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Research Article

Graphs with Constant Sum of Domination and Inverse Domination Numbers

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A subset D of the vertex set of a graph G, is a *dominating set* if every vertex in $V - D$ is adjacent to at least one vertex in D. The domination number $\gamma(G)$ is the minimum cardinality of a dominating set of G. A subset of $V - D$, which is also a dominating set of G is called an *inverse dominating set* of G with respect to D. The inverse domination number $\gamma'(G)$ is the minimum cardinality of the inverse dominating sets.Domke et al. (2004) characterized connected graphs G with $\gamma(G)+\gamma'(G) = n$, where n is the number of vertices in G. It is the purpose of this paper to give a complete characterization of graphs G with minimum degree at least two and $\gamma(G) + \gamma'(G) = n - 1$.

1. Introduction

Let $G = (V, E)$ be a simple graph. For $D \subseteq V$, if every vertex in $V - D$ is adjacent to at least one vertex in D, then D is said to be a *dominating set* of G [1]. A dominating set D is said to be a *minimal dominating set* if no proper subset of D is a dominating set of G. The minimum cardinality among all dominating sets of G is called *domination number* of G, and it is denoted by $\gamma(G)$. Any dominating set of G with cardinality $\gamma(G)$ is noted as a γ set of G [1]. Let D be a γ-set of G. If $V - D$ contains a dominating set D' of G, then D' is called an *inverse dominating set* with respect to D. The minimum cardinality of all inverse dominating sets is called the *inverse domination number* [2] and is denoted by $\gamma'(G)$. An inverse dominating set D' is called a γ'- *set* if $|D'| = \gamma'$. By virtue of the definition of the inverse domination number, $\gamma(G) \leq \gamma'(G)$. The concept of the inverse domination number was introduced by Kulli and Sigarkanti [2]. It is well known by Ore's Theorem [3] that if a graph G has no isolated vertices, then the complement $V - D$ of every γ-set D contains a dominating set. Thus any graph with no isolated vertices contains an inverse dominating set. However, for graphs with isolated vertices, one cannot find an inverse dominating set. For this reason, hereafter, we restrict ourselves to graphs with no isolated vertices.

A Gallai-type theorem has the form $\alpha(G) + \beta(G) = n$, where $\alpha(G)$ and $\beta(G)$ are parameters defined on the graph G, and n is the number of vertices in G. Cockayne et al. [4] proved certain Gallai-type theorems for graphs. In the year 1996, Cockayne et al. characterized graphs with $\delta(G) \geq 2$ and $\gamma(G) = \lfloor n/2 \rfloor$. Since then Baogen et al. [5] and Randerath and Volkmann [6] independently characterized all graphs G satisfying $\gamma(G) = \lfloor n/2 \rfloor$. Next in the year 2004, Domke et al. [7] characterized graphs for which $\gamma(G) + \gamma'(G) = n$. Later on, in the year 2010, Tamizh Chelvam and Grace Prema [8] characterized graphs with $\gamma(G) = \gamma'(G) = (n-1)/2$ where n is an odd positive integer. Now, in this paper we characterize all graphs G with $\delta(G) \geq 2$ for which $\gamma(G) + \gamma'(G) = n - 1$.

Motivated by the inverse domination number, Hedetniemi et al. [9] defined and studied the disjoint domination number $\gamma\gamma(G)$ of a graph G. A pair (D_1, D_2) of disjoint sets of vertices $D_1, D_2 \subseteq V$ is said to dominate a vertex $u \in V$, if D_1 and D_2 dominate u. Further (D_1, D_2) is a dominating pair, if (D_1, D_2) dominates all vertices in V. The total cardinality of a pair (D_1, D_2) is $|D_1| + |D_2|$, and the minimum cardinality of a dominating pair is the disjoint domination number $\gamma\gamma(G)$ of G. As mentioned earlier, by Ore's observation, $\gamma\gamma(G) \leq |V(G)|$ for every graph G without isolated vertices and Hedetniemi et al. characterized all extremal graphs for this bound. In this connection, the existence of two disjoint minimum dominating sets in trees was first studied by Bange et al. [10]. In a related paper, Haynes and Henning [11] studied the existence of two disjoint minimum independent dominating sets in a tree.

