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# COMMON FIXED POINTS FOR NONEXPANSIVE AND NONEXPANSIVE TYPE FUZZY MAPPINGS

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**ABSTRACT**. In this paper we define $g$-nonexpansive and $g$-nonexpansive type fuzzy mappings and prove common fixed point theorems for sequences of fuzzy mappings satisfying certain conditions on a Banach space. Thus we obtain fixed point theorems for nonexpansive type multi-valued mappings.

**KEY WORDS AND PHRASES**: Star-shaped set, Opial's condition, weak convergence, Hausdorff metric, nonexpansive fuzzy mapping, nonexpansive type fuzzy mapping, fixed point, common fixed point.

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## 1. INTRODUCTION

Fixed point theorems for fuzzy mappings were obtained by Chang, Heilpern and others [1-5, 7, 9-13, 16]. Especially, Lee and Cho [10] showed that a sequence of fuzzy mappings with the condition (*) satisfies the condition (**), that a sequence with the condition (**) has a common fixed point and consequently that a sequence of fuzzy mappings with the condition (*) has a common fixed point. These results are fuzzy analogues of common fixed theorems for sequences of $g$-contractive and $g$-contractive type multi-valued mappings [8]. In [11] and [13] Lee et al. also obtained a common fixed point theorem for sequences of fuzzy mappings which generalize the results in [1] and [10] respectively.

In this paper we define $g$-nonexpansive and $g$-nonexpansive type fuzzy mappings and show that a sequence of fuzzy mappings with the condition (****), which are defined on a nonempty weakly compact star-shaped subset of a Banach space $X$ satisfying Opial's condition, has a common fixed point. As corollaries, firstly we show that similar results are obtained for the conditions (*), (**) or (***). Secondly we obtain fixed point theorems for nonexpansive type fuzzy [respectively, compact-valued] mappings $F$ [resp., $f$] from $K(\subset X)$ to $W(K)$ [resp., $2^K$]. Thirdly we show that similar results are obtained for nonexpansive fuzzy [resp., compact-valued] mappings.

## 2. PRELIMINARIES

We review briefly some definitions and terminologies needed.

A fuzzy set $A$ in a metric space $X$ is a function with domain $X$ and values in [0,1]. (In particular, if $A$ is an ordinary (crisp) subset of $X$, its characteristic function $\chi_A$ is a fuzzy set with domain $X$ and values $\{0,1\}$). Especially $\{x\}$ is a fuzzy set with a membership function equal to a characteristic function of the set $\{x\}$. The $\alpha$-level set of $A$, denoted by $A_\alpha$, is defined by

$$A_\alpha = \{x : A(x) \geq \alpha\} \quad \text{if} \quad \alpha \in (0,1],$$

$$A_0 = \overline{\{x : A(x) > 0\}}$$

where $\overline{B}$ denotes the closure of the (nonfuzzy) set $B$.

$W(X)$ denotes the collection of all fuzzy sets $A$ in $X$ such that (i) $A_\alpha$ is compact in $X$ for each $\alpha \in [0,1]$ and (ii) $A_1$ is a nonempty subset of $X$. For $A, B \in W(X), A \subset B$ means $A(x) \leq B(x)$ for each $x \in X$.

Let $A$ and $B$ be two nonempty bounded subsets of a Banach space $X$. The Hausdorff distance between $A$ and $B$ is

$$d_H(A,B) = \max\left[\sup_{a \in A} \inf_{b \in B} \|a-b\|\ ,\ \sup_{b \in B} \inf_{a \in A} \|a-b\|\right].$$

**DEFINITION 2.1.** Let $A, B \in W(X)$ and $\alpha \in [0,1]$. Then we define

$$D(A,B) = \sup_\alpha d_H(A_\alpha, B_\alpha).$$

We note that $D$ is a metric on $W(X)$ such that $D(\{x\},\{y\}) = \|x-y\|$, where $x, y \in X$.

**DEFINITION 2.2.** Let $X$ be an arbitrary set and $Y$ be any metric space. $F$ is called a fuzzy mapping iff $F$ is a mapping from the set $X$ into $W(Y)$.

A fuzzy mapping $F$ is a fuzzy subset on $X \times Y$ with a membership function $F(x)(y)$. The function value $F(x)(y)$ is the grade of membership of $y$ in $F(x)$. In case $X = Y, F(x)$ is a function from $X$ into $[0,1]$. Especially for a multi-valued mapping $f: X \to 2^X$, $\chi_{f(x)}$ is a function from $X$ to $\{0,1\}$. Hence a fuzzy mapping $F: X \to W(X)$ is another extension of a multi-valued mapping $f: X \to 2^X$.

