Research Article

Robust Speed Tracking of Networked PMSM Servo Systems with Uncertain Feedback Delay and Load Torque Disturbance

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This paper considers a class of networked Permanent Magnet Synchronous Motors (PMSMs), whose feedback loops are closed over a shared data network. Although the installation and maintenance cost of the networked PMSM system can be lowered by replacing the conventional point-to-point feedback cables with a network, the network packet dropouts and transmission delay may degrade the system's performance and even destabilize it. The load torque disturbance is another source to deteriorate the PMSM system's performance. To investigate the effects of the data network and the torque disturbance on the speed tracking of a PMSM system is one major task of this paper. In particular, we derive a sufficient stability condition for this system in an LMI (linear matrix inequality) form and provides a way to bound the system's H_∞ performance. Moreover, we adopt an iterative LMI method to design the speed controller of the PMSM system, which can robustly guarantee stability and performance against the network-induced delays, packet dropouts, and the torque disturbance. Simulations are done to verify the effectiveness of the obtained results.

1. Introduction

In the high-performance applications, such as robotics, aeronautic devices, and precision machine tools, the positioning accuracy is required to be higher and higher so that an alternative better than the traditional induction motors is needed. The permanent magnet synchronous motor (PMSM) is one wise choice to meet this accuracy challenge in the low-to-medium power servo systems. Because the PMSM's rotor is a permanent magnet and the flux linkage is constant [1], it possesses many advantages, like superior power density, large torque-to-inertia ratio, and high efficiency. Consequently, the PMSM has received

widespreading acceptance in industrial applications and is recognized as one of the key components in automation applications.

In PMSM applications, speed tracking is of great importance [2]. The existence of the load torque disturbance, however, makes speed tracking a rather tough task. In fact, it is impossible to measure the disturbance directly in real application. One way is to estimate the disturbance and actively cancel it [3]. Such methods may complicate the system design and increase the system's cost. We take another way, that is, designing a robust controller that can guarantee the system's performance is less sensitive to the disturbance.

Conventionally, a PMSM servo system includes a small number of PMSMs, which are connected to controllers by point-to-point cables to close the control loops. The modern industry, however, demands more and more PMSMs that could be located in geographically separated areas. Thus the direct cable connection architecture is inconvenient for the installation and maintenance. Instead, we can close the feedback loops of PMSMs over a digital network to form a networked control system (NCS), which is a newly developed technology being able to reduce the system's wiring cost, simplify the system's diagnosis and maintenance, and improve the system's agility [4]. The use of network will, however, induce intermittent losses and delays of the feedback information, may deteriorate the system's performance, and even cause instability [5]. There are many results on NCSs, some of which are introduced here (see more in the survey [6]). Reference [7] investigates the problem of robust stabilization and disturbance for the NCSs with random communication network-induced delays. Reference [8] Considers H_∞ output tracking for NCSs with delays and data packet dropouts. In [9], the discrete-time NCSs is studied and a state feedback controller with less conservatism is given. Moreover, a modified optimization algorithm is also proposed in [9] to cope with Bilinear Matrix Inequalities (BMIs).

As an important field of NCSs, networked servo systems are also well studied in recent years. In [10], fuzzy logic control is used to enhance the performance of networked servo systems with delays. To compensate the noise in the transmission channels, two-degrees-of-freedom control is utilized in [11]. Predictive control [12] is implemented to suppress the negative effects of delays and dropouts induced by the network. These results mainly focus on servo systems with DC motors or AC induction motors. Nevertheless, PMSM, as one of the main actuators in servo systems, is rarely mentioned in the previous NCS research. Due to its great potential in the industry, the networked PMSM servo systems could be one of the hot research topics in the future. Though a networked PMSM servo system has the same problems of data packet dropouts and transmission delay as typical networked servo systems, it has its own characteristics due to the employment of field-orientation mechanism and the special motor structures [13]. So when we design a controller for such a system, the specific properties should be taken into account to achieve less conservative results. The load torque disturbance, as a common disturbance in applications, should also be considered in the networked PMSM servo system.

This paper is organized as follows. In Section 2, we establish the mathematical model for the networked PMSM servo systems, which suffers the uncertain network-induced delay and data dropouts as well as the load torque disturbance. A sufficient stability condition of the system is given and proven in Section 3 based on the Lyapunov-Krasovskii techniques. Moreover, a bound on the system's H_∞ performance is also provided in Section 3. An iterative LMI method is proposed to design a robust controller in Section 4, which can guarantee both the stability and performance under the torque perturbation and the network-induced delay and dropouts. Simulations are done in Section 5 to verify the effectiveness of the obtained results. Some final remarks are placed in Section 6.

2. Problem Formulation

2.1. PMSM Dynamics

In the synchronously rotating rotor d-q coordinate, a PMSM drive can be modeled as [2]

$$u_d = Ri_d + L_d \frac{di_d}{dt} - n_p \omega L_q i_q,$$

$$u_q = Ri_q + L_q \frac{di_q}{dt} + n_p \omega L_d i_d + n_p \omega \phi_a, \tag{2.1}$$

$$J \frac{d\omega}{dt} + B_0 \omega + T_l = n_p \phi_a,$$

where ω is the rotating speed; i_d and i_q are the d- and q-axes stator currents, respectively; u_d and u_q are the d- and q-axes stator voltages, respectively; R is the stator resistance per phase; L_d and L_q are the d- and q-axes stator inductances, respectively, and $L_d = L_q = L$ in the surface-mounted PMSM; n_p is the number of poles; $\phi_a = \sqrt{3/2}\phi_f$, with ϕ_f being the flux linkage of the permanent magnet rotor; J is the total moment of inertia of the motor and load, and B_0 is the friction coefficient of the motor; T_l is the load torque.

