ON UNSTEADY TWO-PHASE FLUID FLOW DUE TO ECCENTRIC ROTATION OF A DISK

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We examine the unsteady flow of a two-phase fluid generated by the nontorsional oscillations of a disk when the disk and the fluid at infinity rotate noncoaxially with the same angular velocity. The solutions are obtained for both the fluid and the particle velocities in closed form. It is found that the solutions remain valid for all values of the frequency of oscillations of the disk including the resonant frequency, which is equal to the angular velocity of rotation. But, in absence of particles, only in the case of resonance no oscillatory solution is possible, which is similar to that of solid-body rotation as pointed out by Thornley (1968). It is also shown that, unlike the case of single-disk configuration, no unique solution exists in a double-disk configuration, a result which is the reverse to that of solid-body rotation. Finally, the results are presented graphically to determine the quantitative response of the particle on the flow.

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1. Introduction. The dynamics of rotating fluids is particularly important in the analysis of flow phenomena associated with the atmospheric, oceanic, geophysical, and astrophysical problems. Thornley [3] investigated the flow generated in a semi-infinite expanse of viscous fluid bounded by the infinite rigid disk in the presence of the particles in the fluid. However, if the fluid is clean, no physically meaningful resonant solutions are possible in the existing flow configuration, which is an event similar to that of Thornley [3]. Moreover, it is found that, contrary to the case of single-disk geometry, infinite number of solutions exist for the flow confined between two noncoaxially rotating parallel disks. Finally, the results are evaluated quantitatively with a view to examine the effect of particles on the flow.

2. Formulation of the problem. We consider the flow of a two-phase fluid due to an oscillating disk in the xy-plane rotating about the z-axis normal to the disk with an angular velocity Ω in Cartesian coordinate system. The particulate fluid at $z = \infty$ rotates, with the same angular velocity, about an axis parallel to the z-axis passing through the point (x_1, y_1). For this type of motion the velocity fields for the fluid and the particles may be taken in the form

$$u_1 = -\Omega[y - g_1(z,t)], \qquad u_2 = \Omega[x - f_1(z,t)], \qquad u_3 = 0,$$
$$v_1 = -\Omega[y - g_2(z,t)], \qquad v_2 = \Omega[x - f_2(z,t)], \qquad v_3 = 0,$$

(2.1)

where $\vec{u} = (u_1, u_2, u_3)$, and $\vec{v} = (v_1, v_2, v_3)$ represent, respectively, the fluid and the particle velocities.

Following Saffman [2], the unsteady motion of a two-phase fluid with uniformly distributed particles, occupying the semi-infinite space $z > 0$, is governed by the equations

$$\frac{\partial u_1}{\partial t} + u_1 \frac{\partial u_1}{\partial x} + u_2 \frac{\partial u_1}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u_1}{\partial x^2} + \frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_1}{\partial z^2}\right) + \frac{k}{\tau}(v_1 - u_1),$$

$$\frac{\partial u_2}{\partial t} + u_1 \frac{\partial u_2}{\partial x} + u_2 \frac{\partial u_2}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left(\frac{\partial^2 u_2}{\partial x^2} + \frac{\partial^2 u_2}{\partial y^2} + \frac{\partial^2 u_2}{\partial z^2}\right) + \frac{k}{\tau}(v_2 - u_2),$$

$$\frac{\partial p}{\partial z} = 0,$$

$$\frac{\partial v_1}{\partial t} + v_1 \frac{\partial v_1}{\partial x} + v_2 \frac{\partial v_1}{\partial y} = \frac{1}{\tau}(u_1 - v_1),$$

$$\frac{\partial v_2}{\partial t} + v_1 \frac{\partial v_2}{\partial x} + v_2 \frac{\partial v_2}{\partial y} = \frac{1}{\tau}(u_2 - v_2),$$

(2.2)

where k and τ are, respectively, the concentration and the relaxation time of the particles in the fluid.