**DEFINITION 2.3.** Let $g$ be a mapping from a Banach space $(X, \|\cdot\|)$ to itself. A fuzzy mapping $F: X \to W(X)$ is $g$-contractive [respectively, $g$-nonexpansive] if $D(F(x), F(y)) \leq k \cdot \|g(x) - g(y)\|$ for all $x, y \in X$, for some fixed $k, 0 \leq k < 1$ [resp., $k = 1$].

**PROPOSITION 2.4** [9]. Let $(X, \|\cdot\|)$ be a Banach space, $F: X \to W(X)$ a fuzzy mapping and $x \in X$, then there exists $u_x \in X$ such that $\{u_x\} \subset F(x)$.

**DEFINITION 2.5.** Let $g$ be a mapping from a Banach space $(X, \|\cdot\|)$ to itself. We call a fuzzy mapping $F: X \to W(X)$ $g$-contractive type [respectively, $g$-nonexpansive type] if for all $x \in X, \{u_x\} \subset F(x)$ there exists $\{v_y\} \subset F(y)$ for all $y \in X$ such that $D(\{u_x\},\{v_y\}) \leq k \cdot \|g(x) - g(y)\|$ for some fixed $k$, $0 \leq k < 1$ [resp., $k = 1$].

**REMARK.** When $g$ is an identity, a $g$-contractive [respectively, $g$-contractive type, $g$-nonexpansive, $g$-nonexpansive type] fuzzy mapping $F$ is said to be contractive [resp., contractive-type, nonexpansive, nonexpansive type].

**LEMMA 2.6.** Let $A, B \in W(X)$. Then for each $\{x\} \subset A$, there exists $\{y\} \subset B$ such that $D(\{x\},\{y\}) \leq D(A,B)$.

**PROOF.** If $\{x\} \subset A$, then $x \in A_1$. By compactness of $B_1$, we can choose a $y \in B_1$, i.e., $\{y\} \subset B$, such that $\|x - y\| \leq d_H(A_1, B_1)$. By the facts $D(\{x\},\{y\}) = \|x-y\|$ and $d_H(A_1, B_1) \leq D(A,B)$, we have $D(\{x\},\{y\}) \leq D(A,B)$.

**PROPOSITION 2.7.** Let $g$ be a mapping from a Banach space $(X, \|\cdot\|)$ to itself. If $F: X \to W(X)$ is a $g$-nonexpansive [respectively, $g$-contractive] fuzzy mapping, then $F$ is $g$-nonexpansive type [resp., $g$-contractive type].

**PROOF.** It can be easily proved by Lemma 2.6.

## 3. COMMON FIXED POINTS FOR FUZZY MAPPINGS

For a mapping $g$ of a Banach space $X$ into itself and a sequence $(F_i)_{i=1}^\infty$ of fuzzy mappings of $X$ into $W(X)$ we consider the following conditions (*), (**), (***) and (****).

(*) there exists a constant $K$ with $0 \leq k < 1$ such that for each pair of fuzzy mappings $F_i, F_j: X \to W(X)$, $D(F_i(x), F_j(y)) \leq k \cdot \|g(x) - g(y)\|$ for all $x, y \in X$.

(**) there exists a constant $k$ with $0 \le k < 1$ such that for each pair of fuzzy mappings $F_i, F_j : X \to W(X)$ and for any $x \in X, \{u_x\} \subset F_i(x)$ implies that there is $\{v_y\} \subset F_j(y)$ for all $y \in X$ with $D(\{u_x\},\{v_y\}) \le k \cdot \| g(x) - g(y) \|$.

(***) for each pair of fuzzy mappings $F_i, F_j : X \to W(X)$, $D(F_i(x), F_j(y)) \le \| g(x) - g(y) \|$ for all $x, y \in X$.

(****) for each pair of fuzzy mappings $F_i, F_j : X \to W(X)$, and for any $x \in X, \{u_x\} \subset F_i(x)$ implies that there is $\{v_y\} \subset F_j(y)$ for all $y \in X$ with $D(\{u_x\},\{v_y\}) \le \| g(x) - g(y) \|$.

