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A STABILITY RESULT FOR PARAMETER IDENTIFICATION PROBLEMS IN NONLINEAR PARABOLIC PROBLEMS

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ABSTRACT. A sequence of identification problems of coefficients in the parabolic equation with nonlinear boundary conditions is considered. The parameter (index of an element of the sequence) appears in the cost functionals as well as boundary data. It is proved that the optimal solutions exist and that under some continuous convergence of the cost functionals and the convergence of the data, the sets of optimal solutions converge in some sense to the set of optimal solutions of the limit problem.

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1. INTRODUCTION.

In the recent years there has been an increasing interest in the parameter identification (or inverse) problems involving differential equation constraints. Such problems arise in particular in the coefficient estimation for partial differential equations (for example in [2-3], [14], [17-18]) as well as in the theory of structural optimization. The identification problems consist in determining of unknown parameters (coefficients) from known observations of the modelled processes.

In this paper we investigate a class of identification problems for the second order nonlinear parabolic system:

$$u' = -\mathcal{A}(t)u \quad \text{in } \Omega \times (0,T), \tag{1.1}$$

$$\frac{\partial u}{\partial \nu_{\mathcal{A}(t)}} + \beta(u) \ni g \quad \text{in } \Gamma \times (0,T), \tag{1.2}$$

$$u(0) = \varphi \quad \text{in } \Omega, \tag{1.3}$$

where $\Omega \subset \mathbf{R}^n$ with boundary Γ, $0 < T < +\infty$, β is a maximal monotone graph in $\mathbf{R} \times \mathbf{R}$, the operator $\mathcal{A}(t)$ has the form

$$\mathcal{A}(t) = -\frac{\partial}{\partial x_i}(a_{ij}(x,t)\frac{\partial}{\partial x_j})$$

and

$$\frac{\partial u}{\partial \nu_{\mathcal{A}(t)}} = a_{ij}(x,t)\frac{\partial u}{\partial x_i}\nu_j$$

is the conormal derivative associated to $\mathcal{A}(t)$. Above, ν is the unit outward normal vector to Γ.

Given the set of admissible parameters $\mathcal{M}$ and the cost functional $\mathcal{J}$ defined on the space

$\mathcal{W}$ (see Notation) of the solutions to (1.1) - (1.3), we are interested in the following parameter identification problem:

$$(\mathcal{P}) \quad \begin{cases} \text{find the parameters } a^* = \{a^*_{ij}(x,t)\} \text{ in } \mathcal{M} \text{ so that} \\ \mathcal{J}(u(a^*, g, \varphi)) \leq \mathcal{J}(u(a, g, \varphi)) \quad \text{for all } a \in \mathcal{M}, \end{cases}$$

where $u(a, g, \varphi)$ denotes the weak solution in $\mathcal{W}$ corresponding to the data a, g and φ. Here and in what follows, we suppose that the graph β is fixed.

Our aim is to prove the existence of optimal solutions (i.e. an element which realizes the minimum) to $(\mathcal{P})$ and to show the stability of optimal solutions under perturbations of the data g and φ as well as of the cost functional $\mathcal{J}$. As indicated in [1], [4-5] and [10-11] the stability of this kind plays an important role in applications.

It should be noticed that a compactness of admissible subset of parameters is the crucial assumption in the identification problems (compare e.g. [5]). Since the cost functional is not convex in general, the uniqueness of optimal solutions is not guaranteed. Therefore the stability is understood in the sense of continuity of multivalued mapping.

We note that the widely known approach to the parameter identification problems used also in numerical methods (see [4-5], [14]) is the output least squares formulation ([10-11], [13]). In this approach the cost functional to be minimized has the form

$$||\mathcal{C}u(a, g, \varphi) - z_d||^2_{\mathcal{Z}}, \tag{1.4}$$

where $\mathcal{C}: \mathcal{W} \to \mathcal{Z}$ is an observation operator defined on the space of solutions, z_d is the desired element (target) in the space of observations $\mathcal{Z}$. Such cases are also included in the frame of the paper. Next, it should be underlined that the identification of coefficients in partial differential equations is, in general, an unstable problem ([16-17], [13]). This is due to the theory of homogenization ([8], [18]) which shows that operators with highly oscillatory coefficients can be "replaced" by very different ones and still giving the same response.

