Hindawi Publishing Corporation
International Journal of Mathematics and Mathematical Sciences
Volume 2008, Article ID 790964, 9 pages
doi:10.1155/2008/790964

Research Article

Weak Forms of Continuity and Associated Properties

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Received 17 July 2008; Accepted 17 September 2008

Recommended by Dalibor Froncek

We introduce slightly p-continuous mapping and almost p-open mapping and investigate the relationships between these mappings and related types of mappings, and also study some properties of these mappings.

1. Introduction and preliminaries

A subset A of a space X is called regular open if $A = \operatorname{Int}\overline{A}$, and regular closed if $X \setminus A$ is regular open, or equivalently, if $A = \overline{\operatorname{Int} A}$. It is well known that a subset A of a space X is regular open if and only if $A = \operatorname{Int} F$, where F is closed and A is regular closed if and only if $A = \overline{U}$, where U is open. A is called semi-open [1] (resp., preopen [2], semi-preopen [3], b-open [4]) if $A \subset \overline{\operatorname{Int} A}$ (resp., $A \subset \operatorname{Int}\overline{A}$, $A \subset \overline{\operatorname{Int}\overline{A}}$, $A \subset \overline{\operatorname{Int} A} \cup \operatorname{Int}\overline{A}$). It is known that a set A is semi-open if and only if $U \subset A \subset \overline{U}$ for some open set U, and that A is preopen (resp., semi-preopen) if and only if $A = U \cap D$, where U is open (resp., semi-open) and D is dense. The concept of a preopen set was introduced in [5], where the term locally dense was used and the concept of a semi-preopen set was introduced in [6] under the name β-open. It was pointed out in [3] that A is semi-preopen if and only if $P \subset A \subset \overline{P}$ for some preopen set P. Clearly, every regular closed set is semi-open, every open set is both semi-open and preopen, semi-open sets as well as preopen sets are b-open and b-open sets are semi-preopen. It is also known that the closure of every semi-preopen set is regular closed and that the arbitrary union of semi-open (resp., preopen, semi-preopen, b-open) sets is semi-open (resp., preopen, semi-preopen, b-open). A is called semi-closed (resp., preclosed, semi-preclosed, b-closed) if $X \setminus A$ is semi-open (resp., preopen, semi-preopen, b-open). It is well known that a subset A is regular closed if and only if A is both closed and semi-open if and only if A is both closed and semi-preopen.

A mapping f from a space X into a space Y is called regular open [7] if it maps regular open subsets onto regular open sets, almost open [8] if $f^{-1}(\overline{U}) \subset \overline{f^{-1}(U)}$ whenever U is open in Y, slightly continuous [7] if $f(\overline{U}) \subset \overline{f(U)}$ whenever U is open in X, semi-continuous [1] if the inverse image of each open set is semi-open, β-continuous [6] if the inverse image of each open set is β-open, weakly continuous [9] if for each $x \in X$ and for each open set V containing $f(x)$ there exists an open set U containing x such that $f(U) \subset \overline{V}$, weakly θ-irresolute [10] if the inverse image of each regular closed set is semi-open, rc-continuous [11] if the inverse image of each regular closed set is regular closed, and wrc-continuous [12] if the inverse image of each regular closed set is semi-preopen. We will use the term semi-precontinuous to indicate β-continuous. Clearly, every semi-continuous mapping is semi-precontinuous, every rc-continuous mapping is weakly θ-irresolute, and every weakly θ-irresolute mapping is wrc-continuous. In [7], it is shown that the properties semi-continuous and slightly continuous are independent of each other.

A space X is called a weak P-space [13] if for each countable family $\{U_n : n \in N\}$ of open subsets of X, $\overline{\bigcup U_n} = \bigcup \overline{U}_n$. Clearly, X is a weak P-space if and only if the countable union of regular closed subsets of X is regular closed (closed).

A space X is called rc-Lindelöf [14] if every regular closed cover of X has a countable subcover, and called almost rc-Lindelöf [15] if every regular closed cover of X has a countable subfamily whose union is dense in X.

