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Research Article

Optimal Inequalities for Power Means

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We present the best possible power mean bounds for the product $M_p^{\alpha}(a,b)M_{-p}^{1-\alpha}(a,b)$ for any $p > 0$, $\alpha \in (0,1)$, and all $a, b > 0$ with $a \neq b$. Here, $M_p(a,b)$ is the pth power mean of two positive numbers a and b.

1. Introduction

For $p \in \mathbb{R}$, the pth power mean $M_p(a,b)$ of two positive numbers a and b is defined by

$$M_p(a,b) = \begin{cases} \left(\dfrac{a^p + b^p}{2}\right)^{1/p}, & p \neq 0, \\ \sqrt{ab}, & p = 0. \end{cases} \tag{1.1}$$

It is well known that $M_p(a,b)$ is continuous and strictly increasing with respect to $p \in \mathbb{R}$ for fixed $a, b > 0$ with $a \neq b$. Many classical means are special cases of the power mean, for example, $M_{-1}(a,b) = H(a,b) = 2ab/(a+b)$, $M_0(a,b) = G(a,b) = \sqrt{ab}$ and $M_1(a,b) = A(a,b) = (a+b)/2$ are the harmonic, geometric and arithmetic means of a and b, respectively. Recently, the power mean has been the subject of intensive research. In particular, many remarkable inequalities and properties for the power mean can be found in literature [1–22].

Let $L(a,b)=(a-b)/(\log a-\log b)$, $P(a,b)=(a-b)/[4\arctan(\sqrt{a/b})-\pi]$ and $I(a,b)=1/e(a^a/b^b)^{1/(a-b)}$ be the logarithmic, Seiffert and identric means of two positive numbers a and b with $a\neq b$, respectively. Then it is well known that

$$\min\{a,b\}<H(a,b)<G(a,b)<L(a,b)<P(a,b)<I(a,b)<A(a,b)<\max\{a,b\}, \tag{1.2}$$

for all $a,b>0$ with $a\neq b$.

In [23–29], the authors presented the sharp power mean bounds for L, I, $(IL)^{1/2}$ and $(L+I)/2$ as follows:

$$\begin{gathered} M_0(a,b)<L(a,b)<M_{1/3}(a,b), \qquad M_{2/3}(a,b)<I(a,b)<M_{\log 2}(a,b), \\ M_0(a,b)<\sqrt{L(a,b)I(a,b)}<M_{1/2}(a,b), \qquad \frac{1}{2}(L(a,b)+I(a,b))<M_{1/2}(a,b), \end{gathered} \tag{1.3}$$

for all $a,b>0$ with $a\neq b$.

Alzer and Qiu [12] proved that the inequality

$$\frac{1}{2}(L(a,b)+I(a,b))>M_p(a,b) \tag{1.4}$$

holds for all $a,b>0$ with $a\neq b$ if and only if $p\leq(\log 2)/(1+\log 2)=0.40938\ldots$.

The following sharp bounds for the sum $\alpha A(a,b)+(1-\alpha)L(a,b)$, and the products $A^\alpha(a,b)L^{1-\alpha}(a,b)$ and $G^\alpha(a,b)L^{1-\alpha}(a,b)$ in terms of power means were proved in [5, 8]:

$$\begin{gathered} M_{\log 2/(\log 2-\log\alpha)}(a,b)<\alpha A(a,b)+(1-\alpha)L(a,b)<M_{(1+2\alpha)/3}(a,b), \\ M_0(a,b)<A^\alpha(a,b)L^{1-\alpha}(a,b)<M_{(1+2\alpha)/3}(a,b), \\ M_0(a,b)<G^\alpha(a,b)L^{1-\alpha}(a,b)<M_{(1-\alpha)/3}(a,b), \end{gathered} \tag{1.5}$$

for any $\alpha\in(0,1)$ and all $a,b>0$ with $a\neq b$.

In [2, 7] the authors answered the questions: for any $\alpha\in(0,1)$, what are the greatest values $p_1=p_1(\alpha)$, $p_2=p_2(\alpha)$, $p_3=p_3(\alpha)$, and $p_4=p_4(\alpha)$, and the least values $q_1=q_1(\alpha)$, $q_2=q_2(\alpha)$, $q_3=q_3(\alpha)$, and $q_4=q_4(\alpha)$, such that the inequalities

$$\begin{gathered} M_{p_1}(a,b)<P^\alpha(a,b)L^{1-\alpha}(a,b)<M_{q_1}(a,b), \\ M_{p_2}(a,b)<A^\alpha(a,b)G^{1-\alpha}(a,b)<M_{q_2}(a,b), \\ M_{p_3}(a,b)<G^\alpha(a,b)H^{1-\alpha}(a,b)<M_{q_3}(a,b), \\ M_{p_4}(a,b)<A^\alpha(a,b)H^{1-\alpha}(a,b)<M_{q_4}(a,b), \end{gathered} \tag{1.6}$$

hold for all $a,b>0$ with $a\neq b$?

