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A NOTE ON RESONANT FREQUENCIES FOR A SYSTEM OF ELASTIC WAVE EQUATIONS

LUIS A. CORTÉS-VEGA

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We present a rather simple proof of the existence of resonant frequencies for the direct scattering problem associated to a system of elastic wave equations with Dirichlet boundary condition. Our approach follows techniques similar to those in Cortés-Vega (2003). The proposed technique relies on a stationary approach of resonant frequencies, that is, the poles of the analytic continuation of the solution.

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1. Introduction. The existence of resonant frequencies (or poles) of the analytic continuation of the so-called S-matrix (which connects the asymptotic behaviors of the incident and scattered waves), associated with symmetric hyperbolic systems of first order in exterior domains and coercive boundary condition and enjoying the unique continuation property is a problem of significant interest in the time-dependent scattering theory of Lax and Phillips; these complex numbers present resonant properties for the wave motion. A good discussion of this problem and important results on the subject may be found in [21, Chapter IV] and the survey articles (see [22, Theorems 5.5 and 5.6]).

Apart from their intrinsic interest, the resonant frequencies are also relevant as a very rich source of interesting problems. For instance, in the so-called inverse problems the existence and location in the complex plane should give some information about the fashion and size of the obstacle. This kind of results for perturbations of the scalar wave equation appears in [19, 20, 25, 27] and the references therein.

Subsequently, extensive attention in this and other aspects for the direct scattering problems associated to the system of elastic waves and the scalar wave equation appears in [2, 4, 6, 7, 15, 16, 17, 25, 29, 30, 33]; see also [1, 3, 4, 5, 8, 11, 12, 13, 20, 23, 26, 27, 29, 30] for recent results.

In this context, our goal in this work is to develop a rather simple proof of the existence of resonant frequencies associated with a phenomenon described by a system of elastic waves with prescribed Dirichlet operator on the boundary $\partial\Omega \in C^2$ of an arbitrary domain $\Omega = \mathbb{R}^3/\overline{D}$:

$$\begin{gathered} b^2\boldsymbol{\Delta}\mathbf{v}(x) + (a^2-b^2)\nabla(\nabla_x\cdot\mathbf{v}(x)) + \sigma^2\mathbf{v}(x) = \mathbf{h}(x), \quad x\in\Omega, \\ \mathbf{v}(x) = \mathbf{0}, \quad x\in\partial\Omega, \end{gathered} \tag{1.1}$$

and the Kupradze-Sommerfeld radiation conditions [18]

$$
\begin{aligned}
&(\mathrm{Kup}_L)\begin{cases} \mathbf{v}^L(x) = o(1), & \text{as } |x| \longrightarrow \infty, \\ \dfrac{\partial}{\partial |x|}\mathbf{v}^L(x) - i\sigma_L \mathbf{v}^L(x) = o\left(\dfrac{1}{|x|}\right), & \text{as } |x| \longrightarrow \infty, \end{cases} \\
&(\mathrm{Kup}_T)\begin{cases} \mathbf{v}^T(x) = o(1), & \text{as } |x| \longrightarrow \infty, \\ \dfrac{\partial}{\partial |x|}\mathbf{v}^T(x) - i\sigma_T \mathbf{v}^T(x) = o\left(\dfrac{1}{|x|}\right), & \text{as } |x| \longrightarrow \infty, \end{cases}
\end{aligned}
\tag{1.2}
$$

uniformly for all directions $\hat{x} = (1/|x|)x$, where $\mathbf{v} = \mathbf{v}^L + \mathbf{v}^T$ is a sum of an irrotational (lamellar) vector $\mathbf{v}^T$ and a solenoidal vector $\mathbf{v}^L$. Here the variable $\sigma_L \in \mathbb{C}$ is the longitudinal (dilational) wave number, $\sigma_L = \sigma/b$, transverse (shear) wave number $\sigma_T = \sigma/a \in \mathbb{C}$, with $a^2 > (4/3)b^2 > 0$, and D is an open bounded region in $\mathbb{R}^3$.

In this context, a resonant frequency is a complex number σ for which system (1.1) and (1.2) with $\mathbf{h} \equiv \mathbf{0}$ has a nontrivial solution $\mathbf{v}$. Our proof follows similar lines to the arguments in [6, 7], the analysis is based on a stationary approach of resonant frequencies, that is, the poles of the analytic continuation of the solution operator. In my view, the technique combines the attributes of both simplicity and flexibility. Indeed, as pointed out in [6], this method can be used in situations not included in the time-dependent theory of Lax and Phillips [21], for instance, the impedance problem with absorbing boundary conditions [20] or acoustic resonators [11].

The linearized system equations of the time-dependent problem from which one obtains (1.1) and (1.2) are the following (a mathematical formulation of this problem in terms of semigroups of linear operators is studied in [2]):

$$
\begin{gathered}
\mathbf{v}_{tt} - b^2 \boldsymbol{\Delta}\mathbf{v} - (a^2 - b^2)\nabla(\nabla_x \cdot \mathbf{v}) = e^{i\sigma t}\mathbf{h}, \quad t \in \mathbb{R},\ x \in \Omega, \\
\mathbf{v}(x,t) = \mathbf{0}, \quad (x,t) \in \partial\Omega \times \mathbb{R}, \\
\mathbf{v}(x,0) = \mathbf{f}_0(x), \quad \mathbf{v}_t(x,0) = \mathbf{f}_1(x), \quad x \in \Omega,
\end{gathered}
\tag{1.3}
$$

where $\mathbf{v}(x,t) = (v_1(x,t), v_2(x,t), v_3(x,t))$ is the displacement at the time t and location $x \in \mathbb{R}^3$ scattered by the obstacle D, $\mathbf{f} = (\mathbf{f}_0, \mathbf{f}_1)$ is the initial value for this Cauchy problem, $\mathbf{h} = (h_1, h_2, h_3)$ is a given function, and $\sigma \in \mathbb{C}$.

