Hindawi Publishing Corporation
Journal of Applied Mathematics
Volume 2012, Article ID 381503, 25 pages
doi:10.1155/2012/381503

Research Article

Multi-State Dependent Impulsive Control for Pest Management

Huidong Cheng, Fang Wang, and Tongqian Zhang

College of Science, Shandong University of Science and Technology, Qingdao 266510, China

Correspondence should be addressed to Huidong Cheng, chd900517@sdust.edu.cn

Received 17 April 2012; Accepted 6 June 2012

Academic Editor: Zhiwei Gao

According to the integrated pest management strategies, we propose a model for pest control which adopts different control methods at different thresholds. By using differential equation geometry theory and the method of successor functions, we prove the existence of order one periodic solution of such system, and further, the attractiveness of the order one periodic solution by sequence convergence rules and qualitative analysis. Numerical simulations are carried out to illustrate the feasibility of our main results. Our results show that our method used in this paper is more efficient and easier than the existing ones for proving the existence of order one periodic solution.

1. Introduction

It is of great value to study pest management method applied in agricultural production; entomologists and the whole society have been paying close attention to how to control pests effectively and to save manpower and material resources. In agricultural production, pesticides-spraying (chemical control) and release of natural enemies (biological control) are the ways commonly used for pest control. But if we implement chemical control as soon as pests appear, many problems are caused: the first is environmental pollution; the second is increase of costs including human and material resources and time; the third is killing natural enemies, such as parasitic wasp; the last is pests' resistance to pesticides, which brings great negative effects instead of working as well as had been expected [1–3]. The second way, which controls pests with the help of the increasing natural enemies, can avoid problems caused by chemical control and gets more and more attention. So many scholars have been studying and discussing it [4–8]. Considering the effectiveness of the chemical control and nonpollution and limitations of the biological one, people have proposed the method of integrated pest management (IPM), which is a pest management system integrating all appropriate ways and technologies to control economic injury level (EIL) caused by pest populations in view of population dynamics and its relevant environment. In the process of practical application, people usually implement the following two schemes for the integrated pest management: one is to implement control at a fixed time to eradicate pests [9, 10]; the other is to implement measures only when the amount of pests reaches a critical level, which is to make the amount less than certain economic impairment level, not to wipe out pests [11–13]. Salazar conducted an experiment of broad bean being damaged by bean sprouts worm in 1976 and found "crops' compensation to damage of pests", that is, yields of crops which had been damaged a little by pests in the early growth are actually higher than those without damage. In other words, we do not want to wipe out pests but to control them to a certain economic injury level (EIL). So, the second is used most in the process of agricultural industry. Tang and Cheke [14] first proposed the "Volterra" model in the from of a state-dependent impulsive model:

$$\begin{aligned} &\left.\begin{aligned} x'(t) &= x(t)\big(a - by(t)\big), \\ y'(t) &= y(t)(-d + cx(t)), \end{aligned}\right\} \quad x \neq ET, \\ &\left.\begin{aligned} \Delta x(t) &= -\alpha x(t), \\ \Delta y(t) &= q, \end{aligned}\right\} \quad x = ET, \end{aligned} \tag{1.1}$$

and they applied this model to pest management and proved existence and stability of periodic solution of first and second order. Then Tang and Cheke [14] also proposed bait-dependent digestive model with state pulse:

$$\begin{aligned} &\left.\begin{aligned} x'(t) &= x(t)\big(a - by(t)\big), \\ y'(t) &= y(t)\left(\frac{\lambda bx(t)}{1 + bhx(t)} - d\right), \end{aligned}\right\} \quad x \neq h_{\max}, \\ &\left.\begin{aligned} \Delta x(t) &= -\alpha x(t), \\ \Delta y(t) &= q, \end{aligned}\right\} \quad x = h_{\max}, \end{aligned} \tag{1.2}$$

they had the existence of positive periodic solution and stability of orbit. Recently Jiang and Lu et al. [15–17] have proposed pest management model with state pulse and phase structure and several predator-prey models with state pulse and had the existence of semi-trivial periodic solution and positive periodic solution and stability of orbit.

It is worth mentioning that the vast majority of research on population dynamics system with state pulse considers single state pulse, which is to say, only when the amount of population reaches the same economic threshold can measures be taken (e.g., chemical control and biological control); but this single state-pulse control does not confirm to reality. In fact, we often need to use different control methods under different states in real life. For example, in the process of pest management, when the amount of pests is small, biological control is implemented; when the amount is large, combination control is applied. Tang et al. [18] have investigated and developed a mathematical model with hybrid impulsive model:

$$\begin{aligned}
&x'(t) = rx(t)(1-\delta x(t)) - bx(t)y(t), \quad x < ET,\\
&y'(t) = y(t)(cx(t) - a), \quad t = \lambda_m,\\
&\left.\begin{aligned} x(t^+) &= (1-p_1)x(t),\\ y(t^+) &= (1-p_2)y(t), \end{aligned}\right\} \quad x(t) = ET,\\
&y(\lambda_m^+) = (1+p_3)y(\lambda_+) + q, \quad t = \lambda_m.
\end{aligned} \tag{1.3}$$

Motivated by Tang, on the basis of the above analysis, we set up the following predicator-prey system with different control methods in different thresholds:

$$\begin{aligned}
&\left.\begin{aligned} x'(t) &= x(t)(a - by(t)),\\ y'(t) &= y(t)\left(\frac{\lambda bx(t)}{1+bhx(t)} - d\right), \end{aligned}\right\} \quad x \neq h_1, h_2 \text{ or } x = h_1,\ y > y^*,\\
&\left.\begin{aligned} \Delta x(t) &= 0,\\ \Delta y(t) &= \delta, \end{aligned}\right\} \quad x = h_1,\ y \leqslant y^*,\\
&\left.\begin{aligned} \Delta x(t) &= -\alpha x(t),\\ \Delta y(t) &= -\beta y(t) + q, \end{aligned}\right\} \quad x = h_2,
\end{aligned} \tag{1.4}$$

where $x(t)$ and $y(t)$ represent, respectively, the prey and the predator population densities at time t; a, b, λ, h_1, h_2 and d are all positive constants and $h_1 < h_2$; $y^* = a/b$. $\alpha, \beta \in (0,1)$ represent the fraction of pest and predator, respectively, which die due to the pesticide when the amount of prey reaches economic threshold h_2 and q is the release amount of predator. $\lambda bx(t)/(1+bhx(t))$ is the per capita functional response of the predator. When the amount of the prey reaches the threshold h_1 at time t_{h_1}, controlling measures are taken (releasing natural enemies) and the amount of predator abruptly turns to $y(t_{h_1}) + \delta$. When the amount of the prey reaches the threshold h_2 at time t_{h_2}, spraying pesticide, and releasing natural enemies and the amount of prey and predator abruptly turn to $(1-\alpha)x(t_{h_2})$ and $(1-\beta)y(t_{h_2}) + q$, respectively. Refer to [17] Liu et al. for details.

2. Preliminaries

First, we give some basic definitions and lemmas.

Definition 2.1. A triple (X, Π, R^+) is said to be a semidynamical system if X is a metric space, R^+ is the set of all nonnegative real, and $\Pi(P,t) : X \times R^+ \to X$ is a continuous map such that:

(i) $\Pi(P,0) = P$ for all $P \in X$;

(ii) $\Pi(P,t)$ is continuous for t and s;

(iii) $\Pi(\Pi(P,t)) = \Pi(P, t+s)$ for all $P \in X$ and $t, s \in R^+$. Sometimes a semi-dynamical system (X, Π, R^+) is denoted by (X, Π).

Definition 2.2. Assuming that

(i) (X, Π) is a semi-dynamical system;

(ii) M is a nonempty subset of X;

(iii) function $I : M \rightarrow X$ is continuous and for any $P \in M$, there exists a $\varepsilon > 0$ such that for any $0 < |t| < \varepsilon$, $\Pi(P, t) \notin M$.

Then, (X, Π, M, I) is called an impulsive semi-dynamical system.

For any P, the function $\Pi_P : R^+ \rightarrow X$ defined as $\Pi_P(t) = \Pi(P, t)$ is continuous, and we call $\Pi_P(t)$ the trajectory passing through point P. The set $C^+(P) = \{\Pi(P, t)/0 \le t < +\infty\}$ is called positive semitrajectory of point P. The set $C^-(P) = \{\Pi(P, t)/ -\infty < t \le 0\}$ is called the negative semi-trajectory of point P.

Definition 2.3. One considers state-dependent impulsive differential equations:

$$\begin{aligned} &\begin{matrix} x'(t) = P(x, y), \\ y'(t) = Q(x, y), \end{matrix} \quad (x, y) \notin M(x, y), \\ &\begin{matrix} \Delta x(t) = \alpha(x, y), \\ \Delta y(t) = \beta(x, y), \end{matrix} \quad (x, y) \in M(x, y), \end{aligned} \tag{2.1}$$

where $M(x, y)$ and $N(x, y)$ represent the straight line or curve line on the plane, $M(x, y)$ is called impulsive set. The function I is continuous mapping, $I(M) = N$, I is called the impulse function. $N(x, y)$ is called the phase set. We define "dynamic system" constituted by the definition of solution of state impulsive differential equation (2.1) as "semicontinuous dynamic systems", which is denoted as (Ω, f, I, M).

Definition 2.4. Suppose that the impulse set M and the phase set N are both lines, as shown in Figure 1. Define the coordinate in the phase set N as follows: denote the point of intersection Q between N and x-axis as O, then the coordinate of any point A in N is defined as the distance between A and Q and is denoted by y_A. Let C denote the point of intersection between the trajectory starting from A and the impulse set M, and let B denote the phase point of C after impulse with coordinate y_B. Then, we define B as the successor point of A, and then the successor function of point A is that $f(A) = y_B - y_A$.

Definition 2.5. A trajectory $\widetilde{\Pi}(P_0, t)$ is called order one periodic solution with period T if there exists a point $P_0 \in N$ and $T > 0$ such that $P = \Pi(P_0, t) \in M$ and $P^+ = I(P) = P_0$.

We get these lemmas from the continuity of composite function and the property of continuous function.

Lemma 2.6. *Successor function defined in Definition 2.1 is continuous.*

Lemma 2.7. *In system* (1.4), *if there exist $A \in N$, $B \in N$ satisfying successor function $f(A)f(B) < 0$, then there must exist a point P ($P \in N$) satisfying $f(P) = 0$ the function between the point of A and the point of B, thus there is an order one periodic solution in system* (1.4).

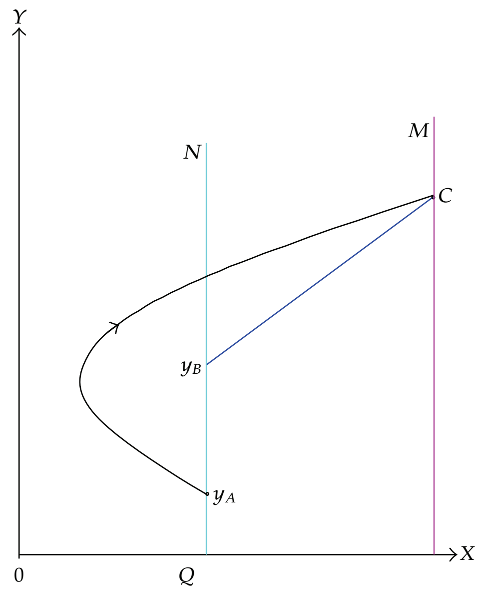


Figure 1: Successor function defined.