It is the aim of this paper to present the best possible power mean bounds for the product $M_p^\alpha(a,b)M_{-p}^{1-\alpha}(a,b)$ for any $p>0$, $\alpha\in(0,1)$ and all $a,b>0$ with $a\neq b$.

2. Main Result

Theorem 2.1. *Let $p > 0$, $\alpha \in (0,1)$ and $a, b > 0$ with $a \neq b$. Then*

(1) $M_{(2\alpha-1)p}(a,b) = M_p^{\alpha}(a,b)M_{-p}^{1-\alpha}(a,b) = M_0(a,b)$ *for* $\alpha = 1/2$,

(2) $M_{(2\alpha-1)p}(a,b) > M_p^{\alpha}(a,b)M_{-p}^{1-\alpha}(a,b) > M_0(a,b)$ *for* $\alpha > 1/2$ *and* $M_{(2\alpha-1)p}(a,b) < M_p^{\alpha}(a,b)M_{-p}^{1-\alpha}(a,b) < M_0(a,b)$ *for* $\alpha < 1/2$, *and the bounds* $M_{(2\alpha-1)p}(a,b)$ *and* $M_0(a,b)$ *for the product* $M_p^{\alpha}(a,b)M_{-p}^{1-\alpha}(a,b)$ *in either case are best possible.*

Proof. From (1.1) we clearly see that $M_p(a,b)$ is symmetric and homogenous of degree 1. Without loss of generality, we assume that $b = 1$, $a = x > 1$.

(1) If $\alpha = 1/2$, then (1.1) leads to

$$\begin{aligned} M_p^{\alpha}(x,1)M_{-p}^{1-\alpha}(x,1) &= \left(\frac{1+x^p}{2}\right)^{1/p}\left(\frac{1+x^{-p}}{2}\right)^{-1/p} \\ &= \left(\frac{1+x^p}{2}\right)^{1/p}\left(\frac{2x^p}{1+x^p}\right)^{1/p} = x = M_0^2(x,1) = M_{(2\alpha-1)p}^2(x,1). \end{aligned} \tag{2.1}$$

(2) Firstly, we compare the value of $M_{(2\alpha-1)p}(x,1)$ to the value of $M_p^{\alpha}(x,1)M_{-p}^{1-\alpha}(x,1)$ for $\alpha \in (0,1/2) \cup (1/2,1)$. From (1.1) we have

$$\begin{aligned} &\log\left[M_p^{\alpha}(x,1)M_{-p}^{1-\alpha}(x,1)\right] - \log M_{(2\alpha-1)p}(x,1) \\ &\quad = \frac{\alpha}{p}\log\frac{1+x^p}{2} - \frac{1-\alpha}{p}\log\frac{1+x^{-p}}{2} - \frac{1}{(2\alpha-1)p}\log\frac{1+x^{(2\alpha-1)p}}{2}. \end{aligned} \tag{2.2}$$

Let

$$f(x) = \frac{\alpha}{p}\log\frac{1+x^p}{2} - \frac{1-\alpha}{p}\log\frac{1+x^{-p}}{2} - \frac{1}{(2\alpha-1)p}\log\frac{1+x^{(2\alpha-1)p}}{2}, \tag{2.3}$$

then simple computations lead to

$$f(1) = 0, \tag{2.4}$$

$$f'(x) = \frac{g(x)}{x(1+x^p)\left(1+x^{(2\alpha-1)p}\right)}, \tag{2.5}$$

where

$$\begin{gathered} g(x) = (\alpha-1)x^{2\alpha p} + \alpha x^p - \alpha x^{(2\alpha-1)p} + 1 - \alpha, \\ g(1) = 0, \end{gathered} \tag{2.6}$$

$$g'(x) = \alpha p x^{p-1} h(x), \tag{2.7}$$

where

$$h(x) = 2(\alpha - 1)x^{(2\alpha-1)p} - (2\alpha - 1)x^{2(\alpha-1)p} + 1,$$
$$h(1) = 0, \tag{2.8}$$

$$h'(x) = -2p(1-\alpha)(2\alpha - 1)x^{2(\alpha-1)p-1}(x^p - 1). \tag{2.9}$$

If $\alpha \in (1/2, 1)$, then (2.9) implies that $h(x)$ is strictly decreasing in $[1, +\infty)$. Therefore, $M_{(2\alpha-1)p}(x,1) > M_p^{\alpha}(x,1)M_{-p}^{1-\alpha}(x,1)$ follows easily from (2.2)–(2.8) and the monotonicity of $h(x)$.