A well-known strategy for a PMSM drive is the field-oriented vector control approach. Under this scheme, a practical structure of cascaded control loops, including a speed loop and two current loops, is usually employed [14]. In order to approximately eliminate the coupling between the d- and q-axes currents, the d-axis reference current i_d^* is set at zero and i_d is regulated via a PI controller, as is shown in Figure 1.

Design the current controller of the q-axis as

$$u_q = Ri_{qr} + n_p \phi_a \omega, \tag{2.2}$$

where i_{qr} is the reference current of the q-axis, which is computed by the speed controller. Then the state-space equation of a PMSM can be represented as

$$\begin{bmatrix} \dfrac{di_q}{dt} \\ \dfrac{d\omega}{dt} \end{bmatrix} = \begin{bmatrix} -\dfrac{R}{L} & 0 \\ \dfrac{n_p \phi_a}{J} & -\dfrac{B_0}{J} \end{bmatrix} \begin{bmatrix} i_q \\ \omega \end{bmatrix} + \begin{bmatrix} \dfrac{R}{L} \\ 0 \end{bmatrix} i_{qr} + \begin{bmatrix} 0 \\ -\dfrac{T_l}{J} \end{bmatrix}. \tag{2.3}$$

To proceed further, we need to make the following two definitions.

Definition 2.1. The tracking errors of speed, the q-axis current i_q, and the reference current i_{qr} are defined as

$$e(t) = \omega^* - \omega, \qquad e_q(t) = i_q^* - i_q, \qquad e_{qr}(t) = i_{qr}^* - i_{qr}, \tag{2.4}$$

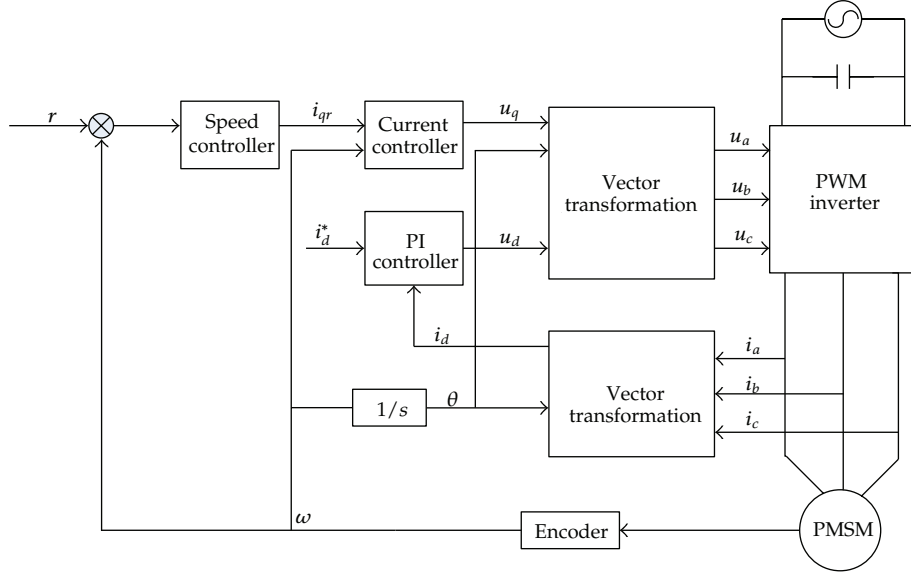where ω^*, i_q^*, i_{qr}^* are the corresponding reference values.

Figure 1: Configuration of a PMSM Servo System.

By Definition 2.1, the state-space equation (2.3) can be rewritten into a compact form as

$$\dot{x}(t) = Ax(t) + B(u(t) + d(t)), \tag{2.5}$$

where

$$x(t) = \begin{bmatrix} e_q(t) \ e(t) \end{bmatrix}^T, \qquad u(t) = \begin{bmatrix} e_{qr}(t) \ 0 \end{bmatrix}^T, \qquad d(t) = \begin{bmatrix} 0 \ \Delta T_l(t) \end{bmatrix}^T,$$

$$A = \begin{bmatrix} -\dfrac{R}{L} & 0 \\ \dfrac{n_p \phi_a}{J} & -\dfrac{B_0}{J} \end{bmatrix}, \qquad B = \begin{bmatrix} \dfrac{R}{L} & 0 \\ 0 & \dfrac{1}{J} \end{bmatrix}. \tag{2.6}$$

$\Delta T_l(t)$ represents the load torque disturbance, which is bounded and perhaps time-varying.

Definition 2.2. The disturbance suppressing performance of the system is evaluated by the following signal:

$$z(t) = e(t). \tag{2.7}$$

We want to design a robust controller for a networked PMSM to satisfy the following requirements.