Another application of finding two disjoint γ sets is the one in respect of networks. In any network (or graphs), dominating sets are central sets, and they play a vital role in routing problems in parallel computing [12]. Also finding efficient dominating sets is always concern in finding optimal central sets in networks [13]. Suppose that S is a γ-set in a graph (or network) G, when the network fails in some nodes in S, the inverse dominating set in $V - S$ will take care of the role of S. In this aspect, it is worthwhile to concentrate on dominating and inverse dominating sets. Note that $\gamma'(G) \geq \gamma(G)$. From the point of networks, one may demand $\gamma'(G) = \gamma(G)$, where as many graphs do not enjoy such a property. For example consider the star graph $K_{1,n}$. Clearly $\gamma(K_{1,n}) = 1$ where as $\gamma'(K_{1,n}) = n$. If we consider the graph $G = K_{1,n} \square K_2$ with $n \geq 3$, then $\gamma(G) = 2$ and $\gamma'(G) = n$. In both the cases if n is large, then $\gamma'(G)$ is sufficiently large compare to $\gamma(G)$.

The purpose of this paper is to characterize all graphs G with $\delta(G) \geq 2$ for which $\gamma(G) + \gamma'(G) = n - 1$. In this regards, it may be possible that $\gamma'(G)$ is larger than $\gamma(G)$ and $\gamma(G) + \gamma'(G) = n - 1$. But we prove that graphs G with $\gamma(G) + \gamma'(G) = n - 1$ having exactly two disjoint minimum dominating sets. Hereafter G denotes a simple graph on n vertices with no isolated vertices. The minimum degree of a graph G is denoted by $\delta(G)$. The set of neighbors of a vertex v in a graph G is denoted by $N_G(v)$, and the set of neighbors of v in an induced subgraph of G induced by $A \subseteq V(G)$ is denoted by $N_A(v)$. Also P_n and C_n denote the path and cycle on n vertices, respectively. The *Cartesian product* of graphs G_1 and G_2 is the graph $G_1 \square G_2$ whose vertex set is $V(G_1) \times V(G_2)$ and whose edge set is the set of all pairs $(u_1, v_1)(u_2, v_2)$ such that either $u_1u_2 \in E(G_1)$ and $v_1 = v_2$ or $v_1v_2 \in E(G_2)$ and $u_1 = u_2$.

Let us first recall the following characterizations of graphs for which $\gamma(G) + \gamma'(G) = n$.

Theorem 1.1 (see [7, Theorem 2]). *Let G be a connected graph on n vertices with $\delta(G) \geq 2$. Then $\gamma(G) + \gamma'(G) = n$ if and only if $G = C_4$.*

Theorem 1.2 (see [7, Theorem 3]). *Let G be a connected graph on n vertices with $n \geq 1$ and $\delta(G) = 1$. Let $L \subseteq V$ be the set of all degree one vertices and $S = N(L)$. Then $\gamma(G) + \gamma'(G) = n$ if and only if the following two conditions hold:*

(1) *$V - S$ is an independent set and*

(2) *for every vertex $x \in V - (S \cup L)$, every stem in $N(x)$ is adjacent to at least two leaves.*

2. Graphs with $\gamma(G) + \gamma'(G) = n - 1$

Tamizh Chelvam and Grace Prema [8] characterized graphs for which $\gamma(G) = \gamma'(G) = (n-1)/2$. In this context, we attempt to characterize graphs G with $\delta(G) \geq 2$ for which $\gamma(G) + \gamma'(G) = n - 1$. To attain this aim, we first present the theorem which is useful in the further discussion. To prove the following theorem, since no better proof technique is available, authors prefer case by case analysis.

