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Research Article

Refinements of Hermite-Hadamard Inequalities for Functions When a Power of the Absolute Value of the Second Derivative Is *P*-Convex

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We extend some estimates of the right-hand side of Hermite-Hadamard-type inequalities for functions whose second derivatives absolute values are P-convex. Applications to some special means are considered.

1. Introduction

Let $f : I \to \mathbb{R}$ be a convex function defined on the interval I of real numbers and $a, b \in I$ with $a < b$. The following double inequality

$$f\left(\frac{a+b}{2}\right) \leq \frac{1}{b-a}\int_a^b f(x)dx \leq \frac{f(a)+f(b)}{2} \tag{1.1}$$

is known in the literature as the Hermite-Hadamard inequality. Both inequalities hold in the reversed direction if f is concave. We note that Hermite-Hadamard inequality may be regarded as a refinement of the concept of convexity and it follows easily from Jensen's inequality. Note that some of the classical inequalities for means can be derived from (1.1) for appropriate particular selections of the function f. Both inequalities hold in the reversed direction if f is concave (see [1]).

It is well known that the Hermite-Hadamard inequality plays an important role in nonlinear analysis. Over the last decade, this classical inequality has been improved and generalized in a number of ways; there have been a large number of research papers written on this subject, (see, [2–13]) and the references therein. In [13] Dragomir and Agarwal established the following results connected with the right-hand side of (1.1) as well as applied them for some elementary inequalities for real numbers and numerical integration.

Theorem 1.1. *Assume that $a, b \in \mathbb{R}$ with $a < b$ and $f : [a,b] \to \mathbb{R}$ is a differentiable function on (a,b). If $|f'|$ is convex on $[a,b]$, then the following inequality holds:*

$$\left|\frac{f(a)+f(b)}{2} - \frac{1}{b-a}\int_a^b f(x)dx\right| \leq \frac{(b-a)\left(|f'(a)| + |f'(b)|\right)}{8}. \tag{1.2}$$

Theorem 1.2. *Assume that $a, b \in \mathbb{R}$ with $a < b$ and $f : [a,b] \to \mathbb{R}$ is a differentiable function on (a,b). Assume $p \in \mathbb{R}$ with $p > 1$. If $|f'|^{p/(p-1)}$ is convex on $[a,b]$, then the following inequality holds:*

$$\left|\frac{f(a)+f(b)}{2} - \frac{1}{b-a}\int_a^b f(x)dx\right| \leq \frac{b-a}{2(p+1)^{1/p}} \cdot \left[\frac{|f'(a)|^{p/(p-1)} + |f'(b)|^{p/(p-1)}}{2}\right]^{(p-1)/p}. \tag{1.3}$$

In [1] Pearce and Pečarić proved the following theorem.

Theorem 1.3. *Let $f : I \to \mathbb{R}$ be a differentiable function on I°, $a, b \in I^\circ$ with $a < b$. If $|f'|^q$ is convex on $[a,b]$, for $q \geq 1$, then the following inequality holds:*

$$\left|\frac{f(a)+f(b)}{2} - \frac{1}{b-a}\int_a^b f(x)dx\right| \leq \frac{(b-a)}{4}\left(\frac{|f'(a)|^q + |f'(b)|^q}{2}\right)^{1/q}. \tag{1.4}$$

Recall that the function $f : [a,b] \to \mathbb{R}$ is said to be quasiconvex if for every $x, y \in I$ we have

$$f(tx + (1-t)y) \leq \max\{f(x), f(y)\}, \quad \forall t \in [0,1]. \tag{1.5}$$

The generalizations of the Theorems 1.1 and 1.2 are introduced by Ion in [14] for quasiconvex functions and are given in [6] to differentiable P-convex functions. Then, Alomari et al. in [2] improved the results in [14] and Theorem 1.3, for twice differentiable quasiconvex functions.

On the other hand, Dragomir et al. in [11] defined the following class of functions.

Definition 1.4. Let $I \subseteq \mathbb{R}$ be an interval. The function $f : I \to \mathbb{R}$ is said to be P-convex (or belong to the class $P(I)$) if it is nonnegative and, for all $x, y \in I$ and $\lambda \in [0,1]$, satisfies the inequality

$$f(\lambda x + (1-\lambda)y) \leq f(x) + f(y). \tag{1.6}$$

Note that $P(I)$ contain all nonnegative convex and quasiconvex functions. Since then numerous articles have appeared in the literature reflecting further applications in this category, see [3, 6, 12, 15, 16] and references therein. Ozdemir and Yildiz in [15] proved the following results.