(1) When the load torque disturbance $d(t) = 0$, the system is asymptotically stable under any initial state, namely,

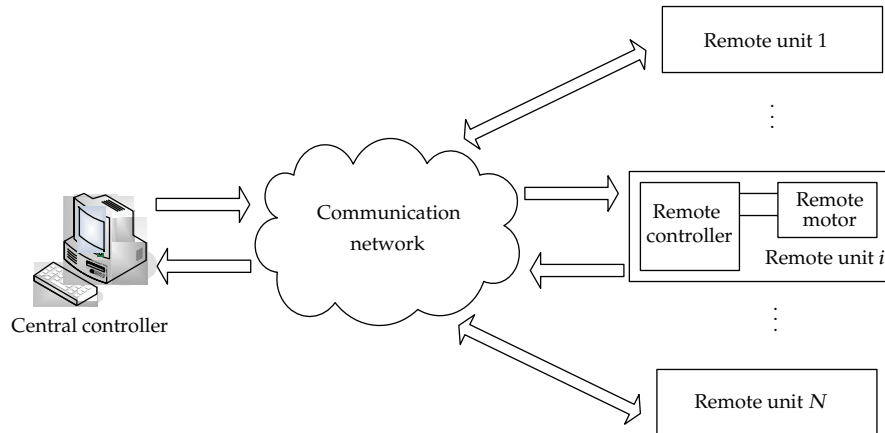$$\lim_{t \to \infty} z(t) = 0. \tag{2.8}$$

Figure 2: The diagram of a typical networked servo system.

(2) When the load torque disturbance $d(t) \neq 0$, the closed-loop system has the ability to suppress disturbance, namely,

$$\|z(t)\|_2 \leq \gamma \|d(t)\|_2, \tag{2.9}$$

where $\|\cdot\|_2$ stands for the L_2 norm of a continuous-time signal and γ is a quantitative measure of the disturbance attenuation. The smaller γ, the better disturbance attenuation.

2.2. Structure of Networked PMSM Servo Systems

A typical networked servo system is shown in Figure 2, which can be divided into three parts:
(1) the remote unit containing a remote controller and a remote motor,
(2) the central controller,
(3) the communication network.
The remote unit and the central controller exchange feedback information through the communication network.

When the remote motors are PMSMs, we get a networked PMSM servo system as shown in Figure 3. Each distributed remote controller receives control signals from the communication network and then convert them into PWM signals to drive the motor. It also sends local measurements, such as rotating speed, motor current, and local environment information, back to the central controller via the shared data network. The central controller is usually a sophisticated controller and can provide advanced real-time control strategies to the remote units.

2.3. System Modeling

Because of the limited bandwidth of the network, data packet dropouts are unavoidable. When a dropout occurs, it might be more advantageous to drop the old packet and transmit a new one than to retransmit the old one [15]. Network-induced delays are also considered here. The model of the concerned networked PMSM servo system is shown in Figure 4.
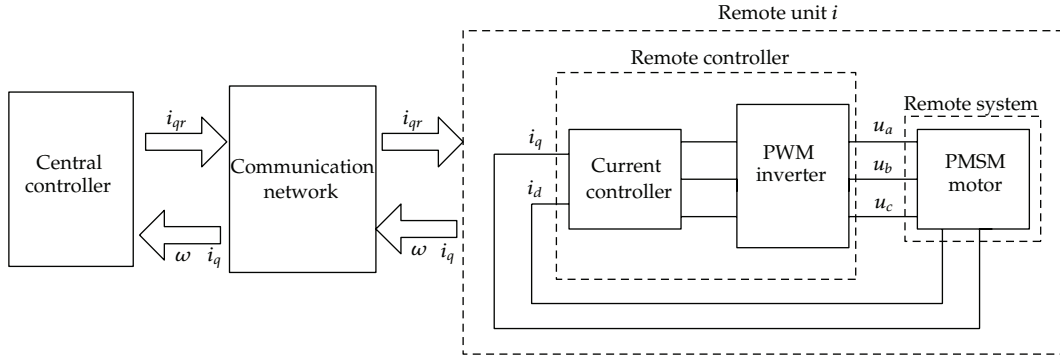
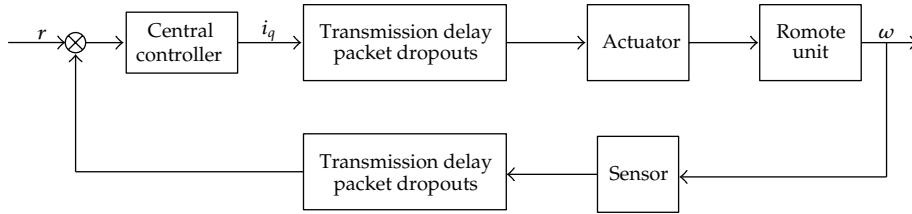**Figure 3:** The diagram of a networked PMSM servo system.

Figure 4: The model of the networked PMSM servo system.

The following assumptions are placed on the networked PMSM servo system.

(1) The sensor is clock driven, and the controller and actuator are event driven.

(2) The sampling period is a positive constant scalar h.

(3) The controller-to-actuator and the sensor-to-controller delays are denoted as τ_{ca} and τ_{sc}, respectively. The state feedback controller is static (see (2.14)). So these two delays can be lumped as

$$\tau_k = \tau_{sc} + \tau_{ca}. \tag{2.10}$$

Moreover, τ_k is less than h, that is, $0 \leq \tau_k \leq \overline{\tau} \leq h$, where $\overline{\tau}$ is the upper bound of delays. Due to the static feedback controller, we can assume that the transmission from the controller to the actuator is delay free and all delays come from the transmissions from the sensor to the controller, that is, $\tau_{sc} = \tau_k$ and $\tau_{ca} = 0$, in Figure 4.

(4) The maximum numbers of the consecutive controller-to-actuator and sensor-to-controller data dropouts are denoted as d_{ca} and d_{sc}, respectively. They can also be lumped as

$$d_k = d_{ca} + d_{sc}. \tag{2.11}$$

d_k is bounded as $0 \leq d_k \leq \overline{d}$. Similar to τ_k, d_k is also assumed to only come from the transmissions from the sensor to the controller, that is, $d_{sc} = d_k$ and $d_{ca} = 0$.