Finally, we point out that the problems of the type (1.1) - (1.3) occur in many mathematical models of phenomena studied in physics. For example, equation (1.1) describes the change of pressure during the flow of viscous fluids in porous media or it governs the heat distribution in a body occupying the volume Ω. It is natural to consider such problems not only with the classical (Dirichlet and/or Neumann) boundary conditions, but also with the more general ones. The boundary condition (1.2) includes some particular cases e.g. the Signorini condition, the Stefan-Boltzmann heat radiation law, the Newton's law, the natural convection, the Michaelis-Menten law. For these and other important examples of the condition (1.2) which appears in mechanics, biology and chemistry, we refer to [12], [7], [9] and the bibliography in them. We also recall that the evolution variational inequalities can be formulated in the form (1.1) - (1.3) (see [6], [9], [12], [15]). For the general identification theory presented in an abstract manner we refer to [13] and [1].

The remainder of this note is divided in three parts. In Section 2 we give a result on the continuous dependence of solution to boundary value problem (1.1) - (1.3) on the data. With this background, in Section 3, we show that the problem $(\mathcal{P})$ has a solution. The last section is devoted to the stability of optimal solutions with respect to variations in the given data and the cost functional.

Notation. Let Ω be a bounded open subset of $\mathbf{R}^n$ with Lipschitz continuous boundary Γ. For a fixed real interval $[0, T]$, we introduce $Q = \Omega \times (0, T)$, $\Sigma = \Gamma \times (0, T)$. Putting $V = H^1(\Omega)$, $H = L^2(\Omega)$, we denote by $||.||$, $|.|$, the norms in V and H, respectively. By V' we denote the dual to V. Following e.g. [15], we define the Banach spaces: $\mathcal{V} = L^2(0, T; V)$, $\mathcal{H} = L^2(0, T; H)$, $\mathcal{V}' = L^2(0, T; V')$ (the spaces of the square summable functions defined on $(0, T)$ with the values, resp., in V, H, V') and $\mathcal{W} = \{v \in \mathcal{V} \ : \ v' \in \mathcal{V}'\}$. Here and subsequently the partial derivative with respect to t, is understood in the distributional sense and dv/dt will be denoted by v'. The pairing of V and V' and also the inner product on H is denoted by $< \cdot, \cdot >$. The symbol $< \cdot, \cdot >_\Gamma$ stands for the inner product on $L^2(\Gamma)$. Given closed sets A and B in a Banach space X, we define the distance function by $d(x, A) = inf\{||x - a||_X : a \in A\}$ and the separation of a set A from a set B by

$$h^*(A, B) = sup\{d(a, B) : a \in A\}. \tag{1.5}$$

Throughout this paper a summation convention over repeated subscripts is adopted.

2. CONTINUOUS DEPENDENCE ON THE DATA.

The goal of this section is to study the question of continuous dependence of solutions to (1.1) - (1.3) on the data a_{ij}, g and φ. First we give the existence result on the solutions to this problem. To this end, we adopt the following

DEFINITION 2.1. (see [7]) A function $u \in \mathcal{W}$ is a weak solution to (1.1) - (1.3) if and only if there exists a function $w \in L^2(\Sigma)$ such that

$$w(\sigma, t) \in \beta(u(\sigma, t)) \quad \text{a.e. on } \Sigma, \tag{2.1}$$

and

$$< u'(t), v > +a(t; u(t), v) + < w(t), v >_\Gamma = < g(t), v >_\Gamma \quad \forall v \in V, \text{a.e. on } (0, T),$$

$$u(0) = \varphi,$$

where we have set

$$a(t; z, v) = \int_\Omega a_{ij}(x, t) \frac{\partial z}{\partial x_i} \frac{\partial v}{\partial x_j} dx, \quad \forall z, v \in V, \quad \text{a.e. } t \in (0, T). \tag{2.2}$$

We need the following hypotheses on the data of the problem (1.1) - (1.3):

(H_1) the coefficients $\{a_{ij}\}$, $i, j = 1, ..., n$ are functions from $C(\overline{Q})$ such that

$$\alpha \xi_i \xi_i \leq a_{ij}(x, t) \xi_i \xi_j, \quad \forall \xi \in \mathbf{R}^n \tag{2.3}$$

a.e. in Q, for some constant $\alpha > 0$,

(H_2) β is a maximal (multivalued) monotone graph in $\mathbf{R} \times \mathbf{R}$ which satisfies the condition $0 \in \beta(0)$,

(H_3) $g \in L^2(\Sigma)$, $\varphi \in H$.

It is well known (see e.g. [6]) that β is a subdifferential of a proper, convex, l.s.c. function $j: \mathbf{R} \to \mathbf{R} \cup \{+\infty\}$, i.e. $\beta = \partial j$.