A subset A of a space X is called an S-set in X [16] if every cover of A by regular closed subsets of X has a finite subcover, and called an rc-Lindelöf set in X (resp., an almost rc-Lindelöf set in X) [17] if every cover of A by regular closed subsets of X admits a countable subfamily that covers A (resp., the closure of the union of whose members contains A). Obviously, every S-set is an rc-Lindelöf set and every rc-Lindelöf set is an almost rc-Lindelöf set. It is also clear that a subset A of a weak P-space X is rc-Lindelöf in X if and only if it is almost rc-Lindelöf in X.

Throughout this paper, N (resp., Q, R) denotes the set of natural (resp., rational, real) numbers. For the concepts not defined here, we refer the reader to [18].

2. Slightly p-continuous mappings

This section is mainly devoted to study several properties of slightly p-continuous mappings. Now, we begin with the following lemma which was pointed out in [19] without proof. We will, however, state and prove it for its special importance in the material of our paper.

Lemma 2.1. (i) *Let $f : X \to Y$ be a semi-continuous and almost open mapping. Then f is weakly θ-irresolute.*

(ii) *Let $f : X \to Y$ be a semi-precontinuous and almost open mapping. Then f is wrc-continuous.*

Proof. (i) Let U be an open subset of Y. Since f is almost open, then $f^{-1}(\overline{U}) \subset \overline{f^{-1}(U)}$. Since f is semi-continuous, then $f^{-1}(U)$ is semi-open, hence there exists an open subset V of X such that $V \subset f^{-1}(U) \subset \overline{V}$, therefore, $V \subset f^{-1}(U) \subset f^{-1}(\overline{U}) \subset \overline{f^{-1}(U)} \subset \overline{V}$. Thus $f^{-1}(\overline{U})$ is semi-open, and f is weakly θ-irresolute.

(ii) Let U be an open subset of Y. Since f is almost open, then $f^{-1}(\overline{U}) \subset \overline{f^{-1}(U)}$. Since f is semi-precontinuous, then $f^{-1}(U)$ is semi-preopen, hence there exists a preopen subset V of X such that $V \subset f^{-1}(U) \subset \overline{V}$, therefore, $V \subset f^{-1}(U) \subset f^{-1}(\overline{U}) \subset \overline{f^{-1}(U)} \subset \overline{V}$. Thus $f^{-1}(\overline{U})$ is semi-preopen, and f is wrc-continuous. □

Corollary 2.2 (see [12]). *Let $f : X \to Y$ be a semi-continuous and almost open mapping. Then f is wrc-continuous.*

Proposition 2.3. *For a mapping $f : X \to Y$, the following are equivalent:*

(i) *f is slightly continuous;*

(ii) *$f(\overline{U}) \subset \overline{f(U)}$ whenever U is semi-open in X.*

Proof. Since every open set is semi-open, it suffices to show that (i) $\to$ (ii). Let U be a semi-open subset of X. Then there exists an open subset V of X such that $V \subset U \subset \overline{V}$. Thus by (i), $f(\overline{U}) = f(\overline{V}) \subset \overline{f(V)} \subset \overline{f(U)}$. □

Proposition 2.4. *Let $f : X \to Y$ be a slightly continuous mapping. Then the following are equivalent:*

(i) *f is weakly θ-irresolute;*

(ii) *f is rc-continuous.*

Proof. Since every regular closed set is semi-open, it suffices to show that (i) $\to$ (ii). Let A be a regular closed subset of Y. By (i), $f^{-1}(A)$ is semi-open, but f is slightly continuous, so by Proposition 2.3, $f(\overline{f^{-1}(A)}) \subset \overline{f(f^{-1}(A))} \subset \overline{A} = A$. Thus $\overline{f^{-1}(A)} \subset f^{-1}f(\overline{f^{-1}(A)}) \subset f^{-1}(A)$, that is, $f^{-1}(A)$ is closed, but $f^{-1}(A)$ is semi-open, so $f^{-1}(A)$ is regular closed. Hence f is rc-continuous. □