If only delays exist in the system, the sampled signal at kh ($\forall k \in \mathcal{N}$) will arrive at the controller at the time $kh + \tau_k$. So the delay is $\eta(t) = t - kh$ and its range is

$$\eta(t) \in [0, h + \overline{\tau}], \tag{2.12}$$

When the data dropouts also exist, they can be treated as delays and yield the following overall delay range:

$$\eta(t) \in \left[0, \left(\overline{d} + 1\right)h + \overline{\tau}\right] = [0, \overline{\eta}]. \tag{2.13}$$

It is well known that time-varying delay is more difficult to handle than constant delay from the control system's perspective. The actuator can know the total delay $\eta(t)$ by the time stamping technique. In the present paper, the actuator is assumed to purposefully postpone to implement the received control variable by the time of $\overline{\eta} - \eta(t)$ and yields a constant overall delay of $\overline{\eta}$, which is easier to deal with. We choose a static state feedback controller. Due to the constant delay strategy, our controller takes the following form:

$$u(t) = Kx(t - \overline{\eta}), \tag{2.14}$$

where K is the feedback gain to be designed. By substituting (2.14) into (2.5), we get the following state-space equation of the networked PMSM servo system:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + BKx(t - \overline{\eta}) + Bd, \\ z(t) &= Cx(t). \end{aligned} \tag{2.15}$$

3. Analysis of Networked PMSM Servo Systems

3.1. A Sufficient Stability Condition

In industry applications, the stability of a servo system is crucial. So we first have to guarantee that the system is stable. Here stability means the asymptotic stability when the disturbance is zero, that is, $d(t) = 0$. Under the delay and dropout conditions in (2.13), we get the following stability condition, which is expressed in an LMI (linear matrix inequality) form and easy to verify.

Theorem 3.1. *Under the given controller gain K and the upper bound $\overline{\eta} > 0$ (in (2.13)), the system (2.15) is asymptotically stable if there exist matrices $P > 0$, $Q > 0$, $Z > 0$, Y and W such that the following matrix inequality (3.1) holds:*

$$\begin{bmatrix} W_1 & W_2 & -\overline{\eta}Y & A^T \\ * & W_3 & -\overline{\eta}W & K^TB^T \\ * & * & -\overline{\eta}Z & 0 \\ * & * & * & -\overline{\eta}^{-1}Z^{-1} \end{bmatrix} < 0, \tag{3.1}$$

where $W_1 = PA + A^TP + Y + Y^T + Q$, $W_2 = PBK - Y + W^T$, and $W_3 = -Q - W - W^T$.

Proof. We construct the following Lyapunov-Krasovskii function:

$$V(t) = V_1(t) + V_2(t) + V_3(t), \tag{3.2}$$

where $V_1(t) = x^T(t)Px(t)$, $V_2(t) = \int_{t-\overline{\eta}}^t x^T(\alpha)Qx(\alpha)d\alpha$, and $V_3(t) = \int_{-\overline{\eta}}^0 \int_{t+\beta}^t \dot{x}^T(s)Z\dot{x}(s)ds\,d\beta$. By the Newton-Leibniz formula, we get

$$x(t - \overline{\eta}) = x(t) - \int_{t-\overline{\eta}}^t \dot{x}(\alpha)d\alpha. \tag{3.3}$$

The derivatives of V_1, V_2, and V_3 are computed as follows:

$$
\begin{aligned}
\dot{V}_1(t) &= 2x^T(t)P\big[Ax(t) + BKx(t - \overline{\eta})\big] \\
&= 2x^T(t)P(A + BK)x(t) - 2x^T(t)PBK\int_{t-\overline{\eta}}^t \dot{x}(\alpha)d\alpha \\
&= 2x^T(t)P(A + BK)x(t) \\
&\quad + 2x^T(t)(Y - PBK)\int_{t-\overline{\eta}}^t \dot{x}(\alpha)d\alpha + 2x^T(t-\overline{\eta})W\int_{t-\overline{\eta}}^t \dot{x}(\alpha)d\alpha \\
&\quad - \left[2x^T(t)Y\int_{t-\overline{\eta}}^t \dot{x}(\alpha)d\alpha + 2x^T(t-\overline{\eta})W\int_{t-\overline{\eta}}^t \dot{x}(\alpha)d\alpha\right] \\
&= 2x^T(t)P(A + BK)x(t) + 2x^T(t)(Y - PBK)\big[x(t) - x(t-\overline{\eta})\big] \\
&\quad + 2x^T(t-\overline{\eta})W\big[x(t) - x(t-\overline{\eta})\big] \\
&\quad - \left[2x^T(t)Y\int_{t-\overline{\eta}}^t \dot{x}(\alpha)d\alpha + 2x^T(t-\overline{\eta})W\int_{t-\overline{\eta}}^t \dot{x}(\alpha)d\alpha\right] \\
&= \frac{1}{\overline{\eta}}\int_{t-\overline{\eta}}^t \Big[2x^T(t)(PA + Y)x(t) + 2x^T(t)\big(PBK - Y + W^T\big)x(t-\overline{\eta}) \\
&\qquad\qquad - 2x^T(t-\overline{\eta})Wx(t-\overline{\eta}) - 2x^T(t)\overline{\eta}Y\dot{x}(\alpha) - 2x^T(t-\overline{\eta})\overline{\eta}W\dot{x}(\alpha)\Big]d\alpha, \\
\dot{V}_2(t) &= x^T(t)Qx(t) - x^T(t-\overline{\eta})Qx(t-\overline{\eta}) \\
&= \frac{1}{\overline{\eta}}\int_{t-\overline{\eta}}^t \Big[x^T(t)Qx(t) - x^T(t-\overline{\eta})Qx(t-\overline{\eta})\Big]d\alpha, \\
\dot{V}_3(t) &= \int_{-\overline{\eta}}^0 \Big[\dot{x}^T(t)Z\dot{x}(t) - \dot{x}(t+\beta)Z\dot{x}(t+\beta)\Big]d\beta \\
&= \int_{t-\overline{\eta}}^t \Big[\dot{x}^T(t)Z\dot{x}(t) - \dot{x}(\alpha)Z\dot{x}(\alpha)\Big]d\alpha \\
&= \frac{1}{\overline{\eta}}\int_{t-\overline{\eta}}^t \Big[x^T(t)\overline{\eta}A^T ZAx(t) + 2x^T(t)\overline{\eta}A^T ZBKx(t-\overline{\eta}) \\
&\qquad\qquad + x^T(t-\overline{\eta})\overline{\eta}K^T B^T ZBKx(t-\overline{\eta}) - \dot{x}^T(\alpha)\overline{\eta}Z\dot{x}(\alpha)\Big]d\alpha.
\end{aligned}
\tag{3.4}
$$