Next, we consider the model (1.4) *without impulse effects:*

$$\begin{aligned} x'(t) &= x(t)\left(a - by(t)\right), \\ y'(t) &= y(t)\left(\frac{\lambda bx(t)}{1 + bhx(t)} - d\right). \end{aligned} \tag{2.2}$$

It is well known that the system (2.2) *possesses*

(I) *two steady states* $O(0,0)$*-saddle point, and* $R(d/b(\lambda - dh), a/b) = R(x^*, y^*)(\lambda > dh)$*-stable centre;*

(II) *a unique closed trajectory through any point in the first quadrant contained inside the point* R*.*

In this paper, we assume that the condition $\lambda > dh$ holds. By the biological background of system (1.4), we only consider $D = \{(x, y) : x \geq 0, y \geq 0\}$. Vector graph of system (2.2) can be seen in Figure 2.

This paper is organized as follows. In the next section, we present some basic definitions and an important lemmas as preliminaries. In Section 3, we prove existence for an order one periodic solution of system (1.4). The sufficient conditions for the attractiveness of order one periodic solutions of system (1.4) are obtained in Section 4. At last, we state conclusion, and the main results are carried out to illustrate the feasibility by numerical simulations.

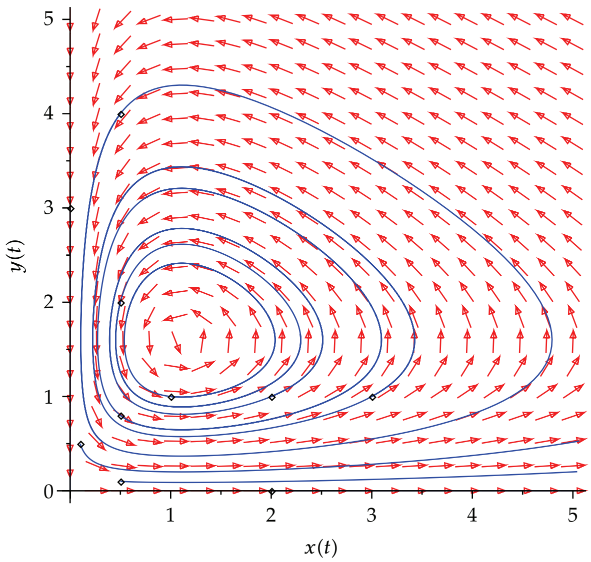


Figure 2: Illustration of vector graph of system (2.2).

3. Existence of the Periodic Solution

In this section, we will investigate the existence of an order one periodic solution of system (1.4) by using the successor function defined in this paper and qualitative analysis. For this goal, we denote that $M_1 = \{(x, y)/x = h_1, 0 \le y \le a/b\}$, and that $M_2 = \{(x, y) \mid x = h_2, y \ge 0\}$. Phase set of set M is that $N_1 = I(M_1) = \{(x, y) \mid x = h_1, a/b < y \le (a/b) + \delta\}$ and that $N_2 = I(M_2) = \{(x, y) \mid x = (1-\alpha)h_2, y \ge q\}$. Isoclinic line is denoted, respectively, by lines: $L_1 = \{(x, y) \mid y = a/b, x \ge 0\}$ and $L_2 = \{(x, y) \mid x = d/b(\lambda - dh), y \ge 0\}$.

For the convenience, if $P \in \Omega - M$, $F(P)$ is defined as the first point of intersection of $C^+(P)$ and M, that is, there exists a $t_1 \in R_+$ such that $F(P) = \Pi(P, t_1) \in M$, and for $0 < t < t_1, \Pi(P, t) \notin M$; if $B \in N, R(B)$ is defined as the first point of intersection of $C^-(P)$ and N, that is, there exists a $t_2 \in R_+$ such that $R(B) = \Pi(B, -t_2) \in N$, and for $-t < t < 0, \Pi(B, t) \notin N$. For any point P, we denote y_P as its ordinate. If the point $P(h, y_P) \in M$, then pulse occurs at the point P, the impulsive function transfers the point P into $P^+ \in N$. Without loss of generality, unless otherwise specified we assume the initial point of the trajectory lies in phase set N.

Due to the practical significance, in this paper we assume the set always lies in the left side of stable centre R, that is, $h_1 < d/b(\lambda - dh)$ and $(1-\alpha)h_2 < d/b(\lambda - dh)$.

In the light of the different position of the set N_1 and the set N_2, we consider the following three cases.

Case 1 ($0 < h_1 < d/b(\lambda - dh)$). In this case, set M_1 and N_1 are both in the left side of stable center R (as shown in Figure 3). Take a point $B_1(h_1, (a/b) + \varepsilon) \in N_1$ above A, where $\varepsilon > 0$ is small enough, then there must exist a trajectory passing through B_1 which intersects with M_1 at point $P_1(h_1, y_{p_1})$, we have $y_{p_1} < a/b$. Since $p_1 \in M_1$, pulse occurs at the point P_1, the impulsive function transfers the point P_1 into $P_1^+(h_1, y_{p_1} + \delta)$ and P_1^+ must lie above B_1, therefore inequation $(a/b) + \varepsilon < y_{p_1} + \delta$ holds, thus the successor function of B_1 is $f(B_1) = y_{p_1} + \delta - ((a/b) + \varepsilon) > 0$.

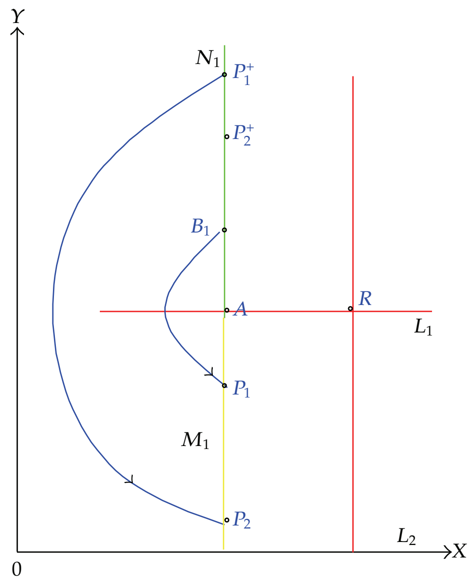


Figure 3: $0 < h_1 < (d/b(1-dh))(a/b) < y_{P_2} + \delta + < y_{P_1} + \delta$.

On the other hand, the trajectory with the initial point P_1^+ intersects M_1 at point $P_2(h_1, y_{p_2})$, in view of vector field and disjointness of any two trajectories, we know $y_{p_2} < y_{p_1} < a/b$. Supposing the point P_2 is subject to impulsive effects to point $P_2^+(h_1, y_{p_2^+})$, where $y_{p_2^+} = y_{p_2} + \delta$, the position of P_2^+ has the following two cases.

Subcase 1.1 ($a/b < y_{p_2} + \delta < y_{p_1} + \delta$). In this case, the point P_2^+ lies above the point A and below P_1^+, then we have $f(P_1^+) = y_{p_2} + \delta - (y_{p_1} + \delta) < 0$.

By Lemma 2.7, there exists an order one periodic solution of system (1.4), whose initial point is between B_1 and P_1^+ in set N_1.

Subcase 1.2 ($a/b \geq y_{p_2} + \delta$ (as shown in Figure 4)). The point P_2^+ lies below the point A, that is, $P_2^+ \in M_1$, then pulse occurs at the point P_2^+, the impulsive function transfers the point P_2^+ into $P_2^{++}(h_1, y_{p_2} + 2\delta)$.

> If $a/b < y_{p_2} + 2\delta < y_{p_1} + \delta$, like the analysis of Subcase 1.1, there exists an order one periodic solution of system (1.4).
>
> If $a/b > y_{p_2} + 2\delta$, that is, $P_2^{++} \in M_1$, then we repent the above process until there exists $k \in Z_+$ such that P_2^{++} jumps to $P_2^{i+}((h_1, y_{p_2} + (k+2)\delta)$ after k times' impulsive effects which satisfies $a/b < y_{p_2} + (k+2)\delta < y_{p_1} + \delta$. Like the analysis of Subcase 1.1, there exists an order one periodic solution of system (1.4).

Now, we can summarize the above results as the following theorem.

Theorem 3.1. *If $\lambda > dh, 0 < h_1 < d/b(\lambda - dh)$, then there exists an order one periodic solution of the system* (1.4).

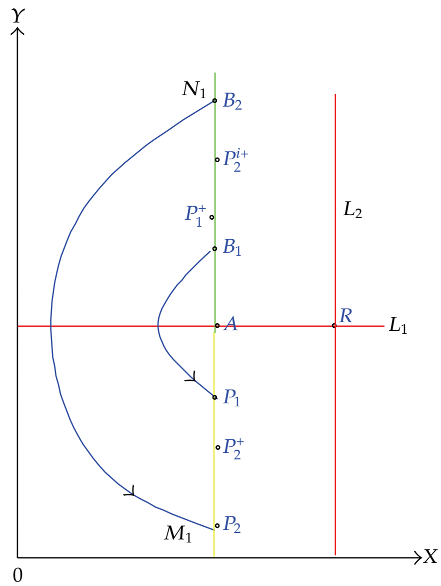


Figure 4: $0 < h_1 < (d/b(1-dh))(a/b) > y_{P_2} + \delta$.

Remark 3.2. It shows from the proved process of Theorem 3.1 that the number of natural enemies should be selected appropriately, which aims to reduce releasing impulsive times to save manpower and resources.

Case 2 ($h_2 < d/b(\lambda - dh)$). In this case, set M_2 and N_2 are both in the left side of stable center R, in the light of the different position of the set N_2, we consider the following two cases.

Subcase 2.1 ($0 < h_1 < (1-\alpha)h_2 < h_2 < d/b(\lambda-dh)$). In this case, the set N_2 is in the right side of M_1 (as shown in Figure 5). The trajectory passing through point A which tangents to N_2 at point A intersects with M_2 at point $P_0(h_2, y_{p_0})$. Since the point $P_0 \in M_2$, then impulse occurs at point P_0. Supposing the point P_0 is subject to impulsive effects to point $P_0^+((1-\alpha)h_2, y_{P_0^+})$, where $y_{P_0^+} = (1-\beta)y_{P_0} + q$, the position of P_0^+ has the following three cases:

(1) ($(1-\beta)y_{P_0}+q > a/b$). Take a point $B_1((1-\alpha)h_2, \varepsilon + a/b) \in N_2$ above A, where $\varepsilon > 0$ is small enough. Then there must exist a trajectory passing through the point B_1 which intersects with the set M_2 at point $P_1(h_2, y_{P_1})$. In view of continuous dependence of the solution on initial value and time, we know $y_{P_1} < y_{P_0}$ and the point P_1 is close to P_0 enough, so we have the point P_1^+ is close to P_0^+ enough and $y_{P_1^+} < y_{P_0^+}$, then we obtain $f(B_1) = y_{P_1^+} - y_{B_1} > 0$.

On the other hand, the trajectory passing through point B tangents to N_1 at point B. Set $F(S) = P_2(h_2, y_{P_2}) \in M_2$. Denote the coordinates of impulsive point $P_2^+((1-\alpha)h_2, y_{P_2^+})$ corresponding to the point $P_2(h_2, y_{P_2})$.

If $y_S \geq y_{P_0^+}$ then $y_{P_2^+} < y_{P_0^+} < y_S$. So we obtain $f(S) = y_{P_2^+} - y_S < 0$. There exists an order one periodic solution of system (1.4), whose initial point is between the point B_1 and the point S in set N_2 (Figure 5).

If $y_S < y_{P_0^+}$ and $y_{P_2^+} \leq y_S$, we have $f(S) = y_{P_2^+} - y_S \leq 0$, we conclude that there exists an order one periodic solution of system (1.4).

