligned} D(y, Ty) &\preceq ad(x, y) + bD(x, Tx) + cD(y, Tx) \\ &\preceq (aq + b + ceq)D(x, Tx) = kD(x, Tx). \end{aligned} \tag{2.5}$$

Let $x_0 \in M$, there exist $x_1 \in Tx_0$ such that $D(x_1, Tx_1) \preceq kD(x_0, Tx_0)$ and $d(x_0, x_1) \preceq qD(x_0, Tx_0)$. Continuing this process, we can iteratively choose a sequence $\{x_n\}$ in M such that $x_{n+1} \in Tx_n$, $D(x_n, Tx_n) \preceq k^n D(x_0, Tx_0)$, and $d(x_n, x_{n+1}) \preceq qD(x_n, Tx_n) \preceq qk^n D(x_0, Tx_0)$. So, for $n > m$, we have,

$$\begin{aligned} \{d(x_n, x_m)\} &\preceq \{d(x_n, x_{n-1}) + d(x_{n-1}, x_{n-2}) + \cdots + d(x_{m+1}, x_m)\} \\ &\preceq q\left(k^{n-1} + k^{n-2} + \cdots + k^m\right)D(x_0, Tx_0) \\ &\preceq qk^m\left(1 + k + k^2 + \cdots\right)D(x_0, Tx_0) \\ &\preceq q\frac{k^m}{1 - k}D(x_0, Tx_0). \end{aligned} \tag{2.6}$$

Therefore, for every $u_0 \in D(x_0, Tx_0)$, $d(x_n, x_m) \le q(k^m/(1-k))u_0$. Let $c \in E$ and $c \gg 0$ be given. Choose $\delta > 0$ such that $c + N_\delta(0) \subseteq P$, where $N_\delta(0) = \{x \in E : \|x\| < \delta\}$. Also, choose a $N \in \mathbb{N}$ such that $q(k^m/(1-k))u_0 \in N_\delta(0)$, for all $m \ge N$. Then $q(k^m/(1-k))u_0 \ll c$, for all $m \ge N$. Thus $d(x_n, x_m) \le q(k^m/(1-k))u_0 \ll c$ for all $n > m$. Namely, $\{x_n\}$ is Cauchy sequence in complete cone metric space, therefore $x_n \to x^*$ for some $x^* \in$ M. Now we show that $x^* \in Tx^*$.

Let $u_n \in D(x_n, Tx_n)$ hence there exists $t_n \in Tx_n$ such that $0 \le u_n = d(x_n, t_n) \le k^n u_0$ for all $u_0 \in D(x_0, Tx_0)$. Now $k^n u_0 \to 0$ as $n \to \infty$ so for all $0 \ll c$ there exists $N \in \mathbb{N}$ such that $0 \le u_n = d(x_n, t_n) \le k^n u_0 \ll c$ for all $n \ge N$.

According to *lsc property* of f, for all $c \gg 0$ there exists $N \in \mathbb{N}$ such that for all $n \ge N$

$$f(x^*) \le f(x_n) = \inf_{y \in Tx_n} d(x_n, y) \le d(x_n, t_n) \ll c. \tag{2.7}$$

So $0 \le f(x^*) \ll c$ for all $c \gg 0$. Namely, $f(x^*) = 0$ thus $d(y_n, x^*) \to 0$ for some $y_n \in Tx^*$, and by the closedness of Tx^* we have $x^* \in Tx^*$. □

We notice that $d(x_n, x) \to 0$ implies that for all $c \gg 0$ there exists $N \in \mathbb{N}$ such that $d(x_n, x) \ll c$ for all $n \ge N$, but the inverse is not true.

Example 2.6. Let $M = E := C^2([0,1], \mathbb{R}^+)$ with norm $\|f\| = \|f\|_\infty + \|f'\|_\infty$ and $P := \{f \in E : f \ge 0\}$ that is not normal cone by [12]. Consider $x_n := (1 - \sin nt)/(n+2)$ and $y_n := (1 + \sin nt)/(n+2)$ so $0 \le x_n \le x_n + y_n \to 0$ and $\|x_n\| = \|y_n\| = 1$, (see [10]) Define cone metric $d : M \times M \to E$ with $d(f,g) = f + g$, for $f \ne g, d(f,f) = 0$. Since $0 \le x_n \ll c$, namely, $d(x_n, 0) \ll c$ but $d(x_n, 0) \nrightarrow 0$. Indeed $x_n \to 0$ in (M, d) but $x_n \nrightarrow 0$ in E. Even for $n > m, d(x_n, x_m) = x_n + x_m \ll c$ and $\|d(x_n, x_m)\| = \|x_n + x_m\| = 2$ in particular $d(x_n, x_{n+1}) \ll c$ but $d(x_n, x_{n+1}) \nrightarrow 0$.