Substituting (2.1) in (2.2), we get

$$\Omega\left\{\nu\frac{\partial^2 g_1}{\partial z^2} - \frac{\partial g_1}{\partial t} - \Omega f_1 + \frac{k}{\tau}(g_2 - g_1)\right\} = \frac{1}{\rho}\frac{\partial p}{\partial x} - \Omega^2 x,$$

(2.3)

$$\Omega\left\{\nu\frac{\partial^2 f_1}{\partial z^2} - \frac{\partial f_1}{\partial t} + \Omega g_1 + \frac{k}{\tau}(f_2 - f_1)\right\} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \Omega^2 y,$$

(2.4)

$$\frac{\partial g_2}{\partial t} + \Omega f_2 = \Omega x + \frac{1}{\tau}(g_1 - g_2),$$

(2.5)

$$\frac{\partial f_2}{\partial t} - \Omega g_2 = \Omega y + \frac{1}{\tau}(f_1 - f_2),$$

(2.6)

$$\frac{\partial p}{\partial z} = 0.$$

(2.7)

From (2.7), it follows that p is independent of z. Hence, on eliminating p from (2.3) and (2.4), we get

$$\nu\frac{\partial^3 w_1}{\partial z^3} - \frac{\partial^2 w_1}{\partial z \partial t} - i\Omega\frac{\partial w_1}{\partial z} + \frac{k}{\tau}\left(\frac{\partial w_2}{\partial z} - \frac{\partial w_1}{\partial z}\right) = 0$$

(2.8)

with $w_1 = f_1 + ig_1$ and $w_2 = f_2 + ig_2$.

Similarly, from (2.5) and (2.6), we get

$$\frac{\partial^3 w_2}{\partial z \partial t} - i\Omega\frac{\partial w_2}{\partial z} = \frac{1}{\tau}\left(\frac{\partial w_1}{\partial z} - \frac{\partial w_2}{\partial z}\right).$$

(2.9)

On eliminating w_2 from (2.8) with the help of (2.9), we have

$$\nu \frac{\partial^4 w_1}{\partial t \partial z^3} - \nu \left(i\Omega - \frac{1}{\tau} \right) \frac{\partial^3 w_1}{\partial z^3} - \frac{\partial^3 w_1}{\partial z \partial t^2} - \frac{1+k}{\tau} \frac{\partial^2 w_1}{\partial t \partial z} - \Omega \left\{ \Omega + \frac{i(1-k)}{\tau} \right\} \frac{\partial w_1}{\partial z} = 0.$$
(2.10)

Equation (2.10) is to be solved with the boundary conditions

$$w_1 = a e^{\text{int}} + b e^{-\text{int}} \quad \text{at } z = 0,$$
(2.11a)

$$w_1 = x_1 + i y_1 \qquad \text{at } z = \infty$$
(2.11b)

along with the assumption that the solutions are bounded at infinity.

3. Solution of the problem. In view of the boundary condition (2.11a), we suggest the solution of (2.10) as

$$W_1 = F_0(z) + a F_1(z) e^{\text{int}} + b F_2(t) e^{-\text{int}}.$$
(3.1)

Substituting (3.1) in (2.10) and utilizing the boundary conditions, the fluid velocity for the case $\sigma \ (= n/\Omega) < 1$ can be obtained as

$$w_1(z,t) = (x_1 + i y_1)(1 - e^{-m_0 z}) + a e^{\text{int} - m_1 z} + b e^{-\text{int} - m_2 z},$$
(3.2)

where

$$m_j z = \xi_j (A_j + i B_j),$$

$$A_j = \left\{ (P_j^2 + 1)^{1/2} + P_j \right\}^{1/2}, \qquad B_j = \left\{ (P_j^2 + 1)^{1/2} - P_j \right\}^{1/2},$$

$$\xi_j = C_j \xi, \quad j = 0, 1, 2,$$

$$\xi = \left(\frac{\Omega}{2\nu} \right)^{1/2} z, \qquad C_0 = \left(\frac{1 - k + \Omega^2 \tau^2}{1 + \Omega^2 \tau^2} \right)^{1/2},$$

$$C_1 = \left(\frac{1 + \sigma - k(1 - \sigma) + \Omega^2 \tau^2 (1 + \sigma)(1 - \sigma)^2}{1 + \Omega^2 \tau^2 (1 - \sigma)^2} \right)^{1/2},$$
(3.3)