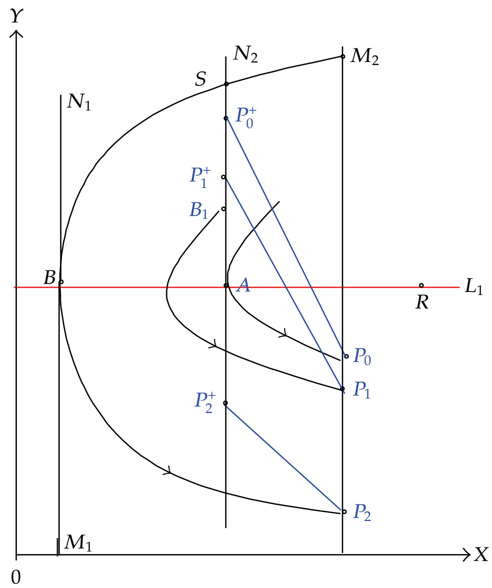


Figure 5: $h_2 < d/b(1-dh)$, $h_1 < (1-\alpha)h_2 < h_2$.

If $y_S < y_{P_0^+}$ and $y_{P_2^+} > y_S$, from the vector field of system (1.4), we know the trajectory of system (1.4) with any initiating point on the N_2 will ultimately stay in $\Omega_1 = \{(x,y)/0 \le x \le h_1, y \ge 0\}$ after one impulsive effect. Therefore there is no an order one periodic solution of system (1.4).

(2) $((1-\beta)y_{P_0} + q < a/b$ (as shown in Figure 6)). In this case, the point P_0^+ lies below the point A, that is, $(1-\beta)y_{P_0} + q < a/b$, thus the successor function of the point A is $f(A) = (1-\beta)y_{P_0} + q - a/b < 0$.

Take another point $B_1((1-\alpha)h_2, \varepsilon) \in N_2$, where $\varepsilon > 0$ is small enough. Then there must exist a trajectory passing through the point B_1 which intersects M_2 at point $P_1(h_2, y_{P_1}) \in M_2$. Suppose the point $P_1(h_2, y_{P_1})$ is subject to impulsive effects to point $P_1^+((1-\alpha)h_2, y_{P_1^+})$, then we have $y_{P_1^+} > \varepsilon$. So we have $f(B_1) = y_{P_1^+} - \varepsilon > 0$.

From Lemma 2.7, there exists an order one periodic solution of system (1.4), whose initial point is between B_1 and A in set N_2.

(3) $((1-\beta)y_{P_0} + q = a/b)$. P_0^+ coincides with A, and the successor function of A is that $f(A) = 0$, so there exists an order one periodic solution of system (1.4) which is just a part of the trajectory passing through the A.

Now, we can summarize the above results as the following theorem.

Theorem 3.3. *Assuming that $\lambda > dh$ and $0 < h_1 < (1-\alpha)h_2 < h_2 < d/b(\lambda - dh)$.*

If $y_{P_0^+} \le y_A$, there exists an order one periodic solutions of the system (1.4)*.*

If $y_{P_0^+} > y_A$, if $y_S \ge y_{P_0^+}$ or $y_S < y_{P_0^+}$, and $y_S > y_{P_2^+}$, there exists an order one periodic solutions of the system (1.4)*.*

If $y_{P_0^+} > y_A$, $y_S < y_{P_0^+}$ and $y_S < y_{P_2^+}$, there is no an order one periodic solutions of the system (1.4)*. The trajectory of system* (1.4) *with any initiating point on the N_2 will ultimately stay in $\Omega_1 = \{(x,y)/0 \le x \le h_1, y \ge 0\}$ after one impulsive effect.*

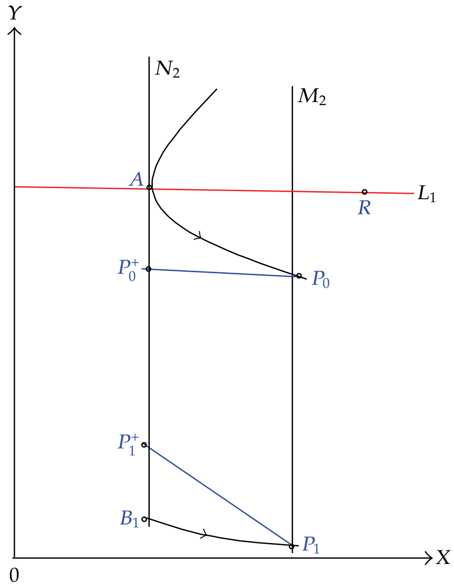


Figure 6: $h_2 < d/b(1 - dh)$, $(1 - \beta)y_{P_0} + q < a/b$.

Subcase 2.2 ($0 < (1-\alpha)h_2 < h_1 < h_2 < d/b(\lambda - dh)$). In this case, the set N_2 is on the left side of N_1. Any trajectory from initial point $(x_0^+, y_0^+) \in N_2$ will intersect with M_1 at some point with time increasing. By the analysis of Case 1, the trajectory from initial point $(x_0^+, y_0^+) \in N_2$ on the set N_2 will stay in the region $\Omega_1 = \{(x, y) \mid x \geq 0, y \geq 0, x \leq h_1\}$. Similarly, any trajectory from initial point $(x_0^+, y_0^+) \in \Omega_0 = \{(x, y) \mid x \geq 0, y \geq 0, x \leq h_2\}$ will stay in the region $\Omega_1 = \{(x, y) \mid x \geq 0, y \geq 0, x \leq h_1\}$ after one impulsive effect or free from impulsive effect.

Theorem 3.4. *If $\lambda > dh$, $0 < (1 - \alpha)h_2 < h_1 < h_2 < d/b(\lambda - dh)$, there is no an order one periodic solutions to the system* (1.4)*, and the trajectory with initial point $(x_0^+, y_0^+) \in \Omega_0 = \{(x, y) \mid x \geq 0, y \geq 0, x \leq h_2\}$ will stay in the region $\Omega_1 = \{(x, y) \mid x \geq 0, y \geq 0, x \leq h_1\}$.*

Case 3 ($d/b(\lambda - dh) < h_2$). In this case, the set M_2 is on the right side of stable center R. In the light of the different position of N_2, we consider the following two subcases.

Subcase 3.1 ($h_1 < (1 - \alpha)h_2 < d/b(\lambda - dh) < h_2$). In this case, the set M_2 is in the right side of R. Then there exists a unique closed trajectory Γ_1 of system (1.4) which contains the point R and is tangent to M_2 at the point A.

Since Γ_1 is closed trajectory, we take their the minimal value $\delta_{\min}$ of abscissas at the trajectory Γ_1, namely, $\delta_{\min} \leq x$ holds for any abscissas of Γ_1.

(1) ($h_1 < (1 - \alpha)h_2 < \delta_{\min} < d/b(\lambda - dh) < h_2$). In this case, there is a trajectory, which contains the point $R(d/b(\lambda - dh), a/b)$ and is tangent to the N_2 at the point B intersects M_2 at a point $P_1(h_2, y_{P_1}) \in M_2$. Suppose point P_1 is subject to impulsive effects to point $P_1^+((1-\alpha)h_2, y_{P_1^+})$, here $y_{P_1^+} = (1-\beta)y_{P_1}+q$. The position of P_1^+ has the following three sub-cases.

If $(1 - \beta)y_{P_1} + q < a/b$ (Figure 7), the point P_1^+ lies below the point B. Like the analysis of Subcase 2.1(2), we can prove there exists an order one periodic solution to the system (1.4) in this case.

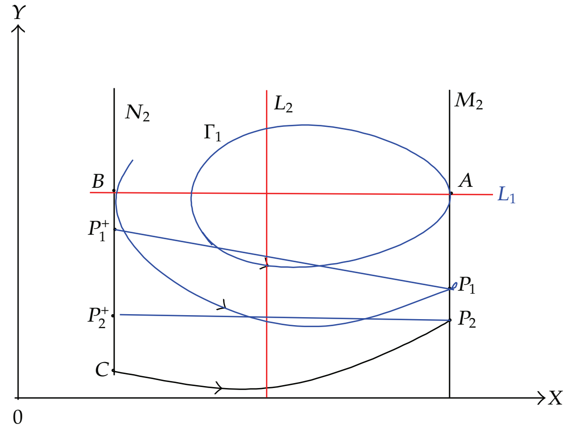


Figure 7: $h_1 < (1-\alpha)h_2 < \delta_{\min} < d/(\lambda - hd)(1-\beta)y_{P_1} + q < a/b$.

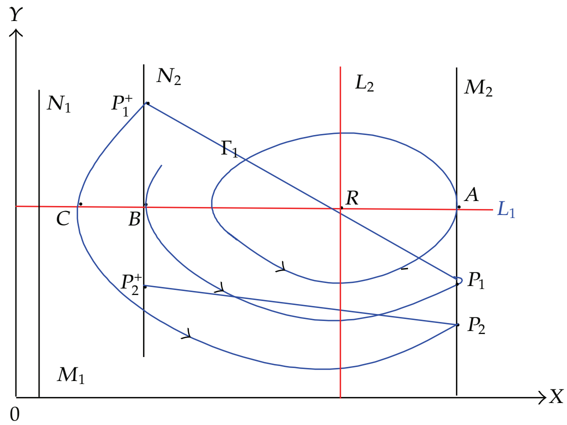


Figure 8: $h_1 < (1-\alpha)h_2 < \delta_{\min} < d/(\lambda - hd)(1-\beta)y_{P_1} + q > a/b$.

If $(1-\beta)y_{P_1} + q > a/b$, the point P_1^+ lies above the point B; the trajectory from initiating point P_1^+ intersects with the line L_1 at point C. If $h_1 \le y_C$ (Figure 8), we have $y_{P_1} > y_{P_2}$ and $y_{P_1^+} > y_{P_2^+}$, then the successor function of P_1^+ is that $f(P_1^+) = y_{P_2^+} - y_{P_1^+} < 0$. Then, we know that there exists an order one periodic solution of system (1.4), whose initial point is between the point P_1^+ and B in set N_2. If $h_1 > y_C$ (Figure 9), there is a trajectory which is tangent to the N_1 at a point D intersects with M_2 at a point $P_3(h_2, y_{P_3}) \in M_2$, P_3 jumps to P_3^+ after the impulsive effects. If $y_{P_3^+} \le y_{B_1}$, we can easily know that there exists an order one periodic solution of system (1.4). If $y_{P_3^+} > y_{B_1}$, by the qualitative analysis of the system (1.4), we know that trajectory with any initiating point on the N_2 will ultimately stay in Γ_1 after a finite number of impulsive effects.

If $(1-\beta)y_{P_1} + q = a/b$, the point P_1^+ coincides with the point B, and the successor function of the point B is that $f(B) = 0$; then there exists an order one periodic solution which is just a part of the trajectory passing through the point B.

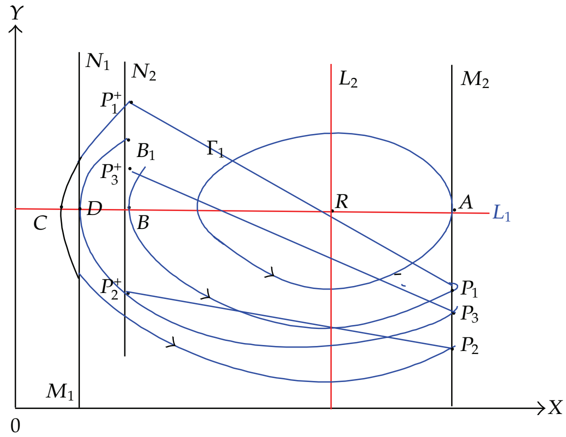


Figure 9: $h_1 < (1-\alpha)h_2 < \delta_{\min} < d/(\lambda - hd) < h_2(1-\beta)y_{P_1} + q > a/b,\ h_1 > y_c$.

Now, we can summarize the above results as the following theorem.

