IJMMS 27:5 (2001) 261–267
PII. S0161171201010778
http://ijmms.hindawi.com

INTUITIONISTIC FUZZY INTERIOR IDEALS OF SEMIGROUPS

KYUNG HO KIM and YOUNG BAE JUN

(Received 18 August 2000 and in revised form 11 December 2000)

Abstract. We consider the intuitionistic fuzzification of the concept of interior ideals in a semigroup S, and investigate some properties of such ideals. For any homomorphism f from a semigroup S to a semigroup T, if $B = (\mu_B, \gamma_B)$ is an intuitionistic fuzzy interior ideal of T, then the preimage $f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\gamma_B))$ of B under f is an intuitionistic fuzzy interior ideal of S.

2000 Mathematics Subject Classification. 20M12, 03E72, 94D05.

1. Introduction. The idea of "intuitionistic fuzzy set" was first published by Atanassov [1, 2], as a generalization of the notion of fuzzy set. Jun et al. considered the fuzzification of interior ideals in semigroups [3]. In this paper, we introduce the notion of an intuitionistic fuzzy interior ideal of a semigroup S, and then some related properties are investigated. Characterizations of intuitionistic fuzzy interior ideals are given. Also for any homomorphism f from a semigroup S to a semigroup T, if $B = (\mu_B, \gamma_B)$ is an intuitionistic fuzzy interior ideal of T, then the preimage $f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\gamma_B))$ of B under f is an intuitionistic fuzzy interior ideal of S.

2. Preliminaries. Let X be a nonempty fixed set. An *intuitionistic fuzzy set* (IFS for short) A is an object having the form

$$A = \{(x, \mu_A(x), \gamma_A(x)) : x \in X\}, \tag{2.1}$$

where the functions $\mu_A : X \to [0,1]$ and $\gamma_A : X \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of nonmembership (namely $\gamma_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \le \mu_A(x) + \gamma_A(x) \le 1$ for all $x \in X$ (see Atanassov [1, 2]). For the sake of simplicity, we use the symbol $A = (\mu_A, \gamma_A)$ for the IFS $A = \{(x, \mu_A(x), \gamma_A(x)) : x \in X\}$.

Let S be a semigroup. By a *subsemigroup* of S we mean a nonempty subset A of S such that $A^2 \subseteq A$. A subsemigroup A of a semigroup S is called an *interior ideal* of S if $SAS \subseteq A$. A mapping f from a semigroup S to a semigroup T is called a *homomorphism* if $f(xy) = f(x)f(y)$ for all $x, y \in S$.

A fuzzy set μ in a semigroup S is called a *fuzzy subsemigroup* of S (see [3]) if $\mu(xy) \ge \mu(x) \wedge \mu(y)$ for all $x, y \in S$.

A fuzzy subsemigroup μ of a semigroup S is called a *fuzzy interior ideal* of S (see [3]) if $\mu(xay) \ge \mu(a)$ for all $a, x, y \in S$.

3. Intuitionistic fuzzy interior ideals. In what follows, S denotes a semigroup unless otherwise specified.

Definition 3.1. An IFS $A = (\mu_A, \gamma_A)$ in S is called an *intuitionistic fuzzy subsemigroup* of S if it satisfies
(IF1) $\mu_A(xy) \geq \mu_A(x) \wedge \mu_A(y)$,
(IF2) $\gamma_A(xy) \leq \gamma_A(x) \vee \gamma_A(y)$,
for all $x, y \in S$.

Example 3.2. Let $S = \{0, e, f, a, b\}$ be a set with the following Cayley table:

$\cdot$	0	e	f	a	b
0	0	0	0	0	0
e	0	e	0	a	0
f	0	0	f	0	b
a	0	a	0	0	e
b	0	0	b	f	0

Then S is a semigroup (see [4]). Define an IFS $A = (\mu_A, \gamma_A)$ in S by $\mu_A(0) = \mu_A(e) = \mu_A(f) = 1$, $\mu_A(a) = \mu_A(b) = 0$, $\gamma_A(0) = \gamma_A(e) = \gamma_A(f) = 0$, and $\gamma_A(a) = \gamma_A(b) = 1$. By routine calculations we know that $A = (\mu_A, \gamma_A)$ is an intuitionistic fuzzy subsemigroup of S.