If $\alpha \in (0, 1/2)$, then (2.9) leads to the conclusion that $h(x)$ is strictly increasing in $[1, +\infty)$. Therefore, $M_{(2\alpha-1)p}(x,1) < M_p^{\alpha}(x,1)M_{-p}^{1-\alpha}(x,1)$ follows easily from (2.2)–(2.8) and the monotonicity of $h(x)$.

Secondly, we compare the value of $M_0(x,1)$ to the value of $M_p^{\alpha}(x,1)M_{-p}^{1-\alpha}(x,1)$. It follows from (1.1) that

$$\log\left[M_p^{\alpha}(x,1)M_{-p}^{1-\alpha}(x,1)\right] - \log M_0(x,1)$$
$$= \frac{\alpha}{p}\log\frac{1+x^p}{2} - \frac{1-\alpha}{p}\log\frac{1+x^{-p}}{2} - \frac{1}{2}\log x. \tag{2.10}$$

Let

$$F(x) = \frac{\alpha}{p}\log\frac{1+x^p}{2} - \frac{1-\alpha}{p}\log\frac{1+x^{-p}}{2} - \frac{1}{2}\log x, \tag{2.11}$$

then simple computations lead to

$$F(1) = 0, \tag{2.12}$$

$$F'(x) = \frac{(2\alpha-1)(x^p - 1)}{x(1+x^p)\left(1+x^{(2\alpha-1)p}\right)}. \tag{2.13}$$

If $\alpha \in (1/2, 1)$, then (2.13) implies that $F(x)$ is strictly increasing in $[1, +\infty)$. Therefore, $M_p^{\alpha}(x,1)M_{-p}^{1-\alpha}(x,1) > M_0(x,1)$ follows easily from (2.10)–(2.12) and the monotonicity of $F(x)$.

If $\alpha \in (0, 1/2)$, then (2.13) leads to the conclusion that $F(x)$ is strictly decreasing in $[1, +\infty)$. Therefore, $M_p^{\alpha}(x,1)M_{-p}^{1-\alpha}(x,1) < M_0(x,1)$ follows easily from (2.10)–(2.12) and the monotonicity of $F(x)$.

Next, we prove that the bound $M_{(2\alpha-1)p}(a,b)$ for the product $M_p^{\alpha}(a,b)M_{-p}^{1-\alpha}(a,b)$ in either case is best possible.

If $\alpha \in (0, 1/2)$, then for any $\epsilon \in (0, (1-2\alpha)p)$ and $x > 0$ we have

$$\begin{aligned} &M_p^{\alpha}(1+x,1)M_{-p}^{1-\alpha}(1+x,1) - M_{(2\alpha-1)p+\epsilon}(1+x,1) \\ &\quad = \left[\frac{1+(1+x)^p}{2}\right]^{\alpha/p}\left[\frac{1+(1+x)^{-p}}{2}\right]^{(\alpha-1)/p} \\ &\qquad - \left[\frac{1+(1+x)^{(2\alpha-1)p+\epsilon}}{2}\right]^{1/[(2\alpha-1)p+\epsilon]}. \end{aligned} \tag{2.14}$$

Letting $x \to 0$ and making use of Taylor's expansion, one has

$$\begin{aligned} &\left[\frac{1+(1+x)^p}{2}\right]^{\alpha/p}\left[\frac{1+(1+x)^{-p}}{2}\right]^{(\alpha-1)/p} - \left[\frac{1+(1+x)^{(2\alpha-1)p+\epsilon}}{2}\right]^{1/[(2\alpha-1)p+\epsilon]} \\ &\quad = \left[1+\frac{\alpha}{2}x+\frac{\alpha(p+\alpha-2)}{8}x^2+o\left(x^2\right)\right] \\ &\qquad \times \left[1+\frac{1-\alpha}{2}x-\frac{(1-\alpha)(p+\alpha+1)}{8}x^2+o\left(x^2\right)\right] \\ &\qquad - \left[1+\frac{1}{2}x+\frac{(2\alpha-1)p+\epsilon-1}{8}x^2+o\left(x^2\right)\right] \\ &\quad = \left[1+\frac{1}{2}x+\frac{(2\alpha-1)p-1}{8}x^2+o\left(x^2\right)\right] \\ &\qquad - \left[1+\frac{1}{2}x+\frac{(2\alpha-1)p+\epsilon-1}{8}x^2+o\left(x^2\right)\right] \\ &\quad = -\frac{\epsilon}{8}x^2+o\left(x^2\right). \end{aligned} \tag{2.15}$$