Theorem 2.1. *Let G be a connected graph on n vertices with $\delta(G) \geq 2$. Then $\gamma(G) + \gamma'(G) = n - 1$ implies that $\gamma(G) = \gamma'(G)$.*

Proof. Let D be a γ-set of G and D' be a γ'-set of G with respect to D. Note that $\gamma(G) \leq \gamma'(G)$. Assume that $\gamma(G) + \gamma'(G) = n - 1$, and let $V(G) - \{D \cup D'\} = \{w\}$. Let $S \subseteq D$ be those vertices that are adjacent to more than one vertex in D'. Suppose that $\gamma(G) < \gamma'(G)$. Then $|D| < |D'|$ and so $S \neq \emptyset$. Let $S' = N(S) \cap D'$.

Claim 1. There is at most one vertex in S' which is adjacent to a vertex in $D - S$.

Suppose not, there are at least two vertices t', r' in S' and $t, r \in D - S$ such that t' is adjacent to t and r' is adjacent to r. Then either both $t, r \in D - S$ are adjacent to w or at least one of t, r is not adjacent to w.

Suppose that both $t, r \in D - S$ are adjacent to w. Since t', r' are the only vertices in $V(G) - \{D \cup \{w\}\}$ which are adjacent to t, r, and t', r' are dominated by some vertices in S, $D_1 = D \cup \{w\} - \{t, r\} \subset D$ is a dominating set of G, which is a contradiction to the fact that D is a γ-set of G.

When at least one of them, say t, is not adjacent to w. Since $t \in D - S$ and $\delta(G) \geq 2$, t is adjacent to a vertex u in D. Therefore $D_1 = D - \{t\}$ is a dominating set of G, which is a contradiction to the fact that D is a γ-set of G. Hence, at most one vertex $t' \in S'$ which is adjacent to a vertex in $D - S$. By similar argument as given above, one can prove that t' is adjacent to exactly one vertex in $D - S$. Let us take

$$S'_1 = \begin{cases} S' - \{t'\} & \text{if } t' \text{ exists}, \\ S' & \text{otherwise}. \end{cases} \tag{2.1}$$

Note that each vertex in S has at least two neighbors in S' and so $S'_1 \neq \emptyset$.

Claim 2. (S'_1 is independent.)

Suppose that there exists a vertex $x' \in S_1'$ which is adjacent to $y' \in S_1'$. Suppose that w is not adjacent to both x' and y'. By the fact that each vertex in S has at least two neighbors in D' and Claim 1, $D' - \{x'\}$ is a γ'-set of G, a contradiction. If w is adjacent to one of x' or y', say x', then $D' - \{y'\}$ is a γ'-set of G, which is a contradiction. Hence S_1' is independent.

Now we have the following three possibilities.

(1) w is not adjacent to any of the vertices in S_1'.

(2) w is adjacent to exactly one vertex in S_1'.

(3) w is adjacent to more than one vertex in S_1'.

Case 1. Suppose that w is not adjacent to any of the vertices of S_1'. If there exists $x' \in S_1'$ which is adjacent to a vertex in $D' - S_1'$, then $D' - \{x'\}$ is a γ'-set with respect to D, a contradiction. Therefore, Claim 2 along with $\delta(G) \geq 2$ together implies that each vertex in S_1' has at least two neighbors in S. Suppose that there exists a vertex $x \in S$ which is adjacent to $y \in S$, then, as in the proof of Claim 2, we get that either $D - \{x\}$ or $D - \{y\}$ is a γ-set of G, a contradiction. Thus S is independent.

Case 1.1. Suppose that there exists a pair of vertices $u, v \in S$ such that $N_{S_1'}(u) \cap N_{S_1'}(v) = \{u'\}$ for some $u' \in S_1'$.

Case 1.1.1. If w is adjacent to a vertex in $D - \{u, v\}$, then, by the assumption in Case 1.1, the vertices in $S_1' - \{u'\}$, dominated by either u or v, are also adjacent to some vertex in $S - \{u, v\}$, and so $D - \{u, v\} \cup \{u'\}$ is a γ-set of G, which is a contradiction.

Case 1.1.2. If w is adjacent to one of u or v, say u, and let $u' \neq v' \in N_{S_1'}(v)$. Note that by assumption in Case 1, w is not adjacent to u' as well as v'.