Definition 3.3. An intuitionistic fuzzy subsemigroup $A = (\mu_A, \gamma_A)$ of S is called an *intuitionistic fuzzy interior ideal* of S if
(IF3) $\mu_A(xay) \geq \mu_A(a)$,
(IF4) $\gamma_A(xay) \leq \gamma_A(a)$,
for all $x, y, a \in S$.

Example 3.4. The IFS $A = (\mu_A, \gamma_A)$ in Example 3.2 is an intuitionistic fuzzy interior ideal of S.

Theorem 3.5. *If $\{A_i\}_{i\in\Lambda}$ is a family of intuitionistic fuzzy interior ideals of S, then $\cap A_i$ is an intuitionistic fuzzy interior ideal of S, where $\cap A_i = (\wedge\mu_{A_i}, \vee\gamma_{A_i})$ and $\wedge\mu_{A_i}$ and $\vee\gamma_{A_i}$ are defined as follows:*

$$\begin{aligned} \wedge\mu_{A_i}(x) &= \inf\{\mu_{A_i}(x) \mid i \in \Lambda,\ x \in S\}, \\ \vee\gamma_{A_i}(x) &= \sup\{\gamma_{A_i}(x) \mid i \in \Lambda,\ x \in S\}. \end{aligned} \tag{3.1}$$

Proof. Let $x, y, a \in S$. Then

$$\begin{gathered} \wedge\mu_{A_i}(xy) \geq \wedge\left(\mu_{A_i}(x) \wedge \mu_{A_i}(y)\right) = \left(\wedge\mu_{A_i}(x)\right) \wedge \left(\wedge\mu_{A_i}(y)\right), \\ \vee\gamma_{A_i}(xy) \leq \vee\left(\gamma_{A_i}(x) \vee \gamma_{A_i}(y)\right) = \left(\vee\gamma_{A_i}(x)\right) \vee \left(\vee\gamma_{A_i}(y)\right), \\ \wedge\mu_{A_i}(xay) \geq \wedge\mu_{A_i}(a), \qquad \vee\gamma_{A_i}(xay) \leq \vee\gamma_{A_i}(a). \end{gathered} \tag{3.2}$$

Hence $\cap A_i$ is an intuitionistic fuzzy interior ideal of S. □

Theorem 3.6. *If an IFS $A = (\mu_A, \gamma_A)$ in S is an intuitionistic fuzzy interior ideal of S, then so is $\Box A := (\mu_A, \bar{\mu}_A)$, $\bar{\mu}_A = 1 - \mu_A$.*

Proof. It is sufficient to show that $\bar{\mu}_A$ satisfies conditions (IF2) and (IF4). For any $a, x, y \in S$, we have

$$\begin{aligned} \bar{\mu}_A(xy) &= 1 - \mu_A(xy) \leq 1 - (\mu_A(x) \wedge \mu_A(y)) \\ &= (1 - \mu_A(x)) \vee (1 - \mu_A(y)) = \bar{\mu}_A(x) \vee \bar{\mu}_A(y) \end{aligned} \tag{3.3}$$

and $\bar{\mu}_A(xay) = 1 - \mu_A(xay) \leq 1 - \mu_A(a) = \bar{\mu}_A(a)$. Therefore, A is an intuitionistic fuzzy interior ideal of S. □

Definition 3.7. Let $A = (\mu_A, \gamma_A)$ be an IFS in S and let $\alpha \in [0,1]$. Then the sets

$$\mu_{A,\alpha}^{\geq} := \{x \in S : \mu_A(x) \geq \alpha\}, \qquad \gamma_{A,\alpha}^{\leq} := \{x \in S : \gamma_A(x) \leq \alpha\} \tag{3.4}$$

are called a *μ-level α-cut* and a *γ-level α-cut* of A, respectively.

