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ON THE ECONOMIC RISK CAPITAL OF PORTFOLIO INSURANCE

WERNER HÜRLIMANN

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A formula for the conditional value-at-risk of classical portfolio insurance is derived and shown to be constant for sufficiently small loss probabilities. As illustrations, we discuss portfolio insurance for an equity market index using empirical data, and analyze the more general multivariate situation of a portfolio of risky assets.

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1. Introduction. Portfolio insurance, introduced by Leland in the night of September 11, 1976, is a simple financial instrument used to protect capital against future adverse falls. A collection of seminal papers, which study some of its main properties, is Luskin [12]. Combined with classical actuarial contingencies like mortality risk, portfolio insurance leads to unit-linked insurance contracts, which under the economic risk capital viewpoint have been analyzed in Hürlimann [6].

In the present note, the focus on classical portfolio insurance reveals a new remarkable feature. It turns out that the economic risk capital as measured by value-at-risk or conditional value-at-risk (CVaR) remains constant, provided the loss probability is sufficiently small. In practice, confidence levels α around 70%–90% often suffice to guarantee this stability property.

In Section 2, we derive a formula for the CVaR of portfolio insurance, and show that it is constant for small loss probabilities. Two specific examples illustrate our results. In Section 3, we discuss portfolio insurance for an equity market index on the basis of empirical data material. The more general multivariate situation of a portfolio of risky assets is exemplified in Section 4 in the bivariate case.

2. Conditional value-at-risk. Suppose that the random variable S represents the market value of a portfolio of assets at some future date T. The goal of portfolio insurance is to protect this future market value in such a way that the fixed value or limit L is guaranteed. For this, an investor can either hold the assets and buy a put option with exercise price L or hold cash at the risk-free continuous interest rate δ and buy a call option with exercise price L. The future values of these equivalent option strategies satisfy the identity

$$S+(L-S)_+=L+(S-L)_+. \tag{2.1}$$

Let S_0 denote the present value of the portfolio of assets, and let $P(L)$, $C(L)$ be the put and call option prices with exercise price L, which are to be paid for these option strategies. Then the total cost $K(L)$ of portfolio insurance satisfies the put-call parity relation

$$K(L) = S_0 + P(L) = L \cdot e^{-\delta T} + C(L). \tag{2.2}$$

The financial gain at time T per unit of invested capital is described by the random return

$$R = \frac{L + (S-L)_+ - K(L)}{K(L)}. \tag{2.3}$$

The potential investor decides upon investment by looking at the tradeoff between expected return and risk. Since the distribution of return is here asymmetrical, the usual variance as measure of risk cannot be recommended. Indeed, a "good" risk measure for the one-sided positively skewed return (2.3) should preserve the usual stochastic order between returns, that is, if the returns R_1, R_2 satisfy $R_1 \le R_2$ with probability one, a relation denoted by $R_1 \le_{\mathrm{st}} R_2$, then $\rho(R_1) \le \rho(R_2)$, where $\rho(\cdot)$ denotes the risk measure. But $R_1 \le_{\mathrm{st}} R_2$ does not imply $\mathrm{Var}[R_1] \le \mathrm{Var}[R_2]$, from which it follows that the variance is not an acceptable risk measure. In the present note, risk is measured in terms of economic risk capital, which is determined using the conditional value-at-risk measure. The latter is defined as follows. First, consider the upper CVaR to the confidence level α defined by

$$\mathrm{CVaR}_\alpha^+[X] = E[X \mid X > \mathrm{VaR}_\alpha[X]], \tag{2.4}$$

where the negative return $X = -R$ represents the financial loss at time T per unit of invested capital, and $\mathrm{VaR}_\alpha[X] = \inf\{x : F_X(x) \ge \alpha\}$ is the value-at-risk, with $F_X(x) = \Pr(X \le x)$ the probability distribution of the random variable X. The VaR quantity represents the maximum possible loss, which is not exceeded with the probability α (in practice $\alpha = 95\%, 99\%, 99.75\%$). The CVaR^+ quantity is the conditional expected loss, given the loss strictly exceeds its value-at-risk. Next, consider the *α-tail transform* X^α of X with distribution function

$$F_{X^\alpha}(x) = \begin{cases} 0, & x < \mathrm{VaR}_\alpha[X], \\ \dfrac{F_X(x) - \alpha}{1-\alpha}, & x \ge \mathrm{VaR}_\alpha[X]. \end{cases} \tag{2.5}$$