In general context, the resonant frequencies associated to the model (1.3) are complex numbers, which are, in some sense, eigenfrequencies of the generator and characterize the asymptotic behavior of the solutions as time approaches infinity.

To state our main result, we introduce some notations which will be used throughout the note: let $\Omega = \mathbb{R}^3 \backslash \overline{D}$ be the exterior of $\overline{D}$ with boundary $\partial\Omega \in C^2$. Also, we denote by ∇ the gradient, by $\nabla_x \times \mathbf{v}$ the rotational vector of $\mathbf{v}$, $\nabla_x \cdot \mathbf{v}$ is the usual divergence of $\mathbf{v}$ (see, above), and

$$
\boldsymbol{\Delta}\mathbf{v} = (\triangle v_1, \triangle v_2, \triangle v_3),
\tag{1.4}
$$

where $\triangle$ is the usual Laplacian operator. For any positive integer p and $1 \le s \le \infty$, we consider the Sobolev space $W^{p,s}(\Omega)$ of (classes of) functions in $L^s(\Omega)$ which together with their derivatives up to order p belong to $L^s(\Omega)$. The norm of $W^{p,s}(\Omega)$ will be denoted by $\|\cdot\|_{p,s}$ in the case $s=2$. We write $H^p(\Omega)$ instead of $W^{p,2}(\Omega)$. If E is a vector space, then we denote $[E]^3=\oplus_{i=1}^3 E$ and the norm of a vector $\mathbf{v}$ which belongs to $[E]^3$ will be denoted by $\|\cdot\|_{[E]^3}$. $C_0^\infty(\mathbb{R}^3)$ denotes the space of all C^∞ functions defined on $\mathbb{R}^3$ with compact support. If E is a Banach space, we consider the space $B(E,E)$ of linear bounded operators in E. If $\mathbf{h}:\mathbb{R}^3\to\mathbb{R}^3$, $\mathbf{h}=(h_1,h_2,h_3)$, then we denote by $\operatorname{supp}\mathbf{h}=\cap_{i=1}^3\operatorname{supp}h_i$ the support of $\mathbf{h}$ and

$$\int_{\mathbb{R}^3}\mathbf{h}\,dx=\left(\int_{\mathbb{R}^3}h_1\,dx,\int_{\mathbb{R}^3}h_2\,dx,\int_{\mathbb{R}^3}h_3\,dx\right). \tag{1.5}$$

If $R>0$, then $B(R)$ is the ball centered at zero and of radius R. Also, we denote by $\partial\overline{B}(R)=\{x\in\mathbb{R}^3:|x|=R\}$ and by $[L_R^2(\mathbb{R}^3)]^3$ the space

$$[L_R^2(\mathbb{R}^3)]^3=\{\mathbf{v}\in[L^2(\mathbb{R}^3)]^3:\mathbf{v}=\mathbf{0},\ \text{if } |x|\geq R\}. \tag{1.6}$$

For any two vectors $\mathbf{A}$ and $\mathbf{B}$ of $\mathbb{R}^3$, we denote by $\mathbf{A}\cdot\mathbf{B}$ the usual inner product between $\mathbf{A}$ and $\mathbf{B}$. If $v:\mathbb{R}^3\to\mathbb{R}$ has partial derivatives and $x\neq 0$, then $\partial v/\partial|x|$ denotes the radial derivative of v, that is,

$$\frac{\partial v}{\partial|x|}=\frac{x}{|x|}\cdot\nabla v. \tag{1.7}$$

Now, if $\mathbf{w}:\mathbb{R}^3\to\mathbb{R}^3$ is such that each component has partial derivatives, then

$$\begin{gathered}\frac{\partial\mathbf{w}}{\partial|x|}=\left(\frac{\partial w_1}{\partial|x|},\frac{\partial w_2}{\partial|x|},\frac{\partial w_3}{\partial|x|}\right),\\ \nabla_x\cdot\mathbf{w}=\frac{\partial w_1}{\partial x_1}+\frac{\partial w_2}{\partial x_2}+\frac{\partial w_3}{\partial x_3},\\ \nabla_x\times\mathbf{w}=\left(\frac{\partial w_3}{\partial x_2}-\frac{\partial w_2}{\partial x_3},\frac{\partial w_1}{\partial x_3}-\frac{\partial w_3}{\partial x_1},\frac{\partial w_2}{\partial x_1}-\frac{\partial w_1}{\partial x_2}\right).\end{gathered} \tag{1.8}$$

Outline of the work. In Section 2, we present the formulation of the main result. Section 3 contains the proof of the main theorem. Finally, in Section 4, we present the meromorphic extension of the solution for every $\sigma\in\mathbb{C}$ with $\Im(\sigma)\leq 0$. With the notations above, we establish our main theorem.

2. Formulation of result.

In this section, we will establish the existence and uniqueness of the solution to a system of elastic waves that is presented in (1.1) and by the radiation conditions (Kup_L) and (Kup_T) in (1.2).

This will be done based on [6, 7]. Our starting point is the following lemma whose proof appears in [6, 18].

Lemma 2.1. *Let $\sigma\in C$ with $\Im(\sigma)>0$ and take $v\in[H^2(\mathbb{R}^3)]^3$ the solution of system*

$$b^2\Delta\mathbf{v}(x)+(a^2-b^2)\nabla(\nabla_x\cdot\mathbf{v}(x))+\sigma^2\mathbf{v}(x)=\mathbf{0},\quad x\in\mathbb{R}^3, \tag{2.1}$$

satisfying the Kupradze-Sommerfeld radiation condition for $a^2>(4/3)b^2>0$. Then

$$\int_{|x|=R}\overline{\mathbf{v}}\cdot\mathbf{T}_{\hat{x}}\mathbf{v}\,ds=0,\quad \text{as } R\longrightarrow\infty, \tag{2.2}$$

where "$\cdot$" is the inner product in $\mathbb{R}^3$ and $\mathbf{T}_{\hat{x}}$ is the stress vector calculated on the surface element

$$\mathbf{T}_{\hat{x}}\mathbf{v} = 2b^2 \frac{\partial \mathbf{v}}{\partial |x|} + (a^2 - 2b^2)\hat{x}(\nabla \cdot \mathbf{v}) + b^2 \hat{x} \times (\nabla \times \mathbf{v}). \tag{2.3}$$