Theorem 3.8. *If an IFS $A = (\mu_A, \gamma_A)$ in S is an intuitionistic fuzzy interior ideal of S, then the μ-level α-cut $\mu_{A,\alpha}^{\geq}$ and γ-level α-cut $\gamma_{A,\alpha}^{\leq}$ of A are interior ideals of S for every $\alpha \in \operatorname{Im}(\mu_A) \cap \operatorname{Im}(\gamma_A) \subseteq [0,1]$.*

Proof. Let $\alpha \in \operatorname{Im}(\mu_A) \cap \operatorname{Im}(\gamma_A) \subseteq [0,1]$ and let $x, y \in \mu_{A,\alpha}^{\geq}$. Then $\mu_A(x) \geq \alpha$ and $\mu_A(y) \geq \alpha$. It follows from (IF1) that

$$\mu_A(xy) \geq \mu_A(x) \wedge \mu_A(y) \geq \alpha \quad \text{so that } xy \in \mu_{A,\alpha}^{\geq}. \tag{3.5}$$

If $x, y \in \gamma_{A,\alpha}^{\leq}$, then $\gamma_A(x) \leq \alpha$ and $\gamma_A(y) \leq \alpha$, and so

$$\gamma_A(xy) \leq \gamma_A(x) \vee \gamma_A(y) \leq \alpha, \quad \text{that is, } xy \in \gamma_{A,\alpha}^{\leq}. \tag{3.6}$$

Hence $\mu_{A,\alpha}^{\geq}$ and $\gamma_{A,\alpha}^{\leq}$ are subsemigroups of S. Now let $x, y \in S$ and $a \in \mu_{A,\alpha}^{\geq}$. Then $\mu_A(xay) \geq \mu_A(a) \geq \alpha$ and so $xay \in \mu_{A,\alpha}^{\geq}$. If $a \in \gamma_{A,\alpha}^{\leq}$, then $\gamma_A(xay) \leq \gamma_A(a) \leq \alpha$ and thus $xay \in \gamma_{A,\alpha}^{\leq}$. Therefore $\mu_{A,\alpha}^{\geq}$ and $\gamma_{A,\alpha}^{\leq}$ are interior ideals of S. □

Theorem 3.9. *Let $A = (\mu_A, \gamma_A)$ be an IFS in S such that the nonempty sets $\mu_{A,\alpha}^{\geq}$ and $\gamma_{A,\alpha}^{\leq}$ are interior ideals of S for all $\alpha \in [0,1]$. Then $A = (\mu_A, \gamma_A)$ is an intuitionistic fuzzy interior ideal of S.*

Proof. Let $\alpha \in [0,1]$ and suppose that $\mu_{A,\alpha}^{\geq} (\neq \emptyset)$ and $\gamma_{A,\alpha}^{\leq} (\neq \emptyset)$ are interior ideals of S. We must show that $A = (\mu_A, \gamma_A)$ satisfies conditions (IF1)–(IF4). If condition (IF1) is false, then there exist $x_0, y_0 \in S$ such that $\mu_A(x_0 y_0) < \mu_A(x_0) \wedge \mu_A(y_0)$. Taking

$$\alpha_0 := \frac{1}{2}(\mu_A(x_0 y_0) + \mu_A(x_0) \wedge \mu_A(y_0)), \tag{3.7}$$

we have $\mu_A(x_0 y_0) < \alpha_0 < \mu_A(x_0) \wedge \mu_A(y_0)$. It follows that $x_0, y_0 \in \mu_{A,\alpha_0}^{\geq}$ and $x_0 y_0 \notin \mu_{A,\alpha_0}^{\geq}$, which is a contradiction. Hence condition (IF1) is true. The proof of other conditions are similar to the case (IF1), we omit the proof. □

Theorem 3.10. *Let M be an interior ideal of S and let $A = (\mu_A, \gamma_A)$ be an IFS in S defined by*

$$\mu_A(x) := \begin{cases} \alpha_0 & \text{if } x \in M, \\ \alpha_1 & \text{otherwise,} \end{cases} \qquad \gamma_A(x) := \begin{cases} \beta_0 & \text{if } x \in M, \\ \beta_1 & \text{otherwise,} \end{cases} \tag{3.8}$$

for all $x \in S$ and $\alpha_i, \beta_i \in [0,1]$ such that $\alpha_0 > \alpha_1$, $\beta_0 < \beta_1$, and $\alpha_i + \beta_i \le 1$ for $i = 0,1$. Then $A = (\mu_A, \gamma_A)$ is an intuitionistic fuzzy interior ideal of S and $\mu^{\gtrsim}_{A,\alpha_0} = M = \gamma^{\lesssim}_{A,\beta_0}$.