Rockafellar and Uryasev [14] define CVaR to the confidence level α as expected value of the α-tail transform, that is, by

$$\mathrm{CVaR}_\alpha[X] = E[X^\alpha]. \tag{2.6}$$

The obtained measure is a coherent risk measure in the sense of Arztner et al. [1, 2] and coincides with CVaR^+ in the case of continuous distributions. For technical simplicity, we restrict ourselves to the latter situation. As pointed out in Hürlimann [8], several equivalent formulas exist for the evaluation of (2.6). We use the stop-loss transform representation

$$\mathrm{CVaR}_\alpha[X] = Q_X(\alpha) + \frac{1}{\varepsilon} \cdot \pi_X[Q_X(\alpha)], \tag{2.7}$$

where $Q_X(\alpha)$ is the α-quantile of X, $\pi_X(x) = E[(X-x)_+]$ is the stop-loss transform, and $\varepsilon = 1-\alpha$ is interpreted as loss probability. Assuming the mean of X exists, we derive the following formula.

Proposition 2.1. *The* CVaR *associated to the negative return of portfolio insurance is determined by*

$$\mathrm{CVaR}_\alpha[X] = \frac{1}{K(L)}\left\{K(L) - L - (Q_S(\varepsilon)-L)_+ + \frac{1}{\varepsilon}\cdot(\overline{\pi}_S[Q_S(\varepsilon)] - \overline{\pi}_S[L])_+\right\}, \tag{2.8}$$

where $\overline{\pi}_X(x) = E[(x-X)_+]$ denotes the conjugate stop-loss transform.

Proof. The function $I(E)$ of the event E denotes an indicator such that $I(E) = 1$ if E is true and $I(E) = 0$ otherwise. The evaluation of the distribution and stop-loss transform of X is done using the following separation into two steps:

$$F_X(x) = \Pr(\{X \le x\} \cap \{S > L\}) + \Pr(\{X \le x\} \cap \{S \le L\}), \tag{2.9}$$

$$\pi_X(x) = E[(X-x)_+ \cdot I\{S > L\}] + E[(X-x)_+ \cdot I\{S \le L\}]. \tag{2.10}$$

To simplify notations, one sets $\gamma = K(L)^{-1}$ and $\beta(x) = (1-x)\cdot K(L)$. Using (2.3), one sees that $X \le x$ if and only if $\beta(x) \le L + (S-L)_+$. It follows without difficulty that

$$\{X \le x\} \cap \{S > L\} = \begin{cases} \{S > \beta(x)\}, & \beta(x) > L, \\ \{S > L\}, & \beta(x) \le L, \end{cases} \tag{2.11}$$

$$\{X \le x\} \cap \{S \le L\} = \begin{cases} \varnothing, & \beta(x) > L, \\ \{S \le L\}, & \beta(x) \le L. \end{cases} \tag{2.12}$$

Inserting in (2.9), one gets

$$F_X(x) = \begin{cases} \overline{F}_S[\beta(x)], & \beta(x) > L, \\ 1, & \beta(x) \le L, \end{cases} \tag{2.13}$$

from which one derives the α-quantile expression

$$Q_X(\alpha) = \begin{cases} \dfrac{K(L)-L}{K(L)}, & Q_S(\varepsilon) \le L, \\ \dfrac{K(L)-Q_S(\varepsilon)}{K(L)}, & Q_S(\varepsilon) > L. \end{cases} \tag{2.14}$$

