THERMOELASTIC WAVES WITHOUT ENERGY DISSIPATION IN AN UNBOUNDED BODY WITH A SPHERICAL CAVITY

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ABSTRACT. The linear theory of thermoelasticity without energy dissipation is employed to study waves emanating from the boundary of a spherical cavity in a homogeneous and isotropic unbounded thermoelastic body. The waves are supposed to be spherically symmetric and caused by a constant step in temperature applied to the stress-free boundary of the cavity. Small-time solutions for the displacement, temperature, and stress fields are obtained by using the Laplace transform technique. It is found that there exist two coupled waves, of which one is predominantly elastic and the other is predominantly thermal, both propagating with finite speeds but with no exponential attenuation. Exact expressions for discontinuities in the field functions that occur at the wavefronts are computed and analysed. The results are compared with those obtained earlier in the contexts of some other models of thermoelasticity.

Keywords and phrases. Thermoelasticity without energy dissipation, generalized thermoelasticity, thermoelastic spherical waves.

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1. Introduction. Thermoelasticity theories that admit finite speeds for thermal signals have aroused much interest in the last three decades. In contrast to the conventional coupled thermoelasticity theory based on a parabolic heat equation [1], which predicts an infinite speed for the propagation of heat, these theories involve hyperbolic heat equations and are referred to as generalized thermoelasticity theories. For details about the physical relevance of these theories and a review of the relevant literature, see [2].

Recently, Green and Naghdi [14] formulated a new generalized thermoelasticity theory by including the so-called "thermal-displacement gradient" among the independent constitutive variables. An important characteristic feature of this theory, which is not present in other thermoelasticity theories, is that this theory does not accommodate dissipation of thermal energy. In the context of the linearized version of this theory, theorems on uniqueness of solutions have been established in [3, 6], boundary-initiated waves in a half-space and in unbounded body with cylindrical cavity have been studied in [4, 9, 10, 11], and plane harmonic waves and Rayleigh waves have been studied in [5, 7, 12].

In this paper, we employ the theory formulated in [14] (hereafter referred to as the GN-theory) to study spherically symmetric thermoelastic waves in a linear, homogeneous and isotropic unbounded solid body containing a spherical cavity. We suppose that the waves are generated due to the application of a constant step in

temperature on the boundary of the cavity which is held in a stress-free state. We employ the Laplace transform technique to analyse the problem. We restrict ourselves to the derivation of small-time solutions for the field functions. We find that there occur two coupled waves both propagating with finite speeds, of which one is predominantly elastic and the other is predominantly thermal, and that these waves do not experience any attenuation. We further find that the displacement field is continuous whereas the thermal and stress fields are discontinuous at both the wavefronts. We obtain exact expressions for the discontinuities and make a brief analysis thereof.

The counterparts of our problem in the contexts of the uncoupled thermoelasticity theory, the coupled thermoelasticity theory, the Green-Lindsay theory (GL-theory) [13] and the Lord-Shulman theory (LS-theory) [15] have been considered in [16, 17, 18, 20], respectively. At appropriate stages of our analysis, we make a comparison of our results with those obtained in these works. This comparison reveals that, on the whole, the predictions of the GN-theory (as obtained here) are qualitatively similar to those of the LS-theory. More importantly, we notice that certain physically unrealistic features inherent in the conventional coupled thermoelasticity theory and the GL-theory are not present in the GN-theory.

2. Formulation of the problem. In the context of the GN-theory, the field equations for a linear, homogeneous and isotropic thermoelastic solid body, in the absence of body forces and heat sources, are as follows [14]:

$$\mu \nabla^2 \mathbf{u} + (\lambda + \mu) \nabla \operatorname{div} \mathbf{u} - \gamma \nabla \theta = \rho \ddot{\mathbf{u}},$$
$$c \ddot{\theta} + \gamma T_0 \operatorname{div} \ddot{\mathbf{u}} = \kappa^* \nabla^2 \theta. \tag{2.1}$$

In these equations, $\mathbf{u}$ is the displacement vector; θ is the temperature-change above a uniform reference temperature T_0; ρ is the mass density; c is the specific heat; λ and μ are the Lame' constants; $\gamma = (3\lambda + 2\mu)\beta^*$, β^* being the coefficient of volume expansion; and κ^* is a material constant characteristic of the theory.