Proof. Let $x, y \in S$. If anyone of x and y does not belong to M, then

$$\begin{aligned} \mu_A(xy) &\ge \alpha_1 = \mu_A(x) \wedge \mu_A(y), \\ \gamma_A(xy) &\le \beta_1 = \gamma_A(x) \vee \gamma_A(y). \end{aligned} \tag{3.9}$$

Other cases are trivial, and we omit the proof. Hence $A = (\mu_A, \gamma_A)$ is an intuitionistic fuzzy subsemigroup of S. Now let $x, y, a \in S$. If $a \notin M$, then $\mu_A(xay) \ge \alpha_1 = \mu_A(a)$ and $\gamma_A(xay) \le \beta_1 = \gamma_A(a)$. Assume that $a \in M$. Since M is an interior ideal of S, it follows that $xay \in M$. Hence $\mu_A(xay) = \alpha_0 = \mu_A(a)$ and $\gamma_A(xay) = \beta_0 = \gamma_A(a)$. Therefore $A = (\mu_A, \gamma_A)$ is an intuitionistic fuzzy interior ideal of S. Obviously $\mu^{\gtrsim}_{A,\alpha_0} = M = \gamma^{\lesssim}_{A,\beta_0}$. □

Corollary 3.11. *Let χ_M be the characteristic function of an interior ideal M of S. Then the IFS $\tilde{M} = (\chi_M, \bar{\chi}_M)$ is an intuitionistic fuzzy interior ideal of S.*

Theorem 3.12. *If an IFS $A = (\mu_A, \gamma_A)$ is an intuitionistic fuzzy interior ideal of S, then*

$$\begin{aligned} \mu_A(x) &:= \sup\{\alpha \in [0,1] \mid x \in \mu^{\gtrsim}_{A,\alpha}\}, \\ \gamma_A(x) &:= \inf\{\alpha \in [0,1] \mid x \in \gamma^{\lesssim}_{A,\alpha}\}, \end{aligned} \tag{3.10}$$

for all $x \in S$.

Proof. Let $\delta := \sup\{\alpha \in [0,1] \mid x \in \mu^{\gtrsim}_{A,\alpha}\}$ and let $\varepsilon > 0$ be given. Then $\delta - \varepsilon < \alpha$ for some $\alpha \in [0,1]$ such that $x \in \mu^{\gtrsim}_{A,\alpha}$. It follows that $\delta - \varepsilon < \mu_A(x)$ so that $\delta \le \mu_A(x)$ since ε is arbitrary. We now show that $\mu_A(x) \le \delta$. Let $\mu_A(x) = \beta$. Then $x \in \mu^{\gtrsim}_{A,\beta}$ and so

$$\beta \in \{\alpha \in [0,1] \mid x \in \mu^{\gtrsim}_{A,\alpha}\}. \tag{3.11}$$

Hence $\mu_A(x) = \beta \le \sup\{\alpha \in [0,1] \mid x \in \mu^{\gtrsim}_{A,\alpha}\} = \delta$. Therefore

$$\mu_A(x) = \delta = \sup\{\alpha \in [0,1] \mid x \in \mu^{\gtrsim}_{A,\alpha}\}. \tag{3.12}$$