$$C_2 = \left(\frac{1 - \sigma - k(1 + \sigma) + \Omega^2 \tau^2 (1 - \sigma)(1 + \sigma)^2}{1 + \Omega^2 \tau^2 (1 + \sigma)^2} \right)^{1/2},$$

$$P_0 = \frac{\Omega k \tau}{1 - k + \Omega^2 \tau^2}, \qquad P_1 = \frac{\Omega k \tau (1 - \sigma)^2}{1 + \sigma - k(1 - \sigma) + \Omega^2 \tau^2 (1 + \sigma)(1 - \sigma)^2},$$

$$P_2 = \frac{\Omega k \tau (1 + \sigma)^2}{1 - \sigma - k(1 + \sigma) + \Omega^2 \tau^2 (1 - \sigma)(1 + \sigma)^2}.$$

Equating the real and imaginary parts of (3.2) by taking $a = a_1 + ia_2$ and $b = b_1 + ib_2$, we get

$$
\begin{aligned}
f_1 &= x_1 \left(1 - e^{-A_0 \xi_0} \cos B_0 \xi_0\right) - y_1 e^{-A_0 \xi_0} \sin B_0 \xi_0 \\
&\quad + e^{-A_1 \xi_1} \left\{a_1 \cos \left(B_1 \xi_1 - nt\right) + a_2 \sin \left(B_1 \xi_1 - nt\right)\right\} \\
&\quad + e^{-A_2 \xi_2} \left\{b_1 \cos \left(B_2 \xi_2 + nt\right) + b_2 \sin \left(B_2 \xi_2 + nt\right)\right\}, \\
g_1 &= x_1 e^{-A_0 \xi_0} \sin B_0 \xi_0 + y_1 \left(1 - e^{-A_0 \xi_0} \cos B_0 \xi_0\right) \\
&\quad + e^{-A_1 \xi_1} \left\{a_2 \cos \left(B_1 \xi_1 - nt\right) - a_1 \sin \left(B_1 \xi_1 - nt\right)\right\} \\
&\quad + e^{-A_2 \xi_2} \left\{b_2 \cos \left(B_2 \xi_2 + nt\right) - b_1 \sin \left(B_2 \xi_2 + nt\right)\right\}.
\end{aligned}
\tag{3.4}
$$

In particular, where $k = 0$, the fluid velocity corresponding to clean fluid motion for $\sigma < 1$ is given by

$$
\begin{aligned}
f_1 &= x_1 \left(1 - e^{-\xi} \cos \xi\right) - y_1 e^{-\xi} \sin \xi \\
&\quad + e^{-\xi \sqrt{1+\sigma}} \left\{a_1 \cos \left(\xi \sqrt{1+\sigma} - nt\right) + a_2 \sin \left(\xi \sqrt{1+\sigma} - nt\right)\right\} \\
&\quad + e^{-\xi \sqrt{1-\sigma}} \left\{b_1 \cos \left(\xi \sqrt{1-\sigma} + nt\right) + b_2 \sin \left(\xi \sqrt{1-\sigma} + nt\right)\right\}, \\
g_1 &= x_1 e^{-\xi} \sin \xi + y_1 \left(1 - e^{-\xi} \cos \xi\right) \\
&\quad + e^{-\xi \sqrt{1+\sigma}} \left\{a_2 \cos \left(\xi \sqrt{1+\sigma} - nt\right) - a_1 \sin \left(\xi \sqrt{1+\sigma} - nt\right)\right\} \\
&\quad + e^{-\xi \sqrt{1-\sigma}} \left\{b_2 \cos \left(\xi \sqrt{1-\sigma} + nt\right) - b_1 \sin \left(\xi \sqrt{1-\sigma} + nt\right)\right\}.
\end{aligned}
\tag{3.5}
$$

The distinctive feature of the solutions (3.4) is that the flow essentially consists of three distinct boundary layers on the disk. The thickness of these layers are of orders

$$
\delta_r = \left(\frac{2\nu}{\Omega}\right)^{1/2} \left(C_r A_r\right)^{-1}, \quad r = 0, 1, 2,
\tag{3.6}
$$

with $\delta_1 < \delta_0 < \delta_2$. Clearly, the thickness of the layers is modified by the presence of particles in the fluid. In fact, it decreases with increase in particle concentration (k). On the other hand, in the absence of particles $(k = 0)$, the above three layers modify themselves to an Ekman layer of thickness of the order $(2\nu/\Omega)^{1/2}$ surrounded by two more Stokes-Ekman layers of thickness of the orders $(2\nu/(\Omega - n))^{1/2}$ and $(2\nu/(\Omega + n))^{1/2}$. These three layers combine into a single Ekman layer of thickness of the order $(2\nu/\Omega)^{1/2}$ when $n = 0$.