Theorem 3.5. *Assuming that $\lambda > dh$ and $h_1 < (1-\alpha)h_2 < \delta_{\min} < d/b(\lambda - dh) < h_2$.*

If $(1-\beta)y_{P_1} + q \le a/b$, there exists an order one periodic solution to the system (1.4).

If $(1-\beta)y_{P_1} + q > a/b$ and $h_1 \le y_C$, then there exists an order one periodic solution to the system (1.4).

If $(1-\beta)y_{P_1} + q > a/b$, $h_1 > y_C$ and $y_{P_3^+} \le y_{B_1}$, then there exists an order one periodic solution to the system (1.4).

(2) *($h_1 < \delta_{\min} < (1-\alpha)h_2 < d/b(\lambda - dh) < h_2$). In this case, denote the closed trajectory Γ_1 of system* (1.4) *intersects with N_2 two points $A_1((1-\alpha)h_2, y_{A_1})$ and $A_2((1-\alpha)h_2, y_{A_2})$ (as shown in Figure 10). Since $A \in M_2$, impulse occurs at the point A. Suppose point A is subject to impulsive effects to point $P_0^+((1-\alpha)h_2, y_{P_0^+})$, here $y_{P_0^+} = (1-\beta)(a/b) + q$.*

If $(1-\beta)(a/b) + q = y_{A_1}$ or $(1-\beta)(a/b) + q = y_{A_2}$, then P_0^+ coincides with A_1 or P_0^+ coincides with A_2, and the successor function of A_1 or A_2 is that $f(A_1) = 0$ or $f(A_2) = 0$. So, there exists an order one periodic solution of system (1.4) *which is just a part of the trajectory Γ_1.*

If $(1-\beta)(a/b) + q < y_{A_2}$, the point P_0^+ lies below the point A_2. Like the analysis of Subcase 2.1(2), *we can prove there exists an order one periodic solution to the system* (1.4) *in this case.*

If $(1-\beta)(a/b) + q > y_{A_1}$ (as shown in Figure 11), the point P_0^+ is above the point A_1. Like the analysis of Subcase 3.1(1), *we obtain sufficient conditions of existence of order one periodic solution to the system* (1.4).

Theorem 3.6. *Assuming that $\lambda > dh, h_1 < (1-\alpha)h_2 < \delta_{\min} < d/b(\lambda - dh) < h_2$.*

If $(1-\beta)(a/b) + q \le y_{A_2}$, there exists an order one periodic solution to the system (1.4).

If $(1-\beta)(a/b) + q > y_{A_1}$ and $h_1 \le y_C$, then there exists an order one periodic solution to the system (1.4).

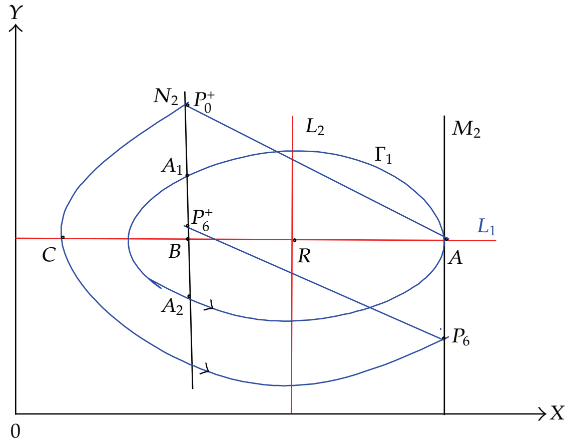


Figure 10: $h_1 < \delta_{\min} < (1-\alpha)h_2 < d/(\lambda - hd) < h_2(1-\beta)(a/b) + q < y_{A_2}$.

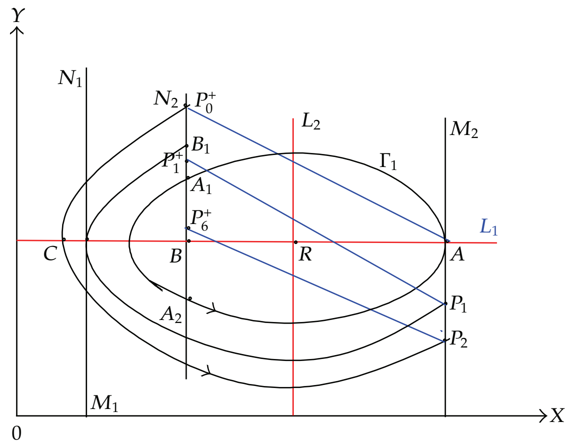


Figure 11: $h_1 < \delta_{\min} < (1-\alpha)h_2 < d/(\lambda - hd) < h_2$.

If $(1-\beta)(a/b) + q > y_{A_1}$, $(1-\beta)y_{P_1} + q > a/b, h_1 > y_C$, *and* $y_{P_3^+} \le y_{B_1}$, *then there exists an order one periodic solution to the system* (1.4).

(3) ($y_{A_2} < (1-\beta)(a/b) + q < y_{A_1}$). *In this case, we note that the point* P_0^+ *must lie between the point* A_1 *and the point* A_2 *(As shown in Figure 12). Taking a point* $B_1 \in M_2$ *such that* B_1 *jumps to* A_2 *after the impulsive effect, denote* $A_2 = B_1^+$. *Since* $y_{P_0^+} > y_{B_1^+}$, *we have* $y_A > y_{B_1}$. *Let* $R(B_1) = B_2^+ \in N_2$, *take a point* $B_2 \in M_2$ *such that* B_2 *jumps to* B_2^+ *after the impulsive effects, then we have* $y_{B_1^+} > y_{B_2^+}$, $y_{B_1} > y_{B_2}$. *This process continues until there exists a* $B_K^+ \in N_2$ $(K \in Z_+)$ *satisfying* $y_{B_k^+} < q$. *So we obtain a sequence* $\{B_k^+\}_{k=1,2,\ldots,K}$ *of the set* M_2 *and a sequence* $\{B_k\}_{k=1,2,\ldots,K}$ *of set* N_2 *satisfying* $R(B_{k-1}) = B_k^+ \in N_2, y_{B_{k-1}^+} > y_{B_k^+}$. *In the following, we will prove the trajectory of system* (1.4) *with any initiating point of set* N_2 *will ultimately stay in* Γ_1.

From the vector field of system (1.4), we know the trajectory of system (1.4) with any initiating point between the point A_1 and A_2 will be free from impulsive effect and ultimately will stay in Γ_1.

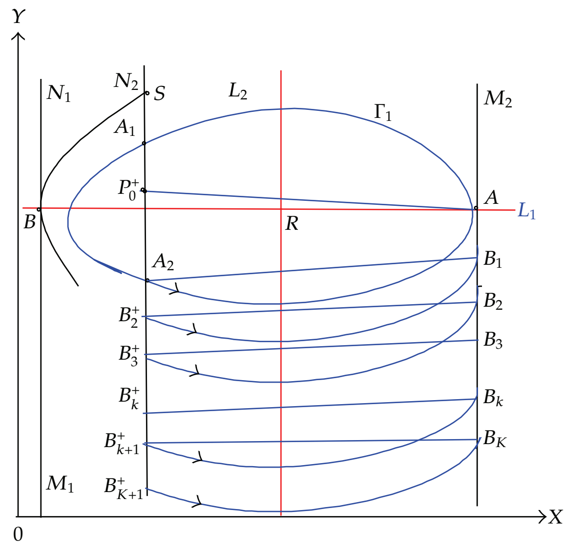


Figure 12: $y_{A_2} < (1-\beta)(a/b) + q < y_{A_1}$.

For any point below A_2, it must lie between B_k^+ and B_{k-1}^+, here $k = 2, 3, \ldots, K+1$ and $A_2 = B_1^+$. After k times' impulsive effects, the trajectory with this initiating point will arrive at some point of the set N_2 which must be between A_1 and A_2, and then ultimately stay in Γ_1.

Denote the intersection of the trajectory passing through the point B which tangents to N_1 at point B with the set N_2 at $S((1-\alpha)h_2, y_S)$. With time increasing, the trajectory of system (1.4) from any initiating point on segment $\overline{A_1 S}$ intersect with the set N_2 at some point which is below A_2; so just like the analysis above we obtain, it will ultimately stay in Γ_1. So for any point below S, will ultimately stay in region Γ_1 with time increasing.

Now, we can summarize the above results as the following theorem.

Theorem 3.7. *Assuming that* $\lambda > dh, h_1 < \delta_{\min} < (1-\alpha)h_2 < d/b(\lambda - dh) < h_2$, *and* $y_{A_2} < (1-\beta)(a/b) + q < y_{A_1}$, *there is no periodic solution in system* (1.4), *and the trajectory with any initiating point on the set* N_2 *will stay in* Γ_1 *or in the region* $\Omega_1 = \{(x, y) \mid x \geq 0, y \geq 0, x \leq h_1\}$.

Subcase 3.2 ($0 < (1-\alpha)h_2 < h_1 < d/b(\lambda - dh) < h_2$). In this case, the set N_2 is on the left side of the set N_1 and M_2 in the right side of R. Like the analysis of Subcase 2.2, we can know that any trajectory with initial point $(x_0^+, y_0^+) \in \Omega_0 = \{(x, y) \mid x \geq 0, y \geq 0,\ x \leq h_2\}$ will stay in the region $\Omega_1 = \{(x, y) \mid x \geq 0, y \geq 0, x \leq h_1\}$ after one impulsive effect or free from impulsive effect.

4. Attractiveness of the Order One Periodic Solution

In this section, under the condition of existence of order one periodic solution to system (1.4) and the initial value of pest population $x(0) \leq h_2$, we discuss its attractiveness. We focus on Case 1 and Case 2; by similar method, we can obtain similar results about Case 3.

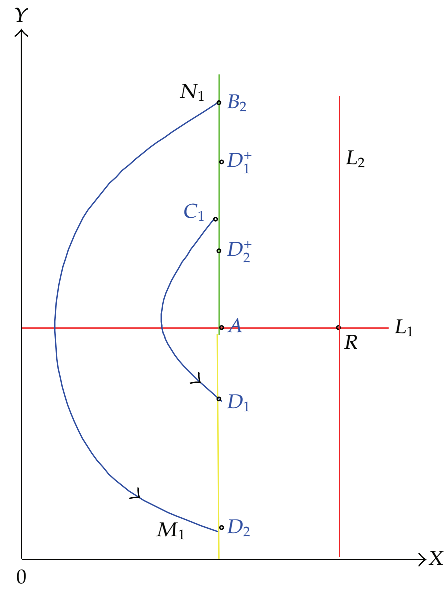


Figure 13: There is a unique order one periodic solution (Theorem 4.1).

Theorem 4.1. *Assuming that* $\lambda > dh, h_1 < d/(b(\lambda - dh))$ *and* $\delta \geq a/b$.
If $y_{P_0^+} > y_{P_2^+} > y_{P^+}$ *or* $y_{P_0^+} < y_{P_2^+} < y_{P^+}$ *(Figure 14), then*

(I) *there exists a unique order one periodic solution of system* (1.4),

(II) *if* $(1-\alpha)h_2 < h_1$, *order one periodic solution of system* (1.4) *is attractive in the region* $\Omega_0 = \{(x,y) \mid x \geq 0, y \geq 0, x \leq h_2\}$.

Proof. By the derivation of Theorem 3.1, we know there exists an order one periodic solution of system (1.4). We assume trajectory $\widehat{PP^+}$ and segment $\overline{PP^+}$ formulate an order one periodic solution of system (1.4), that is, there exists a $P^+ \in N_2$ such that the successor function of P^+ satisfies $f(P^+) = 0$. First, we will prove the uniqueness of the order one periodic solution.