Corollary 2.5. *Let $f : X \to Y$ be a slightly continuous, semi-continuous, and almost open mapping. Then f is rc-continuous.*

Proof. Follows from Lemma 2.1(i) and Proposition 2.4. □

Proposition 2.6. *Let $f : X \to Y$ be a slightly continuous and semi-continuous mapping. Then $\overline{f^{-1}(U)} \subset f^{-1}(\overline{U})$ for every open subset U of Y.*

Proof. Let U be an open subset of Y. Since f is semi-continuous, it follows that $f^{-1}(U)$ is semi-open, but f is slightly continuous, so it follows from Proposition 2.3 that $f(\overline{f^{-1}(U)}) \subset \overline{f(f^{-1}(U))} \subset \overline{U}$. Thus $\overline{f^{-1}(U)} \subset f^{-1}f(\overline{f^{-1}(U)}) \subset f^{-1}(\overline{U})$. □

The following corollary is a slight improvement of Corollary 2.5. This is because the closure of every semi-open set is regular closed.

Corollary 2.7. *Let $f : X \to Y$ be a slightly continuous, semi-continuous, and almost open mapping. Then $\overline{f^{-1}(U)} \subset f^{-1}(\overline{U})$ for every open subset U of Y.*

Proposition 2.8. *Let $f : X \to Y$ be an rc-continuous mapping. If A is rc-Lindelöf in X, then $f(A)$ is rc-Lindelöf in Y.*

Proof. Let $\{U_\alpha : \alpha \in \Lambda\}$ be a cover of $f(A)$ by regular closed subsets of Y. Then $\{f^{-1}(U_\alpha) : \alpha \in \Lambda\}$ is a cover of A by regular closed subsets of X (as f is rc-continuous). Since A is rc-Lindelöf in X, it follows that there exist $\alpha_1, \alpha_2, \ldots \in \Lambda$ such that $A \subset \bigcup_{i=1}^{\infty} f^{-1}(U_{\alpha_i})$, thus it follows that $f(A) \subset \bigcup_{i=1}^{\infty} f(f^{-1}(U_{\alpha_i})) \subset \bigcup_{i=1}^{\infty} U_{\alpha_i}$. Hence $f(A)$ is rc-Lindelöf in Y. □

Corollary 2.9 (see [19]). *Let $f : X \to Y$ be a slightly continuous and weakly θ-irresolute mapping. If A is rc-Lindelöf in X, then $f(A)$ is rc-Lindelöf in Y.*

Proof. Follows from Propositions 2.4 and 2.8. □

Proposition 2.10 (see [20]). *Let $f : X \to Y$ be a weakly continuous and almost open mapping. Then f is rc-continuous.*

Corollary 2.11. *Let $f : X \to Y$ be a weakly continuous and almost open mapping. If A is rc-Lindelöf in X, then $f(A)$ is rc-Lindelöf in Y.*

Proof. Follows from Propositions 2.10 and 2.8. □

Now, we prove the following known result using a slight modification on the previous proof.

Proposition 2.12 (see [7]). *Let $f : X \to Y$ be a slightly continuous and weakly θ-irresolute mapping. If A is almost rc-Lindelöf in X, then $f(A)$ is almost rc-Lindelöf in Y.*

Proof. Let $\{U_\alpha : \alpha \in \Lambda\}$ be a cover of $f(A)$ by regular closed subsets of Y. Since f is slightly continuous and weakly θ-irresolute, it follows from Proposition 2.4 that f is rc-continuous and thus $\{f^{-1}(U_\alpha) : \alpha \in \Lambda\}$ is a cover of A by regular closed subsets of X. Since A is almost rc-Lindelöf in X, it follows that there exist $\alpha_1, \alpha_2, \ldots \in \Lambda$ such that $A \subset \overline{\bigcup_{i=1}^{\infty} f^{-1}(U_{\alpha_i})}$. Now, $f^{-1}(U_{\alpha_i})$ is regular closed and thus semi-open, but the arbitrary union of semi-open sets is semi-open, so $\bigcup_{i=1}^{\infty} f^{-1}(U_{\alpha_i})$ is semi-open. Since f is slightly continuous, it follows from Proposition 2.3 that $f(A) \subset \overline{\bigcup_{i=1}^{\infty} f(f^{-1}(U_{\alpha_i}))} \subset \overline{\bigcup_{i=1}^{\infty} U_{\alpha_i}}$. Hence $f(A)$ is almost rc-Lindelöf in Y. □