If there exists $x \in S$ such that $N_{S_1'}(x) = \{u', v'\}$. Suppose that there exists no vertex in $S_1' - \{u', v'\}$ which is adjacent to only v and x, and then $D - \{v, x\} \cup \{v'\}$ is a γ-set of G, which is a contradiction. If there exists $x' \in S_1' - \{u', v'\}$ such that $N_S(x') = \{v, x\}$, then $D_1 = D - \{v\} \cup \{v'\}$ is a γ-set of G and $D_1' = D' - \{v', x'\} \cup \{v\}$ is a γ'-set of G with respect to D_1, a contradiction. If there exists $y \in S - \{u, v, x\}$ such that y is adjacent to v' and x' only, then by similar argument one can get a contradiction in all the cases.

If there is no vertex $x \in S$ such that $N_{S_1'}(x) = \{u', v'\}$, then, by Claim 2, $D_1 = D - \{v\} \cup \{v'\}$ is a γ-set of G, and so $D_1' = D - \{u', v'\} \cup \{v\}$ is a γ'-set of G, which is a contradiction.

Case 1.2. Suppose that, for each pair of vertices $x, y \in S$, there exist at least two vertices $x', y' \in S_1'$ such that $\{x', y'\} \subseteq N_{S_1'}(x) \cap N_{S_1'}(y)$.

Case 1.2.1. Suppose that, for some $u', v' \in S_1'$, there exists at most one vertex $u \in S$ such that $N_S(u') \cap N_S(v') = \{u\}$. If w is adjacent to a vertex in $D - \{u\}$, then $D_1 = D - \{u\} \cup \{u'\}$ is a γ-set, and so $D_1' = D' - \{u', v'\} \cup \{u\}$ is a γ'-set of G, a contradiction. If $N_D(w) = \{u\}$, then $D_1 = D - \{u\} \cup \{w\}$ is a γ-set and so $D_1' = D' - \{u', v'\} \cup \{u\}$ is a γ'-set of G, a contradiction.

Case 1.2.2. Suppose that, for each pair of vertices $x', y' \in S_1'$, there exist at least two vertices $x, y \in S$ such that $\{x, y\} \subseteq N_S(x') \cap N_S(y')$. Note that $|S| \geq 2$ and $|S_1'| \geq 2$. Assume that $|S| = k$ and $S = \{u_1, u_2, \ldots, u_k\}$. If w is adjacent to some vertex in S, say u_1, then by the assumption in Case 1.2, there exist $u_2 \in S$ and $u_1', u_2' \in S_1'$ such that $\langle \{u_1, u_2, u_1', u_2'\} \rangle = K_{2,2}$ as S, and S_1' are independent.

Assume that $|S| = k \geq 3$. Suppose that $N_{s_1'}(u_3) = \{u_1', u_2'\}$. Since u_2' dominates both u_2, u_3 and u_1', u_2' are the vertices dominated by u_2 and u_3, $D_1 = D - \{u_3, u_2\} \cup \{u_2'\}$ is a

γ-set of G, a contradiction. Thus u_3 is adjacent to vertex in $S - \{u'_1, u'_2\}$, say u'_3. Suppose that $N_S(u'_3) \subseteq \{u_1, u_2, u_3\}$. Since each pair of vertices in S has at least two neighbors, we have $D_1 = D - \{u_2\} \cup \{u'_2\}$ as a γ-set and $D'_1 = D' - \{u'_2, u'_3\} \cup \{u_2\}$ as a γ'-set of G, a contradiction. Thus u'_3 is adjacent to some vertex in $S - \{u_1, u_2, u_3\}$, take u_4. Proceed like this up to u_k, and let $u'_k \in N_{s'_1}(u_k)$. If $N_S(u'_k) \subseteq S$, then $D_1 = D - \{u_{k-1}\} \cup \{u'_{k-1}\}$ is a γ-set, and so $D'_1 = D' - \{u'_{k-1}, u'_k\} \cup \{u_{k-1}\}$ is a γ'-set of G, a contradiction. Hence u'_k is adjacent to at least one vertex in $S - \{u_1, \ldots, u_k\} = \emptyset$, which is not possible.