The fluid velocity for the case $\sigma = (n/\Omega) > 1$ is given by

$$
\begin{aligned}
f_1 = {} & x_1(1 - e^{-\alpha_0 \xi_0} \cos \beta_0 \xi_0) - y_1 e^{-\alpha_0 \xi_0} \sin \beta_0 \xi_0 \\
& + e^{-\alpha_1 \xi_1} \{a_1 \cos(\beta_1 \xi_1 - nt) + a_2 \sin(\beta_1 \xi_1 - nt)\} \\
& + e^{-\alpha_2 \xi_2} \{b_1 \cos(\beta_2 \xi_2 + nt) + b_2 \sin(\beta_2 \xi_2 + nt)\}, \\
g_1 = {} & y_1(1 - e^{-\alpha_0 \xi_0} \cos \beta_0 \xi_0) + x_1 e^{-\alpha_0 \xi_0} \sin \beta_0 \xi_0 \\
& + e^{-\alpha_1 \xi_1} \{a_2 \cos(\beta_1 \xi_1 - nt) - a_1 \sin(\beta_1 \xi_1 - nt)\} \\
& + e^{-\alpha_2 \xi_2} \{b_2 \cos(\beta_2 \xi_2 + nt) - b_1 \sin(\beta_2 \xi_2 + nt)\},
\end{aligned}
\tag{3.7}
$$

where $\alpha_j, \beta_j = \{(q_j^2 + 1) \pm q_j\}^{1/2}$, $\xi_j = D_j \xi$, $\xi = (\Omega/2\nu)^{1/2} z$, $j = 0, 1, 2$,

$$
D_0 = \left(\frac{1 - k + \Omega^2 \tau^2}{1 + \Omega^2 \tau^2} \right)^{1/2}, \qquad
D_1 = \left(\frac{1 + \sigma + k(\sigma - 1) + \Omega^2 \xi^2 (\sigma + 1)(\sigma - 1)^2}{1 + \Omega^2 \tau^2 (\sigma - 1)^2} \right)^{1/2},
$$

$$
D_2 = \left(\frac{\sigma - 1 + k(\sigma + 1) + \Omega^2 \tau^2 (\sigma - 1)(\sigma + 1)^2}{1 + \Omega^2 \tau^2 (\sigma + 1)^2} \right)^{1/2},
$$

$$
q_0 = \frac{\Omega \tau k}{1 - k + \Omega^2 \tau^2}, \qquad
q_1 = \frac{\Omega \tau k(\sigma - 1)^2}{1 + \sigma + k(\sigma - 1) + \Omega^2 \tau^2 (\sigma + 1)(\sigma - 1)^2},
$$

$$
q_2 = \frac{\Omega \tau k(\sigma + 1)^2}{\sigma - 1 + k(\sigma + 1) + \Omega^2 \tau^2 (\sigma - 1)(\sigma + 1)^2}.
\tag{3.8}
$$

When the natural frequency of rotation is equal to the forced frequency n, that is, for $\sigma = 1$, the system resonates and in this case the solution is given by

$$
\begin{aligned}
f_1 = {} & x_1(1 - e^{-A_0 \xi_0} \cos B_0 \xi_0) - y_1 e^{-A_0 \xi_0} \sin B_0 \xi_0 \\
& + e^{-\sqrt{2}\xi} \{a_1 \cos(\sqrt{2}\xi - nt) + a_2 \sin(\sqrt{2}\xi - nt)\} \\
& + e^{-A_2^* \xi_2^*} \{b_1 \cos(B_2^* \xi_2^* + nt) + b_2 \sin(B_2^* \xi_2^* + nt)\}, \\
g_1 = {} & x_1 e^{-A_0 \xi_0} \sin B_0 \xi + y_1(1 - e^{-A_0 \xi_0} \cos B_0 \xi_0) \\
& + e^{-\sqrt{2}\xi} \{a_2 \cos(\sqrt{2}\xi - nt) - a_1 \sin(\sqrt{2}\xi - nt)\} \\
& + e^{-A_2^* \xi_2^*} \{b_2 \cos(B_2^* \xi_2^* + nt) - b_1 \sin(B_2^* \xi_2^* + nt)\},
\end{aligned}
\tag{3.9}
$$