It is easily proved that the condition (*) [respectively, (***)] implies the condition (**) [resp., (****)] by Lemma 2.6, but the following example shows that the converses do not hold in general.

**EXAMPLE 3.1.** Let $g$ be an identity mapping from a Euclidean metric space $([0,\infty), |\cdot|)$ to itself. Let $(F_i)_{i=1}^{\infty}$ be a sequence of fuzzy mappings from $[0,\infty)$ into $W([0,\infty))$, where $F_i(x) : [0,\infty) \to [0,1]$ is defined as follows;

$$\text{if} \quad x = 0, \; F_i(x)(z) = \begin{cases} 1, & z = 0, \\ 0, & z \ne 0, \end{cases}$$

$$\text{otherwise,} \quad F_i(x)(z) = \begin{cases} 1, & 0 \le z \le x/2, \\ 1/2, & x/2 < z \le ix, \\ 0, & z > ix. \end{cases}$$

Then the sequence $(F_i)_{i=1}^{\infty}$ satisfies the condition (****), but does not satisfy the condition (***).

In this section we show that a sequence of fuzzy mappings with the condition (****), which are defined on a nonempty weakly compact star-shaped subset $K$ of a Banach space $X$ which satisfies Opial's condition, has a common fixed point using a common fixed point theorem due to Lee and Cho [10], and consequently a sequence of fuzzy mappings with the condition (*), (**) or (***) has a common fixed point. As corollaries we obtain fixed point theorems for nonexpansive type fuzzy [respectively, compact-valued] mappings $F$ [resp., $f$] from a nonempty weakly compact and star-shaped subset $K$ of a Banach space $X$ which satisfies Opial's condition to $W(X)$ [resp., $2^K$].

The results for the nonexpansive compact-valued mappings are the case of replacing convexity with star-shapedness in Theorem 3.5 due to Husain and Latif [8].

Following Nguyen [14] we define: Let $X$, $Y$ and $Z$ be any nonempty sets, and $A \in \mathcal{F}(X)$ and $B \in \mathcal{F}(Y)$ where $\mathcal{F}(X)$ is the collection of all fuzzy sets in $X$. If $f : X \to Y$, then the fuzzy set $f(A)$ is defined via the extension principle by $f(A) \in \mathcal{F}(Y)$ and $f(A)(y) = \sup_{x \in f^{-1}(y)} A(x)$.

If $f : X \times Y \to Z$, then the fuzzy set $f(A,B)$ is defined via the extension principle by $f(A,B) \in \mathcal{F}(Z)$ and $f(A,B)(z) = \sup_{(x,y) \in f^{-1}(z)} [\min\{A(x), B(y)\}]$.

**PROPOSITION (NGUYEN).** Let $f : X \times Y \to Z$ and $A \in \mathcal{F}(X)$ and $B \in \mathcal{F}(Y)$. Then a necessary and sufficient condition for the equality $[f(A,B)]_\alpha = f(A_\alpha, B_\alpha)$ for all $\alpha \in [0,1]$ is that for all $z \in Z$, $\sup_{(x,y) \in f^{-1}(z)} [\min\{A(x), B(y)\}]$ is attained.

A subset $K$ of a Banach space $X$ is said to be star-shaped if there exists a point $v \in K$ such that $tv + (1-t)x \in K$ for all $x \in K$ and $0 < t < 1$. The point $v$ is called the star center of $K$.

**THEOREM 3.2** [10]. Let $g$ be a nonexpansive mapping from a complete metric linear space $(X, d)$ to itself. If $(F_i)_{i=1}^{\infty}$ is a sequence of fuzzy mappings of $X$ into $W(X)$ satisfying the condition (**), then there exists a point $x \in X$ such that $\{x\} \subset \cap_{i=1}^{\infty} F_i(x)$.

**PROPOSITION 3.3.** Let $K$ be a nonempty bounded star-shaped subset of a Banach space $X$ and $g$ a nonexpansive mapping from $X$ into itself. If $(F_i)_{i=1}^{\infty}$ is a sequence of fuzzy mappings of $K$ into $W(X)$ satisfying the condition (****), then there exist a sequence $(x_n)_{n=1}^{\infty}$ in $K$ and a sequence $(u_n)_{n=1}^{\infty}$ in $X$ satisfying $\{u_n\} \subset F_i(x_n)$ for all $i \in \mathbf{N}$ such that $\| x_n - u_n \| \to 0$ as $n \to \infty$.