PROPOSITION 2.1. In addition to the hypotheses $(H_1) - (H_3)$ we assume that $j(\varphi) \in L^1(\Omega)$. Then the problem (1.1) - (1.3) has a unique weak solution $u \in \mathcal{W}$. Morover, the following estimate holds:

$$||u||_{\mathcal{W}} + ||w||_{L^2(\Sigma)} \leq c(1 + ||g||_{L^2(\Sigma)} + |\varphi|), \tag{2.4}$$

where $w \in L^2(\Sigma)$ is a selection of β which appears in (2.1) and $c \in \mathbf{R}$ is independent of g and φ.

The proof of this result can be found in [7], Proposition 1, where the overall hypotheses on the coefficients a_{ij} were more restrictive. A careful look in that proof can convince the reader that it is still true for the case of coefficients satisfying (H_1).

Let $\{a_{ij}^k\}$, $k \in \mathbf{N}$, be a sequence in $C(\overline{Q})$ satisfying (2.3) uniformly with respect to k and let $\{(g_k, \varphi_k)\}$ be a sequence in $L^2(\Sigma) \times H$. Denoting by $u_k = u(a_{ij}^k, g_k, \varphi_k)$ the solution to (1.1) (1.3) corresponding to $\{a_{ij}^k\}$, $\{(g_k, \varphi_k)\}$, we have

LEMMA 2.1. Under the above notations, let us assume that β satisfies (H_2) and $j(\varphi_k) \in L^1(\Omega)$. If

$$a_{ij}^k \to a_{ij} \quad \text{in } C(\overline{Q}), \quad \forall i, j,$$

$$(g_k, \varphi_k) \to (g, \varphi) \quad \text{in} \quad L^2(\Sigma) \times H,$$

as $k \to +\infty$, then

$$u_k \to u \;\text{ in }\; \mathcal{V} \cap C(0,T;H) \text{ and weakly in } \mathcal{W}, \tag{2.5}$$

as $k \to +\infty$, where $u = u(a_{ij}, g, \varphi)$ is a unique solution (still in the sense of Definition 2.1) to (1.1) - (1.3) corresponding to a_{ij}, g and φ.

PROOF. Note that by lower semicontinuity of j, it follows that $j(\varphi) \in L^1(\Omega)$. Therefore, from Proposition 2.1, we know that there exists a unique solution to (1.1) - (1.3) corresponding to a_{ij}, g and φ. Substracting two equations which are satisfied by u_k and u, we obtain

$$< u_k'(s) - u'(s), v > \; + \; a_k(s; u_k(s), v) \; - a(s; u(s), v) +$$
$$+ \; < w_k(s) - w(s), v >_\Gamma \; = \; < g_k(s) - g(s), v >_\Gamma$$

for all $v \in V$, a.e. $s \in (0,T)$, where $w, w_k \in L^2(\Sigma)$ are such that

$$w(\sigma, s) \in \beta(u(\sigma,s)), \quad w_k(\sigma,s) \in \beta(u_k(\sigma,s)) \;\text{ a.e. on }\; \Sigma$$

and $a(\cdot;\cdot,\cdot), a_k(\cdot;\cdot,\cdot)$ are of the form (2.2) with the coefficients a_{ij}, a_{ij}^k, respectively.

Taking $u_k(s) - u(s)$ as the function v and integrating both sides we get:

$$\frac{1}{2}|u_k(t) - u(t)|^2 \; + \int_0^t a_k(s; u_k(s) - u(s), u_k(s) - u(s)) ds \; +$$
$$+ \int_0^t < w_k(s) - w(s), u_k(s) - u(s) >_\Gamma ds \; = \; \frac{1}{2}|\varphi_k - \varphi|^2 + \int_0^t \Big(a(s; u(s), u_k(s) - u(s)) \; -$$
$$- \; a_k(s; u(s), u_k(s) - u(s)) \Big) ds + \int_0^t < g_k(s) - g(s), u_k(s) - u(s) >_\Gamma ds.$$

Hence, from (H_1) and (H_2) we obtain:

$$|u_k(t) - u(t)|^2 + 2\alpha \int_0^t ||u_k(s) - u(s)||^2 ds \; \le \; |\varphi_k - \varphi|^2 \; +$$
$$+ \int_0^t \sum_{j=1}^n \Big| \sum_{i=1}^n (a_{ij}^k - a_{ij}) \frac{\partial u(s)}{\partial x_i} \Big| \Big| \frac{\partial u_k(s)}{\partial x_j} - \frac{\partial u(s)}{\partial x_j} \Big| ds \; +$$
$$+ 2 \int_0^t |g_k(s) - g(s)|_{L^2(\Gamma)} |u_k(s) - u(s)|_{L^2(\Gamma)} ds.$$