Definition 2.13. A mapping f from a space X into a space Y is said to be slightly p-continuous if $f(\overline{U}) \subset \overline{f(U)}$ whenever U is preopen in X.

Proposition 2.14. *For a mapping $f : X \to Y$, the following are equivalent:*

(i) *f is slightly p-continuous;*

(ii) *$f(\overline{U}) \subset \overline{f(U)}$ whenever U is semi-preopen in X;*

(iii) *$f(\overline{U}) \subset \overline{f(U)}$ whenever U is b-open in X.*

Proof. (i) $\to$ (ii): Let U be a semi-preopen subset of X. Then there exists a preopen subset V of X such that $V \subset U \subset \overline{V}$. Thus by (i), $f(\overline{U}) = f(\overline{V}) \subset \overline{f(V)} \subset \overline{f(U)}$.

(ii) $\to$ (iii) $\to$ (i): follow since every preopen set is b-open and every b-open set is semi-preopen. □

Definition 2.15. A mapping $f : X \to Y$ is called brc-continuous if $f^{-1}(A)$ is b-open for every regular closed subset A of Y.

Clearly, every weakly θ-irresolute mapping is brc-continuous and every brc-continuous mapping is wrc-continuous; the converses are, however, not true as the following two examples tell.

Example 2.16. Let $X = \{a,b,c\}$, $\tau = \{X,\phi,\{a,b\}\}$, and $\tau^* = \{X,\phi,\{a\},\{b,c\}\}$. Then the identity mapping from (X,τ) onto (X,τ^*) is brc-continuous but not weakly θ-irresolute (observe that the regular closed subsets of (X,τ^*) are the members of τ^*, each of which is preopen and thus b-open in (X,τ). However, $\{a\}$ is not semi-open in (X,τ).

Example 2.17. Let τ_u be the usual topology on the set of real numbers R and $\tau = \{R,\phi,A,R\setminus A\}$, where $A = [0,1)\cap Q$. Then the identity mapping from (R,τ_u) onto (R,τ) is wrc-continuous but not brc-continuous (observe that the regular closed subsets of (R,τ) are the members of τ, each of which is semi-preopen in (R,τ_u). However, A is not b-open in (R,τ_u).

Proposition 2.18. *Let $f : X \to Y$ be a slightly p-continuous mapping. Then the following are equivalent:*

(i) *f is weakly θ-irresolute;*

(ii) *f is rc-continuous;*

(iii) *f is wrc-continuous;*

(iv) *f is brc-continuous.*

Proof. (ii) $\to$ (i) $\to$ (iv) $\to$ (iii): follow since every regular closed set is semi-open, every semi-open set is b-open and every b-open set is semi-preopen.

(iii) $\to$ (ii): let A be a regular closed subset of Y. By (iii), $f^{-1}(A)$ is semi-preopen, but f is slightly p-continuous, so by Proposition 2.14, $f(\overline{f^{-1}(A)}) \subset \overline{f(f^{-1}(A))} \subset \overline{A} = A$. Thus $\overline{f^{-1}(A)} \subset f^{-1}f(\overline{f^{-1}(A)}) \subset f^{-1}(A)$, that is, $f^{-1}(A)$ is closed, but $f^{-1}(A)$ is semi-preopen, so $f^{-1}(A)$ is regular closed. Hence f is rc-continuous. □

Corollary 2.19. *Let $f : X \to Y$ be a slightly p-continuous, semi-precontinuous, and almost open mapping. Then f is rc-continuous.*