Now let $\eta = \inf\{\alpha \in [0,1] \mid x \in \gamma^{\lesssim}_{A,\alpha}\}$. Then

$$\inf\{\alpha \in [0,1] \mid x \in \gamma^{\lesssim}_{A,\alpha}\} < \eta + \varepsilon \quad \text{for any } \varepsilon < 0, \tag{3.13}$$

and so $\alpha < \eta + \varepsilon$ for some $\alpha \in [0,1]$ with $x \in \gamma^{\lesssim}_{A,\alpha}$. Since $\gamma_A(x) \le \alpha$ and ε is arbitrary, it follows that $\gamma_A(x) \le \eta$. To prove $\gamma_A(x) \ge \eta$, let $\gamma_A(x) = \zeta$. Then $x \in \gamma^{\lesssim}_{A,\zeta}$ and thus $\zeta \in \{\alpha \in [0,1] \mid x \in \gamma^{\lesssim}_{A,\alpha}\}$. Hence

$$\inf\{\alpha \in [0,1] \mid x \in \gamma^{\lesssim}_{A,\alpha}\} \le \zeta, \quad \text{that is, } \eta \le \zeta = \gamma_A(x). \tag{3.14}$$

Consequently,

$$\gamma_A(x) = \eta = \inf\{\alpha \in [0,1] \mid x \in \gamma^{\leq}_{A,\alpha}\}. \tag{3.15}$$

This completes the proof. □

Theorem 3.13. *Let $\{C_\alpha \mid \alpha \in \Lambda\}$ be a collection of interior ideals of S such that*

(i) $S = \cup_{\alpha\in\Lambda} C_\alpha$,

(ii) $\beta > \alpha$ *if and only if* $C_\beta \subset C_\alpha$ *for all* $\beta, \alpha \in \Lambda$.

Then an IFS $A = (\mu_A, \gamma_A)$ in S defined by

$$\begin{aligned} \mu_A(x) &:= \sup\{\alpha \in \Lambda \mid x \in C_\alpha\}, \\ \gamma_A(x) &:= \inf\{\alpha \in \Lambda \mid x \in C_\alpha\}, \end{aligned} \tag{3.16}$$

for all $x \in S$, is an intuitionistic fuzzy interior ideal of S.

Proof. Following Theorem 3.9, it is sufficient to show that the nonempty level sets $\mu^{\geq}_{A,\alpha}$ and $\gamma^{\leq}_{A,\alpha}$ are interior ideals of S for every $\alpha \in [0,1]$. In order to prove that $\mu^{\geq}_{A,\alpha}(\neq \emptyset)$ is an interior ideal, we have the following two cases:

(i) $\alpha = \sup\{\delta \in \Lambda \mid \delta < \alpha\}$ and

(ii) $\alpha \neq \sup\{\delta \in \Lambda \mid \delta < \alpha\}$.

Case (i) implies that

$$x \in \mu^{\geq}_{A,\alpha} \Longleftrightarrow x \in C_\delta \quad \forall \delta < \alpha \Longleftrightarrow x \in \cap_{\delta<\alpha} C_\delta, \tag{3.17}$$

so that $\mu^{\geq}_{A,\alpha} = \cap_{\delta<\alpha} C_\delta$, which is an interior ideal of S. For the case (ii), we claim that $\mu^{\geq}_{A,\alpha} = \cup_{\delta\geq\alpha} C_\delta$. If $x \in \cup_{\delta\geq\alpha} C_\delta$, then $x \in C_\delta$ for some $\delta \geq \alpha$. It follows that $\mu_A(x) \geq \delta \geq \alpha$, so that $x \in \mu^{\geq}_{A,\alpha}$. This proves that $\cup_{\delta\geq\alpha} C_\delta \subseteq \mu^{\geq}_{A,\alpha}$. Now assume that $x \notin \cup_{\delta\geq\alpha} C_\delta$. Then $x \notin C_\delta$ for all $\delta \geq \alpha$. Since $\alpha \neq \sup\{\delta \in \Lambda \mid \delta < \alpha\}$, there exists $\varepsilon > 0$ such that $(\alpha - \varepsilon, \alpha) \cap \Lambda = \emptyset$. Hence $x \notin C_\delta$ for all $\delta > \alpha - \varepsilon$, which means that if $x \in C_\delta$ then $\delta \leq \alpha - \varepsilon$. Thus $\mu_A(x) \leq \alpha - \varepsilon < \alpha$, and so $x \notin \mu^{\geq}_{A,\alpha}$. Therefore $\mu^{\geq}_{A,\alpha} \subseteq \cup_{\delta\geq\alpha} C_\delta$, and thus $\mu^{\geq}_{A,\alpha} = \cup_{\delta\geq\alpha} C_\delta$ which is an interior ideal of S. Next we prove that $\gamma^{\leq}_{A,\alpha}(\neq \emptyset)$ is an interior ideal of S for all $\alpha \in [0,1]$. We consider the following two cases:

(iii) $\beta = \inf\{\delta \in \Lambda \mid \beta < \delta\}$ and

(iv) $\beta \neq \inf\{\delta \in \Lambda \mid \beta < \delta\}$.