We take any two points $C_1(h_1, y_{C_1}) \in N_1$, $C_2(h_1, y_{C_2}) \in N_1$ satisfying $y_{C_2} > y_{C_1} > y_A$, then we obtain two trajectories whose initiate points are C_1 and C_2 intersects the set M_1 two points $D_1(h_1, y_{D_1})$ and $D_2(h_1, y_{D_2})$, respectively, (Figure 13). In view of the vector field of system (1.4) and the disjointness of any two trajectories without impulse, we know $y_{D_1} > y_{D_2}$. Suppose the points D_1 and D_2 are subject to impulsive effect to points $D_1^+(h_1, y_{D_1^+})$ and $D_2^+(h_2, y_{D_2^+})$, respectively, then we have $y_{D_1^+} > y_{D_2^+}$ and $f(C_1) = y_{D_1^+} - y_{C_1}, f(C_2) = y_{D_2^+} - y_{C_2}$, so we get $f(C_1) - f(C_2) < 0$, thus we obtain the successor function $f(x)$ is decreasing monotonously of N_1, so there is a unique point $P^+ \in N_1$ satisfying $f(P^+) = 0$, and the trajectory $\widehat{P^+PP^+}$ is a unique order one periodic solution of system (1.4).

Next, we prove the attractiveness of the order one periodic solution $\widehat{P^+PP^+}$ in the region $\Omega_0 = \{(x,y) \mid x \geq 0, y \geq 0, x \leq h_2\}$. We focus on the case $y_{P_0^+} > y_{P_2^+} > y_{P^+}$; by similar method, we can obtain similar results about case $y_{P_0^+} < y_{P_2^+} < y_{P^+}$ (Figure 14).

Take any point $P_0^+(h_1, y_{P_0^+}) \in N_1$ above P^+. Denote the first intersection point of the trajectory from initiating point $P_0^+(h_1, y_{P_0^+})$ with the set M_1 at $P_1(h_1, y_{P_1})$, and the corresponding consecutive points are $P_2(h_1, y_{P_2}), P_3(h_1, y_{P_3}), P_4(h_1, y_{P_4}), \ldots$, respectively.

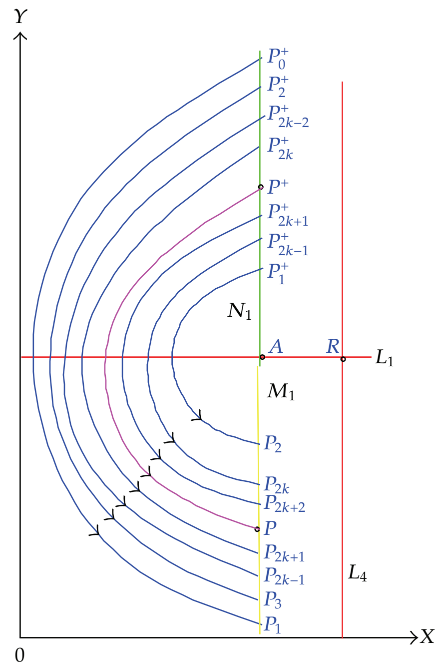


Figure 14: Order one periodic solution is attractive (Theorem 4.1).

Consequently, under the effect of impulsive function, the corresponding points after pulse are $P_1^+(h_1, y_{P_1^+}), P_2^+(h_1, y_{P_2^+}), P_3^+(h_1, y_{P_3^+}), \ldots$.

Due to conditions $y_{P_0^+} > y_{P_2^+} > y_{P^+}$, $y_{P_k^+} = y_{P_k} + \delta$, $\delta \geq a/b$ and disjointness of any two trajectories, then we get a sequence $\{P_k^+\}_{k=1,2,\ldots}$ of the set N_1 satisfying

$$y_{P_1^+} < y_{P_3^+} < \cdots < y_{P_{2k-1}^+} < y_{P_{2k+1}^+} < \cdots < y_{P^+} < \cdots < y_{P_{2k}^+} < y_{P_{2k-2}^+} < \cdots < y_{P_2^+} < y_{P_0^+}. \tag{4.1}$$

So the successor function $f(P_{2k-1}^+) = y_{P_{2k}^+} - y_{P_{2k-1}^+} > 0$ and $f(P_{2k}^+) = y_{P_{2k+1}^+} - y_{P_{2k}^+} < 0$ hold. Series $\{y_{P_{2k-1}}\}_{k=1,2,\ldots}$ increases monotonously and has upper bound, so $\lim_{k\to\infty} y_{P_{2k-1}^+}$ exists. Next, we will prove $\lim_{k\to\infty} y_{P_{2k-1}^+} = y_{P^+}$. Set $\lim_{k\to\infty} P_{2k-1} = C^+$, we will prove $P^+ = C^+$. Otherwise $P^+ \neq C^+$, then there is a trajectory passing through the point C^+ which intersects the set M_1 at point $\widetilde{C}$, then we have $y_{\widetilde{C}} > y_P$, $y_{\widetilde{C}^+} > y_{P^+}$. Since $f(C^+) \geq 0$ and $P^+ \neq C^+$, according to the uniqueness of the periodic solution, then we have $f(C^+) = y_{\widetilde{C}^+} - y_{C^+} > 0$, thus $y_{C^+} < y_{P^+} < y_{\widetilde{C}^+}$ hold. Analogously, let trajectory passing through the point C^+ which intersects the set M_1 at point $\widetilde{\widetilde{C}}$, and the corresponding consecutive points is $\widetilde{\widetilde{C}}$, then $y_{\widetilde{C}} > y_{\widetilde{\widetilde{C}}} > y_p > y_{\widetilde{C}}, y_{\widetilde{C}^+} > y_{\widetilde{\widetilde{C}}^+} > y_{\widetilde{\widetilde{C}}^+} > y_{p^+} > y_{C^+}$, then we have $f(\widetilde{\widetilde{C}}^+) = y_{\widetilde{\widetilde{C}}^+} - y_{\widetilde{\widetilde{C}}^+} > 0$, this is, contradict to the fact that C^+ is a limit of sequence $\{P_{2k-1}^+\}_{k=1,2,\ldots}$, so we obtain $P^+ = C^+$. So, we obtain $\lim_{k\to\infty} y_{P_{2k-1}^+} = y_{P^+}$. Similarly, we can prove $\lim_{k\to\infty} y_{P_{2k}^+} = y_{P^+}$.

From above analysis, we know there exists a unique order one periodic solution in system (1.4), and the trajectory from initiating any point of the N_1 will ultimately tend to be order one periodic solution $\widehat{P^+PP^+}$.

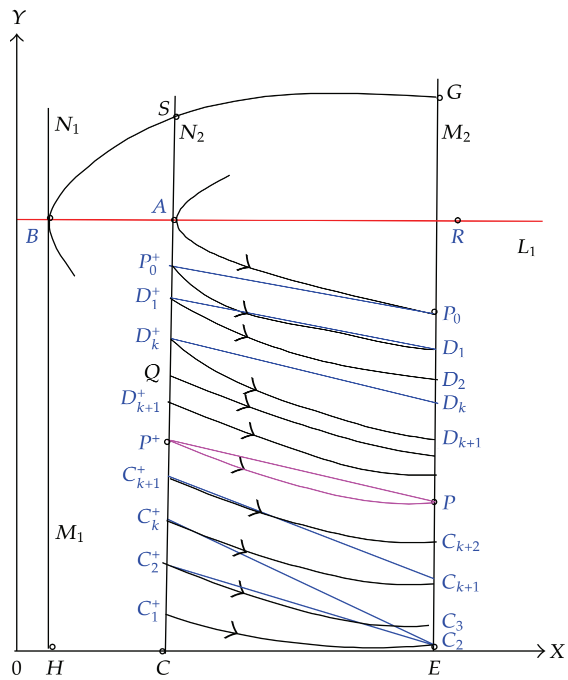


Figure 15: Attractneness of order one periodic solution (Theorem 4.3).

Any trajectory from initial point $(x_0^+, y_0^+) \in \Omega_0 = \{(x, y) \mid x \geq 0, y \geq 0, x \leq h_2\}$ will intersect with N_1 at some point with time increasing on the condition that $(1-\alpha)h_2 < h_1 < h_2 < d/b(\lambda - dh)$; therefore the trajectory from initial point on N_1 ultimately tends to be order one periodic solution $\widehat{P^+PP^+}$. Therefore, order one periodic solution $\widehat{P^+PP^+}$ is attractive in the region Ω_0. This completes the proof. □

Remark 4.2. Assuming that $\lambda > dh, h_1 < h_2 < d/b(\lambda - dh)$ and $\delta \geq a/b$, if $y_{P^+} < y_{P_0^+} < y_{P_2^+}$ or $y_{P^+} > y_{P_0^+} > y_{P_2^+}$ then the order one periodic solution is unattractive.

Theorem 4.3. *Assuming that $\lambda > dh, h_1 < (1-\alpha)h_2 < h_2 < d/b(\lambda - dh)$ and $y_{P_0^+} < y_A$ (as shown in Figure 15), then*

(I) *There exists an odd number of order one periodic solutions of system* (1.4) *with initial value between C_1^+ and A in set N_2.*

(II) *If the periodic solution is unique, then the periodic solution is attractive in region Ω_2, here Ω_2 is open region which is constituted by trajectory $\widehat{GB}$, segment $\overline{BH}$, segment $\overline{HE}$, and segment $\overline{EG}$.*

Proof. (I) According to the Subcase 2.1(2), $f(A) < 0$ and $f(C_1^+) > 0$, and the continuous successor function $f(x)$, there exists an odd number of root satisfying $f(x) = 0$, then we can get there exists an odd number of order one periodic solutions of system (1.4) with initial value between C_1^+ and A in set N_2.

(II) By the derivation of Theorem 3.3, we know there exists an order one periodic solution of system (1.4) whose initial point is between C_1^+ and P_0^+ in the set N_2. Assume trajectory $\widehat{P^+P}$ and segment $\overline{PP^+}$ formulate the unique order one periodic solution of system (1.4) with initial point $P^+ \in N_2$.

On the one hand, take a point $C_1^+((1-\alpha)h_2, y_{C_1^+}) \in N_2$ satisfying $y_{C_1^+} = \varepsilon < q$ and $y_{C_1^+} < y_{P^+}$. The trajectory passing through the point $C_1^+((1-\alpha)h_2, \varepsilon)$ which intersects with the set M_2 at point $C_2(h_2, y_{C_2})$, that is, $F(C_1^+) = C_2 \in M_2$, then we have $y_{C_2} < y_P$, thus $y_{C_2^+} < y_{P^+}$, since $y_{C_2^+} = (1-\beta)y_{C_2} + q > \varepsilon$. So, we obtain $f(C_1^+) = y_{C_2^+} - y_{C_1^+} = y_{C_2^+} - \varepsilon > 0$; Set $F(C_2^+) = C_3 \in M_2$, because $y_{C_1^+} < y_{C_2^+} < y_{P^+}$, we know $y_{C_2} < y_{C_3} < y_P$, then we have $y_{C_2^+} < y_{C_3^+} < y_{P^+}$ and $f(C_2^+) = y_{C_3^+} - y_{C_2^+} > 0$. This process is continuing, then we get a sequence $\{C_k^+\}_{k=1,2,\ldots}$ of the set N_2 satisfying

$$y_{C_1^+} < y_{C_2^+} < \cdots < y_{C_k^+} < \cdots < y_{P^+} \tag{4.2}$$

and $f(C_k^+) = y_{C_{k+1}^+} - y_{C_k^+} > 0$. Series $\{y_{C_k^+}\}_{k=1,2,\ldots}$ increase monotonously and have upper bound, so $\lim_{k\to\infty} y_{C_k^+}$ exists. Like the proof of Theorem 4.1, we can prove $\lim_{k\to\infty} y_{C_k^+} = y_{P^+}$.