Proof. Follows from Lemma 2.1(ii) and Proposition 2.18. □

Proposition 2.20. *Let $f : X \to Y$ be a slightly p-continuous and semi-precontinuous mapping. Then $\overline{f^{-1}(U)} \subset f^{-1}(\overline{U})$ for every open subset U of Y.*

Proof. Let U be an open subset of Y. Since f is semi-precontinuous, it follows that $f^{-1}(U)$ is semi-preopen, but f is slightly p-continuous, so it follows from Proposition 2.14 that $f(\overline{f^{-1}(U)}) \subset \overline{f(f^{-1}(U))} \subset \overline{U}$. Thus $\overline{f^{-1}(U)} \subset f^{-1}f(\overline{f^{-1}(U)}) \subset f^{-1}(\overline{U})$. □

Observing that the closure of every semi-preopen set is regular closed, the following corollary seems a slight improvement of Corollary 2.19.

Corollary 2.21. *Let $f : X \to Y$ be a slightly p-continuous, semi-precontinuous, and almost open mapping. Then $\overline{f^{-1}(U)} = f^{-1}(\overline{U})$ for every open subset U of Y.*

Obviously, every continuous mapping is both semi-continuous and slightly p-continuous and every slightly p-continuous mapping is slightly continuous, the converses are, however, not true as the following two examples tell.

Example 2.22. Let $X = \{a,b,c\}$, $\tau = \{X,\phi,\{a,b\}\}$, and $\tau^* = \{X,\phi,\{a\},\{a,b\}\}$. Then the identity mapping from (X,τ) onto (X,τ^*) is slightly continuous and weakly θ-irresolute (observe that the regular closed subsets of (X,τ^*) are X and ϕ). However, it is not slightly

p-continuous (consider the preopen subset $\{b,c\}$ of (X,τ)). We observe also that this is an example of a mapping that is both slightly continuous and semi-precontinuous but neither slightly p-continuous nor semi-continuous (observe that $\{a\}$, $\{a,b\}$ are both dense and thus preopen in (X,τ). However, $\{a\}$ is not semi-open in (X,τ)). This example also shows that the converses of Propositions 2.6 and 2.20 are not true.

Example 2.23. Let $X = \{a,b,c\}$, $\tau = \{X,\phi,\{a\}\}$, and $\tau^* = \{X,\phi,\{a,b\}\}$. Then the identity mapping from (X,τ) onto (X,τ^*) is slightly p-continuous (observe that the nonempty preopen subsets of (X,τ) are the supersets of $\{a\}$); it is, moreover, semi-continuous and almost open. However, it is not continuous.

Corollary 2.24. *Let $f : X \to Y$ be a slightly p-continuous and wrc-continuous mapping. If A is rc-Lindelöf (resp., almost rc-Lindelöf) in X, then $f(A)$ is rc-Lindelöf (resp., almost rc-Lindelöf) in Y.*

Proof. We observe from Proposition 2.18 that a mapping that is both slightly p-continuous and wrc-continuous is both slightly continuous and weakly θ-irresolute (the converse is not true as Example 2.22 tells). Thus the result follows from Corollary 2.9 and Proposition 2.12. □

Corollary 2.25. *Let $f : X \to Y$ be a slightly p-continuous, semi-precontinuous, and almost open mapping. If A is rc-Lindelöf (resp., almost rc-Lindelöf) in X, then $f(A)$ is rc-Lindelöf (resp., almost rc-Lindelöf) in Y.*

Proof. Follows from Lemma 2.1(ii) and Corollary 2.24. □

Remark 2.26. Since every dense set is preopen, one easily observes that if f is a slightly p-continuous mapping from a space X onto a space Y, then f maps dense subsets of X onto dense subsets of Y.

Recall that a space X is called submaximal (resp., strongly irresolvable) if every dense subset of X is open (resp., semi-open), or equivalently if, every preopen subset of X is open (resp., semi-open).