Lemma 2.2. *Let $\sigma \in \mathbb{C}$ with $\Im(\sigma) > 0$. Then, for any $\mathbf{g} \in [L^2_R(\mathbb{R}^3)]^3$, the system*

$$b^2 \mathbf{\Delta v}(x) + (a^2 - b^2)\nabla(\nabla_x \cdot \mathbf{v}(x)) + \sigma^2 \mathbf{v}(x) = \mathbf{g}(x), \quad x \in \mathbb{R}^3, \tag{2.4}$$

admits a solution $v \in [H^2(\mathbb{R}^3)]^3$ and $v = A(\sigma)g$, where

$$\mathbf{A}(\sigma) : [L^2_R(\mathbb{R}^3)]^3 \longrightarrow [H^2(\mathbb{R}^3)]^3 \tag{2.5}$$

is a linear continuous operator. In particular, if v_1 and v_2 solve (2.4) and satisfy the Kupradze-Sommerfeld radiation condition, then $v_1(x) = v_2(x)$ for all $x \in \mathbb{R}^3$. See [6] for the proof.

Let $\mathbf{f} \in [L^2(\Omega)]^3$ and take $\mathbf{f}_0$ given by

$$\mathbf{f}_0(x) = \begin{cases} \psi(x)\mathbf{f}(x) & \text{if } x \in \Omega, \\ 0 & \text{if } x \in \overline{D}, \end{cases} \tag{2.6}$$

where ψ is the function

$$\psi(x) = \begin{cases} 1 & \text{if } x \in \Omega_R, \\ 0 & \text{if } x \notin \Omega_R, \end{cases} \tag{2.7}$$

and $\Omega_R = \{x \in \Omega : |x| < R\}$.

Lemma 2.3. *Let $\mathbf{g} \in [H^{1/2}(\partial\Omega)]^3$ and suppose $\partial\Omega \in C^2$. Then the system of elastic waves*

$$\begin{gathered} b^2 \mathbf{\Delta w}(x) + (a^2 - b^2)\nabla_x(\nabla_x \cdot \mathbf{w}(x)) = \mathbf{0}, \quad x \in \Omega_R, \\ \mathbf{w}(x) = \mathbf{g}(x), \quad x \in \partial\Omega, \\ \mathbf{w}(x) = \mathbf{0}, \quad x \in \partial\overline{B}(R), \end{gathered} \tag{2.8}$$

has a (unique) solution on $[H^2(\Omega_R)]^3$. See [6] for the proof. For future reference the well-known result given, for example, in [24] is also needed.

Lemma 2.4. *Let $\mathbf{w} \in [H^2(B(R))]^3$ be a solution of the system*

$$\begin{gathered} b^2 \mathbf{\Delta w}(x) + (a^2 - b^2)\nabla(\nabla_x \cdot \mathbf{w}(x)) = \mathbf{0}, \quad x \in B(R), \\ \mathbf{w}(x) = \mathbf{0}, \quad x \in \partial\overline{B}(R). \end{gathered} \tag{2.9}$$

Then $\mathbf{w}(x) = 0$ for every $x \in B(R)$. See [24] or [6] for the proof.

At this point, we derive from the above lemmas the proof of the main theorem.

Theorem 2.5. *Let $\sigma \in \mathbb{C}$ with $\Im(\sigma) > 0$. Then, for any $\mathbf{h} \in [L^2(\Omega)]^3$ with $\operatorname{supp}\mathbf{h} \subset \Omega_R$, the system of elastic waves*

$$\begin{gathered} b^2\boldsymbol{\Delta}\mathbf{v}(x) + (a^2 - b^2)\nabla_x(\nabla_x \cdot \mathbf{v}(x)) + \sigma^2\mathbf{v}(x) = \mathbf{h}(x), \quad x \in \Omega, \\ \mathbf{v}(x) = \mathbf{0}, \quad x \in \partial\Omega, \end{gathered} \tag{2.10}$$

with radiation conditions (Kup_L) *and* (Kup_T)*, has a unique solution $v \in [H^2(\Omega)]^3$. Furthermore, v can be extended in a meromorphic way to $\sigma \in C$ with $\Im(\sigma) \le 0$ except for some countable number of poles (resonant frequencies) in $\Xi = \{\sigma \in \mathbb{C} : \Im(\sigma) \le 0\}$.*

3. Proof of Theorem 2.5.

The proof of Theorem 2.5 is divided into two steps.

Step 3.1 (uniqueness)**.** The uniqueness of the only solution to (2.10) can be established by a standard argument. For the precise details we refer to the appendix.

Step 3.2 (existence)**.** Here, we study the existence of solutions for the system (2.10); to this end, we assume that $\partial\Omega \in C^2$ for the use of the Betti-Green formula. Let $R > 0$ and $R_0 > 0$ be such that $B(R_0) \subset D$, $\partial\Omega \subset B(R)$. We start with an arbitrary function $\zeta \in C_0^\infty(\mathbb{R}^3)$ satisfying

($\zeta 1$) $\operatorname{supp}\zeta \subset B(R)/B(R_0)$,

($\zeta 2$) $\zeta = 1$ in a neighborhood of $\partial\Omega$.

