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Research Article

On the Positive Almost Periodic Solutions of a Class of Nonlinear Lotka-Volterra Type System with Feedback Control

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With the help of the variable substitution and applying the fixed point theorem, we derive the sufficient conditions which guarantee the existence of the positive almost periodic solutions for a class of Lotka-Volterra type system. The main results improve and generalize the former corresponding results.

1. Introduction

Denote $x(t) = (x_1(t), x_2(t), \dots, x_n(t))$, $x_t = (x_{1t}, x_{2t}, \dots, x_{nt})$, $x_{it}(s) = x_i(t+s)$ $(i = 1, 2, \dots, n)$, $s \in [-\tau, 0], \tau$ is a positive constant or $\tau = +\infty$, the norm of a bounded continuous function space $C([-\tau, 0], R^n)$ is defined as $\|\phi\| = \max_{s\in[-\tau,0]}|\phi(s)|$, where $|\phi| = \max\{|\phi_i|,\ i = 1, 2, \dots, n\}$, $C([-\tau, 0], R_+) = \{\phi \in C([-\tau, 0], R) : \text{for all } \theta \in [-\tau, 0],\ \phi(\theta) > 0\}$.

We call an almost periodic function is positive if and only if each component has its positive infimum. Denote $AP(R^n) = \{x(t) \in R^n : x(t)$ is a continuous almost periodic function on $R^n\}$, $AP(R)=\{x(t) \in R : x(t)$ is a continuous almost periodic function on $R\}$, $T_\alpha f(t) = g(t)$ refers to $\lim_{n\to\infty} f(t + \alpha_n) = g(t)$, where $\alpha = \{\alpha_n\}, \{\alpha_n\}$ is a sequence of real numbers, $T(f, \varepsilon) = \{\tau : |f(t+\tau) - f(t)| < \varepsilon, t \in R\}$, for more almost periodic monographs, see references [1, 2].

In [3], Teng first studied the existence of the almost periodic solutions for the following scalar equation:

$$\frac{du}{dt} = u\big(\alpha(t) - \beta(t)\big)u, \tag{1.1}$$

where $t \in R, u \in R$, and $\alpha(t),\ \beta(t) \in AP(R)$, then based on (1.1) and combined Schauder fixed point theorem, he studied the existence of the almost periodic solution for a class of Lotka-Volterra system

$$\frac{dx_i(t)}{dt} = x_i(t)\big(a_i(t) - b_i(t)x_i(t) - f_i(t, x_t)\big), \quad (i = 1, 2, \ldots, n), \tag{1.2}$$

where $a_i(t)$, $b_i(t)$ are continuous almost periodic functions, $f_i(t,\phi)$ is a continuous almost periodic function in t uniformly with respect to $\phi \in C([-\tau, 0], R^n)$, and $f_i(t, 0) \equiv 0$ ($i = 1, 2, \ldots, n$). The author of [4] inherited in the method and ideas in [3], and promoted its conclusions to the following system with feedback control

$$\begin{aligned} \frac{dx_i(t)}{dt} &= x_i(t)\big(a_i(t) - b_i(t)x_i(t) - f_i(t, x_t, u_t)\big), \\ \frac{du_i(t)}{dt} &= -c_i(t)u_i(t) + h_i(t, x_{it}), \end{aligned} \tag{1.3}$$

where $u_i(t)$ is the control variable, $i = 1, 2, \ldots, n$.

The authors of [5] used transformation techniques and fixed point theorem and studied a time-delay system with feedback control which is much wider then the system (1.3),

$$\begin{aligned} \frac{dx_i(t)}{dt} &= x_i(t)\big(a_i(t) - b_i(t)x_i(t) - f_i(t, x_t, u_t)\big), \\ \frac{du_i(t)}{dt} &= -c_i(t)u_i(t) + h_i(t, x_t). \end{aligned} \tag{1.4}$$

In the case of non-Lipschitz condition, they gave a sufficient condition of the existence of the almost periodic solutions for the system (1.4).

However, in the real world, the competition between species is not always shown by the linear relationship, while shown by a certain degree of nonlinearity, therefore, studying the following system becomes more realistic and necessary, over this paper, we study the system which is more extensive than the system (1.4) as follows:

$$\begin{aligned} \frac{dx_i(t)}{dt} &= x_i(t)\big(a_i(t) - b_i(t)x_i^{\alpha_i}(t) - f_i(t, x_t, u_t)\big), \\ \frac{du_i(t)}{dt} &= -c_i(t)u_i(t) + h_i(t, x_t), \end{aligned} \tag{1.5}$$

where $u_i(t)$ is the control variable, α_i is a positive constant, $i = 1, 2, \ldots, n$, by using Schauder's fixed point theorem, we get the sufficient conditions of the existence of the almost periodic solution for the system (1.5), and by using the contraction mapping principle, we give the conclusion of the existence of a unique almost periodic solution for the system (1.5) in the one-dimensional case, some new results are obtained.

2. Some Related Lemmas

Lemma 2.1 (see [6]). *For the equation*

$$\frac{dx}{dt} = a(t)x(t) + b(t), \tag{2.1}$$

where $a(t), b(t) \in AP(R)$ *and are continuous, if* $\mathrm{Re}\ m(a(t)) \neq 0$, *then* (2.1) *exists a unique almost periodic solution* $\eta(t)$, $\mathrm{mod}(\eta) \subset \mathrm{mod}(a, b)$, *and* $\eta(t)$ *can be written as follows*

$$\eta(t) = \begin{cases} \displaystyle\int_{-\infty}^{t} e^{\int_s^t a(\tau)d\tau} b(s)ds, & \mathrm{Re}\ m(a(t)) < 0, \\ \displaystyle -\int_{t}^{+\infty} e^{\int_s^t a(\tau)d\tau} b(s)ds, & \mathrm{Re}\ m(a(t)) > 0, \end{cases} \tag{2.2}$$

where $m(a(t)) = \lim_{T \to +\infty} 1/T \int_0^T a(t)dt$, $\mathrm{Re}\ m(a(t))$ *is the real part of* $m(a(t))$.