The following proposition is a direct consequence of Proposition 2.3.

Proposition 2.27. *Let $f : X \to Y$ be a mapping from a strongly irresolvable space X into a space Y. Then the following are equivalent:*

(i) *f is slightly p-continuous;*

(ii) *f is slightly continuous.*

3. Almost p-open mappings

Definition 3.1. A mapping f from a space X into a space Y is said to be semi-regular open (resp., semi-p-regular open) if it maps regular open subsets onto semi-closed (resp., semi-preclosed) subsets.

Remark 3.2. Since every regular open set is semi-closed and every semi-closed set is semi-preclosed, it is obvious that every regular open mapping is semi-regular open and every semi-regular open mapping is semi-p-regular open. The converses are, however, not true as the following examples show.

Example 3.3. Let $X = \{a,b,c\}$, $\tau = \{X,\phi,\{a,c\},\{b\}\}$, and $\tau^* = \{X,\phi,\{a\},\{b\},\{a,b\}\}$. Then the identity mapping f from (X,τ) onto (X,τ^*) is semi-regular open (observe that the regular open subsets of (X,τ) are the members of τ, each of which is semi-closed in (X,τ^*)); it is, however, not regular open since $\{a,c\}$ is not regular open in (X,τ^*).

Example 3.4. Let $X = \{a,b,c\}$, $\tau = \{X,\phi,\{a,c\},\{b\}\}$, and $\tau^* = \{X,\phi,\{a,b\},\{c\}\}$. Then the identity mapping f from (X,τ) onto (X,τ^*) is semi-p-regular open (observe that $\{a,c\}$ and $\{b\}$ are preopen and thus semi-preopen in (X,τ^*)); it is, however, not semi-regular open since $\{a,c\}$ is not semi-closed in (X,τ^*).

Definition 3.5. A mapping f from a space X into a space Y is said to be almost p-open if $f^{-1}(\overline{U}) \subset \overline{f^{-1}(U)}$ whenever U is preopen in Y.

Proposition 3.6. *For a mapping $f : X \to Y$, the following are equivalent:*

(i) *f is almost open;*

(ii) *$f^{-1}(\overline{U}) \subset \overline{f^{-1}(U)}$ whenever U is semi-open in Y.*

Proof. Since every open set is semi-open, it suffices to show that (i) $\to$ (ii). Let U be a semi-open subset of Y. Then there exists an open subset V of Y such that $V \subset U \subset \overline{V}$. Thus by (i), $f^{-1}(\overline{U}) = f^{-1}(\overline{V}) \subset \overline{f^{-1}(V)} \subset \overline{f^{-1}(U)}$. □

Proposition 3.7. *For a mapping $f : X \to Y$, the following are equivalent:*

(i) *f is almost p-open;*

(ii) *$f^{-1}(\overline{U}) \subset \overline{f^{-1}(U)}$ whenever U is semi-preopen in Y;*

(iii) *$f^{-1}(\overline{U}) \subset \overline{f^{-1}(U)}$ whenever U is b-open in Y.*

Proof. (i) $\to$ (ii): Let U be a semi-preopen subset of Y. Then there exists a preopen subset V of Y such that $V \subset U \subset \overline{V}$. Thus by (i), $f^{-1}(\overline{U}) = f^{-1}(\overline{V}) \subset \overline{f^{-1}(V)} \subset \overline{f^{-1}(U)}$.

(ii) $\to$ (iii) $\to$ (i): follow since every preopen set is b-open and every b-open set is semi-preopen. □

Remark 3.8. Since every open set is preopen, it is obvious that every almost p-open mapping is almost open. However, the converse is not true as the following example tells.

Example 3.9. Let $X = \{a,b,c\}$, $\tau = \{X,\phi,\{a\}\}$, and $\tau^* = \{X,\phi,\{a,b\},\{c\}\}$. Then the identity mapping f from (X,τ) onto (X,τ^*) is almost open and even regular open (observe that the regular open subsets of (X,τ) are X and ϕ); it is, however, not almost p-open since $\{b,c\}$ is dense and thus preopen in (X,τ^*) but not dense in (X,τ).