In order to analyze our existence problem, we introduce here the following function:

$$\mathbf{v}(x) = \mathbf{v}_0(x) + \zeta(x)\tilde{\mathbf{u}}(x), \quad x \in \mathbb{R}^3, \tag{3.1}$$

where $\tilde{\mathbf{u}} \in [H^2(\mathbb{R}^3)]^3$ is the Calderón extension to $\mathbb{R}^3$ of a solution $\mathbf{w} \in [H^2(\Omega_R)]^3$ of the system (see Lemma 2.3)

(w1) $b^2\boldsymbol{\Delta}\mathbf{w}(x) + (a^2 - b^2)\nabla(\nabla_x \cdot \mathbf{w}(x)) = \mathbf{0}$, $x \in \Omega_R$,

(w2) $\mathbf{w}(x) = \mathbf{g}(x)$, $x \in \partial\Omega$,

(w3) $\mathbf{w}(x) = \mathbf{0}$, $x \in \partial\overline{B}(R)$.

Here, $\mathbf{g} = -\mathbf{v}_0 \in [H^{1/2}(\partial\Omega)]^3$ and $\mathbf{v}_0$ satisfies (see Lemma 2.2) the system

$$b^2\boldsymbol{\Delta}\mathbf{v}_0(x) + (a^2 - b^2)\nabla(\nabla_x \cdot \mathbf{v}_0(x)) + \sigma^2\mathbf{v}_0(x) = \mathbf{f}_0(x) \quad \text{on } \mathbb{R}^3, \tag{3.2}$$

and the Kupradze-Sommerfeld radiation conditions. From (3.1) and (w2) we obtain

$$\mathbf{v}(x) = \mathbf{0}, \quad x \in \partial\Omega. \tag{3.3}$$

Furthermore, it is easy to see from ($\zeta 1$) and (3.1) that $\mathbf{v}(x) = \mathbf{v}_0(x)$, for every $x \in \mathbb{R}^3/\overline{B}(R)$. Now, the function $\mathbf{v}_0$ satisfies the Kupradze-Sommerfeld radiation conditions (Kup_L) and (Kup_T). In view of this, the function $\mathbf{v}$ has this property. Thus, for any $\mathbf{h} \in [L^2(\Omega)]^3$ with $\operatorname{supp}\mathbf{h} \subset \Omega_R$ and $\sigma \in \mathbb{C}$ with $\Im(\sigma) > 0$, the function

$$\mathbf{v}(x) = \mathbf{v}_0(x) + \zeta(x)\tilde{\mathbf{u}}(x), \quad x \in \mathbb{R}^3, \tag{3.4}$$

will be a solution of the system (2.10) if and only if, for every $x \in \Omega$, we obtain

$$\begin{aligned}\mathbf{h}(x) &= b^2\boldsymbol{\Delta}\mathbf{v}(x) + (a^2-b^2)\nabla(\nabla_x \cdot \mathbf{v}(x)) + \sigma^2\mathbf{v}(x)\\ &= \mathbf{f}_0(x) + b^2\boldsymbol{\Delta}\zeta(x)\tilde{\mathbf{u}}(x) + (a^2-b^2)\nabla(\nabla_x \cdot \tilde{\mathbf{u}}(x)\zeta(x)) + \sigma^2\zeta(x)\tilde{\mathbf{u}}(x).\end{aligned} \tag{3.5}$$

It is simple to see from (ζ1), (ζ2), (2.6), and (2.7) that (3.5) is valid on the set $\Omega^R = \{x \in \Omega : |x| \geq R\}$, since $\operatorname{supp}\mathbf{h} \subset \Omega_R$. Thus,

$$\mathbf{v}(x) = \mathbf{v}_0(x) + \zeta(x)\tilde{\mathbf{u}}(x), \quad x \in \mathbb{R}^3, \tag{3.6}$$

will be solution of the system (2.10) if and only if, for every $x \in \boldsymbol{\Omega}_R$, we have

$$\mathbf{h}(x) = \mathbf{f}(x) + b^2\boldsymbol{\Delta}\zeta(x)\mathbf{w}(x) + (a^2-b^2)\nabla(\nabla_x \cdot \mathbf{w}(x)\zeta(x)) + \sigma^2\zeta(x)\mathbf{w}(x). \tag{3.7}$$

Applying to $\nabla_x \cdot \mathbf{w}$ the operator ∇ on Ω_R we find

$$\nabla_x \times (\nabla_x \times \mathbf{w}(x)) = -\boldsymbol{\Delta}\mathbf{w}(x) + \nabla(\nabla_x \cdot \mathbf{w}(x)). \tag{3.8}$$

Now, $\mathbf{w}$ on Ω_R solves

$$b^2\boldsymbol{\Delta}\mathbf{w}(x) + (a^2-b^2)\nabla(\nabla_x \cdot \mathbf{w}(x)) = \mathbf{0}. \tag{3.9}$$

Therefore, the ansatz (3.5) takes the form

$$\mathbf{h} = \mathbf{f} + \mathbf{G}_\zeta(\sigma)\mathbf{w}, \tag{3.10}$$

where $\mathbf{G}_\zeta(\sigma)$ is a continuous linear operator

$$\mathbf{G}_\zeta(\sigma) : [H^2(\Omega_R)]^3 \longrightarrow [H^1(\Omega_R)]^3 \tag{3.11}$$

given by the formula

$$\begin{aligned}\mathbf{G}_\zeta(\sigma)\mathbf{w} = (a^2+b^2)&[(\nabla\zeta \cdot \nabla)\mathbf{w}] + [b^2\triangle\zeta + \sigma^2\zeta]\mathbf{w}\\ &+ (a^2-b^2)[(\mathbf{w}\cdot\nabla)\nabla\zeta + \nabla\zeta \times (\nabla_x \times \mathbf{w}) + \nabla\zeta(\nabla_x \cdot \mathbf{w})].\end{aligned} \tag{3.12}$$

On the other hand, the solution operator $P(\sigma)$ associated with the system (w1), (w2), and (w3), that is, $P(\sigma)\mathbf{g} = \mathbf{w}$, where $\mathbf{g} = -\mathbf{v}_0 \in [H^{1/2}(\partial\Omega)]^3$, is well defined, of course; $P(\sigma)$ is a continuous linear operator

$$P(\sigma) : [H^{1/2}(\partial\Omega)]^3 \longrightarrow [H^2(\Omega_R)]^3. \tag{3.13}$$