Lemma 2.2 (see [2]). *Suppose* $f(t), g(t) \in AP(R)$, *then the following conditions are equivalent.*

(1) $\mathrm{mod}(f) \supset \mathrm{mod}(g)$;

(2) *for any* $\varepsilon > 0$, $\exists \delta > 0$, *such that* $T(f, \delta) \subset T(g, \varepsilon)$;

(3) $T_\alpha f$ *exists which implies* $T_\alpha g$ *exists (any sense);*

(4) $T_\alpha f = f$, *implies* $T_\alpha g = g$ *(any sense);*

(5) $T_\alpha f = f$, *implies there is* $\alpha' \subset \alpha$, *so that* $T_{\alpha'} g = g$ *(any sense).*

Consider the equation

$$\frac{dx(t)}{dt} = x(t)[a(t) - b(t)x^{\alpha}(t)], \tag{2.3}$$

where $\alpha > 0$, $a(t)$, $b(t) \in AP(R)$ *and are continuous on* R.

Lemma 2.3. *If* (2.3) *satisfies one of the following conditions*

(1) $m(a(t)) > 0$, $m(b(t)) > 0$, $b(t) \geq 0$

(2) $m(a(t)) < 0$, $m(b(t)) < 0$, $b(t) \leq 0$,

then (2.3) exists a unique positive almost periodic solution $\phi(t)$, and $\mathrm{mod}(\phi) \subset \mathrm{mod}(a, b)$.

Proof. Let $x^{-\alpha}(t) = u(t)$, then (2.3) can be changed as follows:

$$\frac{du(t)}{dt} = -\alpha a(t)u(t) + \alpha b(t), \tag{2.4}$$

by the condition $m(a(t)) > 0,\ \alpha > 0$, we have Re $m(-\alpha a(t)) < 0$, also by the condition $m(b(t)) > 0,\ b(t) \geq 0$, according to Lemma 2.1, it follows that (2.4) exists a unique positive almost periodic solution $\eta(t)$, and $\eta(t)$ can be written as follows:

$$\eta(t) = \alpha \int_{-\infty}^{t} e^{-\alpha \int_s^t a(\tau)d\tau} b(s)ds. \tag{2.5}$$

Next, we prove that $\mathrm{mod}(\eta) \subset \mathrm{mod}(a,b)$. If there is a sequence β' such that $T_{\beta'}a(t) = a(t),\ T_{\beta'}b(t) = b(t)$ are convergent uniformly on R, then there exists a sequence $\beta \subset \beta'$ such that $T_\beta \eta(t) = \zeta(t)$ is convergent uniformly on R, and $\zeta(t)$ is also an almost periodic solution of (2.4), by the uniqueness of the almost periodic solution of (2.4), we can get $T_\beta \eta(t) = \eta(t)$, by Lemma 2.2, it follows $\mathrm{mod}(\eta) \subset \mathrm{mod}(a,b)$. Since $x^{-\alpha}(t) = u(t)$, (2.3) exists a unique positive almost periodic solution $\phi(t)$, and it can be written as follows

$$\phi(t) = \left[\alpha \int_{-\infty}^{t} e^{-\alpha \int_s^t a(\tau)d\tau} b(s)ds\right]^{-1/\alpha}, \tag{2.6}$$

by (2.6), we can easily get $\mathrm{mod}(\phi) \subset \mathrm{mod}(a,b)$.

If the condition (2) holds, similarly, we can prove that (2.3) exists a unique positive almost periodic solution $\phi(t)$, it can be written as follows

$$\phi(t) = \left[-\alpha \int_{t}^{+\infty} e^{-\alpha \int_s^t a(\tau)d\tau} b(s)ds\right]^{-1/\alpha}, \tag{2.7}$$

and $\mathrm{mod}(\phi) \subset \mathrm{mod}(a,b)$. This is the end of the proof of Lemma 2.3. □

Lemma 2.4 (see [2]). *Suppose that an almost periodic sequence $\{f_n(t)\}$ is convergent uniformly on any compact set of R, $f(t)$ is an almost periodic function, and* $\mathrm{mod}(f_n) \subset \mathrm{mod}(f)$ $(n = 1,2,\ldots)$, *then $\{f_n(t)\}$ is convergent uniformly on R.*

3. The Conclusion of the N-dimentional System

Consider the following equation

$$\frac{dx_i(t)}{dt} = x_i(t)\left[a_i(t) - b_i(t)x_i^{\alpha_i}(t)\right], \tag{3.1}$$

where $\alpha_i > 0,\ a_i(t),\ b_i(t) \in AP(R),\ i = 1,2,\ldots,n$.

By Lemma 2.3, it follows if one of the following conditions holds

(A_1) $m(a_i(t)) > 0,\ m(b_i(t)) > 0,\ b_i(t) \geq 0$

(A_2) $m(a_i(t)) < 0,\ m(b_i(t)) < 0,\ b_i(t) \leq 0,$

then (3.1) exists a unique positive almost periodic solution $x^0(t) = (x_1^0(t), x_2^0(t), \ldots, x_n^0(t))$, we can easily get if (A_1) holds, then

$$x_i^0(t) = \left[\alpha_i \int_{-\infty}^{t} e^{-\alpha_i \int_s^t a_i(\tau)d\tau} b_i(s)ds\right]^{-1/\alpha_i}, \tag{3.2}$$

if (A_2) holds, then

$$x_i^0(t) = \left[-\alpha_i \int_{t}^{+\infty} e^{-\alpha_i \int_s^t a_i(\tau)d\tau} b_i(s)ds\right]^{-1/\alpha_i}. \tag{3.3}$$

Now, consider (1.5), suppose

(B_1) $a_i(t),\ b_i(t),\ c_i(t)$ are continuous almost periodic functions, $f_i(t,\phi,\psi)$ is continuous for all variables, and is almost periodic in t uniformly with respect to $(\phi,\psi) \in C([-\tau,0],R^n) \times C([-\tau,0],R^n)$, $h_i(t,\phi)$ is continuous for all variables, and is almost periodic in t uniformly with respect to $\phi \in C([-\tau,0],R^n)$;

(B_2) $h_i(t,\phi) > 0$, for all $\phi_i > 0, t \in R$ $(i = 1,2,\ldots,n)$;

$(\overline{B}_2)$ $h_i(t,\phi) < 0$, for all $\phi_i > 0, t \in R$ $(i = 1,2,\ldots,n)$;

(B_3) $m(a_i(t)) > 0,\ m(b_i(t)) > 0,\ b_i(t) \geq 0,\ m(c_i(t)) > 0\ (i = 1,2,\ldots,n)$;

$(\overline{B}_3)$ $m(a_i(t)) < 0,\ m(b_i(t)) < 0,\ b_i(t) \leq 0,\ m(c_i(t)) < 0\ (i = 1,2,\ldots,n)$.