For the case (iii) we have

$$x \in \gamma^{\leq}_{A,\beta} \Longleftrightarrow x \in C_\delta \quad \forall \beta < \delta \Longleftrightarrow x \in \cap_{\beta<\delta} C_\delta, \tag{3.18}$$

and hence $\gamma^{\leq}_{A,\beta} = \cap_{\beta<\delta} C_\delta$ which is an interior ideal of S. For the case (iv), there exists $\varepsilon > 0$ such that $(\beta, \beta+\varepsilon) \cap \Lambda = \emptyset$. We show that $\gamma^{\leq}_{A,\beta} = \cup_{\beta\geq\delta} C_\delta$. If $x \in \cup_{\beta\geq\delta} C_\delta$, then $x \in C_\delta$ for some $\beta \geq \delta$. It follows that $\gamma_A(x) \leq \delta \leq \beta$ so that $x \in \gamma^{\leq}_{A,\beta}$. Hence $\cup_{\beta\geq\delta} C_\delta \subseteq \gamma^{\leq}_{A,\beta}$. Conversely, if $x \notin \cup_{\beta\geq\delta} C_\delta$ then $x \notin C_\delta$ for all $\delta \leq \beta$, which implies that $x \notin C_\delta$ for all $\delta < \beta + \varepsilon$, that is, if $x \in C_\delta$ then $\delta \geq \beta + \varepsilon$. Thus $\gamma_A(x) \geq \beta + \varepsilon > \beta$, that is, $x \notin \gamma^{\leq}_{A,\beta}$. Therefore $\gamma^{\leq}_{A,\beta} \subseteq \cup_{\beta\geq\delta} C_\delta$ and consequently $\gamma^{\leq}_{A,\beta} = \cup_{\beta\geq\delta} C_\delta$ which is an interior ideal of S. This completes the proof. □

Theorem 3.14. *An IFS $A = (\mu_A, \gamma_A)$ is an intuitionistic fuzzy interior ideal of S if and only if the fuzzy sets μ_A and $\bar{\gamma}_A$ are fuzzy interior ideals of S.*

Proof. Let $A = (\mu_A, \gamma_A)$ be an intuitionistic fuzzy interior ideal of S. Then clearly μ_A is a fuzzy interior ideal of S. Let $x, a, y \in S$. Then

$$\begin{aligned}\bar{\gamma}_A(xy) &= 1-\gamma_A(xy) \geq 1-\gamma_A(x) \vee \gamma_A(y)\\ &= (1-\gamma_A(x)) \wedge (1-\gamma_A(y)) = \bar{\gamma}_A(x) \wedge \bar{\gamma}_A(y),\\ \bar{\gamma}_A(xay) &= 1-\gamma_A(xay) \geq 1-\gamma_A(a) = \bar{\gamma}_A(a).\end{aligned} \tag{3.19}$$

Hence $\bar{\gamma}_A$ is a fuzzy interior ideal of S.

Conversely, suppose that μ_A and $\bar{\gamma}_A$ are fuzzy interior ideals of S. Let $a, x, y \in S$. Then

$$\begin{aligned}1-\gamma_A(xy) &= \bar{\gamma}_A(xy) \geq \bar{\gamma}_A(x) \wedge \bar{\gamma}_A(y)\\ &= (1-\gamma_A(x)) \wedge (1-\gamma_A(y))\\ &= 1-\gamma_A(x) \vee \gamma_A(y),\\ 1-\gamma_A(xay) &= \bar{\gamma}_A(xay) \geq \bar{\gamma}_A(a) = 1-\gamma_A(a),\end{aligned} \tag{3.20}$$

which imply that $\gamma_A(xy) \leq \gamma_A(x) \vee \gamma_A(y)$ and $\gamma_A(xay) \leq \gamma_A(a)$. This completes the proof. □