Let $|S| = k = 2$. If $|S'_1| \geq 3$, then u'_3 is adjacent to u_1 and u_2 only. Therefore $D_1 = D - \{u_2\} \cup \{u'_2\}$ is a γ-set, and $D'_1 = D' - \{u'_2, u'_3\} \cup \{u_2\}$ is a γ'-set of G, a contradiction. If $|S'_1| = 2$, then $D = D'$, which is a contradiction.

Case 2. Suppose that w is adjacent to exactly one vertex $x' \in S'_1$. If $u' \in S'_1 - \{x'\}$ is adjacent to a vertex in D', then $D' - \{u'\}$ is a γ'-set of G, a contradiction. Thus every vertex in $S'_1 - \{x'\}$ has at least two neighbors in S.

If $|S'_1| \geq 3$, then, as in Case 1, replacing S'_1 by $S'_1 - \{x'\}$, we get contradiction in all the possibilities.

Let $|S'_1| = 2$ and $S'_1 = \{x', y'\}$. Since y' has at least two neighbors in S, $|S| \geq 2$ and so $|S| \geq |S'_1|$, which is contradiction to $|D| < |D'|$.

If $|S'_1| = 1$, then since $|D| < |D'|$, $S = \emptyset$, a contradiction to $S \neq \emptyset$.

Case 3. Suppose w is adjacent to more than one vertex in S'_1, say $u', v' \in S'_1$. If no vertex in S is adjacent to only u', v', then $D' - \{u', v'\} \cup \{w\}$ is a γ-set of G, a contradiction. Thus there exists a vertex $u \in S$ such that $N_{S'_1}(u) = \{u', v'\}$. If w is adjacent to u, then $D_1 = D - \{u\} \cup \{w\}$ is a γ-set and $D'_1 = D' - \{u', v'\} \cup \{u\}$ is a γ'-set of G, a contradiction. Now let $u \neq x \in N_D(w)$ and let $x' \in N_{D'}(x)$.

Suppose that there exists $y \in D - \{x\}$ such that $y \in N(x')$. Suppose there exists $z \in D - \{u, x\}$ such that $N_{D'}(z) \subseteq \{u', v', x'\}$. Then $D - \{u, z\} \cup \{u'\}$ is a γ-set of G, a contradiction. Otherwise, $D_1 = D - \{u, x\} \cup \{u', w\}$ is a γ-set and $D'_1 = D' - \{u', v', x'\} \cup \{x, u\}$ is a γ'-set of G, a contradiction.

Suppose that $N_D(x') = \{x\}$. If x' is adjacent to w, then, $D' - \{u', x'\} \cup \{w\}$ is a γ'-set of G. If x' is not adjacent to w, then as $\delta(G) \geq 2$, x' is adjacent to at least a vertex say $y' \in D'$. Since $x' \in N(y')$, $x, u' \in N(w)$ and $N_D(x') = \{x\}$, we get $D' - \{u', x'\} \cup \{w\}$ is a γ'-set of G, which is a contradiction.

Hence, $\gamma(G) = \gamma'(G)$. □

Bange et al. [10] characterized trees with two disjoint minimum dominating sets. In the following corollary, we give the necessary condition for graphs with minimum degree at least two having two disjoint minimum dominating sets.

Corollary 2.2. *Let G be a connected graph on n vertices with $\delta(G) \geq 2$. If $\gamma(G) + \gamma'(G) = n - 1$, then G has two disjoint γ-sets.*

The following example shows that in general Theorem 2.1 is not true whenever $\delta(G) = 1$.

Example 2.3. (i) Consider the graph P_6, the path on 6 vertices. Then $\gamma(P_6) = 2$ and $\gamma'(P_6) = 3$. Therefore, $\gamma(P_6) + \gamma'(P_6) = 5$ but $\gamma(P_6) \neq \gamma'(P_6)$.