Following the front of the $x^0(t)$ defined, suppose that (B_1), (B_2), (B_3) or (B_1), $(\overline{B}_2)$, $(\overline{B}_3)$ hold, and denoted the unique positive almost periodic solution of the following equation

$$\frac{du_i(t)}{dt} = -c_i(t)u_i(t) + h_i\left(t, x_t^0\right), \tag{3.4}$$

by $u^0(t) = (u_1^0(t), u_2^0(t), \ldots, u_n^0(t))$, where

$$u_i^0(t) = \begin{cases} \displaystyle\int_{-\infty}^{t} e^{\int_s^t (-c_i(\tau))d\tau} h_i\left(s, x_s^0\right)ds, & \operatorname{Re}\ m(c_i(t)) > 0, \\ \displaystyle -\int_{t}^{+\infty} e^{\int_s^t (-c_i(\tau))d\tau} h_i\left(s, x_s^0\right)ds, & \operatorname{Re}\ m(c_i(t)) < 0. \end{cases} \tag{3.5}$$

For any $(\phi,\psi) \in C([-\tau,0],R^n) \times C([-\tau,0],R^n)$, we define

$$\begin{gathered} F_1(t,\phi,\psi) = \left(a_1(t) - b_1(t)\phi_1^{\alpha_1}(0) - f_1(t,\phi,\psi), \ldots, a_n(t) - b_n(t)\phi_n^{\alpha_n}(0) - f_n(t,\phi,\psi)\right), \\ H_1(t,\phi,\psi) = \left(-c_1(t)\psi_1(0) + h_1(t,\phi), \ldots, -c_n(t)\psi_n(0) + h_n(t,\phi)\right) \\ X_1 = \left\{ (\phi(t),\psi(t)) \in\ AP\left(R^{2n}\right) : \operatorname{mod}\ (\phi,\psi) \subset \operatorname{mod}\ (F_1, H_1) \right\}, \end{gathered} \tag{3.6}$$

then X_1 is a Banach space with the model of $\|(\phi(t),\psi(t))\| = \sup_{t\in R}(|\phi(t)| + |\psi(t)|)$, construct a bounded closed convex set G_1 as follows:

$$G_1 = \left\{(\phi,\psi) \in X_1 : 0 \le \phi_i(t) \le x_i^0(t),\ 0 \le \psi_i(t) \le u_i^0(t)\right\}. \tag{3.7}$$

(B_4) For any $(\phi,\psi) \in G_1$ and $t \in R$, $0 \le f_i(t,\phi_t,\psi_t) \le f_i(t,x_t^0,u_t^0)$, $0 \le h_i(t,\phi_t) \le h_i\ (t,x_t^0)$, $m(a_i(t) - f_i\ (t,x_t^0,u_t^0)) > 0$, $(i = 1,2,\ldots,n)$

($\overline{B_4}$) For any $(\phi,\psi) \in G_1$ and $t \in R$, $0 \ge f_i(t,\phi_t,\psi_t) \ge f_i(t,x_t^0,u_t^0)$, $0 \ge h_i(t,\phi_t) \ge h_i(t,x_t^0)$, $m(a_i(t) - f_i(t,x_t^0,u_t^0)) < 0$, $(i = 1,2,\ldots,n)$.

Theorem 3.1. *If* (B_1), (B_2), (B_3), (B_4) *or* (B_1), ($\overline{B}_2$), ($\overline{B}_3$), ($\overline{B}_4$) *hold, then* (1.5) *exists at least a positive almost periodic solution in* G_1.

Proof. If (B_1), (B_2), (B_3) hold, for any $(\phi,\psi) \in G_1$, consider the following equation

$$\frac{dx_i(t)}{dt} = x_i(t)\left(a_i(t) - f_i\left(t,\phi_t,\psi_t\right) - b_i(t)x_i^{\alpha_i}(t)\right), \quad (i = 1,2,\ldots,n) \tag{3.8}$$

by the condition (B_4), it follows $m(a_i(t) - f_i(t,\phi_t,\psi_t)) > 0$, by Lemma 2.3, it follows that (3.8) exists a unique positive almost periodic solution $Z_{\phi\psi}(t) = (Z_{\phi\psi_1}(t), Z_{\phi\psi_2}(t),\ldots,Z_{\phi\psi_n}(t))$, and it can be written as follows

$$Z_{\phi\psi i}(t) = \left[\alpha_i \int_{-\infty}^{t} e^{-\alpha_i \int_s^t (a_i(\tau) - f_i(\tau,\phi_\tau,\psi_\tau))d\tau} b_i(s)ds\right]^{-1/\alpha_i}, \quad (i = 1,2,\ldots,n), \tag{3.9}$$

and mod $(Z_{\phi\psi i}(t)) \subset$ mod $(a_i(t) - f_i(t,\phi_t,\psi_t), b_i(t))$, since $f_i(\tau,\phi_\tau,\psi_\tau) - a_i(\tau) > -a_i(\tau)$, thus by (3.2) and (3.9), we can get $0 < Z_{\phi\psi i}(t) \le x_i^0(t)$, $i = 1,2,\ldots,n$, on the other hand, since mod $(a_i(t) - f_i(t,\phi_t,\psi_t), b_i(t)) \subset$ mod (F_1), it follows mod$(Z_{\phi\psi}(t)) \subset$ mod $(F_1)_\circ$.