The stress tensor $\mathbf{T}$ associated with $\mathbf{u}$ and θ is given by [14]

$$\mathbf{T} = \lambda(\operatorname{div} \mathbf{u})\mathbf{I} + \mu\left(\nabla \mathbf{u} + \nabla \mathbf{u}^T\right) - \gamma \theta \mathbf{I}. \tag{2.2}$$

In all the above equations, the direct vector/tensor notation [8] is employed. Also, an over dot denotes partial derivative with respect to the time variable t. Some of our symbols and the notation are slightly different from those employed in [14].

For spherically symmetric interactions, the displacement vector possesses only the radial component $u = u(r, t)$, where r is the radial distance measured from the origin (point of symmetry), and the stress tensor is determined by the radial stress σ_r and the circumferential stress (hoop stress) σ_ϕ. In this case, equations (2.1) yield the following governing equations for u and θ:

$$(\lambda + 2\mu)\left[\frac{\partial}{\partial r}\left(\frac{\partial u}{\partial r} + \frac{2}{r}u\right)\right] - \gamma\frac{\partial \theta}{\partial r} = \rho\frac{\partial^2 u}{\partial t^2},$$
$$c\frac{\partial^2 \theta}{\partial t^2} + \gamma T_0 \frac{\partial^2}{\partial t^2}\left(\frac{\partial}{\partial r} + \frac{2}{r}\right)u = \kappa^*\left(\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r}\right)\theta. \tag{2.3}$$

Also, the relation (2.2) yields

$$\sigma_r = (\lambda + 2\mu)\frac{\partial u}{\partial r} + 2\lambda\frac{u}{r} - \gamma\theta,$$

$$\sigma_\phi = \lambda\frac{\partial u}{\partial r} + 2(\lambda + \mu)\frac{u}{r} - \gamma\theta. \tag{2.4}$$

For a mathematical analysis, it is convenient to have (2.3) and (2.4) rewritten in non-dimensional form. For this purpose, we consider the transformations

$$r' = \frac{r}{L}, \qquad t' = \frac{V}{L}t, \qquad u' = \frac{(\lambda + 2\mu)}{L\gamma T_0}u,$$

$$\theta' = \frac{\theta}{T_0}, \qquad \sigma_r' = \frac{\sigma_r}{\gamma T_0}, \qquad \sigma_\phi' = \frac{\sigma_\phi}{\gamma T_0}, \tag{2.5}$$

where L is a standard length and V is a standard speed. Using these transformations in (2.3) and (2.4) and suppressing primes for simplicity in the notation, we obtain the following equations/relations which are in *non-dimensional form*:

$$C_p^2\left[\frac{\partial}{\partial r}\left(\frac{\partial u}{\partial r} + \frac{2}{r}u\right) - \frac{\partial\theta}{\partial r}\right] = \frac{\partial^2 u}{\partial t^2},$$

$$C_T^2\left(\frac{\partial^2\theta}{\partial r^2} + \frac{2}{r}\frac{\partial\theta}{\partial r}\right) = \frac{\partial^2\theta}{\partial t^2} + \varepsilon\frac{\partial^2}{\partial t^2}\left(\frac{\partial u}{\partial r} + \frac{2}{r}u\right), \tag{2.6}$$

$$\sigma_r = \frac{\partial u}{\partial r} + 2\eta\frac{u}{r} - \theta, \tag{2.7}$$

$$\sigma_\phi = \eta\frac{\partial u}{\partial r} + (1 + \eta)\frac{u}{r} - \theta. \tag{2.8}$$

Here,

$$C_P^2 = \frac{(\lambda + 2\mu)}{\rho V^2}, \qquad C_T^2 = \frac{\kappa^*}{cV^2}, \qquad \varepsilon = \frac{\gamma^2 T_0}{c(\lambda + 2\mu)}, \qquad \eta = \frac{\lambda}{\lambda + 2\mu}. \tag{2.9}$$

We note that C_P and C_T respectively represent the non-dimensional speeds of purely elastic dilatational wave and purely thermal wave, and ε is the usual thermoelastic coupling factor.