Example 2.7. Let $M = E := C^2([0,1], \mathbb{R})$ with norm $\|f\| = \|f\|_\infty + \|f'\|_\infty$ and $P := \{f \in E : f \ge 0\}$ that is not normal cone. Define cone metric $d : M \times M \to E$ with $d(f,g) = f^2 + g^2$, for $f \ne g, d(f,f) = 0$ and set-valued mapping $T : M \to C(M)$ by $Tf = \{-f, 0, f\}$. In this space every Cauchy sequence converges to zero. The function $F(f) = d(f, Tf) = \inf_{g \in Tf} d(f,g) = \inf\{0, f^2, 2f^2\} = 0$ have *lsc property*. Also we have $D(f, Tf) = \{0, f^2, 2f^2\}$ and $D(f, Tg) = \{f^2, f^2 + g^2\}$. Now for $q > 1,\ e \ge 1,\ a, b, c \ge 0,\ k = aq + b + ceq < 1$ and for all $f \in M$ take $g := 0 \in Tf$. Therefore, it satisfies in all of the hypothesis of Theorem 2.5. So T has a fixed point $f \in Tf$. For sample take $a = b = c = 1/6,\ e = 1$, and $q = 2$.

Theorem 2.8. *Let (M, d) be a complete cone metric space, $T : M \to K(M)$, a set-valued map, and a function $f : M \to P$ defined by $f(x) = d(x, Tx)$, $x \in M$ with lsc property. The following conditions hold:*

(i) *if there exist real numbers $a, b, c, e \ge 0$ and $q > 1$ with $k := aq + b + ceq < 1$ such that for all $x \in M$, there exists $y \in Tx$:*

$$\begin{aligned} d(x,y) &\preceq qS(x,Tx), \\ S(y,Tx) &\preceq ed(x,y), \\ S(y,Ty) &\preceq ad(x,y) + bS(x,Tx) + cS(y,Tx), \end{aligned} \tag{2.8}$$

then $\mathrm{Fix}(T) \ne \emptyset$,

(ii) *if there exist real numbers* $a, b, c, e \geq 0$ *and* $q > 1$ *with* $k := aq + b + ceq < 1$ *such that for all* $x \in M$ *and* $y \in Tx$:

$$\begin{aligned} d(x,y) &\preceq qS(x,Tx), \\ S(y,Tx) &\preceq ed(x,y), \\ S(y,Ty) &\preceq ad(x,y) + bS(x,Tx) + cS(y,Tx), \end{aligned} \tag{2.9}$$

then $\mathrm{Fix}(T) = End(T) \neq \emptyset$.

Proof. (i) It is obvious that $S(x,Tx) \subseteq D(x,Tx)$. It is enough to show that $S(x,Tx) \neq \emptyset$ for all $x \in M$. However $S(x,Tx) = \emptyset$ for some $x \in M$, it implies $d(x,y) \preceq \emptyset$ for some $y \in Tx$, and this is a contradiction.

(ii) By (i), there exists $x^* \in M$ such that $x^* \in Tx^*$. Then for $y \in Tx^*$ and $0 \in S(x^*, Tx^*)$ we have $d(x^*, y) \preceq (1/b)S(x^*, Tx^*)$. Therefore, $d(x^*, y) \leq (1/b)0 = 0$. This implies that $x^* = y \in Tx^*$. □

Corollary 2.9. *Let* (M, d) *be a complete cone metric space,* $T : M \to C(M)$*, a set-valued map, and the function* $f : M \to P$ *defined by* $f(x) = d(x, Tx)$*, for* $x \in M$ *with lsc property. If there exist real numbers* $a, b \geq 0$ *and* $q > 1$ *with* $k := aq + b < 1$ *such that for all* $x \in M$ *there exists* $y \in Tx$ *with*

$$\begin{aligned} d(x,y) &\preceq qD(x,Tx), \\ D(y,Ty) &\preceq ad(x,y) + bD(x,Tx), \end{aligned} \tag{2.10}$$

then $\mathrm{Fix}(T) \neq \emptyset$.

To have Theorems 3.1 and 3.2 in [9], as the corollaries of our theorems we need the following lemma and remarks.

Lemma 2.10. *Let* (M, d) *be a cone metric space,* P *a normal cone with constant one and* $T : M \to C(M)$*, a set-valued map, then*

$$\|d(x,Tx)\| = \left\| \inf_{y \in Tx} d(x,y) \right\| = \inf_{y \in Tx} \|d(x,y)\|. \tag{2.11}$$

Proof. Put $\alpha := \inf_{y \in Tx} \|d(x,y)\|$ and $\beta := \inf_{y \in Tx} d(x,y)$ we show that $\alpha = \|\beta\|$.