In a similar fashion, the trace

$$\Lambda_n : [H^2(\Omega_R)]^3 \longrightarrow [H^{1/2}(\partial\Omega)]^3, \tag{3.14}$$

where $\Lambda_n\mathbf{v}_0 = \mathbf{g}$ is a continuous linear operator. Thus, with this operator and taking into account the fact that $\mathbf{v}_0 = \mathbf{v}_{0|\Omega_R}$ on Ω_R, (3.10) can be written in the form

$$\mathbf{h} = \mathbf{f} - \mathbf{G}_\zeta(\sigma)P(\sigma)\Lambda_n F_R(\sigma)\widetilde{A}(\sigma)\mathbf{f}, \tag{3.15}$$

where $F_R(\sigma)\mathbf{v}_0 = \mathbf{v}_{0|\Omega_R}$,

$$F_R(\sigma) : [H^2(\mathbb{R}^3)]^3 \longrightarrow [H^2(\Omega_R)]^3 \tag{3.16}$$

is a restrictive, continuous linear operator. Also,

$$\widetilde{A}(\sigma) : [L^2(\Omega_R)]^3 \longrightarrow [H^2(\mathbb{R}^3)]^3 \tag{3.17}$$

is a continuous linear operator given by the composition

$$\widetilde{A}(\sigma)\mathbf{r} = \mathbf{A}(\sigma)M_\psi\mathbf{r}, \tag{3.18}$$

where $A(\sigma)$ is the solution operator of the system

$$b^2\Delta\mathbf{v}_0(x) + (a^2 - b^2)\nabla(\nabla_x \cdot \mathbf{v}_0(x)) + \sigma^2\mathbf{v}_0(x) = \mathbf{f}_0(x), \quad x \in \mathbb{R}^3 \tag{3.19}$$

(see Lemma 2.2), and M_ψ is the multiplication operator

$$(M_\psi\mathbf{r})(x) = \begin{cases} \mathbf{r}(x) & \text{if } x \in \Omega_R, \\ 0 & \text{if } x \notin \Omega_R. \end{cases} \tag{3.20}$$

Note that

$$||M_\psi\mathbf{r}||^2_{[L^2(\mathbb{R}^3)]^3} = \|\psi\mathbf{r}\|^2_{[L^2(\mathbb{R}^3)]^3} = \int_{\mathbb{R}^3} |\psi|^2\|\mathbf{r}\|^2 dx = \int_{\Omega_R} \|\mathbf{r}\|^2 dx < \infty. \tag{3.21}$$

Therefore, M_ψ is a continuous linear operator

$$M_\psi : [L^2(\Omega_R)]^3 \longrightarrow [L^2_R(\mathbb{R}^3)]^3, \tag{3.22}$$

since $M_\psi\mathbf{r} = 0$ if $|x| \geq R$. Thus, $M_\psi\mathbf{r} \in [L^2_R(\mathbb{R}^3)]^3$ for every $\mathbf{r} \in [L^2(\Omega_R)]^3$. Let $B_\zeta(\sigma)$ be the operator defined by

$$B_\zeta(\sigma)\mathbf{f} = -\mathbf{G}_\zeta(\sigma)P(\sigma)\Lambda_n F_R(\sigma)\widetilde{A}(\sigma)\mathbf{f}. \tag{3.23}$$

Thus, (3.17) can be written as

$$\mathbf{h} = \mathbf{f} + B_\zeta(\sigma)\mathbf{f}. \tag{3.24}$$

From these considerations we see that the theorem will be proved if

(I) the set of operators $\{B_\zeta(\sigma)\}$, $\sigma \in \mathbb{C}$, with $\Im(\sigma) > 0$, given in (3.23) is a family of compact operators of $[L^2(\Omega_R)]^3$ onto itself, and the homogeneous equation

$$\mathbf{f} + B_\zeta(\sigma)\mathbf{f} = \mathbf{0} \tag{3.25}$$

has only the trivial solution.

Proof (I). We denote by $S_{v_0} \subset [H^2(\Omega_R)]^3$ the space ($\mathbf{g} = -\mathbf{v}_0 \in [H^{1/2}(\partial\Omega)]^3$) of solutions of the system

$$\begin{gathered} b^2\boldsymbol{\Delta}\mathbf{w}(x) + (a^2 - b^2)\nabla(\nabla_x \cdot \mathbf{w}(x)) = \mathbf{0}, \quad x \in \Omega_R, \\ \mathbf{w}(x) = \mathbf{g}(x), \quad x \in \partial\Omega, \\ \mathbf{w}(x) = \mathbf{0}, \quad x \in \partial\overline{B}(R). \end{gathered} \tag{3.26}$$

Now, note that

$$\begin{gathered} \mathbf{G}_\zeta(\sigma) : S_{\mathbf{v}_0} \subset [H^2(\Omega_R)]^3 \longrightarrow [H^1(\Omega_R)]^3, \\ P(\sigma) : [H^{1/2}(\partial\Omega)]^3 \longrightarrow S_{\mathbf{v}_0} \subset [H^2(\Omega_R)]^3, \\ \Lambda_n : [H^2(\Omega_R)]^3 \longrightarrow [H^{1/2}(\partial\Omega)]^3, \\ F_R(\sigma) : [H^2(\mathbb{R}^3)]^3 \longrightarrow [H^2(\Omega_R)]^3, \\ \widetilde{A}(\sigma) : [L^2(\Omega_R)]^3 \longrightarrow [H^2(\mathbb{R}^3)]^3 \end{gathered} \tag{3.27}$$

are continuous applications. Therefore, item (I) is a simple consequence of (3.23) and of the compactness of $\mathbf{i} : [H^1(\Omega_R)]^3 \to [L^2(\Omega_R)]^3$. See the operators in the following diagram:

$$\begin{tikzcd} {[L^2(\Omega_R)]^3} \arrow[r, "A(\sigma)"] & {[H^2(\mathbb{R}^3)]^3} \arrow[r, "F_R(\sigma)"] \arrow[d, phantom, "B_\zeta(\sigma)\,\circlearrowleft"] & {[H^2(\Omega_R)]^3} \arrow[d, "\Lambda_n"] \\ {[H^1(\Omega_R)]^3} \arrow[u, "i"] & {[H^2(\Omega_R)]^3 \supset S_{\mathbf{v}_0}} \arrow[l, "G_\zeta(\sigma)"'] & {[H^{1/2}(\partial\Omega)]^3} \arrow[l, "P(\sigma)"'] \end{tikzcd} \tag{3.28}$$

$\square$

We are now ready to prove (II).