Similarly, one has $X > x$ if and only if $\beta(x) > L + (S-L)_+$, and one obtains that

$$(X-x)_+ \cdot I\{S > L\} = \gamma \cdot (\beta(x) - S)_+ \cdot I\{S > L\}, \tag{2.15}$$

$$(X-x)_+ \cdot I\{S \le L\} = \gamma \cdot (\beta(x) - S)_+ \cdot I\{S \le L\}. \tag{2.16}$$

If $\beta(x) \le L$, then (2.15) vanishes. Otherwise, one has

$$\begin{aligned} E&[(X-x)_+ \cdot I\{S > L\}] \\ &= \gamma \cdot E[(\beta(x) - L) \cdot I(L < S \le \beta(x))] \\ &= \gamma \cdot \{E[(\beta(x) - S) \cdot I(S \le \beta(x))] - E[(\beta(x) - S) \cdot I(S \le L)]\} \\ &= \gamma \cdot \{\overline{\pi}_S[\beta(x)] - (\beta(x) - L) \cdot F_S(L) - \overline{\pi}_S[L]\} \\ &= \gamma \cdot \{\pi_S[\beta(x)] - \pi_S[L] + (\beta(x) - L) \cdot \overline{F}_S(L)\}. \end{aligned} \tag{2.17}$$

If $\beta(x) \le L$, then (2.16) also vanishes. Otherwise, one has

$$E[(X-x)_+ \cdot I\{S \le L\}] = \gamma \cdot (\beta(x) - L) \cdot F_S(L). \tag{2.18}$$

Inserting (2.17) and (2.18) into (2.10) one obtains

$$\pi_X(x) = \begin{cases} \gamma \cdot \{\beta(x) - L + \pi_S[\beta(x)] - \pi_S[L]\}, & \beta(x) > L, \\ 0, & \beta(x) \le L. \end{cases} \tag{2.19}$$

Now, insert (2.14) into (2.19) to get

$$\pi_X[Q_X(\alpha)] = \begin{cases} \gamma \cdot \{\overline{\pi}_S[Q_S(\varepsilon)] - \overline{\pi}_S[L]\}, & \beta(x) > L, \\ 0, & \beta(x) \le L. \end{cases} \tag{2.20}$$

Finally, put (2.14) and (2.20) into (2.7), and summarize to get the desired formula. □

A remarkable feature of the portfolio insurance strategy is the *constant* amount of required *economic risk capital* as measured by value-at-risk and conditional value-at-risk as long as the loss probability is sufficiently small.

Corollary 2.2. *If $\varepsilon \le F_S(L)$, then*

$$\mathrm{CVaR}_\alpha[X] = \mathrm{VaR}_\alpha[X] = \frac{K(L) - L}{K(L)}. \tag{2.21}$$

Proof. This follows immediately from (2.8) and (2.14). □

It should be emphasized that in practice the condition of Corollary 2.2 is nearly always fulfilled. Even more, a relatively large range of confidence levels may be tolerated. For example, suppose the logarithm return $\ln(S/S_0)$ is normally distributed with mean μ and standard deviation σ. In case $L = S_0$ is "at the money," one should have $\alpha \ge \Phi(k^{-1})$, where $k = \sigma/\mu$ is the coefficient of variation and $\Phi(x)$ is the standard normal distribution. Numerically, if $\mu = 0.1$, $\sigma = 0.2$, one has $\alpha \ge \Phi(1/2) = 0.691$. An empirical study, which confirms these observations, follows in Section 3.

3. Portfolio insurance for a market index. Consider portfolio insurance for the Swiss Market Index (SMI) over the one-month period between 20/11/1998 and 18/12/1998. One has $S_0 = 7138$ and the time horizon is $T = 1/12$. Following Herbert et al. [4, page 68], the long-term logarithm return $\ln(S/S_0)$ can be assumed to be normally distributed

TABLE 3.1. Put and call option prices for the SMI.