Corollary 3.15. *An IFS $A = (\mu_A, \gamma_A)$ is an intuitionistic fuzzy interior ideal of S if and only if $\Box A = (\mu_A, \bar{\mu}_A)$ and $\Diamond A = (\bar{\gamma}_A, \gamma_A)$ are intuitionistic fuzzy interior ideals of S.*

Proof. The proof is straightforward by Theorem 3.14. □

Let f be a map from a set X to a set Y. If $A = (\mu_A, \gamma_A)$ and $B = (\mu_B, \gamma_B)$ are IFSs in X and Y, respectively, then the *preimage* of B under f, denoted by $f^{-1}(B)$, is an IFS in X defined by

$$f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\gamma_B)), \quad \text{where } f^{-1}(\mu_B) = \mu_B(f). \tag{3.21}$$

Theorem 3.16. *Let $f : S \to T$ be a homomorphism of semigroups. If $B = (\mu_B, \gamma_B)$ is an intuitionistic fuzzy interior ideal of T, then the preimage $f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\gamma_B))$ of B under f is an intuitionistic fuzzy interior ideal of S.*

Proof. Assume that $B = (\mu_B, \gamma_B)$ is an intuitionistic fuzzy interior ideal of T and let $x, y \in S$. Then

$$\begin{aligned}f^{-1}(\mu_B)(xy) &= \mu_B(f(xy))\\ &= \mu_B(f(x)f(y))\\ &\geq \mu_B(f(x)) \wedge \mu_B(f(y))\\ &= f^{-1}(\mu_B(x)) \wedge f^{-1}(\mu_B(y)),\\ f^{-1}(\gamma_B)(xy) &= \gamma_B(f(xy))\\ &= \gamma_B(f(x)f(y))\\ &\leq \gamma_B(f(x)) \vee \gamma_B(f(y))\\ &= f^{-1}(\gamma_B(x)) \vee f^{-1}(\gamma_B(y)).\end{aligned} \tag{3.22}$$

Hence $f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\gamma_B))$ is an intuitionistic fuzzy subsemigroup of S. For any $a, x, y \in S$, we have

$$\begin{aligned} f^{-1}(\mu_B)(xay) &= \mu_B(f(xay)) \\ &= \mu_B(f(x)f(a)f(y)) \\ &\geq \mu_B(f(a)) \\ &= f^{-1}(\mu_B(a)), \\ f^{-1}(\gamma_B)(xay) &= \gamma_B(f(xay)) \\ &= \gamma_B(f(x)f(a)f(y)) \\ &\leq \gamma_B(f(a)) \\ &= f^{-1}(\gamma_B(a)). \end{aligned} \tag{3.23}$$

Therefore $f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\gamma_B))$ is an intuitionistic fuzzy interior ideal of S. $\square$

Acknowledgement. The authors are deeply grateful to the referees for the valuable suggestions.

References

[1] K. T. Atanassov, *Intuitionistic fuzzy sets*, Fuzzy Sets and Systems **20** (1986), no. 1, 87–96. MR 87f:03151. Zbl 631.03040.

[2] ———, *New operations defined over the intuitionistic fuzzy sets*, Fuzzy Sets and Systems **61** (1994), no. 2, 137–142. CMP 1 262 464. Zbl 824.04004.

[3] S. M. Hong, Y. B. Jun, and J. Meng, *Fuzzy interior ideals in semigroups*, Indian J. Pure Appl. Math. **26** (1995), no. 9, 859–863. CMP 1 347 533. Zbl 834.20074.

[4] J. M. Howie, *An Introduction to Semigroup Theory*, Academic Press, London, 1976. MR 57#6235. Zbl 355.20056.

Kyung Ho Kim: Department of Mathematics, Chungju National University, Chungju 380-702, Korea

E-mail address: ghkim@gukwon.chungju.ac.kr

Young Bae Jun: Department of Mathematics Education, Gyeongsang National University, Chinju 660-701, Korea

E-mail address: ybjun@nongae.gsnu.ac.kr