Consider the following equation

$$\frac{du_i(t)}{dt} = -c_i(t)u_i(t) + h_i\left(t,\phi_t\right), \tag{3.10}$$

by Lemma 2.1, (3.10) exists a unique positive almost periodic solution

$$V_\phi(t) = \left(V_{\phi_1}(t), V_{\phi_2}(t),\ldots,V_{\phi_n}(t)\right), \tag{3.11}$$

and it can be written as follows:

$$V_{\phi i}(t) = \int_{-\infty}^{t} e^{\int_s^t(-c_i(\tau))d\tau} h_i\left(s,\phi_s\right)ds, \quad \text{Re}\ \ m(c_i(t)) > 0, \tag{3.12}$$

also by the condition $0 \leq h_i(t,\phi_t) \leq h_i(t,x_t^0)$, $(i = 1,2,\ldots,n)$, the first formula of (3.5) and (3.12), we have $0 < V_{\phi i}(t) \leq u_i^0(t)$, and $\operatorname{mod}(V_{\phi i}(t)) \subset \operatorname{mod}(c_i(t),\ \mathrm{h_i}(t,\phi_t))$, also mod $(c_i(t),h_i(t,\phi_t)) \subset \operatorname{mod}(H_1)$, thus $\operatorname{mod}(V_{\phi i}(t)) \subset \operatorname{mod}(H_1)$, therefore, we can get mod $(Z_{\phi\psi},V_\phi) \subset$ mod (F_1,H_1), hence we can define

$$P : G_1 \longrightarrow G_1, \qquad P(\phi,\psi) = (Z_{\phi\psi},V_\phi). \tag{3.13}$$

Now we are committed to prove the continuity of P in G_1. Suppose $\{(\phi^k,\psi^k) \subset G_1\}$, and when $k \longrightarrow \infty$, $(\phi^k,\psi^k) \longrightarrow (\phi,\psi) \in G_1$, define $S = \{\phi^k,\psi^k\} \cup \{(\phi,\psi)\}$, then S is a compact set of X_1, since $m(a_i(t) - f_i(\tau,x_\tau^0,u_\tau^0)) > 0$, $i = 1,2,\ldots,n$, there exist the positive numbers λ and H, such that when $t \geq s, e^{-\alpha_i\int_s^t(a_i(\tau)-f_i(\tau,x_\tau^0,u_\tau^0))d\tau} \leq He^{\lambda(s-t)}$, $i = 1,2,\ldots,n$, similarly, since $m(c_i(t)) > 0$, $i = 1,2,\ldots,n$, there exist the positive numbers μ and I, such that when $t \geq s$, $e^{\int_s^t(-c_i(\tau))d\tau} \leq Ie^{u(s-t)}$, for any $(\phi,\psi) \in G_1$, it follows $0 \leq f_i(t,\phi_t,\psi_t) \leq f_i(t,x_t^0,u_t^0)$.

Therefore, by the mean value theorem

$$\begin{aligned}
&\left|e^{-\alpha_i\int_s^t(a_i(\tau)-f_i(\tau,\phi_\tau^k,\psi_\tau^k))d\tau} - e^{-\alpha_i\int_s^t(a_i(\tau)-f_i(\tau,\phi_\tau,\psi_\tau))d\tau}\right| \\
&\quad = \left|\alpha_i\int_s^t\left(f_i\left(\tau,\phi_\tau^k,\psi_\tau^k\right) - f_i(\tau,\phi_\tau,\psi_\tau)\right)d\tau\right|e^{\xi} \\
&\quad \leq \left|\alpha_i\int_s^t\left(f_i\left(\tau,\phi_\tau^k,\psi_\tau^k\right) - f_i(\tau,\phi_\tau,\psi_\tau)\right)d\tau\right|e^{-\alpha_i\int_s^t(a_i(\tau)-f_i(\tau,x_\tau^0,u_\tau^0))d\tau} \\
&\quad \leq (t-s)\left\|\alpha_i\left(f_i\left(t,\phi_t^k,\psi_t^k\right) - f_i(t,\phi_t,\psi_t)\right)\right\|He^{\lambda(s-t)}, \quad t \geq s,\ i = 1,2,\cdots,n.
\end{aligned} \tag{3.14}$$

Where,

$$\begin{aligned}
\xi &\leq \max\left\{-\alpha_i\int_s^t\left(a_i(\tau) - f_i\left(\tau,\phi_\tau^k,\psi_\tau^k\right)\right)d\tau,\ -\alpha_i\int_s^t(a_i(\tau) - f_i(\tau,\phi_\tau,\psi_\tau))d\tau\right\} \\
&\leq -\alpha_i\int_s^t\left(a_i(\tau) - f_i\left(\tau,x_\tau^0,u_\tau^0\right)\right)d\tau.
\end{aligned} \tag{3.15}$$

Also by the mean value theorem, we can obtain the following:

$$\begin{aligned}
&\left\|P_i\left(\phi^k,\psi^k\right) - P_i(\phi,\psi)\right\| \\
&\quad = \left\|\left(Z_{\phi^k\psi^k i}(t) - Z_{\phi\psi i}(t), V_{\phi^k i}(t) - V_{\phi i}(t)\right)\right\| \\
&\quad = \left\|\left(-\frac{\left[Z_{\phi^k\psi^k i}(t)\right]^{-\alpha_i} - \left[Z_{\phi\psi i}(t)\right]^{-\alpha_i}}{\alpha_i\zeta_i^{-\alpha_i-1}}, V_{\phi^k i}(t) - V_{\phi i}(t)\right)\right\|,
\end{aligned} \tag{3.16}$$

where, ζ_i is between the positive almost periodic function $Z_{\phi^k\psi^k i}(t)$ and $Z_{\phi\psi i}(t)$, thus

$$0 < \ \zeta_i \leq \ x_i^0(t). \tag{3.17}$$

By (3.9), (3.12), and (3.14), it follows that

$$\begin{aligned}
&\left| -\frac{\left[Z_{\phi^k\psi^k i}(t)\right]^{-\alpha_i} - \left[Z_{\phi\psi i}(t)\right]^{-\alpha_i}}{\alpha_i \zeta_i^{-\alpha_i-1}} \right| \\
&\quad= \left| \frac{1}{\zeta_i^{-\alpha_i-1}} \int_{-\infty}^{t} \left(e^{-\alpha_i \int_s^t (a_i(\tau) - f_i(\tau,\phi_\tau^k,\psi_\tau^k))d\tau} - e^{-\alpha_i \int_s^t (a_i(\tau) - f_i(\tau,\phi_\tau,\psi_\tau))d\tau} \right) b_i(s) ds \right| \\
&\quad\le \frac{1}{\zeta_i^{-\alpha_i-1}} \int_{-\infty}^{t} \left| e^{-\alpha_i \int_s^t (a_i(\tau) - f_i(\tau,\phi_\tau^k,\psi_\tau^k))d\tau} - e^{-\alpha_i \int_s^t (a_i(\tau) - f_i(\tau,\phi_\tau,\psi_\tau))d\tau} \right| |b_i(s)| ds \\
&\quad\le \frac{H}{\left(x_i^0(t)\right)^{-\alpha_i-1}} \|\alpha_i\| \|b_i(t)\| \left\| f_i\left(t,\phi_t^k,\psi_t^k\right) - f_i(t,\phi_\tau,\psi_\tau) \right\| \int_{-\infty}^{t} (t-s) e^{\lambda(s-t)} ds \\
&\quad\le \frac{H}{\lambda^2} \left\| \left(x_i^0(t)\right)^{\alpha_i+1} \right\| \|\alpha_i\| \|b_i(t)\| \left\| f_i\left(t,\phi_t^k,\psi_t^k\right) - f_i(t,\phi_t,\psi_t) \right\|, \\
&\left| V_{\phi^k i}(t) - V_{\phi i}(t) \right| \\
&\quad= \left| \int_{-\infty}^{t} e^{\int_s^t (-c_i(\tau))d\tau} h_i\left(s,\phi_s^k\right) ds - \int_{-\infty}^{t} e^{\int_s^t (-c_i(\tau))d\tau} h_i(s,\phi_s) ds \right| \\
&\quad\le \int_{-\infty}^{t} e^{\int_s^t (-c_i(\tau))d\tau} \left| h_i\left(s,\phi_s^k\right) - h_i(s,\phi_s) \right| ds \\
&\quad\le \frac{I}{\mu} \left\| h_i\left(t,\phi_t^k\right) - h_i(t,\phi_t) \right\|,
\end{aligned} \tag{3.18}$$