Finally, we have

$$\dot{V}(t) = \frac{1}{\overline{\eta}} \int_{t-\overline{\eta}}^{t} \zeta^T(t, \alpha) \Phi \zeta(t, \alpha) d\alpha, \tag{3.5}$$

where

$$\zeta(t, \alpha) = \begin{bmatrix} x(t) \\ x(t - \overline{\eta}) \\ \dot{x}(\alpha) \end{bmatrix}, \qquad \Phi = \begin{bmatrix} W_1 + \overline{\eta}A^T Z A & W_2 + \overline{\eta}A^T Z B K & -\overline{\eta}Y \\ * & W_3 + \overline{\eta}K^T B^T Z B K & -\overline{\eta}W \\ * & * & -\overline{\eta}Z \end{bmatrix}. \tag{3.6}$$

By the Schur complement theorem, we get from (3.1) that there must exist $\epsilon > 0$ such that

$$\Phi < -\epsilon I, \tag{3.7}$$

where I represents an identity matrix of an appropriate dimension. Substituting (3.7) into (3.5) yields

$$\dot{V}(t) < -\epsilon \|x(t)\|^2. \tag{3.8}$$

According to Lyapunov-Krasovskii theorem, if there exist $\epsilon > 0$ such that $\dot{V}(t) < -\varepsilon\|x\|^2$, the system (2.15) is asymptotic stable. So if the matrix inequality (3.1) holds, the system (2.15) is asymptotically stable. This completes the proof. $\qquad\square$

3.2. Robust Performance Analysis

Definition 3.2. A stable system in (2.15) is said to satisfy the H_∞ performance index $\gamma > 0$ if under the zero initial condition,

$$\|z(t)\|_2 \le \gamma \|d(t)\|_2. \tag{3.9}$$

We can verify whether the performance requirement in (3.9) is satisfied through the following theorem.

Theorem 3.3. *Under the given controller gain K and the upper bound $\overline{\eta} > 0$ (in (2.13)), the system (2.15) satisfies the performance index γ in (3.9) if there exist matrices $P > 0$, $Q > 0$, $Z > 0$, Y and W such that the following matrix inequality holds:*

$$\begin{bmatrix} W_1 & W_2 & -\overline{\eta}Y & PB & A^T & C^T \\ * & W_3 & -\overline{\eta}W & 0 & K^T B^T & 0 \\ * & * & -\overline{\eta}Z & 0 & 0 & 0 \\ * & * & * & -\gamma^2 I & B^T & 0 \\ * & * & * & * & -\overline{\eta}^{-1}Z^{-1} & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0, \tag{3.10}$$

where $W_1 = PA + A^T P + Y + Y^T + Q$, $W_2 = PBK - Y + W^T$, and $W_3 = -Q - W - W^T$.

Proof. Equation (3.10) implies (3.1). By Theorem 3.1, we know the system is stable.

Define $J_{zd} = \int_0^\infty [z^T(t)z(t) - \gamma^2 d^T(t)d(t)]dt$. J_{zd} can be modified into

$$J_{zd} = \int_0^\infty \left[z^T(t)z(t) - \gamma^2 d^T(t)d(t) + \dot{V}(t) \right]dt + V(t)|_{t=0} - V(t)|_{t=\infty}. \tag{3.11}$$

Under the zero initial condition, $V(t)|_{t=0} = 0$. Because of $V(t)|_{t=\infty} \geq 0$, we get

$$
\begin{aligned}
J_{zd} &\leq \int_0^\infty \left[z^T(t)z(t) - \gamma^2 d^T(t)d(t) + \dot{V}(t) \right]dt \\
&\leq \frac{1}{\overline{\eta}} \int_0^\infty \int_{t-\overline{\eta}}^t
\begin{bmatrix} x(t) \\ x(t-\overline{\eta}) \\ \dot{x}(\alpha) \\ d(t) \end{bmatrix}^T \Xi
\begin{bmatrix} x(t) \\ x(t-\overline{\eta}) \\ \dot{x}(\alpha) \\ d(t) \end{bmatrix},
\end{aligned} \tag{3.12}
$$

where

$$\Xi = \begin{bmatrix} W_1 + C^T C & W_2 & -\overline{\eta}Y & PB \\ * & W_3 & -\overline{\eta}W & 0 \\ * & * & -\overline{\eta}Z & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} + \begin{bmatrix} A^T \\ K^T B^T \\ 0 \\ B^T \end{bmatrix} \overline{\eta}Z \begin{bmatrix} A^T \\ K^T B^T \\ 0 \\ B^T \end{bmatrix}^T. \tag{3.13}$$