Equations (2.14) and (2.15) imply that for any $\alpha \in (0, 1/2)$ and $\epsilon \in (0, (1-2\alpha)p)$ there exists $\delta_1 = \delta_1(\epsilon) > 0$, such that $M_p^{\alpha}(1+x,1)M_{-p}^{1-\alpha}(1+x,1) < M_{(2\alpha-1)p+\epsilon}(1+x,1)$ for $x \in (0, \delta_1)$.

If $\alpha \in (1/2, 1)$, then for any $\epsilon \in (0, (2\alpha-1)p)$ and $x > 0$ we have

$$\begin{aligned} &M_p^{\alpha}(1+x,1)M_{-p}^{1-\alpha}(1+x,1) - M_{(2\alpha-1)p-\epsilon}(1+x,1) \\ &\quad = \left[\frac{1+(1+x)^p}{2}\right]^{\alpha/p}\left[\frac{1+(1+x)^{-p}}{2}\right]^{(\alpha-1)/p} \\ &\qquad - \left[\frac{1+(1+x)^{(2\alpha-1)p-\epsilon}}{2}\right]^{1/[(2\alpha-1)p-\epsilon]}. \end{aligned} \tag{2.16}$$

Letting $x \to 0$ and making use of Taylor's expansion, one has

$$\begin{aligned}
&\left[\frac{1+(1+x)^{p}}{2}\right]^{\alpha/p}\left[\frac{1+(1+x)^{-p}}{2}\right]^{(\alpha-1)/p}-\left[\frac{1+(1+x)^{(2\alpha-1)p-\epsilon}}{2}\right]^{1/[(2\alpha-1)p-\epsilon]}\\
&\quad=\left[1+\frac{\alpha}{2}x+\frac{\alpha(p+\alpha-2)}{8}x^{2}+o\left(x^{2}\right)\right]\\
&\qquad\times\left[1+\frac{1-\alpha}{2}x-\frac{(1-\alpha)(p+\alpha+1)}{8}x^{2}+o\left(x^{2}\right)\right]\\
&\qquad-\left[1+\frac{1}{2}x+\frac{(2\alpha-1)p-\epsilon-1}{8}x^{2}+o\left(x^{2}\right)\right]\\
&\quad=\left[1+\frac{1}{2}x+\frac{(2\alpha-1)p-1}{8}x^{2}+o\left(x^{2}\right)\right]\\
&\qquad-\left[1+\frac{1}{2}x+\frac{(2\alpha-1)p-\epsilon-1}{8}x^{2}+o\left(x^{2}\right)\right]\\
&\quad=\frac{\epsilon}{8}x^{2}+o\left(x^{2}\right).
\end{aligned} \tag{2.17}$$

Equations (2.16) and (2.17) imply that for any $\alpha \in (1/2, 1)$ and $\epsilon \in (0, (2\alpha-1)p)$ there exists $\delta_2 = \delta_2(\epsilon) > 0$, such that $M_p^{\alpha}(1+x,1)M_{-p}^{1-\alpha}(1+x,1) > M_{(2\alpha-1)p-\epsilon}(1+x,1)$ for $x \in (0, \delta_2)$.

Finally, we prove that the bound $M_0(a,b)$ for the product $M_p^{\alpha}(a,b)M_{-p}^{1-\alpha}(a,b)$ in either case is best possible.

If $\alpha \in (0, 1/2)$, then for any $\epsilon > 0$ we clearly see that

$$\lim_{x\to+\infty}\frac{M_p^{\alpha}(x,1)M_{-p}^{1-\alpha}(x,1)}{M_{-\epsilon}(x,1)}=+\infty. \tag{2.18}$$

Equation (2.18) implies that for any $\alpha \in (0, 1/2)$ and $\epsilon > 0$ there exists $T_1 = T_1(\epsilon) > 1$, such that $M_p^{\alpha}(x,1)M_{-p}^{1-\alpha}(x,1) > M_{-\epsilon}(x,1)$ for $x \in (T_1, +\infty)$.