The body being considered for our analysis here is an unbounded thermoelastic solid with a spherical cavity. We choose the origin at the centre of the cavity and denote the dimensionless radius of the cavity by a. If initially the body is at rest in an undeformed state and has its temperature-change and temperature-rate equal to zero, then the following initial conditions hold:

$$u = \frac{\partial u}{\partial t} = \theta = \frac{\partial\theta}{\partial t} = 0 \quad \text{at } t = 0 \text{ for } r \geq a. \tag{2.10}$$

If the thermoelastic interactions are caused by a uniform step in temperature applied to the boundary of the cavity which is held in the stress-free state, then the following boundary conditions hold:

$$\theta = \chi H(t), \qquad \sigma_r = 0 \quad \text{for } r = a, \, t > 0, \tag{2.11}$$

where χ is a positive constant and $H(t)$ is the Heaviside unit step function. With the aid of (2.7), these boundary conditions can be rewritten as

$$\frac{\partial u}{\partial r} + 2\eta \frac{u}{r} = \chi H(t) \quad \text{for } r = a, \, t > 0. \tag{2.12}$$

Thus, for our problem here, (2.6) are the governing differential equations, (2.10) are the initial conditions, and (2.12) is the boundary condition. Once u and θ are determined by solving this initial-boundary value problem, then σ_r and σ_ϕ can be computed by using (2.7) and (2.8).

3. Transform solution. Taking the Laplace transforms of (2.6) under the homogeneous initial conditions (2.10), we obtain the following equations:

$$[C_p^2 DD_1 - s^2]\overline{u} = C_p^2 D\overline{\theta}, \tag{3.1}$$

$$[C_T^2 D_1 D - s^2]\overline{\theta} = \varepsilon s^2 D_1 \overline{u}. \tag{3.2}$$

Here, an over bar denotes the Laplace transform of the corresponding function, s is the transform parameter, and

$$D = \frac{d}{dr}, \qquad D_1 = D + \frac{2}{r}. \tag{3.3}$$

The coupled (3.1) and (3.2) can be decoupled by eliminating $\overline{\theta}$ or $\overline{u}$, and put in the form

$$(DD_1 - m_1^2)(DD_1 - m_2^2)\overline{u} = 0,$$
$$(D_1 D - m_1^2)(D_1 D - m_2^2)\overline{\theta} = 0, \tag{3.4}$$

where m_1^2 and m_2^2 satisfy the quadratic equation

$$C_p^2 C_T^2 x^2 - [C_T^2 + (1 + \varepsilon)C_p^2]s^2 x + s^4 = 0. \tag{3.5}$$

Under the regularity conditions that $\overline{u}$ and $\overline{\theta} \to 0$ as $r \to \infty$, the general solutions of (3.4) are given by

$$\overline{u} = \sum_{\alpha=1}^{2} A_\alpha \left(\frac{1}{r} + \frac{1}{m_\alpha r^2} \right) e^{-m_\alpha r}, \tag{3.6}$$

$$\overline{\theta} = \frac{1}{r} \sum_{\alpha=1}^{2} B_\alpha e^{-m_\alpha r}, \tag{3.7}$$

where A_α and B_α, $\alpha = 1, 2$, are functions of s, of which only A_α or B_α are arbitrary, and m_α is the square root, with positive real part, of m_α^2. By solving (3.5), we obtain the following *exact expressions* for m_α:

$$m_\alpha = \frac{s}{V_\alpha}, \tag{3.8}$$

where

$$V_\alpha = \frac{1}{\sqrt{2}}[C_T^2 + (1 + \varepsilon)C_p^2 + (-1)^{\alpha+1}\Delta]^{1/2} \tag{3.9}$$

with

$$\Delta = \{[C_T^2 - (1+\varepsilon)C_P^2]^2 + 4\varepsilon C_P^2 C_T^2\}^{1/2}. \tag{3.10}$$

Here and in the expressions that follow, the suffix α takes values 1 and 2.

Substituting for $\overline{u}$ and $\overline{\theta}$ from (3.6) and (3.7) in (3.2) and equating the corresponding coefficients, we get

$$B_\alpha = \frac{\varepsilon s^2 m_\alpha}{s^2 - C_T^2 m_\alpha^2} A_\alpha. \tag{3.11}$$