On the other hand, set $F(P_0^+) = D_1 \in M_2$, then D_1 jumps to $D_1^+ \in N_2$ under the impulsive effects. Since $y_{P^+} < y_{P_0^+} < y_A$, we have $y_P < y_{D_1} < y_{P_0}$, thus we obtain $y_{P^+} < y_{D_1^+} < y_{P_0^+}$, $f(P_0^+) = y_{D_1^+} - y_{P_0^+} < 0$. Set $F(D_1^+) = D_2 \in M_2$, then D_2 jumps to $D_2^+ \in N_2$ under the impulsive effects. We have $y_{P^+} < y_{D_2^+} < y_{D_1^+}$; this process is continuing, we can obtain a sequence $\{D_k^+\}_{k=1,2,\ldots,}$ of the set N_2 satisfying

$$y_{P_0^+} > y_{D_1^+} > y_{D_2^+} > \cdots > y_{D_k^+} > \cdots > y_{P^+} \tag{4.3}$$

and $f(D_k^+) = y_{D_{k+1}^+} - y_{D_k^+} < 0$. Series $\{y_{D_k^+}\}_{k=1,2,\ldots}$ decreases monotonously and has lower bound, so $\lim_{k\to\infty} y_{D_k^+}$ exists. Similarly, we can prove $\lim_{k\to\infty} y_{D_k^+} = y_{P^+}$.

Any point $Q \in N_2$ below A must be in some interval $[y_{D_{k+1}^+}, y_{D_k^+})_{k=1,2,\ldots}$, $[y_{D_1^+}, y_{P_0^+})$, $[y_{P_0^+}, y_A)$, $[y_{C_k^+}, y_{C_{k+1}^+})_{k=1,2,\ldots}$. Without loss of generality, we assume the point $Q \in [y_{D_{k+1}^+}, y_{D_k^+})$. The trajectory with initiating point Q moves between trajectory $\widehat{D_k^+D_{k+1}}$ and $\widehat{D_{k+1}^+D_{k+2}}$ and intersects with M_2 at some point between D_{k+2} and D_{k+1}; under the impulsive effects, it jumps to the point of N_2 which is between $[y_{D_{k+2}^+}, y_{D_{k+1}^+})$, then trajectory $\widetilde{\Pi}(Q,t)$ continues to move between trajectory $\widehat{D_{k+1}^+D_{k+2}}$ and $\widehat{D_{k+2}^+D_{k+3}}$. This process can be continued unlimitedly. Since $\lim_{k\to\infty} y_{D_k^+} = y_{P^+}$, the intersection sequence of trajectory $\widetilde{\Pi}(Q,t)$, and the set N_2 will ultimately tend to the point P^+. Similarly, if $Q \in [y_{C_k^+}, y_{C_{k+1}^+})$, we also can get the intersection sequence of trajectory $\widetilde{\Pi}(Q,t)$ and the set N_2 will ultimately tend to point P^+. Thus, the trajectory initiating any point below A ultimately tend to the unique order one periodic solution $\widehat{P^+PP^+}$.

Denote the intersection of the trajectory passing through the point B which tangents to N_1 at the point B with the set N_2 at a point $S((1-\alpha)h_2, y_S)$. The trajectory from any initiating point on segment $\overline{AS}$ will intersect with the set N_2 at some point below A with time increasing. So like the analysis above, we obtain the trajectory from any initiating point on segment $\overline{AS}$ will ultimately tend to be the unique order one periodic solution $\widehat{P^+PP^+}$.

Since the trajectory with any initiating point of the Ω_2 will certainly intersect with the set N_2, then from the above analysis, we know the trajectory with any initiating point on segment $\overline{AS}$ will ultimately tend to be order one periodic solution $\widehat{P^+PP^+}$. Therefore, the unique order one periodic solution $\widehat{P^+PP^+}$ is attractive in the region Ω_2. This completes the proof. □

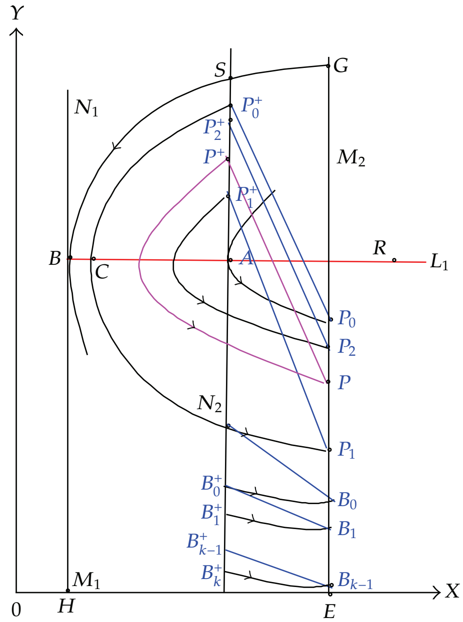


Figure 16: Attractneness of order one periodic solution (Theorem 4.5).

Remark 4.4. Assuming that $\lambda > dh$, $h_1 < (1-\alpha)h_2 < h_2 < d/b(\lambda - dh)$, and $y_{C_1^+} < y_A < y_{P_0^+}$, then the order one periodic solution with initial point between A and P_0^+ is unattractive.

Theorem 4.5. *Assuming that $\lambda > dh$, $h_1 < (1-\alpha)h_2 < h_2 < d/b(\lambda - dh)$, $y_{P_0^+} > y_{P_1^+} > y_A$ (Figure 16) then, there exists a unique order one periodic solution of system* (1.4) *which is attractive in the region Ω_2, here Ω_2 is open region which enclosed by trajectory $\widehat{GB}$, segment $\overline{BH}$, segment $\overline{HE}$ and segment $\overline{EG}$.*

Proof. By the derivation of Theorem 3.3, we know there exists an order one periodic solution of system (1.4). We assume trajectory $\widehat{P^+P}$ and segment $\overline{PP^+}$ formulate an order one periodic solution of system (1.4), that is, $P^+ \in N_2$ is its initial point satisfying $f(P^+) = 0$. Like the proof of Theorem 4.1, we can prove the uniqueness of the order one periodic solution of system (1.4).

Next, we prove the attractiveness of the order one periodic solution $\widehat{P^+PP^+}$ in the region Ω_2.

Denote the first intersection point of the trajectory from initiating point P_0^+ with the impulsive set M_2 at $P_1(h, y_{P_1})$, and the corresponding consecutive points are $P_2(h, y_{P_2}), P_3(h, y_{P_3}), P_4(h, y_{P_4}) \cdots$ respectively. Consequently, under the effect of impulsive function I, the corresponding points after pulse are $P_1^+(h, y_{P_1^+}), P_2^+(h, y_{P_2^+}), P_3^+(h, y_{P_3^+}), \ldots$. In view of $y_{P_0^+} > y_{P_1^+} > y_A$ and disjointness of any two trajectories, we have

$$y_{P_1^+} < y_{P_3^+} < \cdots < y_{P_{2k-1}^+} < y_{P_{2k+1}^+} < \cdots y_{P_{2k}^+} < y_{P_{2k-2}^+} < y_{P_2^+} < y_{P_0^+}. \tag{4.4}$$

So $f(P_{2k-1}^+) = y_{P_{2k}^+} - y_{P_{2k-1}^+} > 0$ and $f(P_{2k}^+) = y_{P_{2k+1}^+} - y_{P_{2k}^+} < 0$ hold. Like the proof of Theorem 4.1, we can prove $\lim_{k\to\infty} y_{P_{2k-1}^+} = \lim_{k\to\infty} y_{P_{2k}^+} = y_{P^+}$.

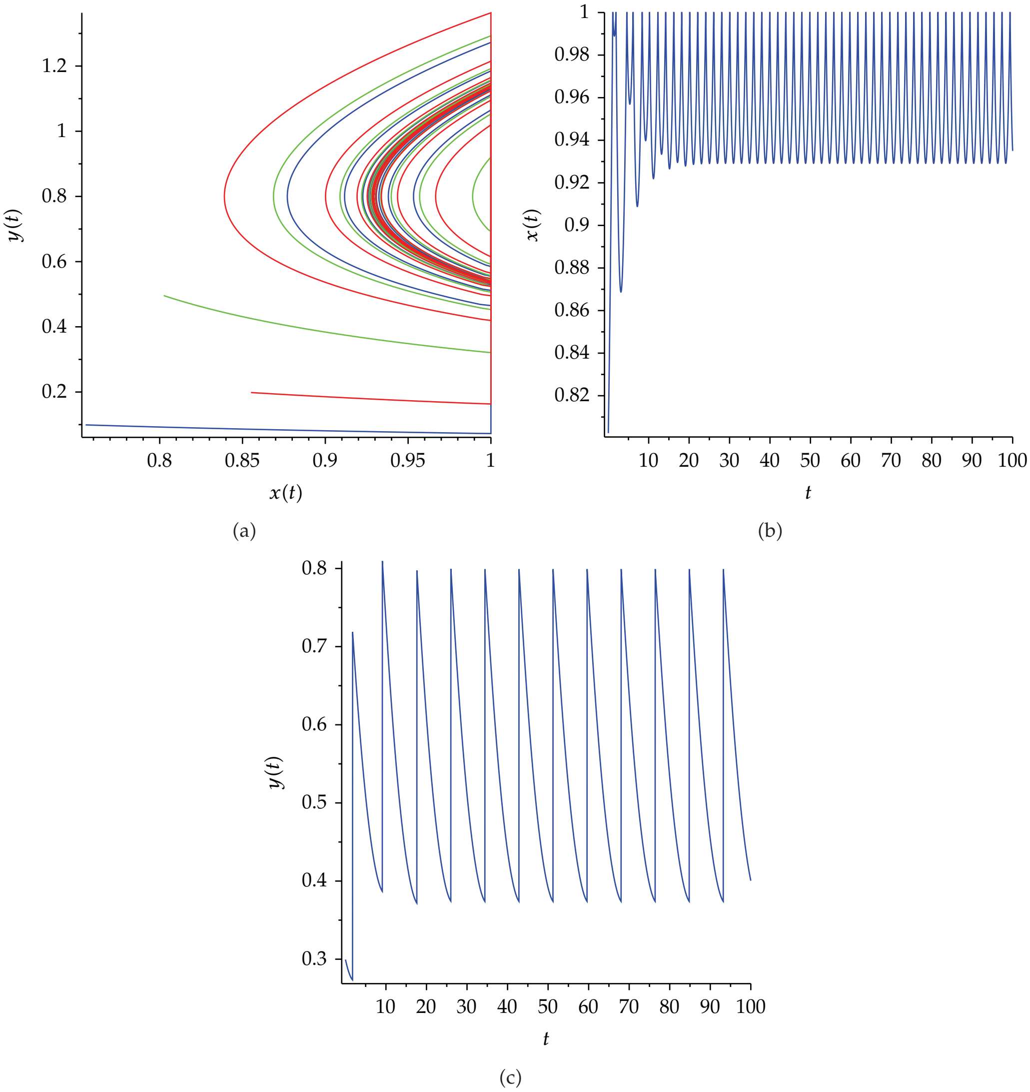


Figure 17: The time series and phase diagram for system (1.4) starting from initial value $(0.85, 0.2)$ (red), $(0.8, 0.5)$ (green), and $(0.75, 0.11)$ (blue), $\delta = 0.6$, $h_1 = 1 < x^*$.

The trajectory from initiating point between B_0^+ and P_0^+ will intersect with impulsive set N_2 with time increasing, under the impulsive effects it arrives at a point of N_2 which is between $[y_{P_{2k-1}^+}, y_{P_{2k+1}^+})$ or $[y_{P_{2k}^+}, y_{P_{2k-2}^+})$. Then like the analysis of Theorem 4.3, we know the trajectory from any initiating point between B_0^+ and P_0^+ will ultimately tend to be order one periodic solution $\widehat{P^+PP^+}$.