If $\alpha \in (1/2, 1)$, then for any $\epsilon > 0$ we have

$$\lim_{x\to+\infty}\frac{M_p^{\alpha}(x,1)M_{-p}^{1-\alpha}(x,1)}{M_{\epsilon}(x,1)}=0. \tag{2.19}$$

Equation (2.19) implies that for any $\alpha \in (1/2, 1)$ and $\epsilon > 0$ there exists $T_2 = T_2(\epsilon) > 1$, such that $M_p^{\alpha}(x,1)M_{-p}^{1-\alpha}(x,1) < M_{\epsilon}(x,1)$ for $x \in (T_2, +\infty)$. □

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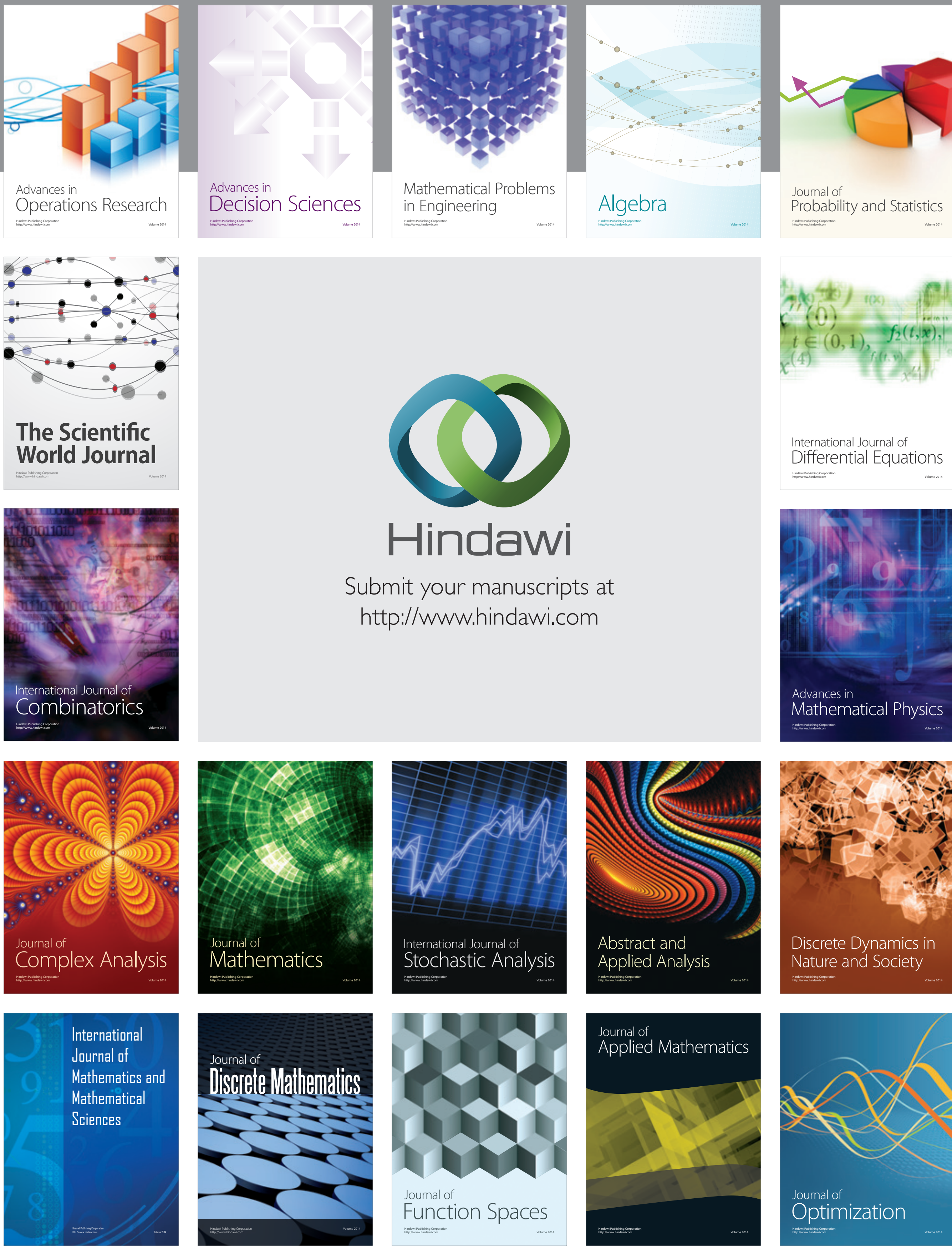
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