where

$$
\xi_2^* = C_2^* \xi, \qquad C_2^* = \left(\frac{2k}{1 + 4\Omega^2 \tau^2} \right)^{1/2},
$$

$$
A_2^*, B_2^* = \left\{ (1 + 4\Omega^2 \tau^2)^{1/2} - 2\Omega \tau \right\}^{1/2}.
\tag{3.10}
$$

It is worth noting that, when $k \neq 0$, the results (3.9) provide a meaningful resonant solution satisfying all boundary conditions. But when $k = 0$, the last terms of (3.9) do not satisfy the boundary condition at infinity. Accordingly, in the case of clean viscous fluids, no oscillatory solution exists at a resonant

frequency $n = \Omega$. This phenomenon is similar to that pointed out by Thornley [3] in the case of solid-body rotation.

To determine the particle velocity satisfying (2.9), we assume that

$$W_2(z,t) = G_0(z) + aG_1(z)e^{\text{int}} + bG_2(z)e^{-\text{int}}, \tag{3.11}$$

with

$$W_2(z,t) = ae^{\text{int}} + be^{-\text{int}} \quad \text{at } z = 0. \tag{3.12}$$

Substituting (3.1) and (3.11) in (2.9) and utilizing the boundary conditions, the particle velocity is given by

$$
\begin{aligned}
W_2 &= \frac{F_0}{1 - i\Omega\tau} + a\left\{\frac{F_1 - i(\Omega\tau - n\tau)}{1 - i(\Omega\tau - n\tau)}\right\}e^{\text{int}} + b\left\{\frac{F_2 - i(\Omega\tau + n\tau)}{1 - i(\Omega\tau + n\tau)}\right\}e^{-\text{int}}, \quad n < \sigma \\
&= \frac{F_0}{1 - i\Omega\tau} + aF_1 e^{\text{int}} + be^{-\text{int}}, \quad n = \sigma \\
&= \frac{F_0}{1 - i\Omega\tau} + a\left\{\frac{F_1 + i(n\tau - \Omega\tau)}{1 + i(n\tau - \Omega\tau)}\right\}e^{\text{int}} + b\left\{\frac{F_2 - i(n\tau + \Omega\tau)}{1 - i(n\tau + \Omega\tau)}\right\}e^{-\text{int}}, \quad n > \sigma.
\end{aligned}
\tag{3.13}
$$

It follows that the particles at infinity are unable to follow the fluid motion due to the presence of Ω and n. But when Ω and n equal zero, the particle and the fluid move in unison and we have $W_1 = W_2$.

We next turn our attention to the case of another disk introduced at $z = d$ which rotates with the angular velocity Ω about an axis parallel to the z-axis and passing through the point (x_1, y_1) so that the boundary condition of $F_0(z) = \infty$ is replaced by $W = x_1 + iy_1$ at $z = d$. We focus our attention on the solution of $F_0(z)$ only because the essential nature of $F_1(z)$ and $F_2(z)$, for the unsteady case in (3.1), is similar to $F_0(z)$. The solution for $F_0(z)$ satisfying (2.10), (2.11a), and the boundary condition at $z = d$ is given by

$$F_0(z) = C\left(1 - \frac{\sinh m_0(d-z) + \sinh m_0 z}{\sinh m_0 d}\right) + (x_1 + iy_1)\frac{\sinh m_0 z}{\sinh m_0 d}, \tag{3.14}$$

where

$$m_0 = \left(\left(\frac{\Omega}{\nu}\right)\frac{\Omega\tau + i(1-k)}{1 - i\Omega\tau}\right)^{1/2}. \tag{3.15}$$

The result (3.14) contains an arbitrary constant C which remains undetermined under the stated boundary conditions, giving a possibility of infinite number of solutions. Similarly, the solutions of F_1 and F_2 also contain arbitrary constants each is undetermined. Thus, instead of getting a unique solution as in the case of single-disk configuration, the double-disk configuration provides an infinite number of solutions.