Using the continuity of the trace operator from V to $L^2(\Gamma)$ and the inequality $2ab \le \frac{2}{\alpha} a^2 + \frac{\alpha}{2} b^2$, we have

$$|u_k(t) - u(t)|^2 + \alpha \int_0^t ||u_k(s) - u(s)||^2 ds \; \le \; |\varphi_k - \varphi|^2 \; +$$
$$+ c_1 ||u||_{\mathcal{V}}^2 \sum_{i,j=1}^n ||a_{ij}^k - a_{ij}||_{C(\overline{Q})} \; + \; c_2 \int_0^t |g_k(s) - g(s)|_{L^2(\Gamma)} ds,$$

where c_i, $i = 1,2$ are positive constants independent of k. From this, (2.4) and from the hypotheses we deduce that

$$u_k \to u \;\text{ in }\; \mathcal{V} \cap C(0,T;H), \;\text{ as }\; k \to +\infty.$$

On the other hand, owing to the estimate (2.4), which is uniform with respect to k, we obtain

$$u_k \to u \text{ weakly in } \mathcal{W}, \text{ as } k \to +\infty.$$

The uniqueness of solutions of problem (1.1) - (1.3) implies that the whole sequence $\{u_k\}$ converges to u in the sense of (2.5). This completes the proof of the lemma.

3. EXISTENCE RESULT IN IDENTIFICATION.

Using the result of Lemma 2.1, we can prove the existence of solution of the identification problem $(\mathcal{P})$. This is given in the following

THEOREM 3.1. Let the assumptions of Proposition 2.1 hold. We suppose that the set of admissible parameters satisfies

(H_4) the set $\mathcal{M}$ is a compact subset of $(C(\overline{Q}))^{n^2}$ such that $\alpha\xi_i\xi_i \le a_{ij}(x,t)\xi_i\xi_j$ for each $\xi \in \mathbf{R}^n$ and for all $a = \{a_{ij}(x,t)\}$ in $\mathcal{M}$.

Let the cost functional $\mathcal{J}: \mathcal{W} \to \mathbf{R}$ be weakly sequentially lower semicontinuous on $\mathcal{W}$. Then the problem $(\mathcal{P})$ has a solution.

PROOF. We apply the direct method of calculus of variations (see e.g. [1]). Let g, φ satisfy (H_3) and let $\{a_k\}$ be a minimizing sequence from $\mathcal{M}$ such that

$$\lim_{k\to\infty} \mathcal{J}(u(a_k,g,\varphi)) = \inf\{\mathcal{J}(u(a,g,\varphi)) : a \in \mathcal{M}\} = m.$$

Since $\mathcal{M}$ is compact, there exists a subsequence of the sequence $\{a_k\}$, relabeled again as $\{a_k\}$, and a $a_0 \in \mathcal{M}$ such that $a_k \to a_0$. It follows from Lemma 2.1 that in particular

$$u(a_k,g,\varphi) \to u(a_0,g,\varphi) \text{ weakly in } \mathcal{W}, \quad \text{as } k \to +\infty.$$

Therefore, by lower semicontinuity of $\mathcal{J}$ we have

$$m \le \mathcal{J}(u(a_0,g,\varphi)) \le \liminf_{k\to\infty} \mathcal{J}(u(a_k,g,\varphi)) = \lim_{k\to\infty} \mathcal{J}(u(a_k,g,\varphi)) = m.$$

Hence the result follows.

REMARK 3.1. In general, without convexity assumptions, we do not expect uniqueness of the optimal solution in identification ([1], [16]).

4. STABILITY RESULT.

In this section we give the main result of this paper on the dependence (hence also the stability) of the optimal elements for the problem $(\mathcal{P})$ on the data as well as on the cost functional.

We consider the sequence (indexed by the parameter $k \in \mathbf{N}$) of the identification problems:

$$(\mathcal{P}_k)\begin{cases} \text{find } a^* = \{a^*_{ij}\} \text{ in } \mathcal{M} \text{ so that} \\ \mathcal{J}_k\big(u(a^*,g_k,\varphi_k)\big) \le \mathcal{J}_k\big(u(a,g_k,\varphi_k)\big) \text{ for all } a \in \mathcal{M}, \end{cases}$$

where $u(a,g_k,\varphi_k)$ are the solutions in $\mathcal{W}$ to (1.1) - (1.3) corresponding to the perturbed data g_k, φ_k and $\mathcal{J}_k$ are the perturbed functionals. We show that the set of optimal solutions to $(\mathcal{P}_k)$ converges in some sense, to the set of optimal solution to $(\mathcal{P})$.