L	6900	7000	7100	7200	7300	7400
$P(L)$	154	187	218	272	406.8	476.8
$C(L)$	370	308	253	194	152.5	114

TABLE 3.2. CVaR for portfolio insurance of SMI, $\alpha \geq 0.7067$.

Exercise price	Put option strategy		
	$100 \cdot E[R_P]$	$100 \cdot \mathrm{CVaR}_\alpha[X_P]$	$100 \cdot \mathrm{RAROC}_\alpha[X_P]$
6900	0.692	5.376	0.129
7000	0.682	4.437	0.154
7100	0.786	3.48	0.226
7200	0.666	2.834	0.235
7300	−0.441	3.245	−0.136
7400	−0.585	2.821	−0.207
Exercise price	Call option strategy		
	$100 \cdot E[R_C]$	$100 \cdot \mathrm{CVaR}_\alpha[X_C]$	$100 \cdot \mathrm{RAROC}_\alpha[X_C]$
6900	1.194	4.904	0.244
7000	1.115	4.026	0.277
7100	1.027	3.249	0.316
7200	1.086	2.428	0.447
7300	0.995	1.849	0.538
7400	0.953	1.317	0.724

with mean $\mu = (r - (1/2)\nu^2) \cdot T$ and volatility $\sigma = \nu \cdot \sqrt{T}$. According to Hürlimann [7, Table 7.2], a valid parameter estimation over the one-year period between 29/9/1998 and 24/9/1999 is $r = 0.1727$, $\nu = 0.2863$. Possible exercise prices with corresponding put and call option prices as published in newspaper from 21/11/1998 are found in Table 3.1.

One notes that the put-call parity relation (2.2) is empirically violated for all constant choices of the risk-free rate. This phenomenon is not new and well known in the literature (see, e.g., Chance [3]). For the put and call option strategies, the different empirical total costs are denoted, respectively, by

$$K_P(L) = S_0 + P(L), \qquad K_C(L) = L \cdot e^{-\delta T} + C(L). \tag{3.1}$$

The corresponding random returns and negative random returns are denoted, respectively, by R_P, X_P, R_C, X_C. The numerical percentage figures of our evaluation are summarized in Table 3.2. The risk-free rate is chosen to be $\delta = \ln(1.025)$. For $\varepsilon \leq 0.2933$, one sees that $Q_S(\varepsilon) = S_0 \cdot \exp(\mu + \Phi^{-1}(\varepsilon)\sigma) \leq 6900$, hence Corollary 2.2 applies.

The CVaR risk measure is of great importance in decision-making, because it can be used as a tool in risk-adjusted performance measurement. Consider the random return of portfolio insurance per unit of CVaR to a fixed confidence level α, called CVaR *return*

TABLE 3.3. Maximum percentage one-year expected return for constant CVaR by varying confidence level and volatility.

ε	ν				
	0.10	0.15	0.20	0.25	0.30
0.20	0	4.2	19.6	35.3	51.3
0.10	4.2	27.0	50.1	73.4	97.0
0.05	16.8	45.9	75.3	104.9	134.7
0.01	40.4	81.3	122.5	163.9	205.6

ratio, which is defined by

$$\frac{R}{\mathrm{CVaR}_\alpha[-R]}, \tag{3.2}$$

where the random return R has been defined in (2.3). The expected value of the CVaR return ratio measures the risk-adjusted return on capital. This way of computing the return is commonly called RAROC (see, e.g., Matten [13, page 59]), and is defined by

$$\mathrm{RAROC}_\alpha[R] = \frac{E[R]}{\mathrm{CVaR}_\alpha[-R]}. \tag{3.3}$$

Now, if an investor has to decide upon the more profitable of two portfolio insurance strategies with different exercise prices and random returns R_1 and R_2, a decision in favor of the first strategy is taken if and only if one has $\mathrm{RAROC}_\alpha[R_1] \geq \mathrm{RAROC}_\alpha[R_2]$ at given confidence levels α. This preference criterion tells us that a return is preferred to another if its expected value per unit of economic risk capital is greater. In Table 3.2 the exercise price $L = 7400$ for the call option strategy is preferred to the other ones under this RAROC criterion.