Finally, the fluid velocity near $z = 0$, corresponding to the case $\sigma < 1$, is obtained from (3.4) as

$$
\begin{aligned}
f_1 &\sim x_1 A_0 \xi_0 - y_1 B_0 \xi_0 \\
&\quad + \{a_1(1-A_1\xi_1) + b_1(1-A_2\xi_2) + a_2 B_1 \xi_1 + b_2 B_2 \xi_2\}\cos nt \\
&\quad + \{b_2(1-A_2\xi_2) - a_2(1-A_1\xi_1) + a_1 B_1 \xi_1 - b_1 B_2 \xi_2\}\sin nt, \\
g_1 &\sim x_1 A_0 \xi_0 - y_1 B_0 \xi_0 \\
&\quad + \{a_2(1-A_1\xi_1) + b_2(1-A_2\xi_2) - a_1 B_1 \xi_1 - b_1 B_2 \xi_2\}\cos nt \\
&\quad + \{a_1(1-A_1\xi_1) - b_1(1-A_2\xi_2) + a_2 B_1 \xi_1 - b_2 B_2 \xi_2\}\sin nt.
\end{aligned}
\tag{3.16}
$$

The above results (3.16) indicate that the velocity vector near the disk is inclined at an angle $\tan^{-1}(-f/g)$ to the disk. For a special case in which $a_1 = a_2 = b_1 = b_2 = y_1 = 1$, $x_1 = 0$, and $nt = \pi/2$, we have

$$
\begin{aligned}
f_1 &\sim -B_0 \xi_0 + (A_1 + B_1)\xi_1 - (A_2 + B_2)\xi_2, \\
g_1 &\sim B_0 \xi_0 - (A_1 - B_1)\xi_1 + (A_2 - B_2)\xi_2
\end{aligned}
\tag{3.17}
$$

which, when $k \to 0$, gives

$$
\begin{aligned}
f_1 &\sim 2(\xi_1 - \xi_2) - \xi_0, \\
g_1 &\sim \xi_0
\end{aligned}
\tag{3.18}
$$

so that the velocity vector is inclined at an angle $\tan^{-1}(1 - 2N)$, where

$$
N = (1+\sigma)^{1/2} - (1-\sigma)^{1/2}.
\tag{3.19}
$$

Thus, the angle of inclination of the velocity vector near the disk not only depends on the particles but also on $\sigma = n/\Omega$. However, when both $k \to 0$ and $n \to 0$, the velocity vector is inclined at an angle $45°$ to the disk.

4. Conclusion. The analysis given above clearly indicates that in a noncoaxial system of rotation the resonance occurs at a frequency equal to angular velocity of rotation of the disk which is not the case in a coaxial system of solid-body rotation where the resonance occurs at a frequency equal to twice the angular velocity of rotation of the disk as pointed out by Thornley [3].

Secondly, the difficulty in obtaining the resonant solution in the case of clean fluid is resolved automatically in presence of the particles in the fluid.

Finally, the infinite number of solutions existing for the flow in the geometry of two parallel disks given by Berker [1] reduce to a single unique solution for the case of a single disk.