Next, taking the Laplace transform of the boundary condition (2.12) and substituting for $\overline{u}$ from (3.6) in the resulting expression, we obtain

$$A_\alpha = (-1)^{\alpha+1}\frac{\Omega_\alpha}{\Gamma}, \tag{3.12}$$

where

$$\begin{aligned}
\Omega_\alpha &= \frac{a\chi}{m_\alpha}(C_T^2 m_\alpha^2 - s^2)[\varepsilon s^2 a^2 m_1^2 m_2^2 - (C_T^2 m_{3-\alpha}^2 - s^2)\Lambda_\alpha]e_{\alpha a}^m, \\
\Gamma &= \varepsilon s^3[(C_T^2 m_2^2 - s^2)\Lambda_1 - (C_T^2 m_1^2 - s^2)\Lambda_2], \\
\Lambda_\alpha &= m_\alpha^2[m_{3-\alpha}^2 a^2 + 2(1-\eta)m_{3-\alpha}a + 2(1-\eta)].
\end{aligned} \tag{3.13}$$

Substituting for m_α, A_α and B_α from (3.8) and (3.11), (3.12), and (3.13) in (3.6) and (3.7), we obtain expressions for $\overline{u}$ and $\overline{\theta}$ in terms of r and s. The forms of these expressions indicate that each of the displacement and thermal fields (and consequently the stress field) consists of two parts and that each part corresponds to a wave propagating with a *finite* speed, the wave corresponding to the first part having its speed equal to V_1 and that corresponding to the second part equal to V_2. We find that the expressions giving these speeds, namely, (3.9) and (3.10), are identical with the expressions giving the speeds of boundary-initiated plane waves in a half-space [4]. As in [4], it can be shown that $V_1 > V_2$ and that the faster wave is predominantly elastic or predominantly thermal and the slower wave is predominantly thermal or predominantly elastic according as $C_P > C_T$ or $C_T > C_P$.

4. Small-time solution. We notice that A_α, given by (3.12) and (3.13), and therefore B_α, given by (3.11), are complicated functions of s. Consequently, the Laplace transform inversion of the expressions (3.6) and (3.7) for all s is a formidable task. We confine ourselves to the derivation of small-time solutions for the field functions. This is done by taking s to be large (as in [16, 17, 18], for example).

When s is large, the results (3.8) and (3.11), (3.12), and (3.13) yield the following approximate expressions for A_α and B_α (after some straightforward but lengthy calculations):

$$\begin{aligned}
A_\alpha &\approx (-1)^{\alpha+1}\chi V_\alpha\left[\frac{L_\alpha}{s^2} + \frac{M_\alpha}{s^3}\right]\exp\left(\frac{sa}{V_\alpha}\right), \\
B_\alpha &\approx (-1)^{\alpha+1}\chi\left[\frac{N_\alpha}{s} + \frac{P_\alpha}{s^2}\right]\exp\left(\frac{sa}{V_\alpha}\right),
\end{aligned} \tag{4.1}$$

where

$$L_\alpha = \frac{a}{\varepsilon C_T^2 (V_1^2 - V_2^2)} (V_\alpha^2 - C_T^2)[C_T^2 - (1+\varepsilon)V_{3-\alpha}^2],$$

$$M_\alpha = \frac{2(1-\eta)}{\varepsilon C_T^2 (V_1^2 - V_2^2)} (C_T^2 - V_\alpha^2)\left\{V_{3-\alpha}(V_{3-\alpha}^2 - C_T^2) - \frac{(V_1 V_2 + C_T^2)}{C_T^2 (V_1 + V_2)}[(1+\varepsilon)V_{3-\alpha}^2 - C_T^2]\right\},$$

$$(N_\alpha, P_\alpha) = \frac{\varepsilon V_\alpha^2}{(V_\alpha^2 - C_T^2)} (L_\alpha, M_\alpha).$$

$$(4.2)$$

Now, using (3.8) and (4.1) in (3.6) and (3.7), and inverting the resulting expressions with the aid of standard formulas of the Laplace transform theory [19], we obtain the following solutions for u and θ valid for *small values* of t:

$$u = \frac{\chi}{r} \sum_{\alpha=1}^{2} (-1)^{\alpha+1} V_\alpha \left[L_\alpha \tau_\alpha + \frac{1}{2}\left(\frac{1}{r}V_\alpha L_\alpha + M_\alpha\right)\tau_\alpha^2\right] H(\tau_\alpha),$$

$$\theta = \frac{\chi}{r} \sum_{\alpha=1}^{2} (-1)^{\alpha+1} (N_\alpha + P_\alpha \tau_\alpha) H(\tau_\alpha).$$