Denote the intersection of the trajectory passing through point B which tangents to N_1 at point B with the set N_2 at S. Since the trajectory from initiating any point below S of the set N_2 will certain intersect with set N_2, next we only need to prove the trajectory with any initiating point below S of the set N_2 will ultimately tend to be order one periodic solution $\widehat{P^+PP^+}$.

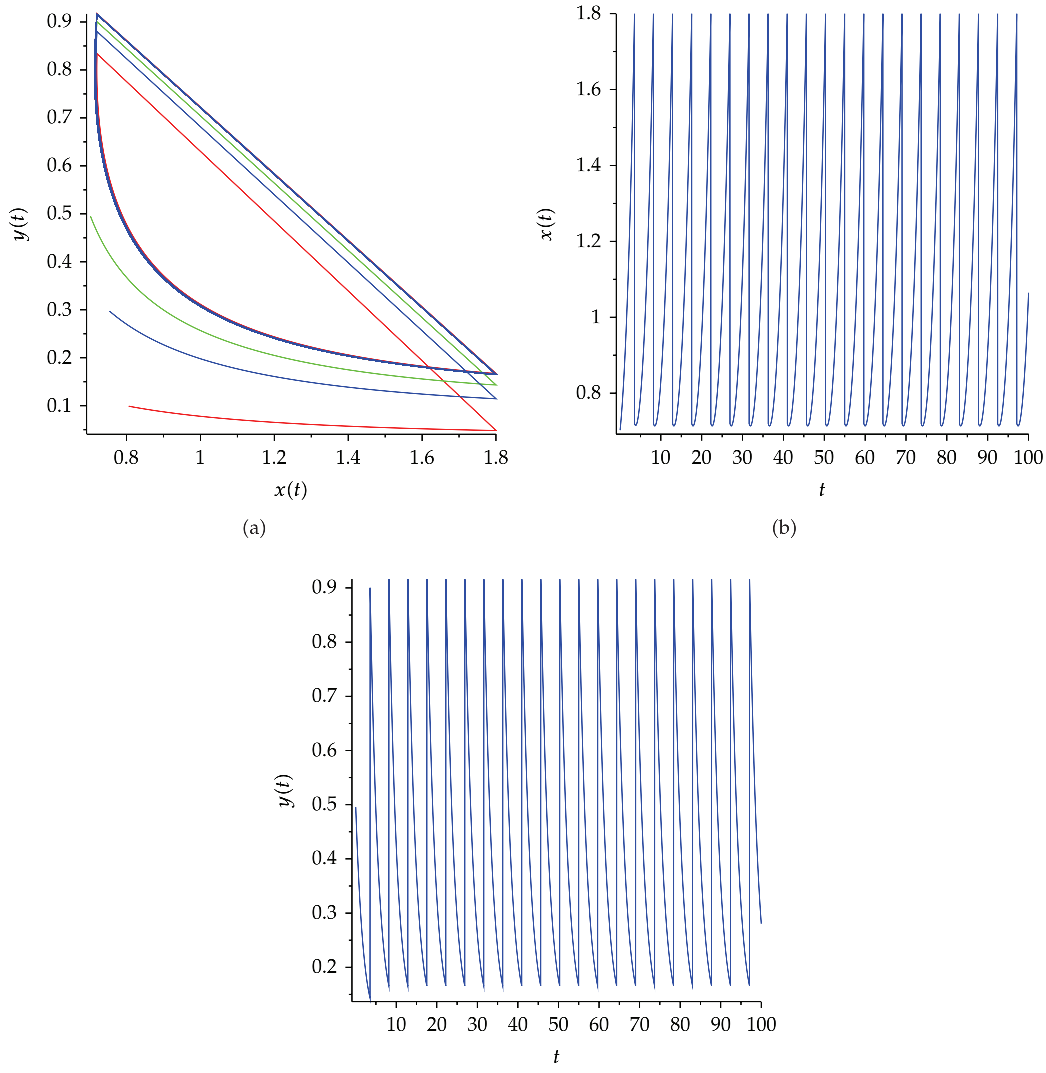


Figure 18: The time series and phase diagram for system (1.4) starting from initial value $(0.8, 0.1)$ (red), $(0.7, 0.5)$ (green), and $(0.75, 0.3)$ (blue) $\alpha = 0.6$, $\beta = 0.3$, $q = 0.8$, $h_2 = 1.8$, $h_1 < h_2 < x^*$.

Assume a point B_0 of set M_2 jumps to B_0^+ under the impulsive effect. Set $R(B_0) = B_1^+ \in N_2$. Assume point B_1 of set N_2 jumps to B_1^+ under the impulsive effect. Set $R(B_1) = B_2^+ \in N_2$. This process is continuing until there exists a $B_{K_0^+} \in N$ $(K_0^+ \in N_2)$ satisfying $y_{B_{K_0^+}} < q$. So we obtain a sequence $\{B_k\}_{k=0,1,2,\dots,K_0}$ of set M_2 and a sequence $\{B_k^+\}_{k=0,1,2,\dots,K_0}$ of set N_2 satisfying $R(B_{k-1}) = B_k^+$, $y_{B_k^+} < y_{B_{k-1}^+}$. For any point of set N_2 below B_0^+, it must lie between B_{k+1}^+ and B_k^+ here $k = 1, 2, \dots, K_0$. After $K_0 + 1$ times' impulsive effects, the trajectory with this initiating point will arrive at some point of the set N_2 which must be between B_0^+ and P_0^+, and then will ultimately tend to order one periodic solution $\widehat{P^+PP^+}$. There is no order one periodic solution with the initial point below B_0^+.

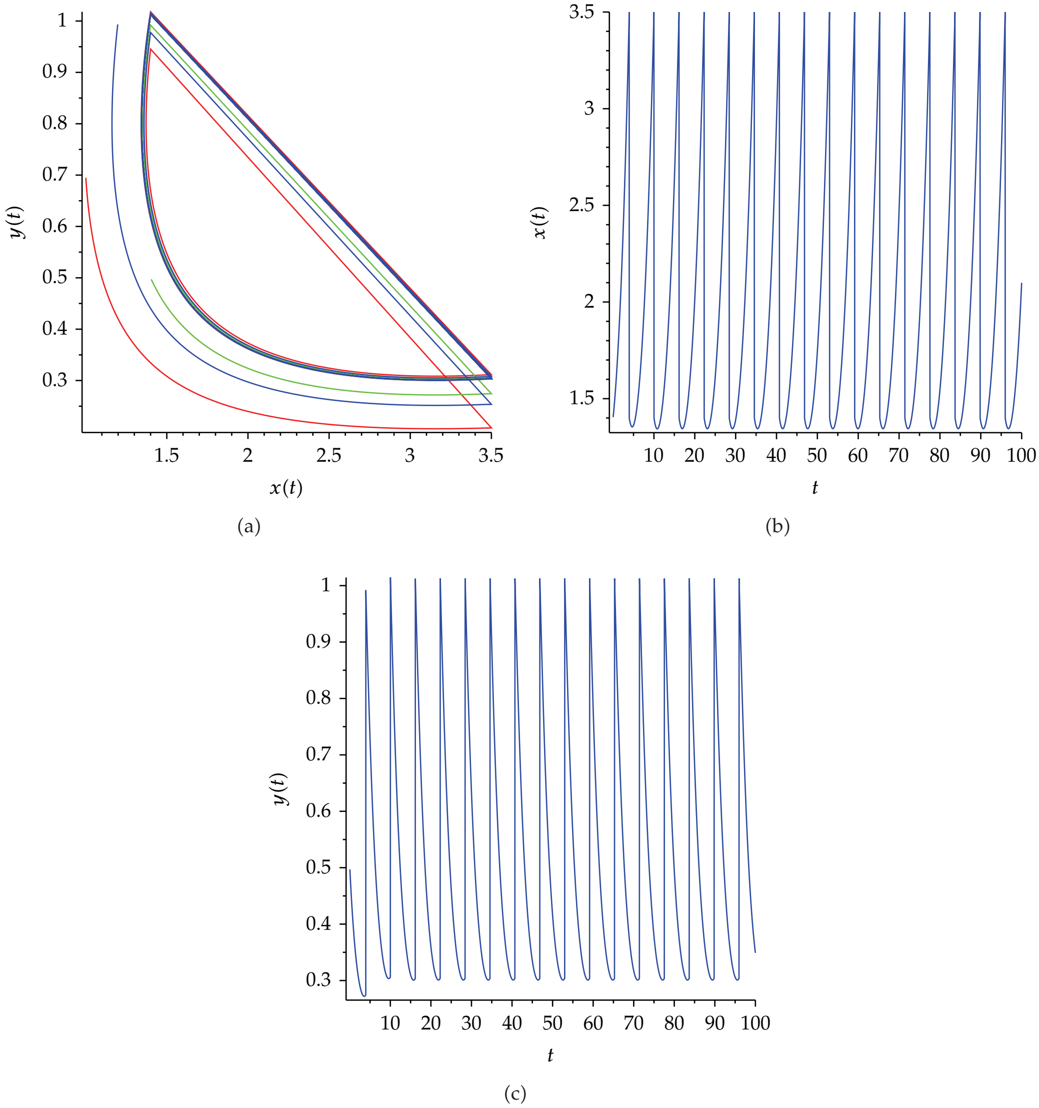


Figure 19: The time series and phase diagram for system (1.4) starting from initial value (1, 0.7) (red), (1.4, 0.5) (green), and (1.2,1) (blue) $\alpha = 0.6$, $\beta = 0.3$, $q = 0.8$, $h_2 = 3.5$, $h_1 < x^* < h_2$.

The trajectory with any initiating point in segment $\overline{AS}$ will intersect with the set N_2 at some point below B_0^+ with time increasing. Like the analysis above, we obtain the trajectory initiating any point on segment $\overline{AS}$ will ultimately tend to be the unique order one periodic solution $\widehat{P^+PP^+}$.

From above analysis, we know there exists a unique order one periodic solution in system (1.4), and the trajectory from any initiating point below S will ultimately tend to be order one periodic solution $\widehat{P^+PP^+}$. Therefore, order one periodic solution $\widehat{P^+PP^+}$ is attractive in the region Ω_2. This completes the proof. $\square$

5. Conclusion

In this paper, a state-dependent impulsive dynamical model concerning different control methods at different thresholds is proposed, we find a new method to study existence and attractiveness of order one periodic solution of such system. We define semicontinuous dynamical system and successor function, demonstrate the sufficient conditions that system (1.4) exists order one periodic solution with differential geometry theory and successor function. Besides, we successfully prove the attractiveness of the order one periodic solution by sequence convergence rules and qualitative analysis.The method can be also extended to mechanical dynamical systems with impacts, for example [19, 20].

These results show that the state-dependent impulsive effects contribute significantly to the richness of the dynamics of the model. The conditions of existence of order one periodic solution in this paper have more extensively applicable scope than the conditions given in [14]. Our results show that, in theory, a pest can be controlled such that its population size is no larger than its ET by applying effects impulsively once, twice, or at most, a finite number of times, or according to a periodic regime. The methods of the theorems are proved to be new in this paper, and these methods are more efficient and easier to operate than the existing research methods which have been applied the models with impulsive state feedback control [16–18, 21], so they are deserved further promotion. In this paper, according to the integrated pest management strategies, we propose a model for pest control which adopts different control methods at different thresholds, the corresponding control is exerted, which leads to the two state impulses in model. Certainly, many biological systems will always be described by three or more state variables, which are the main work in the future.