Proof (II). Take $\mathbf{f} \in [L^2(\Omega_R)]^3$ such that

$$\mathbf{f} + B_\zeta(\sigma)\mathbf{f} = \mathbf{0}. \tag{3.29}$$

Then, (3.24) yields $\mathbf{h} = 0$. Therefore, the function $\mathbf{v}$ is a solution of the homogeneous system

$$\begin{gathered} b^2\boldsymbol{\Delta}\mathbf{v}(x) + (a^2 - b^2)\nabla(\nabla_x \cdot \mathbf{v}(x)) + \sigma^2\mathbf{v}(x) = \mathbf{0}, \quad x \in \Omega, \\ \mathbf{v}(x) = \mathbf{0}, \quad x \in \partial\Omega, \end{gathered} \tag{3.30}$$

with radiation conditions (Kup_L) and (Kup_T). This implies that $\mathbf{v} = 0$ on Ω (see uniqueness in the appendix). Hence, in particular, we obtain

$$-\zeta\tilde{\mathbf{u}} = \mathbf{v}_0, \quad \text{on } \Omega. \tag{3.31}$$

Now, from (3.31) we can conclude that

$$\mathbf{v}_0 = 0 \quad \text{on } \Omega^R = \{x \in \mathbb{R}^3 : |x| \geq R\}, \tag{3.32}$$

since $\operatorname{supp}\zeta \subset B(R)/B(R_0)$. Moreover, $\zeta = 0$ on $\partial\overline{B}(R)$ implies $\mathbf{v}_0 = \mathbf{0}$ on $\partial\overline{B}(R)$. We now introduce on $\overline{B}(R)$ the following function:

$$\vartheta(x) = \chi(x)\mathbf{v}_0(x) + (1-\chi(x))\tilde{\mathbf{u}}(x), \tag{3.33}$$

where

$$\chi(x) = \begin{cases} 1 & \text{if } x \in \overline{D}, \\ 0 & \text{if } x \in \Omega_R \cup \partial\overline{B}(R). \end{cases} \tag{3.34}$$

We note that $\vartheta \in [H^2(B(R))]^3$. Furthermore,

$$b^2\boldsymbol{\Delta}\vartheta(x) + (a^2-b^2)\nabla(\nabla_x\cdot\vartheta(x)) = -\sigma^2\chi(x)\mathbf{v}_0(x) \quad \text{on } B(R). \tag{3.35}$$

Note also that $\vartheta(x) = 0$ on $\partial\overline{B}(R)$, because $\mathbf{v}_0(x) = \tilde{\mathbf{u}}(x) = 0$ on $\partial\overline{B}(R)$. Now, using the Betti-Green formula on $B(R)$, we obtain

$$\int_{B(R)} \overline{\vartheta}\cdot\tilde{\boldsymbol{\Delta}}\vartheta\,dx + \int_{B(R)} e(\overline{\vartheta},\vartheta)dx = \int_{\partial\overline{B}(R)} \overline{\vartheta}\cdot\mathbf{T}_n\vartheta\,ds = 0. \tag{3.36}$$

So

$$\int_{B(R)} e(\overline{\vartheta},\vartheta)dx = \sigma^2\int_{B(R)}\chi(x)\|\mathbf{v}_0\|^2dx. \tag{3.37}$$

This yield

$$0 = 2\Re(\sigma)\Im(\sigma)\int_{B(R)}\chi(x)\|\mathbf{v}_0\|^2dx, \tag{3.38}$$

$$\int_{B(R)} e(\overline{\vartheta},\vartheta)dx = [\Re(\sigma)^2-\Im(\sigma)^2]\int_{B(R)}\chi(x)\|\mathbf{v}_0\|^2dx. \tag{3.39}$$

Now, $\Re(\sigma) = 0$, $\sigma\in\mathbb{C}$, with $\Im(\sigma) > 0$ and the formula (3.39), implies that $\mathbf{v}_0 = 0$, on $\overline{D}$, since $\int_{B(R)} e(\overline{\vartheta},\vartheta)dx \geq 0$. Also, $\Re(\sigma) \neq 0$, $\sigma\in\mathbb{C}$, with $\Im(\sigma) > 0$ and (3.38) yields $\mathbf{v}_0 = 0$, on $\overline{D}$. Therefore, for any $\sigma\in\mathbb{C}$ with $\Im(\sigma) > 0$, the function $\vartheta\in[H^2(B(R))]^3$ in the ansatz (3.33) solves the system

$$\begin{gathered} b^2\boldsymbol{\Delta}\vartheta(x) + (a^2-b^2)\nabla(\nabla_x\cdot\vartheta(x)) = \mathbf{0}, \quad x\in B(R), \\ \boldsymbol{\vartheta}(x) = \mathbf{0}, \quad x\in\partial\overline{B}(R). \end{gathered} \tag{3.40}$$