thus we have

$$\begin{aligned}
\left\| P_i\left(\phi^k,\psi^k\right) - P_i(\phi,\psi) \right\| \le{}& \frac{H}{\lambda^2} \left\| \left(x_i^0(t)\right)^{\alpha_i+1} \right\| \|\alpha_i\| \|b_i(t)\| \left\| f_i\left(t,\phi_t^k,\psi_t^k\right) - f_i(t,\phi_t,\psi_t) \right\| \\
&+ \frac{I}{\mu} \left\| h_i\left(t,\phi_t^k\right) - h_i(t,\phi_t) \right\|.
\end{aligned} \tag{3.19}$$

Hence

$$\begin{aligned}
\left\| P\left(\phi^k,\psi^k\right) - P(\phi,\psi) \right\| \le{}& \frac{H}{\lambda^2} \left\| \left(x^0(t)\right)^{\alpha+1} \right\| \|\alpha\| \|b(t)\| \left\| f\left(t,\phi_t^k,\psi_t^k\right) - f(t,\phi_t,\psi_t) \right\| \\
&+ \frac{I}{\mu} \left\| h\left(t,\phi_t^k\right) - h(t,\phi_t) \right\|.
\end{aligned} \tag{3.20}$$

In addition, taking into account that $f(t,\phi,\psi)$, $h(t,\phi)$ are continuous uniformly on $R \times S$, when $k \to \infty$, it follows $\|f(t,\phi_t^k,\psi_t^k) - f(t,\phi_t,\psi_t)\| \to 0$, $\|h(t,\phi_t^k) - h(t,\phi_t)\| \to 0$, thus when $k \to \infty$, we have

$$\left\| P\left(\phi^k,\psi^k\right) - P(\phi,\psi) \right\| \longrightarrow 0, \tag{3.21}$$

therefore, P is continuous. □

Then to prove P is relatively compact in G_1. By the boundedness of G_1 and (3.8), (3.10), we can obtain that there exists a positive number A such that $\|d(P(\phi,\psi))/dt\| \leq A$, for all $(\phi,\psi) \in G_1$, thus $P(G_1)$ is uniformly bounded and equicontinuous on R, by the theorem of Ascoli-arzela, for any sequence $\{(\xi^k,\zeta^k)\}$ in G_1, there exist a subsequences (also denoted by $\{(\xi^k,\zeta^k)\}$) such that $\{(\xi^k,\zeta^k)\}$ is convergent uniformly in any compact set of R, also combined with Lemma 2.4, $\{(\xi^k,\zeta^k)\}$ is convergent uniformly on R, that is to say P is relatively compact in G_1. According to Schauder's fixed point theorem, P exists as a fixed point in G_1, that is to say (1.5) exists at least a positive almost periodic solution in G_1. Similarly, when (B_1), (B_2), $(\overline{B}_3)$, $(\overline{B}_4)$ hold, we can prove that (1.5) exists at least a positive almost periodic solution in G_1.

Remark 3.2. When $\alpha_i \equiv 1$, then (1.5) turns into (1.4), obviously, Theorem 3.1 is the generalization of Theorems 2.1 and 2.2 in [5];

Remark 3.3. In Theorem 2.2 of [5], it requires the condition $f_i(t,0,0) \equiv 0$, in this paper, we do not require $f_i(t,0,0) \equiv 0$, this can be seen in the weak conditions we get the similar results, thus Theorem 3.1 extends the results of Theorem 2.2 of the paper [5].

4. The Conclusions Of The One-dimentional System

Here we will discuss the system (1.5) in the one-dimensional case

$$\begin{aligned} \frac{dx(t)}{dt} &= x(t)\big(a(t) - b(t)x^{\alpha}(t) - f(t,x_t,u_t)\big), \\ \frac{du(t)}{dt} &= -c(t)u(t) + h(t,x_t), \end{aligned} \tag{4.1}$$

where $\alpha > 0$, $t \in R$, $a(t)$, $b(t)$, $c(t)$ are continuous almost periodic functions, $f(t,\phi,\psi)$ is almost periodic in t uniformly with respect to $(\phi,\psi) \in C([-\tau,0],R_+) \times C([-\tau,0],R_+)$, and $f\ (t,0,0) \equiv 0, h(t,\phi)$ is almost periodic in t uniformly with respect to $\phi \in C([-\tau,0],R_+)$, and $h(t,0) \equiv 0$. For convenience, we introduce the following notations. For a continuous bounded function $f(t)$, denote $f_L = \inf_{t\in R} f(t), f_M = \sup_{t\in R} f(t)$,

(C_1) $a_L > 0,\ b_L > 0,\ c_L > 0$;

(C_2) $f(t,\phi,\psi) \geq 0,\ h(t,\phi) \geq 0$ for $\phi \geq 0,\ \psi \geq 0$ and $t \in R$;

(C_3) $|f(t,x_1,y_1) - f(t,x_2,y_2)| \leq L_1(|x_1-x_2| + |y_1-y_2|),\ |h(t,x_1) - h(t,x_2)| \leq L_2|x_1-x_2|$

for $t \in R, x_1,\ x_2, y_1, y_2 \in R_+, L_1, L_2$ are positive constants;

(C_4) $L_1\gamma_1/\alpha a_L\ \beta^{1/\alpha+1} + L_1/a_L\ \beta^{1/\alpha} + L_1\gamma_2/a_L + L_2/\alpha c_L\ \beta^{1/\alpha+1} + L_1\gamma_1/a_L < 1$, where, $\beta = \min\{b_L/a_M(\alpha^2+\alpha+1)\ , [b_L/a_M\ (\alpha^2+\alpha+1)]^{\alpha/2\alpha+1}\}$, $\gamma_1 = (b_M+\alpha a_L\ (\beta+\alpha\beta^{2+(1/\alpha)}))/a_L, \gamma_2 = \alpha\beta$.