We need the following continuous convergence of functionals. Let $(\mathcal{X},\tau)$ be a topological space and $\mathcal{J}_k: \mathcal{X} \to \mathbf{R}$.

DEFINITION 4.1. We say that a sequence of functionals $\{\mathcal{J}_k\}$, $k \in \mathbf{N}$, is sequential continuously convergent (shortly, C_{seq}-converges) to $\mathcal{J}$, and we write $\mathcal{J} = C_{seq}(\tau - \mathcal{X})\lim_{k\to\infty} \mathcal{J}_k$, if for every $x \in \mathcal{X}$ and for every $\{x_k\} \subset \mathcal{X}$ which τ-converges to x, the sequence $\mathcal{J}_k(x_k)$ converges to $\mathcal{J}(x)$.

For each $k \in \mathbf{N}$, we denote by $\mathcal{S}$, $\mathcal{S}_k$ the sets of optimal elements to the problem $(\mathcal{P})$, $(\mathcal{P}_k)$, respectively, i.e.

$$\mathcal{S} = \mathcal{S}(g,\varphi,\mathcal{J}) = \{a^* \in \mathcal{M} : \mathcal{J}\big(u(a^*,g,\varphi)\big) \le \mathcal{J}\big(u(a,g,\varphi)\big), \ \forall a \in \mathcal{M}\},$$

$$\mathcal{S}_k = \mathcal{S}(g_k,\varphi_k,\mathcal{J}_k) = \{a^* \in \mathcal{M} : \mathcal{J}_k\big(u(a^*,g_k,\varphi_k)\big) \le \mathcal{J}_k\big(u(a,g_k,\varphi_k)\big), \ \forall a \in \mathcal{M}\}.$$

With the above notation we have

THEOREM 4.1. In addition to the hypotheses (H_2) and (H_4), we assume that $\mathcal{J}, \mathcal{J}_k: \mathcal{W} \to \mathbf{R}$ are given weakly sequentially lower semicontinuous functionals. Let $g, g_k \in L^2(\Sigma)$ and $\varphi, \varphi_k \in H$ be such that $j(\varphi_k) \in L^1(\Omega)$. If

$$\{(g_k, \varphi_k)\} \to (g, \varphi) \ \text{ in } \ L^2(\Sigma) \times H, \ \text{ as } \ k \to +\infty, \tag{4.1}$$

$$\mathcal{J} = C_{seq}(w - \mathcal{W}) \lim_{k \to \infty} \mathcal{J}_k, \tag{4.2}$$

(where $w - \mathcal{W}$ stands for the weak topology in $\mathcal{W}$), then

$$\lim_{k \to \infty} h^*(\mathcal{S}_k, \mathcal{S}) = 0, \tag{4.3}$$

where $h^*(\cdot, \cdot)$ is defined in (1.5).

PROOF. We argue by contradiction. If (4.3) is not verified there exist $\bar{\varepsilon} > 0$ and a sequence $\{k_\nu\}$, $k_\nu \to +\infty$ such that

$$\bar{\varepsilon} < h^*(\mathcal{S}_{k_\nu}, \mathcal{S}), \ \forall\, k_\nu \in \mathbf{N}.$$

Clearly, there exist $a^*_{k_\nu} \in \mathcal{S}_{k_\nu}$ such that

$$\bar{\varepsilon} < d(a^*_{k_\nu}, \mathcal{S}), \ \forall\, k_\nu \in \mathbf{N}. \tag{4.4}$$

In view of compactness of $\mathcal{M}$, we deduce that there exists a subsequence of $\{a^*_{k_\nu}\}$, that we will denote in the same way, such that

$$a^*_{k_\nu} \to a^* \ \text{ in } \ \left(C(\overline{Q})\right)^{n^2}, \ \text{ as } \ k_\nu \to +\infty, \tag{4.5}$$

for some $a^* \in \mathcal{M}$.