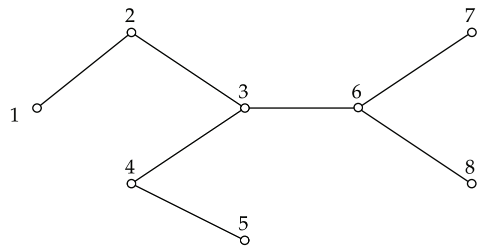

Figure 1: Graph G.

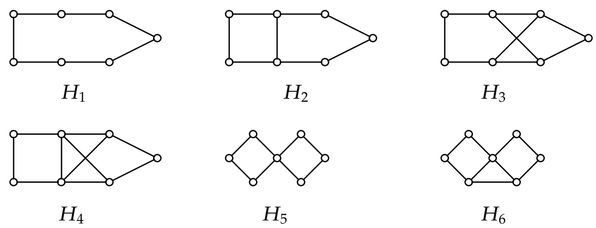

Figure 2: Graphs in family $\mathcal{C}$.

(ii) Consider the graph G in Figure 1. Clearly, $\gamma(G) = 3$ and $\gamma'(G) = 4$. Therefore $\gamma(G) + \gamma'(G) = 7 = n - 1$ whereas $\gamma(G) \neq \gamma'(G)$.

Lemma 2.4. *Let G be a connected graph with $\delta(G) \geq 2$. Then $\gamma(G) + \gamma'(G) = n - 1$ if and only if $\gamma(G) = \gamma'(G) = \lfloor n/2 \rfloor$, and n is odd.*

Proof. If $\gamma(G) + \gamma'(G) = n - 1$, then, by Theorem 2.1, $\gamma(G) = \gamma'(G)$. Therefore $\gamma(G) = \gamma'(G) = (n-1)/2$, and hence n is odd. Conversely, assume that $\gamma(G) = \gamma'(G) = \lfloor n/2 \rfloor$ and n is odd. Since n is an odd integer, we get $\gamma(G) + \gamma'(G) = n - 1$. □

Let $\mathcal{C}$ and $\mathfrak{D}$ be the families of graphs given in Figures 2 and 3, respectively.

Note that the class $\mathcal{C}$ is a subclass of the class $\mathcal{A}$, and the class $\mathfrak{D}$ is same as the class $\mathcal{B}$ where $\mathcal{A}$ and $\mathcal{B}$ are classes given in Theorem 2.6 [1, Page 45].

The next theorem characterizes all connected graphs G with $\delta(G) \geq 2$ for which $\gamma(G) + \gamma'(G) = n - 1$. By Lemma 2.4 and Lemma 2.4 [1], we get the main theorem of this paper.

Theorem 2.5. *Let G be a connected graph with $\delta(G) \geq 2$. Then $\gamma(G) + \gamma'(G) = n - 1$ if and only if $G \in \mathcal{C} \cup \mathfrak{D}$.*

It may be worth noting that, for any graph G, the disjoint domination number $\gamma\gamma(G) \leq \gamma(G) + \gamma'(G)$. Due to this, we get an upper bound for $\gamma\gamma(G)$ and $\gamma(G) + \gamma'(G)$ which is better than the Ore's observation for disjoint domination number and sum of domination number and inverse domination number [7].

Corollary 2.6. *Let G be a connected graph with $\delta(G) \geq 2$ and $G \notin \{H_1, \ldots, H_{11}\}$. Then $\gamma\gamma(G) \leq \gamma(G) + \gamma'(G) \leq n - 2$.*

We suggest the following problems for further study in this direction.

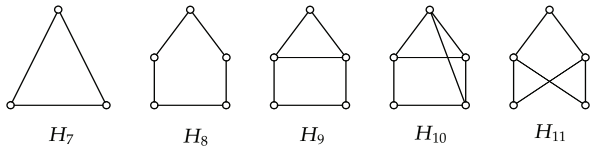

Figure 3: Graphs in family $\mathfrak{D}$.

Open Problems

(1) Find a necessary and sufficient condition for a graph G with $\delta(G) = 1$ and $\gamma(G) + \gamma'(G) = n - 1$.