In the next paper, we suppose that initial values $(x(\theta),u(\theta))$ of system (4.1) satisfy $x(\theta) \geq 0$, $u(\theta) \geq 0$, $\theta \in [-\tau,0]$, for the solution $(x(t),u(t))$ of the system (4.1), we only consider its positive initial value, that is to say

$$x(0) > 0, \qquad u(0) > 0. \tag{4.2}$$

It is not difficult for us to see the solution $(x(t), u(t))$ of (4.1) with the positive initial value is always positive.

Theorem 4.1. *If* (C_1), (C_2), (C_3), (C_4) *hold, then* (4.1) *exists a unique positive almost periodic solution.*

Proof. Let $y(t) = 1/x^\alpha(t)$, then (4.1) can be changed into the following equation:

$$\begin{aligned}\frac{dy(t)}{dt} &= -\alpha a(t)y(t) + \alpha b(t) + \alpha y(t)\mathrm{f}\left(t, y_t^{-1/\alpha}, u_t\right),\\ \frac{du(t)}{dt} &= -c(t)u(t) + h\left(t, y_t^{-1/\alpha}\right).\end{aligned} \tag{4.3}$$

For $(\phi, \psi) \in C([-\tau, 0], R_+) \times C([-\tau, 0], R_+)$, define

$$\begin{gathered}F(t, \phi, \psi) = -a(t)\phi(0) + b(t) + f\left(t, \phi^{-1/\alpha}, \psi\right)\phi(0),\\ H(t, \phi, \psi) = -c(t)\psi(0) + h\left(t, \phi^{-1/\alpha}\right),\\ X = \left\{(\phi, \psi) \in AP\left(R^2\right) : \beta \le \phi \le \gamma_1,\ 0 < \psi \le \gamma_2,\ \operatorname{mod}(\phi, \psi) \subset \operatorname{mod}(F, H)\right\},\end{gathered} \tag{4.4}$$

then X is a Banach space with the model of $\|\phi, \psi\| = \sup_{t\in R}(|\psi(t)| + |\psi(t)|)$, for all $(\phi, \psi) \in X$, consider the following equation:

$$\frac{dy(t)}{dt} = -\alpha a(t)y(t) + \alpha b(t) + \alpha\phi(t)f\left(t, \phi_t^{-1/\alpha}, \psi_t\right), \tag{4.5}$$

by Lemma 2.1, it follows that (4.5) exists a unique positive almost periodic solution $Z_{\phi\psi}(t)$, it can be written as follows:

$$Z_{\phi\psi}(t) = \alpha\int_{-\infty}^{t} e^{-\alpha\int_s^t a(\tau)d\tau}\left[b(s) + \phi(s)f\left(s, \phi_s^{-1/\alpha}, \psi_s\right)\right]ds, \tag{4.6}$$

similar to the proof of Lemma 2.3 we can obtain

$$\operatorname{mod}(Z_{\phi\psi}) \subset \operatorname{mod}\left(a(t), b(t) + f\left(t, \phi_t^{-1/\alpha}, \psi\right)\phi(t)\right) \subset \operatorname{mod}(F), \tag{4.7}$$

in addition,

$$\begin{aligned} Z_{\phi\psi}(t) &\geq \alpha \int_{-\infty}^{t} e^{-\alpha \int_s^t a(\tau)d\tau} \left(b_L - |\phi(s)| \left| f\left(s, \phi_s^{-1/\alpha}, \psi_s\right) \right| \right) ds \\ &= \alpha \int_{-\infty}^{t} e^{-\alpha \int_s^t a(\tau)d\tau} \left(b_L - |\phi(s)| \left| f\left(s, \phi_s^{-1/\alpha}, \psi_s\right) - f(s,0,0) \right| \right) ds \\ &\geq \alpha \int_{-\infty}^{t} e^{-\alpha \int_s^t a(\tau)d\tau} \left(b_L - L_1 |\phi(s)| \left(\left|\phi_s^{-1/\alpha}\right| + |\psi_s| \right) \right) ds \\ &\geq \frac{b_L - \gamma_1 L_1 (\beta^{-1/\alpha} + \gamma_2)}{a_M}. \end{aligned} \tag{4.8}$$

Note that the condition (C_4) holds, thus we have $L_1 < \alpha a_L \beta^{1+(1/\alpha)}/\gamma_1$, hence

$$\begin{aligned} Z_{\phi\psi}(t) &\geq \frac{b_L - \gamma_1 \alpha a_L \ \beta^{1+1/\alpha}/\gamma_1 \ (\beta^{-1/\alpha} + \alpha\beta)}{a_M} \\ &\geq \frac{b_L - \alpha a_L \beta^{1+1/\alpha}(\beta^{-1/\alpha} + \alpha\beta)}{a_M} \\ &= \frac{b_L - \alpha a_L \beta - \alpha^2 a_L \beta^{2+(1/\alpha)}}{a_M}. \end{aligned} \tag{4.9}$$

Note that $\beta = \min\{b_L/a_M(\alpha^2+\alpha+1),\ [b_L/a_M(\alpha^2+\alpha+1)]^{\alpha/(2\alpha+1)}\}$, if $0 < b_L/(a_M(\alpha^2+\alpha+1)) \leq 1$, then $\beta = b_L/a_M(\alpha^2+\alpha+1)$, thus we can get

$$\begin{aligned} Z_{\phi\psi}(t) &\geq \frac{b_L - \alpha a_L \beta - \alpha^2 a_L \beta^{2+(1/\alpha)}}{a_M} \\ &\geq \frac{b_L - \alpha a_M \beta - \alpha^2 a_M \beta}{a_M} \\ &= \beta, \end{aligned} \tag{4.10}$$

if $b_L/a_M(\alpha^2+\alpha+1) > 1$, then $\beta = [b_L/a_M(\alpha^2+\alpha+1)]^{\alpha/(2\alpha+1)}$, therefore

$$\begin{aligned} Z_{\phi\psi}(t) &\geq \frac{b_L - \alpha a_L \beta - \alpha^2 a_L \beta^{2+(1/\alpha)}}{a_M} \\ &\geq \frac{b_L - \alpha a_M \beta^{2+(1/\alpha)} - \alpha^2 a_M \beta^{2+(1/\alpha)}}{a_M} \\ &= \frac{b_L}{a_M(\alpha^2+\alpha+1)} \\ &\geq \beta. \end{aligned} \tag{4.11}$$