By the Schur complement theorem, we know that (3.10) implies $\Xi < 0$. Therefore, $J_{z\omega} < 0$, that is,

$$\int_0^\infty \left[z^T(t)z(t) - \gamma^2 d^T(t)d(t) \right]dt < 0. \tag{3.14}$$

After simple manipulations, the above equation yields

$$\|z(t)\|_2^2 \leq \gamma^2 \|d(t)\|_2^2. \tag{3.15}$$

Equation (3.15) is equivalent to (3.9). □

In (3.9), the left side variable $z(t)$ and the right one $d(t)$ have different units. So the ratio between them, γ, does not have a clear physical meaning. In order to overcome this difficulty, we introduce the following relative sensitive functions.

Definition 3.4. The sensitive functions of the speed tracking error and the load torque disturbance are defined as

$$S_z = \frac{\|Z\|_2}{\omega_{\text{ref}}}, \qquad S_T = \frac{\|\Delta T_l\|_2}{T_{l0}}, \tag{3.16}$$

where ω_{ref} is the reference tracking speed and T_{l0} is the nominal load torque.

Based on the above definition, (3.15) can be rewritten into

$$S_z \leq \tilde{\gamma} S_T, \tag{3.17}$$

where $\tilde{\gamma} = (T_{l0}/\omega_{\text{ref}})\gamma$.

Remark 3.5. In (3.17), both units of S_z and S_T are percentage. So (3.17) means how much percent of load torque disturbance yields how much percent of speed tracking error. $\tilde{\gamma}$ is exactly the gain between two percentage variables (S_z and S_T) and can quantitatively reflect the capability to attenuate the load torque disturbance attenuation. $\tilde{\gamma}$ is determined by the system's delay in (2.13) and the controller gain K in (2.14). Although we cannot change the system's delay, we do have freedom to choose an appropriate K to yield a better (smaller) $\tilde{\gamma}$, which is the major task of the next section.

4. The Design of the Robust Controller

When we design the robust controller, the controller gain K is unknown. So the matrix inequalities in Theorems 3.1 and 3.3 are bilinear matrix inequalities (BMIs). As a result, we cannot find a maximum $\bar{\eta}$ or the minimum γ using convex optimization algorithms. In the subsequent part, we propose some methods to resolve this issue.

Define $X = P^{-1}$ and $\Delta = \text{diag}\{X, X, X, I, I, I\}$. Pre- and postmultiply (3.10) by Δ, we obtain

$$\begin{bmatrix} \widetilde{W}_1 & \widetilde{W}_2 & -\bar{\eta}\tilde{Y} & B & XA^T & XC^T \\ * & \widetilde{W}_3 & -\bar{\eta}\widetilde{W} & 0 & F^TB^T & 0 \\ * & * & -\bar{\eta}XS^{-1}X & 0 & 0 & 0 \\ * & * & * & -\gamma^2 I & B^T & 0 \\ * & * & * & * & -\bar{\eta}^{-1}S & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0, \tag{4.1}$$

where $\widetilde{W}_1 = AX + XA^T + \tilde{Y} + \tilde{Y}^T + \tilde{Q}$, $\widetilde{W}_2 = BF - \tilde{Y} + \widetilde{W}^T$, $\widetilde{W}_3 = -\tilde{Q} - \widetilde{W} - \widetilde{W}^T$, $\tilde{Y} = XYX$, $\tilde{Q} = XQX$, $\widetilde{W} = XWX$, $F = KX$, and $S = Z^{-1}$.

Define a matrix variable $M < XS^{-1}X$, then matrix inequality (4.1) is equivalent to the combination of matrix inequalities (4.2), (4.3) and (4.4)

$$\begin{bmatrix} \widetilde{W}_1 & \widetilde{W}_2 & -\bar{\eta}\tilde{Y} & B & XA^T & XC^T \\ * & \widetilde{W}_3 & -\bar{\eta}\widetilde{W} & 0 & F^TB^T & 0 \\ * & * & -\bar{\eta}M & 0 & 0 & 0 \\ * & * & * & -\gamma^2 I & B^T & 0 \\ * & * & * & * & -\bar{\eta}^{-1}S & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0, \tag{4.2}$$

$$\begin{bmatrix} -Z & P \\ P & -N \end{bmatrix} < 0, \tag{4.3}$$

$$S = Z^{-1}, \qquad N = M^{-1}, \qquad P = X^{-1}. \tag{4.4}$$

Based on the above transformation, a robust controller can be designed as follows.

Theorem 4.1. *Under the given $\bar{\eta} > 0$ and $\gamma > 0$, the system (2.15) satisfies the performance index γ if there exist matrices $X > 0$, $P > 0$, $Q > 0$, $Z > 0$, $M > 0$, $S > 0$, $N > 0$, Y and W such that matrix inequalities (4.2), (4.3) and (4.4) hold. In this case, a robust H_∞ state feedback controller gain can be chosen as $K = FX^{-1}$.*

It is noted that the conditions in Theorem 4.1 are not LMI because of the inverse matrix constraints in (4.3) and (4.4). Fortunately, there are some methods to efficiently solve these inequalities. In [16], a method is given to obtain the suboptimal delay $\bar{\eta}$ or the suboptimal γ by setting $S = X$. With more computational efforts, better results can be obtained by an iterative algorithm in [17]. That iterative algorithm is called cone complementary linearization (CCL) method. By adopting the CCL method, we get the following algorithm to cope with the nonlinear minimization problem subject to LMIs.