It is remarkable that the above results hold for all confidence levels $\alpha \geq 0.7067$. The CVaR measure remains also stable under variation of the volatility. Indeed, in our setting, one has $Q_S(\varepsilon) \leq L$ if and only if

$$r \leq \frac{1}{2}\nu^2 - \Phi^{-1}(\varepsilon) \cdot \frac{\nu}{\sqrt{T}} + \frac{1}{T} \cdot \ln\left(\frac{L}{S_0}\right). \tag{3.4}$$

Since $\Phi^{-1}(\varepsilon) \leq 0$ for $\varepsilon \leq 1/2$, the right-hand side is monotone increasing in the volatility parameter. Therefore, by fixed ε and ν, the condition of Corollary 2.2 holds provided the one-year expected return r does not exceed the value reported in Table 3.3.

4. Multivariate portfolio insurance. In general, investors do not hold a market index but a portfolio of risky assets stemming from various asset categories in different financial markets. Let $S_1, \dots, S_m$ be the initial prices of m risky assets to be held over a period of time $[0,T]$. Then $S_0 = \sum_{i=1}^m S_i$ is the present value of the portfolio of assets, and $w_i = S_i/S_0$ is the proportion of wealth invested in the ith risky asset, $i = 1, \dots, m$. Let R_i, $i = 1, \dots, m$, be the random accumulated returns over $[0,T]$. Then the random accumulated return of the portfolio choice $w = (w_1, \dots, w_m)$ is given and denoted by $R_w = \sum_{i=1}^m w_i R_i$. The random market value $S = S_0 R_w$ of the portfolio at time T satisfies the relationship $S = \sum_{i=1}^{m} X_i$, with $X_i = S_i R_i$, $i = 1,\dots,m$. To evaluate CVaR following Section 2, one needs a specification of the multivariate distribution of $R = (R_1,\dots,R_m)$ as well as option pricing formulas corresponding to the put and call random payoffs in (2.1).

Suitable multivariate distributions with arbitrary marginals are obtained through the method of copulas. However, according to Joe [10, Section 4.13, page 138], and until quite recently, it has been an open problem to construct analytically tractable parametric families of copulas that satisfy some desirable properties. Based on mixtures of independent conditional distributions and bivariate margins from a Fréchet copula, a multivariate Fréchet copula with the desired properties has been constructed in Hürlimann [5, 9]. To maintain technicalities at a minimum level, our presentation is restricted here to the bivariate case.

With the option pricing model of Black-Scholes in mind, assume the margin R_i is lognormally distributed with parameters $\mu_i = (r_i - (1/2)\nu_i^2)\cdot T$ and $\sigma_i = \nu_i \cdot \sqrt{T}$, where r_i, ν_i represent the one-year expected return and volatility, respectively. For option pricing, we use the transformed margin R_i^{δ}, which is R_i but with r_i replaced by the risk-free rate δ. Then $X_i = S_i R_i$ and $X_i^{\delta} = S_i R_i^{\delta}$ are also lognormally distributed. The corresponding distributions are denoted, respectively, by $F_i(x)$ and $F_i^{\delta}(x)$, $i = 1,2$. The bivariate distributions of the random couples (X_1,X_2) and $(X_1^{\delta},X_2^{\delta})$ are denoted and defined, respectively, by

$$F(x_1,x_2) = C\left[F_1(x_1),F_2(x_2)\right], \qquad F^{\delta}(x_1,x_2) = C\left[F_1^{\delta}(x_1),F_2^{\delta}(x_2)\right], \tag{4.1}$$

where $C(u,v)$ is the linear Spearman copula

$$\begin{gathered} C(u,v) = (1-|\theta|)\cdot C_0(u,v) + |\theta|\cdot C_{\operatorname{sgn}(\theta)}(u,v), \quad \theta \in [-1,1], \\ C_0(u,v) = uv, \quad C_1(u,v) = \min(u,v), \quad C_{-1}(u,v) = \max(u+v-1,0). \end{gathered} \tag{4.2}$$