**PROOF.** Let $x_0$ be the star-center of $K$. Choose a real sequence $(k_n)_{n=1}^{\infty}$ such that $0 < k_n < 1$ and $k_n \to 0$ as $n \to \infty$. Then for each $x \in K$, $k_n x_0 + (1-k_n)x \in K$. Define a fuzzy mapping $F_i^n$ of $K$ into $W(X)$ by setting $F_i^n(x) = k_n\{x_0\} + (1-k_n)F_i(x)$ for all $i \in \mathbf{N}$, then by Proposition 3.3 in [14] it follows that $[F_i^n(x)]_\alpha = k_n x_0 + (1-k_n)[F_i(x)]_\alpha$ for all $i \in \mathbf{N}$ and each $\alpha \in [0,1]$. Now we show that for each $n \in \mathbf{N}$, $(F_i^n)_{i=1}^{\infty}$ is a sequence of fuzzy mappings satisfying the condition (**). If we let $\{u_x\} \subset F_i^n(x)$ for each $x \in K$, we get $u_x = k_n x_0 + (1-k_n)v_x$ for some $v_x \in K$ such that $\{v_x\} \subset F_i(x)$. Since $(F_i)_{i=1}^{\infty}$ satisfies the condition (****), there exists a $\{v_y\} \subset F_j(y)$ for all $y \in K$ such that $\| v_x - v_y \| \le \| g(x) - g(y) \| \le \| x - y \|$. Put $u_y = k_n x_0 + (1-k_n)v_y$, clearly by definition of $F_j(y)$ we get $\{u_y\} \subset F_j^n(y)$ and $\| u_x - y_y \| = \| (1-k_n)(v_x - v_y) \| \le (1-k_n)\| g(x) - g(y) \| \le (1-k_n)\| x - y \|$ which proves that $(F_i^n)_{i=1}^{\infty}$ is a sequence of fuzzy mappings satisfying the condition (**). The common fixed point theorem for a sequence of fuzzy mappings due to Lee and Cho [10] i.e., Theorem 3.2 guarantees that for each fixed $n \in \mathbf{N}$, $(F_i^n)_{i=1}^{\infty}$ has a common fixed point in $K$, say $\{x_n\} \subset F_i^n(x_n) \in W(K)$ for all $i \in \mathbf{N}$. From the definition of $F_i^n(x_n)$ there exists a $\{u_n\} \subset F_i(x_n)$ such that $x_n = k_n x_0 + (1-k_n)u_n$ for all $i \in \mathbf{N}$ and each fixed $n \in \mathbf{N}$. Thus $\| x_n - u_n \| = \| k_n x_0 (1-k_n)u_n - u_n \| = k_n \| x_0 - u_n \|$. By the definition of $W(K)$, $\{u_n\} \subset F_i(x_n) \in W(K)$ implies $u_n \in K$. Thus $\{\| u_n - x_0 \|\}$ is bounded. So by the fact that $k_n \to 0$ as $n \to \infty$, we have $\| x_n - u_n \| \to 0$ as $n \to \infty$.

We use the following notion due to Opial [15]. A Banach space $X$ is said to satisfy Opial's condition [15] if for each $x \in X$ and each sequence $(x_n)_{n=1}^{\infty}$ weakly convergent to $x$,

$$\frac{\lim}{n \to \infty} \| x_n - y \| > \frac{\lim}{n \to \infty} \| x_n - x \|$$

for all $y \ne x$.

**PROPOSITION 3.4.** Let $K$ be a nonempty subset of a Banach space $X$ which satisfies Opial's condition and $F$ a $g$-nonexpansive type fuzzy mapping of $K$ into $W(K)$. Let $(x_n)_{n=1}^{\infty}$ be a sequence in $K$ which converges weakly to an element $x \in K$. If $(y_n)_{n=1}^{\infty}$ is a sequence in $X$ such that $\{x_n - y_n\} \subset F(x_n)$ and converges to $y \in X$, then $\{x - y\} \subset F(x)$.