Algorithm 4.2. There are 4 steps.

Step 1. Choose a sufficiently large initial variable $\gamma > 0$ such that there exists a feasible solution to matrix inequalities (4.2) and (4.3) and (4.4). Set $\gamma_{min} = \gamma$.

Step 2. Find a feasible set $(X_0, P_0, Q_0, Z_0, M_0, S_0, N_0, Y_0, W_0)$ satisfying (4.2) and (4.3), and the matrix inequality in (4.5) (One way to get a feasible set is by setting $S = X$ as the aforementioned suboptimal solution). Set $k = 0$,

$$\begin{bmatrix} S & I \\ I & Z \end{bmatrix} < 0, \qquad \begin{bmatrix} N & I \\ I & M \end{bmatrix} < 0, \qquad \begin{bmatrix} P & I \\ I & X \end{bmatrix} < 0. \tag{4.5}$$

Step 3. Solve the LMI problem in (4.6),

$$\text{Min}_{X,P,Q,Z,M,S,N,Y,W} \ \ \text{tr}(S_k Z + S Z_k + N M_k + P_k X + P X_k) \tag{4.6}$$

s.t.   equations (4.2),(4.3), and (4.5).

Set $X_{k+1} = X$, $P_{k+1} = P$, $Q_{k+1} = Q$, $Z_{k+1} = Z$, $M_{k+1} = M$, $S_{k+1} = S$, $N_{k+1} = N$, $Y_{k+1} = Y$, and $W_{k+1} = W$.

Step 4. If (3.10) holds, then set $\gamma_{min} = \gamma$ and return to Step 2 after decreasing γ to some extent. If (3.10) is not satisfied within a specified number of iterations, to say k_{max}, then we stop the above iterations. Otherwise, set $k = k + 1$ and go back to Step 3.

Table 1: The nominal parameters of a PMSM.

R	L	ϕ_f	B_0
$2.875\,\Omega$	$0.0085\,H$	$0.0816\,Wb$	$0.00185\,N \cdot m \cdot s$
J	n_p	T_l	
$0.0008\,N \cdot m \cdot s$	4	$2\,N \cdot m$	

Table 2: The relationship between the speed tracking error and $\overline{\eta}$.

Speed error percentage	6%	5.6%	5.2%	4.8%	4.4%	4.0%
$\overline{\eta}$	0.082	0.075	0.068	0.061	0.055	0.049
Speed error percentage	3.6%	3.2%	2.8%	2.4%	2.0%	
$\overline{\eta}$	0.043	0.037	0.032	0.026	0.005	

5. Simulation Results

To verify the results in Theorems 3.1, 3.3, and 4.1, a MATLAB/SIMULINK simulation platform of networked PMSM servo systems is built up, which is shown in Figure 1. The sampling period h is 10 ms. The tracking reference speed is set to 1500 r/min. The nominal parameters of a PMSM is shown in Table 1.

In the following simulations, the load torque disturbance is no larger than 30% of the nominal value. According to Theorem 4.1 and Definition 3.4, we obtain the relationship between the speed tracking error and $\overline{\eta}$, which is demonstrated in Table 2.

Remark 5.1. The data in Table 2 shows the relationship between the speed tracking error and the maximum delay $\overline{\eta}$. When the networked PMSM servo system needs higher tracking accuracy, it can tolerate less transmission delay and data packet dropouts. If the speed tacking error is less than 2.0%, $\overline{\eta}$ is equal to 0.005 s, which means that the system cannot tolerate even one data packet dropout under this circumstance.

In the sequel, we simulate 3 cases to demonstrate the effectiveness of the obtained results. In these simulations, the maximum relative speed tracking error is 5%, and the load torque disturbance is no larger than 30% of the nominal value. According to Definition 3.4, we get

$$\gamma \leq \gamma_{\min} = 125. \tag{5.1}$$

By Theorem 4.1 (with $\gamma = \gamma_{\min}$), we try to maximize $\overline{\eta}$ and reach a suboptimal solution with $\overline{\eta} = 0.064$ s and the corresponding controller gain of $K = [0, -0.0212]$.

5.1. Stability of System

Case 1. The first simulation is to verify the stability of the networked PMSM servo system. $\overline{\eta}$ is set to 0.064 s. Figure 5 shows the speed tracking error trajectory. After approximate 0.15 s, the speed tracking error is almost zero. So this result demonstrates the correctness of the stability condition in Theorem 3.1.
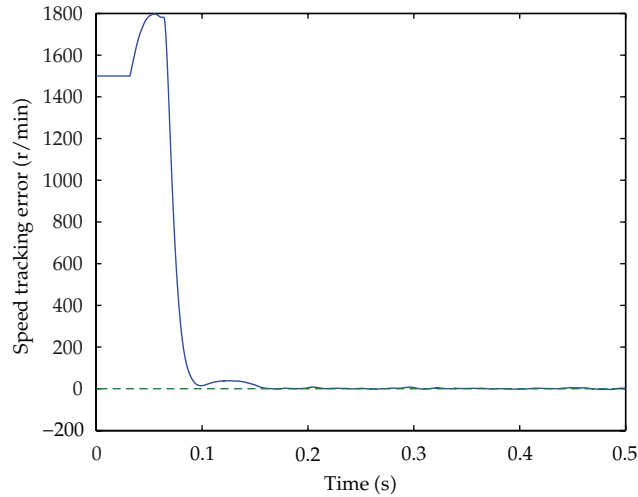
Figure 5: The speed tracking error.