No matter what $0 < b_L/a_M(\alpha^2+\alpha+1) \leq 1$ or $b_L/a_M(\alpha^2+\alpha+1) > 1$, we always have

$$Z_{\phi\psi}(t) \geq \beta. \tag{4.12}$$

Also we have

$$
\begin{aligned}
Z_{\phi\psi}(t) &\leq \alpha \int_{-\infty}^{t} e^{-\alpha\int_s^t a(\tau)d\tau}\left(|b(s)| + |\phi(s)|\left|f\left(s,\phi_s^{-1/\alpha},\psi_s\right)\right|\right)ds \\
&= \alpha \int_{-\infty}^{t} e^{-\alpha\int_s^t a(\tau)d\tau}\left(|b(s)| + |\phi(s)|\left|f\left(s,\phi_s^{-1/\alpha},\psi_s\right) - f(s,0,0)\right|\right)ds \\
&\leq \alpha \int_{-\infty}^{t} e^{-\alpha\int_s^t a(\tau)d\tau}\left(|b(s)| + |\phi(s)|L_1\left(\left|\phi_s^{-1/\alpha}\right| + |\psi_s|\right)\right)ds \\
&\leq \frac{b_M + \|\phi(t)\|L_1\left(\left\|\phi_t^{-1/\alpha}\right\| + \|\psi_t\|\right)}{a_L} \\
&\leq \frac{b_M + \gamma_1 L_1\left(\beta^{-1/\alpha} + \gamma_2\right)}{a_L} \\
&\leq \frac{b_M + \gamma_1 \alpha a_L \beta^{1+1/\alpha}/\gamma_1\left(\beta^{-1/\alpha} + \gamma_2\right)}{a_L} \\
&= \frac{b_M + \alpha a_L \beta^{1+1/\alpha}\left(\beta^{-1/\alpha} + \alpha\beta\right)}{a_L} \\
&= \gamma_1.
\end{aligned}
\tag{4.13}
$$

Consider the following equation:

$$
\frac{du(t)}{dt} = -c(t)u(t) + h\left(t,\phi_t^{-1/\alpha}\right).
\tag{4.14}
$$

By Lemma 2.1 we can see that (4.14) has a unique positive almost periodic solution $V_\phi(t)$, it can be written as follows

$$
V_\phi(t) = \int_{-\infty}^{t} e^{-\int_s^t c(\tau)d\tau} h\left(s,\phi_s^{-1/\alpha}\right)ds.
\tag{4.15}
$$

Similar to the proof of Lemma 2.3, we can get

$$
\begin{aligned}
\operatorname{mod}\left(V_\phi(t)\right) \subset \operatorname{mod}&\left(c(t), h\left(t,\phi_t^{-1/\alpha}\right)\right) \subset \operatorname{mod}(H), \\
V_\phi(t) &\leq \int_{-\infty}^{t} e^{-\int_s^t c(\tau)d\tau}\left|h\left(s,\phi_s^{-1/\alpha}\right)\right|ds \\
&= \int_{-\infty}^{t} e^{-\int_s^t c(\tau)d\tau}\left|h\left(s,\phi_s^{-1/\alpha}\right) - h(s,0)\right|ds \\
&\leq L_2 \int_{-\infty}^{t} e^{-\int_s^t c(\tau)d\tau}\left|\phi_s^{-1/\alpha}\right|ds \\
&\leq \frac{L_2\beta^{-1/\alpha}}{c_L}.
\end{aligned}
\tag{4.16}
$$

Note that the condition (C_4) holds, it follows that $L_2 < \alpha c_L \beta^{1+(1/\alpha)}$, hence

$$V_{\phi}(t) \leq \frac{\alpha c_{\mathrm{L}} \beta^{1+1/\alpha} \beta^{-1/\alpha}}{c_L} = \alpha\beta = \gamma_2. \tag{4.17}$$

Now, define a mapping

$$T(\phi,\psi)(t) = \left(T_1(\phi,\psi)(t), T_2(\phi,\psi)(t)\right) = \left(Z_{\phi\psi}(t), V_{\phi}(t)\right). \tag{4.18}$$

From the above discussion we can see $T(\phi,\psi)(t) \in X$, thus $T : X \to X$, for given any $(\phi_1,\psi_1), (\phi_2,\psi_2) \in X$, we have