**PROOF.** Since $F$ is a $g$-nonexpansive type fuzzy mapping, there exists a $\{v_n\} \subset F(x)$ such that $\| x_n - y_n - v_n \| \le \| g(x_n) g(x) \| \le \| x_n - x \|$. It follows that $\frac{\lim}{n\to\infty} \| x_n - y_n - v_n \| \le \frac{\lim}{n\to\infty} \| x_n - x \|$. Since every weakly convergent sequence is necessarily bounded, limits in the proceeding expression are finite. Since $(v_n)_{n=1}^{\infty}$ is a sequence in a compact subset $[F(x)]_\alpha$ of $X$ for each $\alpha \in [0,1]$, there is a subsequence of $(v_n)_{n=1}^{\infty}$, also denoted by $(v_n)_{n=1}^{\infty}$, converging to $v \in [F(x)]_\alpha$ for each $\alpha \in [0,1]$. Hence $\{v\} \subset F(x)$, therefore

$$\begin{aligned}
\frac{\lim}{n \to \infty} \| x_n - y_n - v_n \| &= \frac{\lim}{n \to \infty} \| x_n - y_n - v_n - (y+v) + (y+v) \| \\
&\ge \frac{\lim}{n \to \infty} (\| x_n - (y+v) \| - \| (y_n + v_n) - (y+v) \|) \\
&\ge \frac{\lim}{n \to \infty} \| x_n - (y+v) \| + \frac{\lim}{n \to \infty} (-\| y_n + v_n - y - v \|) \\
&= \frac{\lim}{n \to \infty} \| x_n - (y+v) \|
\end{aligned}$$

Thus we have shown that $\frac{\lim}{n\to\infty} \| x_n - x \| \ge \frac{\lim}{n\to\infty} \| x_n - (y+v) \|$.

Since $(x_n)_{n=1}^{\infty}$ converges to $x$ weakly, Opial's condition implies that $x = y + v$, so $x - y = v \in [F(x)]_\alpha$ for each $\alpha \in [0,1]$. Hence $\{x - y\} \subset F(x)$ and the proposition is proved.

**REMARK.** From the above proof it follows that the weak limit of fixed points of a nonexpansive-type fuzzy mapping $F$ defined on a nonempty subset $K$ of a Banach space $X$ satisfying Opial's condition, in particular for a Hilbert space is also a fixed point of $F$.

**THEOREM 3.5.** Let $K$ be a nonempty weakly compact star-shaped subset of a Banach space $X$ which satisfies Opial's condition. If $(F_i)_{i=1}^{\infty}$ is a sequence of fuzzy mappings of $K$ into $W(K)$ satisfying the condition (****), then $(F_i)_{i=1}^{\infty}$ has a common fixed point.

**PROOF.** Since $K$ is weakly compact, it is a bounded subset of $X$. By the Proposition 3.3 there exist a sequence $(x_n)_{n=1}^{\infty}$ in $K$ and a sequence $(u_n)_{n=1}^{\infty}$ in $X$ satisfying $\{u_n\} \subset F_i(x_n)$ for all $i \in \mathbf{N}$ such that $\|x_n - u_n\| \to 0$ as $n \to \infty$. Put $y_n = x_n - u_n$. $K$ being weakly compact, we can find a weakly convergent subsequence $(x_m)_{m=1}^{\infty}$ of $(x_n)_{n=1}^{\infty}$. Let $x_0$ be the weak limit of the sequence $(x_m)_{m=1}^{\infty}$. Clearly $x_0 \in K$ and we have $y_m = x_m - u_m, \{u_m\} \subset F_i(x_m)$ for all $i \in \mathbf{N}$. Then it follows that $y_m \to 0$ and by Proposition 3.4 there exists a fixed point $x_0 \in X$ such that $\{x_0\} \subset F_i(x_0)$ for all $i \in \mathbf{N}$.

**THEOREM 3.6.** Let $K$ be a nonempty weakly compact star-shaped subset of a Banach space $X$ which satisfies Opial's condition. If $(F_i)_{i=1}^{\infty}$ is a sequence of fuzzy mappings of $K$ into $W(K)$ satisfying the condition (*), (**) or (***), then $(F_i)_{i=1}^{\infty}$ has a common fixed point.

**PROOF.** It is proved by the fact that the condition (***) [respectively, (*)] implies the condition (****) [resp., (**)].

If we put $F_i = F$ for all $i \in \mathbf{N}$ in Proposition 3.3, then the sequence of fuzzy mappings $(F_i)_{i=1}^{\infty} = (F)$ in the condition (****) is a sequence of $g$-nonexpansive type fuzzy mappings. Thus we obtain the following corollary for $g$-nonexpansive type fuzzy mappings.

**COROLLARY 3.7.** Let $K$ be a nonempty weakly compact star-shaped subset of a Banach space $X$ which satisfies Opial's condition. Then each $g$-nonexpansive type fuzzy mapping $F : K \to W(K)$ has a fixed point.