In order to testify the validity of our results, we consider the following example.

$$\begin{aligned}
&\left.\begin{aligned} x'(t) &= x(t)\big(0.4 - 0.5y(t)\big), \\ y'(t) &= y(t)\left(\frac{0.25x(t)}{1+0.1x(t)} - 0.6\right), \end{aligned}\right\} \quad x \neq h_1, h_2 \text{ or } x = h_1,\ y > y^*, \\
&\left.\begin{aligned} \Delta x(t) &= 0, \\ \Delta y(t) &= \delta, \end{aligned}\right\} \quad x = h_1,\ y \leqslant y^*, \\
&\left.\begin{aligned} \Delta x(t) &= -\alpha x(t), \\ \Delta y(t) &= -\beta y(t) + q, \end{aligned}\right\} \quad x = h_2,
\end{aligned} \tag{5.1}$$

where $\alpha, \beta \in (0,1)$, $\delta > 0$, $q > 0$, $0 < h_1 < h_2$. Now, we consider the impulsive effects influences on the dynamics of system (5.1).

Example 5.1. Existence and attractiveness of order one periodic solution.

We set $h_1 = 1$, $\alpha = 0.6$, $\beta = 0.8$, $q = 0.8$, $h_2 = 1.8$, initiating points are $(0.85, 0.2)$ (red), $(0.8, 0.5)$ (green), and $(0.75, 0.11)$ (blue), respectively. Figure 17 shows that the conditions of Theorems 3.1 and 4.1 hold, system (5.1) exists order one periodic solution. The trajectory from different initiating must ultimately tend to be the order one periodic solution. Therefore, order one periodic solution is attractive.

Example 5.2. Existence and attractiveness of positive periodic solution.

We set $h_1 = 0.7$, $\alpha = 0.6$, $\beta = 0.8$, $q = 0.8$, $h_2 = 1.8$, $h_1 < (1-\alpha)h_2 < x^*$, initiating points are $(0.8, 0.1)$ (red), $(0.7, 0.5)$ (green), and $(0.75, 0.3)$ (blue), respectively. Figure 18 shows that the conditions of Theorems 3.3 and 4.3 hold, there exists order one periodic solution of the system (5.1), and the trajectory from different initiating must ultimately tend to be the order one periodic solution. Therefore, order one periodic solution is attractive.

Example 5.3. Existence and attractive of positive periodic solutions.

We set $h_1 = 0.7$, $\alpha = 0.6$, $\beta = 0.8$, $q = 0.8$, $h_2 = 3.5$, $h_1 < (1-\alpha)h_2 < x^* < h_2$, initiating points are $(1, 0.7)$ (red), $(1.4, 0.5)$ (green), and $(1.2, 1)$ (blue) as shown in Figure 19. Therefore, the conditions of Theorems 3.6 and 4.5 hold, then system (5.1) exists order one periodic solution, and it is attractive.

Acknowledgment

This Project supported by the National Natural Science Foundation of China (no. 10872118).

References

[1] B. Liu, Y. J. Zhang, L. S. Chen, and L. H. Sun, "The dynamics of a prey-dependent consumption model concerning integrated pest management," *Acta Mathematica Sinica*, vol. 21, no. 3, pp. 541–554, 2005.

[2] X. Song, M. Hao, and X. Meng, "A stage-structured predator-prey model with disturbing pulse and time delays," *Applied Mathematical Modelling*, vol. 33, no. 1, pp. 211–223, 2009.

[3] P. Georgescu, H. Zhang, and L. Chen, "Bifurcation of nontrivial periodic solutions for an impulsively controlled pest management model," *Applied Mathematics and Computation*, vol. 202, no. 2, pp. 675–687, 2008.

[4] Z. Xiong, Y. Xue, and S. Li, "A food chain system with Holling IV functional responses and impulsive effect," *International Journal of Biomathematics*, vol. 1, no. 3, pp. 361–375, 2008.

[5] B. Liu, Y. Zhang, and L. Chen, "Dynamic complexities of a Holling I predator-prey model concerning periodic biological and chemical control," *Chaos, Solitons and Fractals*, vol. 22, no. 1, pp. 123–134, 2004.

[6] G. Jiang, Q. Lu, and L. Peng, "Impulsive ecological control of a stage-structured pest management system," *Mathematical Biosciences and Engineering*, vol. 2, no. 2, pp. 329–344, 2005.

[7] Z. Gao, "PD observer parametrization design for descriptor systems," *Journal of the Franklin Institute*, vol. 342, no. 5, pp. 551–564, 2005.

[8] Z. W. Gao and D. W. C. Ho, "On state-space realization of Bezout factorizations in singular systems," *Dynamics of Continuous, Discrete & Impulsive Systems A*, vol. 13, no. 3-4, pp. 387–410, 2006.

[9] A. D'Onofrio, "Pulse vaccination strategy in the SIR epidemic model: global asymptotic stable eradication in presence of vaccine failures," *Mathematical and Computer Modelling*, vol. 36, no. 4-5, pp. 473–489, 2002.

[10] X. Z. Meng and L. S. Chen, "A stage-structured SI eco-epidemiological model with time delay and impulsive controlling," *Journal of Systems Science & Complexity*, vol. 21, no. 3, pp. 427–440, 2008.

[11] L. Nie, J. Peng, Z. Teng, and L. Hu, "Existence and stability of periodic solution of a Lotka-Volterra predator-prey model with state dependent impulsive effects," *Journal of Computational and Applied Mathematics*, vol. 224, no. 2, pp. 544–555, 2009.

[12] D. Baĭnov and P. Simeonov, *Impulsive Differential Equations: Periodic Solutions and Applications*, Longman Scientific & Technical, New York, NY, USA, 1993.

[13] X. Meng and Z. Li, "The dynamics of plant disease models with continuous and impulsive cultural control strategies," *Journal of Theoretical Biology*, vol. 266, no. 1, pp. 29–40, 2010.

[14] S. Tang and R. A. Cheke, "State-dependent impulsive models of integrated pest management (IPM) strategies and their dynamic consequences," *Journal of Mathematical Biology*, vol. 50, no. 3, pp. 257–292, 2005.

[15] S. Tang, Y. Xiao, L. Chen, and R. A. Cheke, "Integrated pest management models and their dynamical behaviour," *Bulletin of Mathematical Biology*, vol. 67, no. 1, pp. 115–135, 2005.

[16] G. Jiang, Q. Lu, and L. Peng, "Impulsive ecological control of a stage-structured pest management system," *Mathematical Biosciences and Engineering*, vol. 2, no. 2, pp. 329–344, 2005.

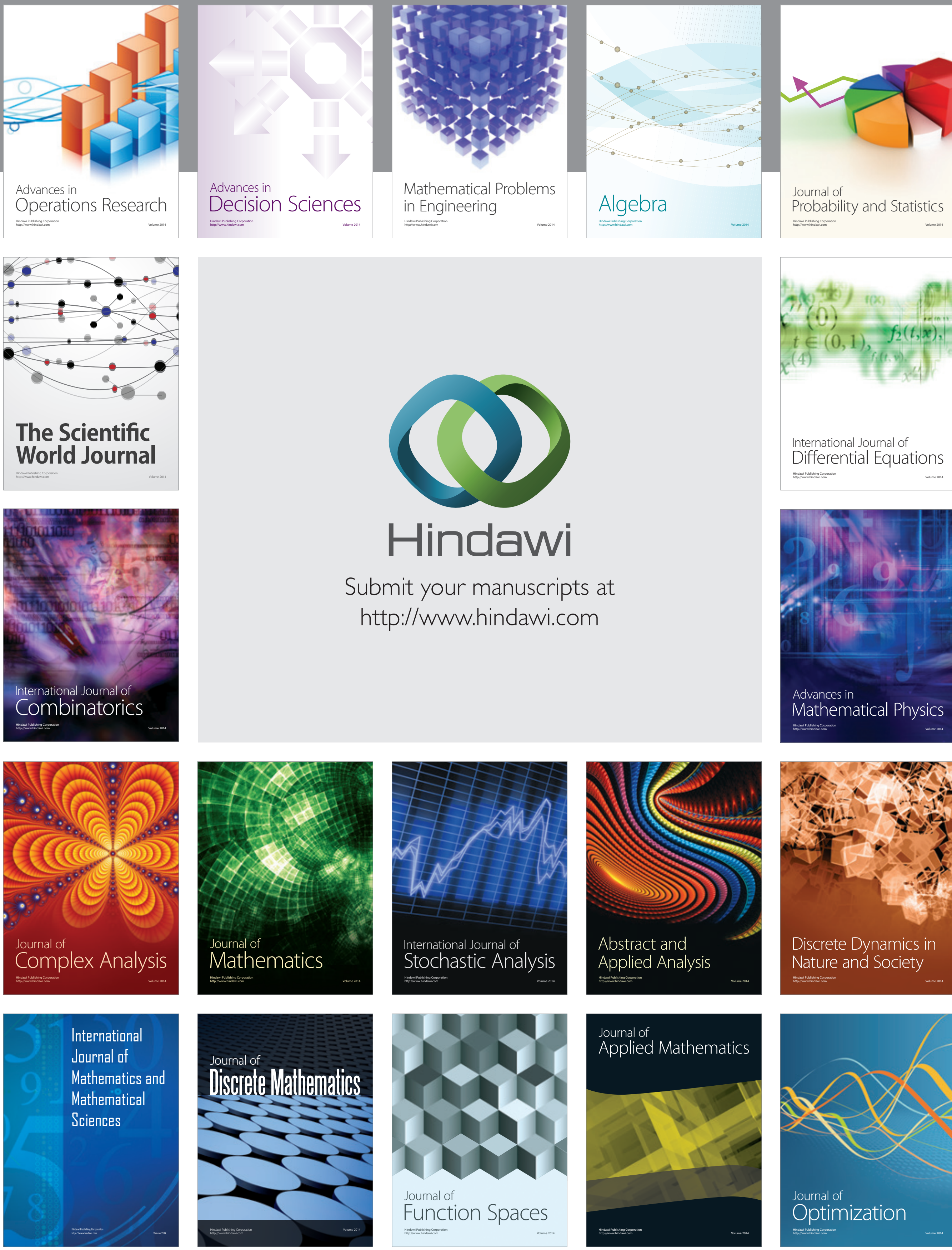
[17] B. Liu, Y. Zhang, and L. Chen, "Dynamic complexities of a Holling I predator-prey model concerning periodic biological and chemical control," *Chaos, Solitons and Fractals*, vol. 22, no. 1, pp. 123–134, 2004.

[18] S. Tang, G. Tang, and R. A. Cheke, "Optimum timing for integrated pest management: modelling rates of pesticide application and natural enemy releases," *Journal of Theoretical Biology*, vol. 264, pp. 623–638, 2010.

[19] J. Awrejcewicz, K. Tomczak, and C.-H. Lamarque, "Controlling system with impacts," *International Journal of Bifurcation and Chaos*, vol. 9, no. 3, pp. 547–553, 1999.

[20] J. Awrejcewicz, M. Fečkan, and P. Olejnik, "On continuous approximation of discontinuous systems," *Nonlinear Analysis*, vol. 62, no. 7, pp. 1317–1331, 2005.

[21] J. Jiao and L. Chen, "Global attractivity of a stage-structure variable coefficients predator-prey system with time delay and impulsive perturbations on predators," *International Journal of Biomathematics*, vol. 1, no. 2, pp. 197–208, 2008.


Advances in Operations Research
Advances in Decision Sciences
Mathematical Problems in Engineering
Algebra
Journal of Probability and Statistics
The Scientific World Journal
International Journal of Differential Equations
Hindawi
Submit your manuscripts at
http://www.hindawi.com
International Journal of Combinatorics
Advances in Mathematical Physics
Journal of Complex Analysis
Journal of Mathematics
International Journal of Stochastic Analysis
Abstract and Applied Analysis
Discrete Dynamics in Nature and Society
International Journal of Mathematics and Mathematical Sciences
Journal of Discrete Mathematics
Journal of Function Spaces
Journal of Applied Mathematics
Journal of Optimization