$$\begin{aligned}
&\|T(\phi_1,\psi_1) - T(\phi_2,\psi_2)\| \\
&\quad = \left\|\left(T_1(\phi_1,\psi_1) - T_1(\phi_2,\psi_2), T_2(\phi_1,\psi_1) - T_2(\phi_2,\psi_2)\right)\right\| \\
&\quad = \left\|\left(Z_{\phi_1\psi_1}(t) - Z_{\phi_2\psi_2}(t), V_{\phi_1}(t) - V_{\phi_2}(t)\right)\right\| \\
&\quad = \sup_{t\in R}\left(\left|\alpha\int_{-\infty}^{t} \mathrm{e}^{-\alpha\int_s^t a(\tau)d\tau}\left[\phi_1(s) f\left(s,\phi_{1s}^{-1/\alpha},\psi_{1s}\right) - \phi_2(s) f\left(s,\phi_{2s}^{-1/\alpha},\psi_{2s}\right)\right]ds\right|\right. \\
&\qquad\qquad \left.+\left|\int_{-\infty}^{t} e^{-\int_s^t c(\tau)d\tau}\left[h\left(s,\phi_{1s}^{-1/\alpha}\right) - h\left(s,\phi_{2s}^{-1/\alpha}\right)\right]ds\right|\right) \\
&\quad \leq \sup_{t\in R}\left(\alpha\int_{-\infty}^{t} e^{-\alpha\int_s^t a(\tau)d\tau}\left(|\phi_1(s)|\left|f\left(s,\phi_{1s}^{-1/\alpha},\psi_{1s}\right) - f\left(s,\phi_{2s}^{-1/\alpha},\psi_{2s}\right)\right|\right.\right. \\
&\qquad\qquad\qquad\qquad \left.+\left|f\left(s,\phi_{2s}^{-1/\alpha},\psi_{2s}\right)\right|\left|\phi_1(s) - \phi_2(s)\right|\right)ds \\
&\qquad\qquad \left.+\int_{-\infty}^{t} e^{-\int_s^t c(\tau)d\tau}\left|h\left(s,\phi_{1s}^{-1/\alpha}\right) - h\left(s,\phi_{2s}^{-1/\alpha}\right)\right|ds\right) \\
&\quad \leq \sup_{t\in R}\left(\alpha\int_{-\infty}^{t} e^{-\alpha\int_s^t a(\tau)d\tau}\left(L_1|\phi_1(s)|\left[\left|\phi_{1s}^{-1/\alpha} - \phi_{2s}^{-1/\alpha}\right| + |\psi_{1s} - \psi_{2s}|\right]\right.\right. \\
&\qquad\qquad\qquad\qquad \left.+L_1\left(\left|\phi_{2s}^{-1/\alpha}\right| + |\psi_{2s}|\right)|\phi_1(s) - \phi_2(s)|\right)ds \\
&\qquad\qquad \left.+L_2\int_{-\infty}^{t} e^{-\int_s^t c(\tau)d\tau}\left|\phi_{1s}^{-1/\alpha} - \phi_{2s}^{-1/\alpha}\right|ds\right).
\end{aligned} \tag{4.19}$$

By the mean value theorem (where ξ of the following formula is between ϕ_{1s} and ϕ_{2s}), the above formula can be changed as follows:

$$\begin{aligned}
&\|T(\phi_1,\varphi_1)-T(\phi_2,\varphi_2)\| \\
&\quad\le \sup_{t\in R}\Bigg(\alpha\int_{-\infty}^{t} e^{-\alpha\int_s^t a(\tau)d\tau}\Bigg(L_1|\phi_1(s)|\left[\left|\frac{1}{\alpha}\xi^{(-1/\alpha)-1}\right||\phi_{1s}-\phi_{2s}|+|\varphi_{1s}-\varphi_{2s}|\right] \\
&\qquad\qquad +L_1\left(\left|\phi_{2s}^{-1/\alpha}\right|+|\varphi_{2s}|\right)|\phi_1(s)-\phi_2(s)|\Bigg) \\
&\qquad +L_2\int_{-\infty}^{t} e^{-\int_s^t c(\tau)d\tau}\left|\frac{1}{\alpha}\xi^{(-1/\alpha)-1}\right||\phi_{1s}-\phi_{2s}|ds\Bigg) \\
&\quad\le \sup_{t\in R}\Bigg(\alpha\int_{-\infty}^{t} e^{-\alpha\int_s^t a(\tau)d\tau}\Bigg(L_1\gamma_1\left[\frac{1}{\alpha}\beta^{-1/\alpha-1}|\phi_{1s}-\phi_{2s}|+|\varphi_{1s}-\varphi_{2s}|\right] \\
&\qquad\qquad +L_1\left(\beta^{(-1/\alpha)}+\gamma_2\right)|\phi_1(s)-\phi_2(s)|\Bigg)ds \\
&\qquad +L_2\int_{-\infty}^{t} e^{-\int_s^t c(\tau)d\tau}\frac{1}{\alpha}\beta^{(-1/\alpha)-1}|\phi_{1s}-\phi_{2s}|ds\Bigg) \\
&\quad\le \frac{1}{a_L}\Bigg(L_1\gamma_1\frac{1}{\alpha}\beta^{(-1/\alpha)-1}\|\phi_{1t}-\phi_{2t}\|+L_1\gamma_1\|\varphi_{1t}-\varphi_{2t}\| \\
&\qquad +L_1\left(\beta^{(-1/\alpha)}+\gamma_2\right)\|\phi_1(t)-\phi_2(t)\|\Bigg) \\
&\qquad +\frac{1}{c_L}L_2\frac{1}{\alpha}\beta^{(-1/\alpha)-1}\|\phi_{1t}-\phi_{2t}\| \\
&\quad= \left(\frac{1}{a_L}L_1\gamma_1\frac{1}{\alpha}\beta^{-1/\alpha-1}+\frac{1}{a_L}L_1\beta^{-1/\alpha}+\frac{1}{a_L}L_1\gamma_2+\frac{1}{c_L}L_2\frac{1}{\alpha}\beta^{(-1/\alpha)-1}\right)\|\phi_1-\phi_2\| \\
&\qquad +\frac{1}{a_L}L_1\gamma_1\|\varphi_1-\varphi_2\| \\
&\quad\le \left(\frac{1}{a_L}L_1\gamma_1\frac{1}{\alpha}\beta^{(-1/\alpha)-1}+\frac{1}{a_L}L_1\beta^{-1/\alpha}+\frac{1}{a_L}L_1\gamma_2+\frac{1}{c_L}L_2\frac{1}{\alpha}\beta^{(-1/\alpha)-1}+\frac{1}{a_L}L_1\gamma_1\right) \\
&\qquad \times\|(\phi_1,\varphi_1)-(\phi_2,\varphi_2)\|,
\end{aligned}\tag{4.20}$$

notice that the condition (C_4) holds, it follows

$$\|T(\phi_1,\varphi_1)-T(\phi_2,\varphi_2)\|<\|(\phi_1,\varphi_1)-(\phi_2,\varphi_2)\|,\tag{4.21}$$

by (4.21), T is a contraction mapping, thus T exists a unique fixed point in X, this fixed point is the only positive almost periodic solution $(\varphi^*(t),u^*(t))$ of (4.3), and $\beta\le\varphi^*(t)\le\gamma_1,\ 0<u^*(t)\le\gamma_2$. note that $y(t)=1/x^{\alpha}(t)$, $x(t)=u^{-1/\alpha}(t)$, thus (4.1) exists a positive almost periodic solution $(\varphi^*(t),u^*(t))$, and $1/(\gamma_1^{1/\alpha})\le\varphi^*(t)\le 1/\beta^{1/\alpha},\ 0<u^*(t)\le\gamma_2$. This completes the proof of Theorem 4.1. □

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