**COROLLARY 3.8.** Let $K$ be a nonempty weakly compact star-shaped subset of a Banach space $X$ which satisfies Opial's condition. Then each nonexpansive type, compact-valued mapping $f : K \to 2^K$ has a fixed point.

**PROOF.** Define $F : K \to W(K)$ by $F(x) = \chi_{f(x)}$ then $F$ is a nonexpansive-type fuzzy mapping. By Corollary 3.7 there exists a point $x \in X$ such that $\{x\} \subset F(x) = \chi_{f(x)}$ i.e., $x \in f(x)$.

Corollary 3.8 is a generalization of the following theorem due to Husain and Latif [8].

**THEOREM 3.9.** Let $K$ be a nonempty weakly compact convex subset of a Banach space $X$ which satisfies Opial's condition. Then each nonexpansive type, compact-valued mapping $f : K \to 2^K$ has a fixed point.

**COROLLARY 3.10.** Let $K$ be a nonempty weakly compact star-shaped subset of a Banach space $X$ which satisfies Opial's condition. Then each nonexpansive fuzzy mapping $F : K \to W(K)$ has a fixed point.

**COROLLARY 3.11.** Let $K$ be a nonempty weakly compact star-shaped subset of a Banach space $X$ having a weakly continuous duality mapping. Then each nonexpansive-type fuzzy mapping $F : K \to W(K)$ has a fixed point.

**PROOF.** If a Banach space $X$ admits a weakly continuous duality mapping, then it satisfies Opial's condition [6].

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## REFERENCES

[1] BOSE, R. K. and SAHANI, D., Fuzzy mappings and fixed point theorems, *Fuzzy Sets and Systems* 21 (1987), 53-58.

[2] CHANG, S. S. and HUANG, N. J., Fixed point theorems for generalized fuzzy mappings, *Acta of Engineering Math.* No. 2 (1984), 135-137 (in Chinese).

[3] CHANG, S. S., Fixed point theorems for fuzzy mappings, *Kexue Tongbao* 14 (1984), 833-836 (in Chinese).

[4] CHANG, S. S., Fixed degree for fuzzy mappings and a generalization of Ky Fan's theorem, *Fuzzy Sets and Systems* 24 (1987), 103-112.

[5] CHITRA, A., A note on the fixed points of fuzzy maps on partially ordered topological spaces, *Fuzzy Sets and Systems* 19 (1986), 305-308.

[6] GOSSEZ, J. P. and LAMI DOZO, E., Some geometric properties related to the fixed point theory of non-expansive mappings, *Pacific J. Math.* 40 (1972), 565-573.

[7] HEILPERN, S., Fuzzy mappings and fixed point theorem, *J. Math. Anal. Appl.* 83 (1981), 566-569.

[8] HUSAIN, T. and LATIF, A., Fixed points of multivalued nonexpansive maps, *Internat. J. Math. & Math. Sci.*, Vol. 14, No. 3 (1991), 421-430.

[9] LEE, B. S. and CHO, S. J., A fixed point theorem for contractive-type fuzzy mappings, *Fuzzy Sets and Systems* 61 (1994), 309-312.

[10] LEE, B. S. and CHO, S. J., Common fixed point theorems for sequences of fuzzy mappings, to appear in *Internat. J. Math. & Math. Sci.*

[11] LEE, B. S., CHO, S. J., LEE, G. M. and KIM, D. S., On the common fixed point theorems for a sequence of fuzzy mappings, submitted.

[12] LEE, B. S., LEE, G. M., CHO, S. J. and KIM, D. S., A common fixed point theorem for a pair of fuzzy mappings, submitted.

[13] LEE, B. S., LEE, G. M., CHO, S. J. and KIM, D. S., Generalized common fixed point theorems for a sequence of fuzzy mappings, to appear in *Internat. J. Math. & Math. Sci.*

[14] NGUYEN, H. T., A note on the extension principle for fuzzy set, *J. Math. Anal. Appl.* 64 (1978), 369-380.

[15] OPIAL, Z., Weak convergence of the sequence of successive approximations for nonexpansive mappings, *Bull. Amer. Math. Soc.* 73 (1967), 591-597.

[16] SOM, T. and MUKHERJEE, R. N., Some fixed point theorems for fuzzy mappings, *Fuzzy Sets and Systems* 33(1989), 213-219.