5.2. Robustness against Transmission Delay and Data Dropouts

Case 2. In the second case, performance comparison is made between two approaches, the robust controller proposed in this paper and the PID controller.

The PID Controller

Its parameters are tuned to perform well under the network-delay-free situation. In the simulation, the network-induced delay is set to 0.1 h (= 0.001 s).

Robust Controller

The robust controller is then applied as the central controller to regulate the speed loop. The network-induced delay $\bar{\eta}$ is set to be 0.064 s. $\bar{\eta} = 0.064\,\mathrm{s} = 6.4\,\mathrm{h}$ means that the system can tolerate 5 dropouts among any 6 consecutive data packets.

The results of this case are shown in Figure 6.

Although the PID controller suffers a much smaller delay than the robust controller, it generates much worse tracking performance than the robust controller. The reason lies in that the design of the robust controller takes the delay into account, while the design of the PID controller does not. So the effectiveness of Theorems 3.3 and 4.1 is confirmed.

5.3. Robustness against Load Torque Disturbance

Case 3. The purpose of the third simulation is to verify that the proposed robust controller can effectively suppress the load torque disturbance. Under the worst condition of five consecutive data packet dropouts and network-induced delay up to 0.4 h, a 30% load torque disturbance occurs at time 0.5 s and disappears at time 0.55 s. Figure 7 shows the simulation results.
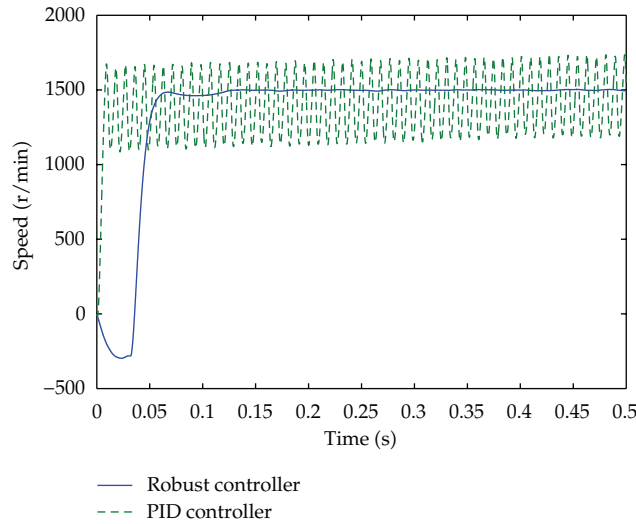
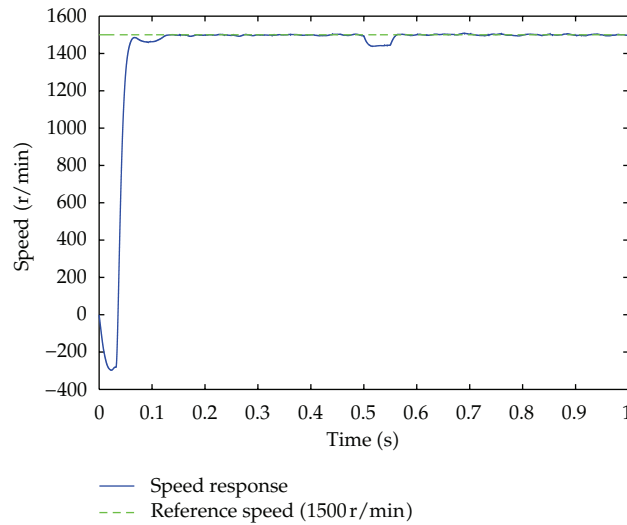**Figure 6:** Speed response under the network delays and dropouts.

Figure 7: Speed response under the network delays and dropouts and the load torque disturbance.

As we see in Figure 7, when the load torque disturbance starts at time 0.5 s, the speed deviates from the reference speed of 1500 r/min. However, after the disturbance disappears at time 0.55 s, the speed of PMSM quickly returns back to the reference speed in about 0.03 s. From the speed response curve, we can also see that the speed tracking error is less than 5% during the whole process. These simulation results confirm that the robust controller designed in this paper satisfies the accuracy demand and can effectively suppress the load torque disturbance.

6. Conclusion

The networked PMSM servo system has a promising future in industry applications. However, its performance may be degraded by the network delays and data packet dropouts.

The load disturbance is also a detrimental factor for the control performance. In this paper, we propose a sufficient stability condition by the Lyapunov-Krasovskii method, quantitatively investigate the robustness of the system's performance against the load torque disturbance, and give a way to design a robust controller, which can either tolerate larger network delay or give better H_∞ performance. The simulations are done to verify the correctness of the stability result and demonstrate the superiority of the obtained controller in terms of performance robustness against the data packet dropouts and transmission delay as well as the load torque disturbance.
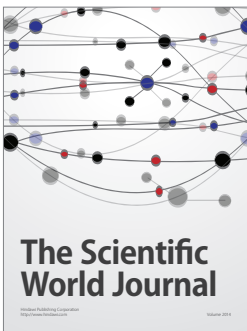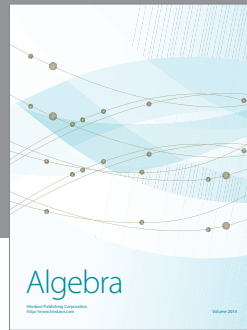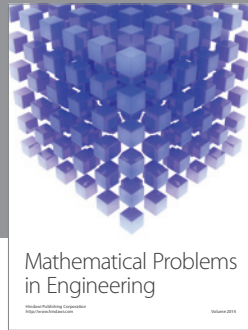
Acknowledgments

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