Now, thanks to Lemma 2.4, we obtain $\boldsymbol{\vartheta}(x) = 0$, for any $x\in B(R)$, that is, $\tilde{\mathbf{u}}(x) = 0$, on Ω_R. From this together with

$$-\zeta(x)\tilde{\mathbf{u}}(x) = \mathbf{v}_0(x), \quad x\in\Omega_R\subset\Omega, \tag{3.41}$$

we get

$$0 = b^2\boldsymbol{\Delta}\mathbf{v}_0(x) + (a^2-b^2)\nabla(\nabla_x\cdot\mathbf{v}_0(x)) + \sigma^2\mathbf{v}_0(x) = \mathbf{f}(x), \quad x\in\Omega_R. \tag{3.42}$$

Now, from the Fredholm theory, the equation

$$\mathbf{f} + B_\zeta(\sigma)\mathbf{f} = \mathbf{h} \tag{3.43}$$

is uniquely solvable and the proof is finished. □

4. Meromorphic extension. In the previous sections and the appendix, the existence and uniqueness of the solution for the system that is presented in (1.1) and by the radiation conditions (Kup_L) and (Kup_T) in (1.2) with $\sigma \in \mathbb{C}$ with $\Im(\sigma) > 0$ is proved. Now, the goal of this section is to present the extension of the solution for all $\sigma \in \mathbb{C}$ such except for some countable number of complex singularities, called "resonant frequencies." Our approach follows the main ideas of the previous sections and the subject initiated in [6, 7], but it is related to some other works, mainly [1, 3, 2, 4, 8, 9, 11, 25]. The basic tool for the proof is the Steinberg theorem [31] about families of compact operators depending on a complex parameter (see also [28]). With the same notations of Sections 2 and 3, we establish the following.

Lemma 4.1. *Let $\sigma \in \mathbb{C}$ with $\Im(\sigma) > 0$. Fix $\zeta \in C_0^\infty(\mathbb{R}^3)$, with properties $(\zeta 1)$, $(\zeta 2)$ (see Section 3). Then for any $\mathbf{h} \in [L^2(\Omega)]^3$ such that $\operatorname{supp}\mathbf{h} \subset \Omega_R$, the function*

$$\mathbf{v}(x) = \mathbf{v}_0(x) + \zeta(x)\tilde{\mathbf{u}}(x), \quad x \in \mathbb{R}^3, \tag{4.1}$$

solves the system (1.1) and (1.2) if only if $\mathbf{f} \in [L^2(\Omega_R)]^3$ solves

$$\mathbf{h} = \mathbf{f} + B_\zeta(\sigma)\mathbf{f}. \tag{4.2}$$

Here, $B_\zeta(\sigma)$ is given by (3.23) where the operators

$$\mathbf{G}_\zeta(\sigma),\ P(\sigma),\ \Lambda_n,\ F_R(\sigma), \widetilde{A}(\sigma) \tag{4.3}$$

are given in (3.13), (3.14), (3.15), (3.16), and (3.17), respectively.

Proof. The proof is implicit in Theorem 2.5. □

Lemma 4.2. *The set operators $\{B_\zeta(\sigma)\}$, $\sigma \in \mathbb{C}$, with $\Im(\sigma) > 0$, given in (3.23), is an analytic family of compact operators of $[L^2(\Omega_R)]^3$ onto itself.*

Proof. Since the solution $\mathbf{v}_0$ from system

$$b^2\Delta\mathbf{v}_0(x) + (a^2 - b^2)\nabla(\nabla_x \cdot \mathbf{v}_0(x)) + \sigma^2\mathbf{v}_0(x) = \mathbf{f}(x), \quad x \in \Omega_R, \tag{4.4}$$

depends analytically on $\sigma \in \mathbb{C}$ with $\Im(\sigma) > 0$, the operators $\mathbf{G}_\zeta(\sigma)$, $P(\sigma)$, Λ_n, $F_R(\sigma)$, $\widetilde{A}(\sigma)$ given in (3.13), (3.14), (3.15), (3.16), and (3.17) have this property. From this and (3.23), the operators $\{B_\zeta(\sigma)\}$ depend analytically on $\sigma \in \mathbb{C}$. The compactness follows from (I). □

Theorem 4.3. *The inverse operators $[\mathbf{I} + B_\zeta(\sigma)]^{-1}$ have an analytic extension from $\Im(\sigma) > 0$ to all the complex plane except for a countable set of poles, called resonant frequencies. Furthermore, σ is a resonant frequency of the operator $[\mathbf{I} + B_\zeta(\sigma)]^{-1}$ if and only if the system (1.1) and (1.2) with $\mathbf{h} = 0$ has nonzero solutions.*

Proof. From Lemma 4.2, we have that the set $\{B_\zeta(\sigma)\}$ with $\sigma \in \mathbb{C}$ and $\Im(\sigma) > 0$ is an analytic family of compact operators of $[L^2(\Omega_R)]^3$ onto itself. By the Steinberg theorem [31], either (a) the operators $[\mathbf{I}+B_\zeta(\sigma)]^{-1}$ are never invertible for $\sigma \in \mathbb{C}$, or (b) there is $\sigma_0 \in \mathbb{C}$ such that the operator

$$[\mathbf{I}+B_\zeta(\sigma_0)]^{-1} \tag{4.5}$$

is invertible. From Theorem 2.5 we have the existence and uniqueness of the solution for the system (1.1) and (1.2) for all $\sigma \in \mathbb{C}$ with $\Im(\sigma) > 0$; by the equivalence established in Lemma 4.2 we are in case (b). In this case, Steinberg's theorem also states that

$$[\mathbf{I}+B_\zeta(\sigma)]^{-1} \tag{4.6}$$

is defined analytically on $\mathbb{C}$ except for a countable set of poles. Now, Lemma 4.2 yields the equivalence statement. □

Final remark. It can be thought that there is a reasonable parallelism between my former paper [7] and this note, but it is necessary to remark that a complete parallelism does not hold if we consider the imposed boundary conditions in both problems. Indeed, in [7] we studied the system of elastic waves with the Neumann boundary condition, whereas the condition that we impose here is of Dirichlet type. The results obtained in [7] and those of this work are analogous (which is a virtue of the technique). However, the models are different, because it is a known fact that in the Neumann case an interesting phenomenon related to the existence of surface waves exists (Rayleigh surface waves, who mathematically predicted the existence of this kind of waves in 1885) ; such waves remain near the border of the obstacle; this fact too stimulated the interest of many researchers in the influence of surface waves near scattering objects; see [32, 33]. A first implication of this fact is that the uniform decay (in the local energy) of the solution is not preserved. As was already proved in [16, 17], for the case of the isotropic elastic wave with Neumann boundary condition, the solution does not have uniform local energy decay. Important contributions in this direction appear in [29, 30] and the references therein. In contrast to the Neumann case, it is a well-known fact that for the system studied here the phenomenon of Rayleigh is absent, and as a consequence, the uniform decay of the solutions may be guaranteed; see, for instance, [14, 15]. Thus, independent of the differences in both models, the method presented here is still successful.