The quantitative evaluation of the results for f_1 and g_1 for various values of flow parameters is presented in Figures 4.1, 4.2, and 4.3.
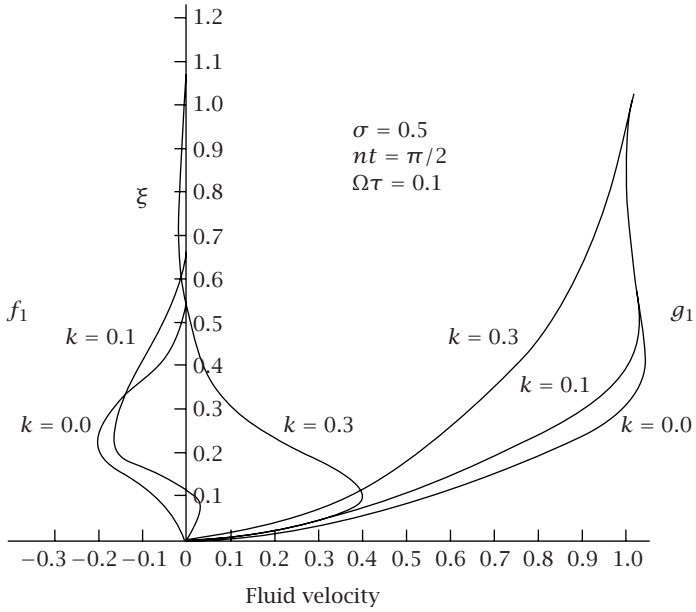
FIGURE 4.1. Variations of f_1 and g_1 for different values of particle concentration k and for fixed values of nt and $\Omega\tau$ when $\sigma < 1$.
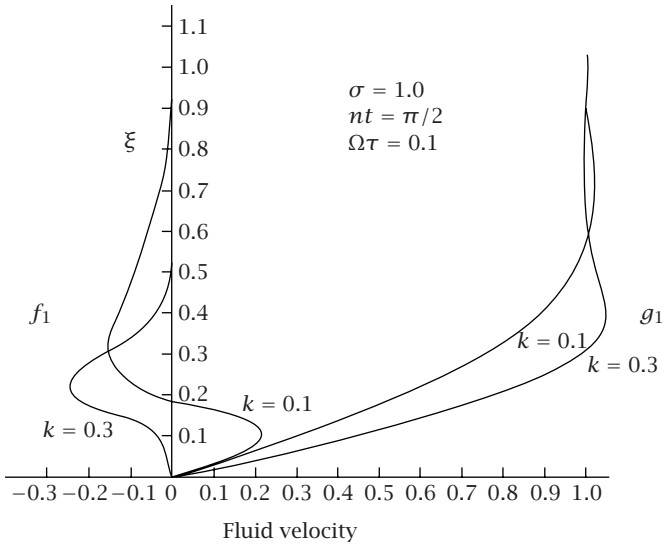

FIGURE 4.2. Variations of f_1 and g_1 for different values of particle concentration k in the resonant case and for fixed values of nt and $\Omega\tau$ when $\sigma = 1$.
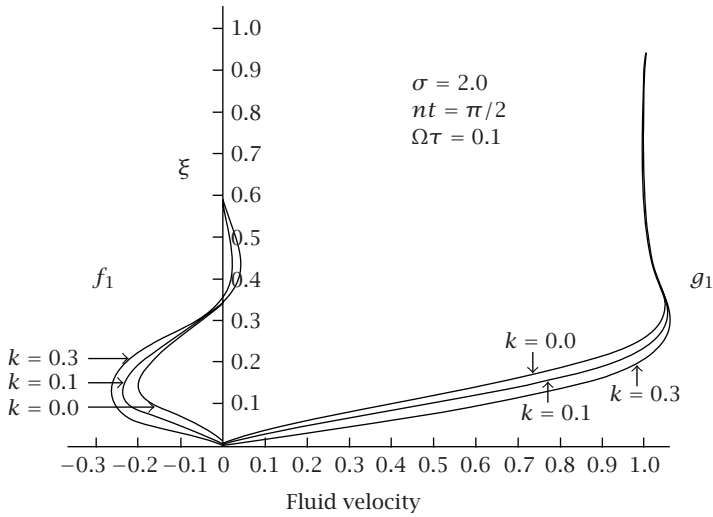
FIGURE 4.3. Variations of f_1 and g_1 for different values of particle concentration k and for fixed values of nt and $\Omega\tau$ when $\sigma > 1$.

References

[1]   R. Berker, *Intégration des équations du mouvement d'un fluide visqueux incompressible*, Handbuch der Physik, Vol. VIII/2, Springer-Verlag, Berlin, 1963, pp. 1–384 (French).

[2]   P. G. Saffman, *On the stability of laminar flow of a dusty gas*, J. Fluid Mech. **13** (1962), 120–128.

[3]   C. Thornley, *On Stokes and Rayleigh layers in a rotating system*, Quart. J. Mech. Appl. Math. **21** (1968), 451–461.

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