$$(4.3)$$

Here,

$$\tau_\alpha = \left(t - \frac{r-a}{V_\alpha}\right).$$

$$(4.4)$$

Substituting the above solutions for u and θ in the relations (2.7) and (2.8), we obtain the following small-time solutions for σ_r and σ_ϕ:

$$\sigma_r = \frac{\chi}{r} \sum_{\alpha=1}^{2} (-1)^\alpha \left\{(L_\alpha + N_\alpha) + \left[M_\alpha + P_\alpha + \frac{2}{r}(1-\eta)V_\alpha L_\alpha\right]\tau_\alpha\right\} H(\tau_\alpha), \qquad (4.5)$$

$$\sigma_\phi = \frac{\chi}{r} \sum_{\alpha=1}^{2} (-1)^\alpha \left\{\eta L_\alpha + N_\alpha + \left[\eta M_\alpha + P_\alpha + \frac{1}{r}(\eta-1)V_\alpha L_\alpha\right]\tau_\alpha\right\} H(\tau_\alpha). \qquad (4.6)$$

The forms of the solutions (4.3), (4.5), and (4.6) confirm the existence of two coupled waves emanating from the boundary of the cavity and propagating outward with constant speeds V_α, and having $\tau_\alpha = 0$ as wavefronts. Bearing in mind that $V_1 > V_2$, we check that these solutions are identically zero for $r > a + tV_1$. This means that the effects of the waves are localized at a given instant of time $t^* > 0$, the points of the body that are beyond the faster wavefront ($r = a + V_1 t^*$) do not experience any disturbance. This observation verifies that the GN-theory is indeed a generalized thermoelasticity theory.

As mentioned earlier, the counterparts of our problem in the contexts of the conventional coupled thermoelasticity theory, the GL-theory and the LS-theory have been considered in [16, 18, 17]. In these works also, like in our analysis here, only small-time solutions have been derived. By comparing the solutions obtained in these works with

our solutions, we find that whereas the solutions in the contexts of the three theories just mentioned decay exponentially with increasing r, our solutions do not exhibit such an exponential decay. This is due to the fact that the GN-theory does not admit dissipation of thermal energy. In this connection, it may be mentioned that in the context of the uncoupled thermoelasticity theory also, the exponential decay is not observed [20].

5. Analysis of discontinuities. By direct inspection of solutions (4.3), (4.5), and (4.6) we can determine the discontinuities experienced by the field functions across the wavefronts. These discontinuities are

$$[u]_\alpha = 0, \tag{5.1}$$

$$[\theta]_\alpha = \frac{\chi}{r}(-1)^{\alpha+1}N_\alpha,$$

$$[\sigma_r]_\alpha = \frac{\chi}{r}(-1)^\alpha(L_\alpha + N_\alpha), \tag{5.2}$$

$$[\sigma_\phi]_\alpha = \frac{\chi}{r}(-1)^\alpha(\eta L_\alpha + N_\alpha).$$

Here, $[\cdots]_\alpha$ denotes the discontinuity of the function across the wavefront $\tau_\alpha = 0$.

It should be noted that, since the discontinuities in the field functions are precisely the coefficients of $H(\tau_\alpha)$ that do not contain τ_α in the corresponding solutions, the discontinuities documented above are *exact*, although they are derived from small-time solutions.

Expression (5.1) shows that the displacement is continuous across both the wavefronts. This is the situation in the context of the LS-theory also [16]. But in the contexts of the conventional coupled theory and the GL-theory, the displacement has been found to be discontinuous at the wavefronts [18]. A discontinuity in displacement implies that one portion of matter penetrates into another, and this phenomenon violates the continuum hypothesis [8]. Thus, as far as the behaviour of the displacement near the wavefronts is concerned, the predictions of the conventional coupled thermoelasticity theory and the GL-theory are physically absurd.

Expressions (5.2) show that the temperature and the radial and circumferential stresses are all discontinuous at both wavefronts and that the magnitudes of discontinuities are finite. The same situation arises in the context of the LS-theory also [16]. But the case with the GL-theory is different according to this theory, the stresses suffer delta function singularities at both wavefronts. This predictions of the GL-theory is also not physically realistic.

Thus, as far as the discontinuities across the wavefronts are concerned, the predictions of the GN-theory (as obtained here) are qualitatively similar to those of the LS-theory. Furthermore, like the LS-theory, the GN-theory also does not possess physically unrealistic features inherent in the conventional coupled thermoelasticity theory and the GL-theory (as described above).

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