Proposition 3.10. *For an almost p-open mapping $f : X \to Y$, the following are equivalent:*

(i) *f is semi-p-regular open;*

(ii) *semi-regular open;*

(iii) *regular open.*

Proof. (i) → (iii): Let A be a regular open subset of X. By assumption, $f(A)$ is semi-preclosed, that is, $Y \setminus f(A)$ is semi-preopen. By Proposition 3.7, $f^{-1}(\overline{Y \setminus f(A)}) \subset \overline{f^{-1}(Y \setminus f(A))} \subset \overline{X \setminus A} = X \setminus A$. Thus $f^{-1}(\overline{Y \setminus f(A)}) \cap A = \phi$ and, therefore, $(\overline{Y \setminus f(A)}) \cap f(A) = \phi$, that is, $f(A) \subset \operatorname{Int} f(A)$, that is, $f(A)$ is open, but $f(A)$ is semi-preclosed, so $f(A)$ is regular open.

(iii) → (ii) → (i): follow since every regular open mapping is semi-regular open and every semi-regular open mapping is semi-p-regular open. □

Proposition 3.11. *For an almost open mapping $f : X \to Y$, the following are equivalent:*

(i) *semi-regular open;*

(ii) *regular open.*

Proof. Since every regular open mapping is semi-regular open, it suffices to show that (i) → (ii). Let A be a regular open subset of X. By assumption, $f(A)$ is semi-closed, that is, $Y \setminus f(A)$ is semi-open. By Proposition 3.6, $f^{-1}(\overline{Y \setminus f(A)}) \subset \overline{f^{-1}(Y \setminus f(A))} \subset \overline{X \setminus A} = X \setminus A$. Thus $f^{-1}(\overline{Y \setminus f(A)}) \cap A = \phi$ and, therefore, $(\overline{Y \setminus f(A)}) \cap f(A) = \phi$, that is, $f(A) \subset \operatorname{Int} f(A)$, that is, $f(A)$ is open, but $f(A)$ is semi-closed, so $f(A)$ is regular open. □

Proposition 3.12 (see [19]). *Let f be an almost open and regular open mapping from a space X onto a space Y. Then the following hold.*

(i) *If for each $y \in Y$, $f^{-1}(y)$ is an S-set in X, then $f^{-1}(A)$ is almost rc-Lindelöf in X whenever A is almost rc-Lindelöf in Y.*

(ii) *If for each $y \in Y$, $f^{-1}(y)$ is rc-Lindelöf in X, then $f^{-1}(A)$ is rc-Lindelöf in X whenever A is almost rc-Lindelöf in Y provided that X is a weak P-space.*

Corollary 3.13. *Let f be an almost p-open and semi-p-regular open mapping from a space X onto a space Y. Then the following hold.*

(i) *If for each $y \in Y$, $f^{-1}(y)$ is an S-set in X, then $f^{-1}(A)$ is almost rc-Lindelöf in X whenever A is almost rc-Lindelöf in Y.*

(ii) *If for each $y \in Y$, $f^{-1}(y)$ is rc-Lindelöf in X, then $f^{-1}(A)$ is rc-Lindelöf in X whenever A is almost rc-Lindelöf in Y provided that X is a weak P-space.*

Proof. We observe from Proposition 3.10 that a mapping that is both almost p-open and semi-p-regular open is both almost open and regular open (the converse is not true as Example 3.9 tells). Thus the result follows from Proposition 3.12. □

Remark 3.14. Since every dense set is preopen, one easily observes that if f is an almost p-open mapping from a space X into a space Y, then the inverse image of a dense subset of Y is a dense subset of X.

The following proposition is a direct consequence of Proposition 3.6.

Proposition 3.15. *Let $f : X \to Y$ be a mapping from a space X into a strongly irresolvable space Y. Then the following are equivalent:*

(i) *f is almost p-open;*

(ii) *f is almost open.*

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