Let now $u^*_{k_\nu} = u(a^*_{k_\nu}, g_{k_\nu}, \varphi_{k_\nu})$ and $u^* = u(a^*, g, \varphi)$ denote the solutions to (1.1) - (1.3) which correspond to the triples $(a^*_{k_\nu}, g_{k_\nu}, \varphi_{k_\nu})$ and (a^*, g, φ), respectively. From (4.1) and (4.5) Lemma 2.1 gives

$$u^*_{k_\nu} \to u^* \text{ weakly in } \mathcal{W}, \ \text{as } k_\nu \to +\infty.$$

Since the functionals $\mathcal{J}_k$ C_{seq}-converges to $\mathcal{J}$, we get

$$\mathcal{J}(u^*) = \lim \mathcal{J}_{k_\nu}(u^*_{k_\nu}). \tag{4.6}$$

Let us fix an arbitrary $a \in \mathcal{M}$. Let $u_{k_\nu} = u(a, g_{k_\nu}, \varphi_{k_\nu})$ and $u = u(a, g, \varphi)$ be the solutions to (1.1) - (1.3) with the indicated data. From the continuous dependence on the data, we conclude that

$$u_{k_\nu} \to u \text{ weakly in } \mathcal{W}, \ \text{ as } \ k_\nu \to +\infty.$$

Hence, as above, we have

$$\mathcal{J}(u) = \lim \mathcal{J}_{k_\nu}(u_{k_\nu}). \tag{4.7}$$

Since $a^*_{k_\nu}$ are in $\mathcal{S}_{k_\nu}$, we have

$$\mathcal{J}_{k_\nu}(u^*_{k_\nu}) \le \mathcal{J}_{k_\nu}(u_{k_\nu}).$$

In the limit, as $k_\nu \to +\infty$, one gets from (4.6), (4.7) :

$$\mathcal{J}\left(u(a^*, g, \varphi)\right) \le \mathcal{J}\left(u(a, g, \varphi)\right), \ \ \forall\, a \in \mathcal{M}.$$

From the arbitrariety of $a \in \mathcal{M}$, we have $a^* \in \mathcal{S}$ and this implies that

$$d(a^*_{k_\nu}, \mathcal{S}) \le \|a^*_{k_\nu} - a^*\|. \tag{4.8}$$

But now from (4.5), we obtain that (4.8) contradicts (4.4). This proves (4.3) and completes the proof of the theorem.

The convergence (4.2) holds, for instance, for the sequence of functionals of the form (1.4). Namely, the following two cases can be considered:

(i) Let $\mathcal{Z} = \mathcal{H}$. Given the functionals $\mathcal{J}_k(u) = ||u - z_d^k||^2_{\mathcal{H}}$, where $\{z_d^k\}$ is a sequence of perturbed distributed observations in $\mathcal{H}$ converging to z_d, we have that (4.2) is satisfied with $\mathcal{J}(u) = ||u - z_d||^2_{\mathcal{H}}$. This follows directly, by the definition of C_{seq}-limit, from the fact that $\mathcal{W} \subset \mathcal{H}$ and this embedding is compact (compare [15]).

(ii) Let $\mathcal{Z} = L^2(\Sigma)$. We consider the functionals $\mathcal{J}_k(u) = ||u - z_d^k||^2_{L^2(\Sigma)}$, where $\{z_d^k\}$ is a sequence of perturbed measurements (observations) performed on Σ. If $z_d^k \to z_d$ in $L^2(\Sigma)$, as $k \to +\infty$, then using the compactness of the trace embedding from $\mathcal{W}$ into $L^2(\Sigma)$, we easily get the convergence (4.2), where $\mathcal{J}(u) = ||u - z_d||^2_{L^2(\Sigma)}$.

REMARK 4.1. One can generalize the results presented above to the case when $\mathcal{A}(t)$ is a differential operator of the form

$$\mathcal{A}(t) = -\frac{\partial}{\partial x_i}(a_{ij}(x,t)\frac{\partial}{\partial x_j}) + a(x,t)u.$$

Our theory, with some minor changes, can handle inverse problems involving the identification of any of the parameters a_{ij}, a, or initial data or boundary function.

REMARK 4.2. A further generalization of our results can be obtained considering the identification problems for (1.1) - (1.3) with the mixed boundary conditions

$$\frac{\partial u}{\partial \nu_{\mathcal{A}(t)}} + \beta_i(u) \ni g_i \quad \text{on} \quad \Sigma_i, \quad i = 1,2,$$

instead of (1.2), where $\Sigma_i = \Gamma_i \times (0,T)$ and Γ_i are the disjoint parts of Γ. The exact formulation of the results with obvious modifications is left to the interested reader.

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