The Spearman grade correlation coefficient and the coefficient of upper tail dependence of this copula are both equal to the dependence parameter θ. The distributions and stop-loss transforms of the dependent sums $S = X_1 + X_2$ and $S^{\delta} = X_1^{\delta} + X_2^{\delta}$ are determined by the following analytical expressions (Hürlimann [5, Theorem 8.1]). For $i = 1,2$, let $Q_i(u) = F_i^{-1}(u)$, $u \in [0,1]$, be the u-quantile of X_i, and set $u_\theta = (1/2)[1-\operatorname{sgn}(\theta)] + \operatorname{sgn}(\theta)u$. Then one has the formulas

$$F_S\left[Q_1(u)+Q_2(u_\theta)\right] = (1-|\theta|)\cdot F_{S^\perp}\left[Q_1(u)+Q_2(u_\theta)\right] + |\theta|\cdot u, \tag{4.3}$$

$$\begin{aligned} &\pi_S\left[Q_1(u)+Q_2(u_\theta)\right] \\ &\quad = (1-|\theta|)\cdot \pi_{S^\perp}\left[Q_1(u)+Q_2(u_\theta)\right] \\ &\qquad + |\theta|\cdot\left\{\pi_1\left[Q_1(u)\right] + \operatorname{sgn}(\theta)\cdot\pi_2\left[Q_2(u_\theta)\right] + \frac{1}{2}\left[1-\operatorname{sgn}(\theta)\right]\cdot\left[E[X_2]-Q_2(u_\theta)\right]\right\}, \end{aligned} \tag{4.4}$$

where $S^\perp = X_1^\perp + X_2^\perp$, with $(X_1^\perp,X_2^\perp)$ an independent version of (X_1,X_2) such that $X_1^\perp$ and $X_2^\perp$ are independent and identically distributed as X_1 and X_2. Similar expressions hold for S^{δ} with $Q_i(u)$ replaced by $Q_i^{\delta}(u) = (F_i^{\delta})^{-1}(u)$, $i = 1,2$. For portfolio insurance valuation, we use the call and put option prices

$$C(L) = e^{-\delta T} \cdot \pi_{S^\delta}(L), \qquad P(L) = e^{-\delta T} \cdot \left(L + \pi_{S^\delta}(L)\right) - S_0, \tag{4.5}$$

and CVaR follows from the representation (2.8):

$$\mathrm{CVaR}_\alpha[-R] = \frac{1}{K(L)} \left\{ K(L) - L - \left(Q_S(\varepsilon) - L\right)_+ + \frac{1}{\varepsilon} \cdot \left(Q_S(\varepsilon) + \pi_S\left[Q_S(\varepsilon)\right] - L - \pi_S[L]\right)_+ \right\}. \tag{4.6}$$

For a concrete implementation of (4.3), (4.4), (4.5), and (4.6), analytical expressions for one of density, distribution, and stop-loss transform of the independent sums $S^\perp = X_1^\perp + X_2^\perp$ and $(S^\delta)^\perp = (X_1^\delta)^\perp + (X_2^\delta)^\perp$ are required. From Johnson et al. [11, page 218], the analytical expression for the density is equal to