Let $y \in Tx$ then $\beta \leq d(x,y)$ and so $\|\beta\| \leq \|d(x,y)\|$, which implies $\|\beta\| \leq \alpha$.

For the inverse, let for all $0 \leq r \leq \alpha$. Then $r \leq \|d(x,y)\|$ for all $y \in Tx$.

Since $\beta := \inf_{y \in Tx} d(x,y)$, for every c that $c \gg 0$ there exists $y \in Tx$ such that $d(x,y) < \beta + c$, so $r \leq \|d(x,y)\| < \|\beta + c\| \leq \|\beta\| + \|c\|$, for all $c \gg 0$. Thus $r \leq \|\beta\|$. □

Remark 2.11. By Proposition 1.7.59, page 117 in [11], if E is an ordered Banach space with positive cone P, then P is a normal cone if and only if there exists an equivalent norm $|\cdot|$ on E which is monotone. So by renorming the E we can suppose P is a normal cone with constant one.

Remark 2.12. Let (M,d) be a cone metric space, P a normal cone with constant one, $T : M \to C(M)$, a set-valued map, the function $f : M \to P$ defined by $f(x) = d(x,Tx)$, $x \in M$ with *lsc property*, and $g : E \to \mathbb{R}^+$ with $g(x) = \|x\|$. Then $gof(x) = \inf_{y\in Tx}\|d(x,y)\|$, is lower semi-continuous.

Now the Theorems 3.1 and 3.2 in [9] is stated as the following corollaries without the assumption of normality, and by Lemma 2.10 and Remarks 2.11, 2.12 we have the same theorems.

Corollary 2.13 (see [9, Theorem 3.1]). *Let (M,d) be a complete cone metric space, $T : M \to C(M)$, a set-valued map and the function $f : M \to P$ defined by $f(x) = d(x,Tx)$, $x \in M$ with lsc property. If there exist real numbers $0 \leq \lambda < 1$, $\lambda < b \leq 1$ such that for all $x \in M$ there exists $y \in Tx$ one has $D(y,Ty) \preceq \lambda d(x,y)$ and $bd(x,y) \preceq D(x,Tx)$ then* $\mathrm{Fix}(T) \neq \emptyset$.

Corollary 2.14 (see [9, Theorem 3.2]). *Let (M,d) be a complete cone metric space, $T : M \to K(M)$, a set-valued map and the function $f : M \to P$ defined by $f(x) = d(x,Tx)$, $x \in M$ with lsc property. The following hold:*

(i) *if there exist real numbers $0 \leq \lambda < 1$, $\lambda < b \leq 1$ such that for all $x \in M$ there exists $y \in Tx$ one has $S(y,Ty) \preceq \lambda d(x,y)$ and $bd(x,y) \preceq S(x,Tx)$, then* $\mathrm{Fix}(T) \neq \emptyset$,

(ii) *if there exist real numbers $0 \leq \lambda < 1$, $\lambda < b \leq 1$ such that for all $x \in M$ and every $y \in Tx$ one has $S(y,Ty) \preceq \lambda d(x,y)$ and $bd(x,y) \preceq S(x,Tx)$, then* $\mathrm{Fix}(T) = \mathit{End}(T) \neq \emptyset$.

Definition 2.15. For $A \subseteq M, T : M \to C(M)$ where T is a set-valued map we define

$$\overline{D}(A,TA) := \bigcup_{x\in A} D(x,Tx), \qquad \underline{D}(A,TA) := \bigcap_{x\in A} D(x,Tx). \tag{2.12}$$

Note that $T^2x = T(Tx)$ for $x \in M$.

The following theorem is a reform of Theorem 2.5.

Theorem 2.16. *Let (M,d) be a complete cone metric space, $T : M \to C(M)$, a set-valued map, and the function $f : M \to P$ defined by $f(x) = d(x,Tx)$, $x \in M$ with lsc property. If there exists $0 \leq k < 1$ such that*

$$\overline{D}\left(Tx,T^2x\right) \preceq k\underline{D}(M,TM). \tag{2.13}$$

for all $x \in M$. Then $\mathrm{Fix}(T) \neq \emptyset$.

Proof. For every $x \in M$, then there exist $y \in Tx$ and $z \in Ty$ such that $d(y,z) \preceq kd(x,t)$, for all $t \in Tx$. Let $x_n \in M$, there exist $x_{n+1} \in Tx_n$ and $x_{n+2} \in Tx_{n+1}$ such that $d(x_{n+1},x_{n+2}) \preceq kd(x_n,x_{n+1})$, since $x_{n+1} \in Tx_n$. Thus $d(x_n,x_{n+1}) \preceq k^n d(x_0,x_1)$. The remaining is same as the proof of Theorem 2.5. □

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