(2) Characterize all connected graphs G with $\delta(G) \geq 2$ for which $\gamma(G) + \gamma'(G) = n - 2$.

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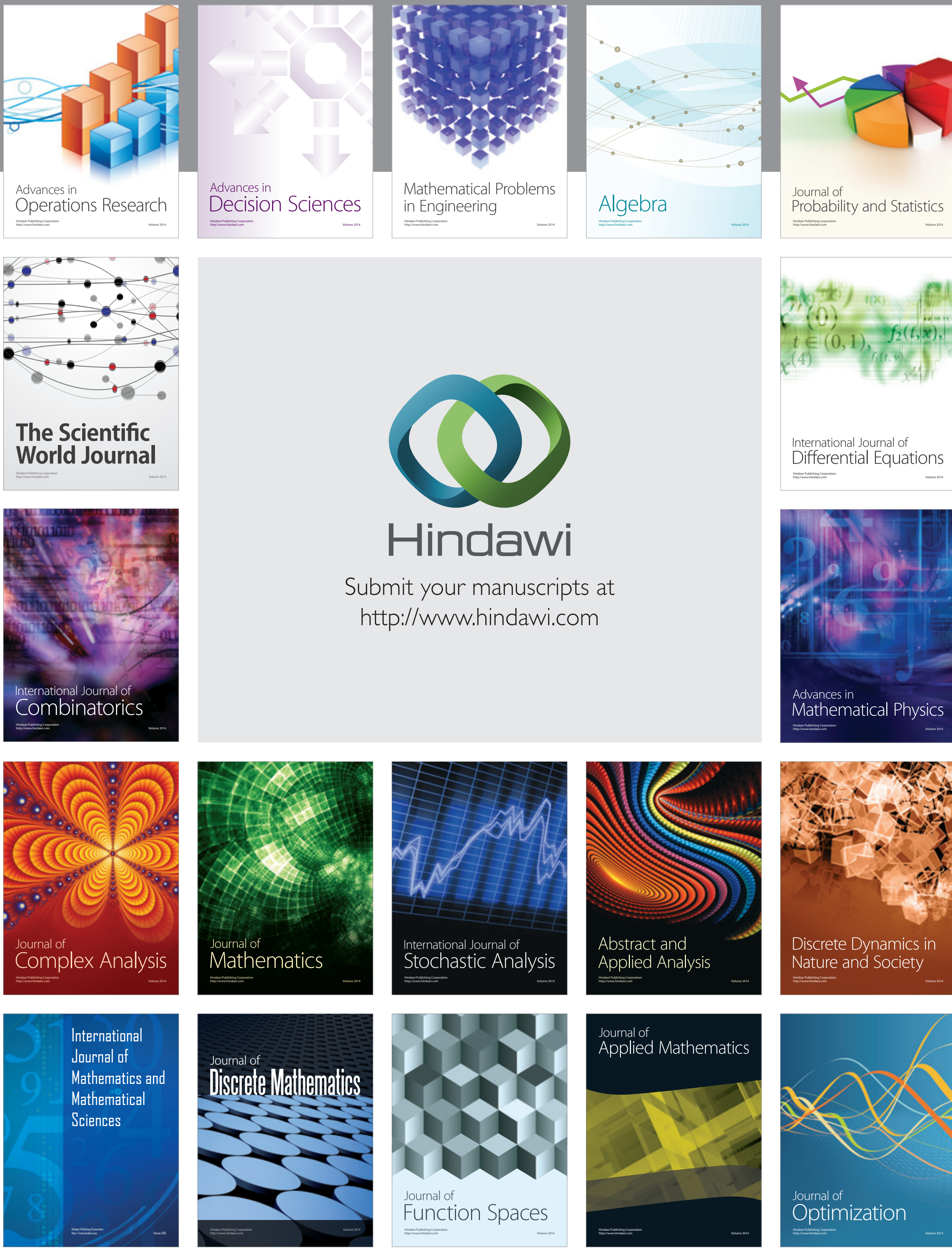
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