Appendix. In this appendix, we prove the uniqueness of the solution to the system that is presented in (1.1) and by the radiation conditions (Kup_L) and (Kup_T) in (1.2).

Proof of Uniqueness. Let $\mathbf{v}$ be the difference between two solutions $\mathbf{v}_1$ and $\mathbf{v}_2$ of (1.1) and (1.2), then $\mathbf{v}$ satisfies (1.1) and (1.2) with $\mathbf{h} = 0$. Now, let $R > 0$ be such that $\partial\overline{B}(R)$ is contained in Ω and denoted by $\overline{\Omega}_R = \{x \in \Omega : |x| \le R\}$; the Betti-Green formula (see, e.g., [18] or [10]) yields

$$\int_{\Omega_R} \overline{\mathbf{v}} \cdot \tilde{\boldsymbol{\Delta}} \mathbf{v}\, dx + \int_{\Omega_R} e(\overline{\mathbf{v}}, \mathbf{v})\, dx = \int_{\partial\Omega_R} \overline{\mathbf{v}} \cdot \mathbf{T}_n \mathbf{v}\, ds, \tag{A.1}$$

where

$$\begin{aligned} e(\overline{\mathbf{v}},\mathbf{v}) = \frac{3a^2-4b^2}{3}\left|\nabla_x\cdot v\right|^2 + \frac{b^2}{2}\sum_{p\neq q}\left|\frac{\partial v_p}{\partial x_q}+\frac{\partial v_q}{\partial x_p}\right|^2 \\ + \frac{b^2}{3}\sum_{p,q=1}^{3}\left|\frac{\partial v_p}{\partial x_p}-\frac{\partial v_q}{\partial x_q}\right|^2, \end{aligned} \tag{A.2}$$

$$\tilde{\mathbf{\Delta}}\mathbf{v} = b^2\mathbf{\Delta}\mathbf{v} + (a^2-b^2)\nabla(\nabla_x\cdot\mathbf{v}). \tag{A.3}$$

Recall that $\tilde{\mathbf{\Delta}}\mathbf{v} = -\sigma^2\mathbf{v}$ on $\Omega_R \subset \Omega$ and $\partial\overline{\Omega}_R = \partial\overline{B}(R)\cup\partial\Omega$. A direct calculation shows that

$$-\sigma^2\int_{\Omega_R}\|\mathbf{v}\|^2dx + \int_{\Omega_R}e(\overline{\mathbf{v}},\mathbf{v})dx = \int_{\partial\overline{B}(R)}\overline{\mathbf{v}}\cdot\mathbf{T}_n\mathbf{v}\,ds + \int_{\partial\Omega}\overline{\mathbf{v}}\cdot\mathbf{T}_n\mathbf{v}\,ds. \tag{A.4}$$

Now, using (A.4) together with Lemma 2.1, the homogeneous boundary condition, and passing to the limit as $R\to\infty$, we get

$$\int_{\Omega}e(\overline{\mathbf{v}},\mathbf{v})dx = \sigma^2\int_{\Omega}\|\mathbf{v}\|^2dx, \tag{A.5}$$

so

$$\int_{\Omega}e(\overline{\mathbf{v}},\mathbf{v})dx = \left[\left(\Re(\sigma)^2-\Im(\sigma)^2\right)+2i\Re(\sigma)\Im(\sigma)\right]\int_{\Omega}\|\mathbf{v}\|^2dx. \tag{A.6}$$

From (A.6) we have that

$$0 = 2\Re(\sigma)\Im(\sigma)\int_{\Omega}\|\mathbf{v}\|^2dx, \tag{A.7}$$

$$\int_{\Omega}e(\overline{\mathbf{v}},\mathbf{v})dx = \left[\Re(\sigma)^2-\Im(\sigma)^2\right]\int_{\Omega}\|\mathbf{v}\|^2dx. \tag{A.8}$$

Thus, we have the following two possibilities.

(a) If $\Re(\sigma)=0$, from (A.8) we get

$$\int_{\Omega}e(\overline{\mathbf{v}},\mathbf{v})dx = -\Im(\sigma)^2\int_{\Omega}\|\mathbf{v}\|^2dx. \tag{A.9}$$

With the formula above, $\Im(\sigma)>0$, and

$$\int_{\Omega}e(\overline{\mathbf{v}},\mathbf{v})dx \geq 0, \tag{A.10}$$

it is easy to see that $\mathbf{v}=0$ on Ω.

(b) If $\Re(\sigma)\neq 0$, taking into account the fact that $\Im(\sigma)>0$, from (A.7) we obtain

$$\int_{\Omega}\|\mathbf{v}\|^2dx = 0. \tag{A.11}$$

Hence $\mathbf{v}=0$ on Ω. Therefore, (a) and (b) imply $\mathbf{v}_1=\mathbf{v}_2$. The uniqueness is proved for all $\sigma\in\mathbb{C}$ with $\Im(\sigma)>0$. □

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Luis A. Cortés-Vega: Departamento de Matemáticas, Facultad de Ciencias Básicas, Universidad de Antofagasta, Casilla 170, Antofagasta, Chile

E-mail address: lcortes@uantof.cl