$$\begin{aligned} f_{S^\perp}(x) = \frac{1}{2\pi\beta_1\beta_2 x} \int_0^1 \frac{1}{t(1-t)} \exp\Bigg\{ &-\frac{1}{2}\left(\frac{\ln(1-t) + \ln(x) - \alpha_1}{\beta_1}\right)^2 \\ &-\frac{1}{2}\left(\frac{\ln(t) + \ln(x) - \alpha_2}{\beta_2}\right)^2 \Bigg\} dt, \end{aligned} \tag{4.7}$$

where the parameters are given by

$$\begin{aligned} \alpha_i &= \ln\left(S_i\right) + \left(r_i - \frac{1}{2}\nu_i^2\right) \cdot T, \\ \beta_i &= \nu_i \cdot \sqrt{T}, \quad i = 1, 2. \end{aligned} \tag{4.8}$$

Assuming finite integrals are implemented, one further obtains

$$F_{S^\perp}(x) = \int_0^x f_{S^\perp}(y)\,dy, \qquad \pi_{S^\perp}(x) = \mu - x + \int_0^x (x-y) f_{S^\perp}(y)\,dy. \tag{4.9}$$

Moreover, the stop-loss transform of the margin $X_i = S_i R_i$ reads

$$\pi_i(x) = S_i e^{r_i T} \left(1 - \Phi\left(\frac{\ln(x) - \alpha_i}{\beta_i} - \beta_i\right)\right) - x \cdot \left(1 - \Phi\left(\frac{\ln(x) - \alpha_i}{\beta_i}\right)\right), \quad i = 1, 2. \tag{4.10}$$

For illustration, we list in Tables 4.1, 4.2, and 4.3 the values of cost, CVaR, and RAROC for different exercise prices by varying the dependence parameter of the bivariate return distribution. The choice of our parameters is $\delta = \ln(1.025)$, $\nu_1 = 0.3$, $\nu_2 = 0.2$, $S_1 = S_2 = 1/2$, $T = 1$, $r_1 = \ln(1.15)$, $r_2 = \ln(1.10)$. For all $\varepsilon \le 0.2$ (or $\alpha \ge 0.8$), one has

$$Q_S(\varepsilon) \le Q_1(\varepsilon) + Q_2(\varepsilon) = \frac{1}{2} \cdot \left\{\exp\left(\mu_1 + \Phi^{-1}(\varepsilon)\sigma_1\right) + \exp\left(\mu_2 + \Phi^{-1}(\varepsilon)\sigma_2\right)\right\} \le 0.88263. \tag{4.11}$$

It follows that Corollary 2.2 applies whenever $L \ge 0.9$.

TABLE 4.1. Cost of bivariate portfolio insurance.

θ	L		
	0.9	1.0	1.1
0	1.02303	1.05962	1.11735
0.25	1.02838	1.06633	1.12397
0.5	1.03374	1.07303	1.13060
0.75	1.03909	1.07973	1.13722
1	1.04445	1.08644	1.14384

TABLE 4.2. CVaR for bivariate portfolio insurance.

θ	L		
	0.9	1.0	1.1
0	0.12026	0.05627	0.01553
0.25	0.12484	0.06220	0.02133
0.5	0.12937	0.06806	0.02706
0.75	0.13386	0.07385	0.03273
1	0.13830	0.07956	0.03833

TABLE 4.3. RAROC for bivariate portfolio insurance.

θ	L		
	0.9	1.0	1.1
0	0.20789	0.44429	1.61024
0.25	0.20028	0.40196	1.17231
0.5	0.19327	0.36740	0.92401
0.75	0.18681	0.33865	0.76409
1	0.18083	0.31436	0.65249

Applying the RAROC criterion, the exercise price $L = 1.1$ is preferred. Moreover, in accordance with the usual standards in finance, low dependence between returns is also preferred. Again, all these results hold under the weak assumption $\alpha \geq 0.8$.

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Werner Hürlimann: Aon Re and IRMG (Switzerland) Ltd., Sternengasse 21, 4010 Basel, Switzerland

E-mail address: werner.huerlimann@aon.ch