Theorem 1.5. *Let $f : I \to \mathbb{R}$ be a twice differentiable function on I° and $a, b \in I^\circ$ with $a < b$. If $|f''|$ is P-convex, $0 \le \lambda \le 1$, then the following inequality holds:*

$$\left|(1-\lambda)f\left(\frac{a+b}{2}\right) - \lambda\frac{f(a)+f(b)}{2} + \frac{1}{b-a}\int_a^b f(x)dx\right| \le \begin{cases} \dfrac{(b-a)^2}{24}(8\lambda^3 - 3\lambda + 1)\{|f''(a)| + |f''(b)|\}, & 0 \le \lambda \le \dfrac{1}{2}, \\ \dfrac{(b-a)^2}{24}(3\lambda - 1)\{|f''(a)| + |f''(b)|\}, & \dfrac{1}{2} \le \lambda \le 1. \end{cases} \quad (1.7)$$

Corollary 1.6. *If in Theorem 1.5 one chooses $\lambda = 1$, one obtains*

$$\left|\frac{f(a)+f(b)}{2} - \frac{1}{b-a}\int_a^b f(x)dx\right| \le \frac{(b-a)^2}{12}\{|f''(a)| + |f''(b)|\}. \quad (1.8)$$

Theorem 1.7. *Let $f : I \to \mathbb{R}$ be a twice differentiable function on I° and $a, b \in I^\circ$ with $a < b$. If $|f''|^q$ is P-convex, $0 \le \lambda \le 1$ and $q \ge 1$, then the following inequality holds:*

$$\left|(1-\lambda)f\left(\frac{a+b}{2}\right) - \lambda\frac{f(a)+f(b)}{2} + \frac{1}{b-a}\int_a^b f(x)dx\right| \le \begin{cases} \dfrac{(b-a)^2}{48}(8\lambda^3 - 3\lambda + 1)\left(\{|f''(a)|^q + |f''(b)|^q\}\right)^{1/q}, & 0 \le \lambda \le \dfrac{1}{2}, \\ \dfrac{(b-a)^2}{48}(3\lambda - 1)\left(\{|f''(a)|^q + |f''(b)|^q\}\right)^{1/q}, & \dfrac{1}{2} \le \lambda \le 1. \end{cases} \quad (1.9)$$

Corollary 1.8. *If in Theorem 1.7 one chooses $\lambda = 1$, one obtains*

$$\left|\frac{f(a)+f(b)}{2} - \frac{1}{b-a}\int_a^b f(x)dx\right| \le \frac{(b-a)^2}{24}\left(\{|f''(a)|^q + |f''(b)|^q\}\right)^{1/q}. \quad (1.10)$$

The main purpose of this paper is to establish the refinements of results in [15]. Applications for special means are considered.

2. Main Results

In order to prove our main theorems, we need the following Lemma in [5] throughout this paper.

Lemma 2.1. *Suppose that $f : I \to \mathbb{R}$ is a twice differentiable function on I°, the interior of I. Assume that $a, b \in I^\circ$, with $a < b$ and f'', is integrable on $[a, b]$. Then, the following equality holds:*

$$
\begin{aligned}
&\frac{f(a)+f(b)}{2} - \frac{1}{b-a}\int_a^b f(x)dx \\
&\quad = \frac{(b-a)^2}{16}\int_0^1 \left(1-t^2\right)\left(f''\left(\frac{1+t}{2}a + \frac{1-t}{2}b\right) + f''\left(\frac{1-t}{2}a + \frac{1+t}{2}b\right)\right)dt.
\end{aligned}
\tag{2.1}
$$

In the following theorem, we will propose some new upper bound for the right-hand side of (1.1) for P-convex functions, which is better than the inequality had done in [15].

Theorem 2.2. *Let $f : I \to \mathbb{R}$ be a twice differentiable function on I° such that $|f''|$ is a P-convex function on I. Suppose that $a, b \in I^\circ$ with $a < b$ and $f'' \in L_1[a, b]$. Then, the following inequality holds:*

$$
\left|\frac{f(a)+f(b)}{2} - \frac{1}{b-a}\int_a^b f(x)dx\right| \le \frac{(b-a)^2}{24}\left[\left|f''(a)\right| + 2\left|f''\left(\frac{a+b}{2}\right)\right| + \left|f''(b)\right|\right]. \tag{2.2}
$$

Proof. Since $|f''|$ is a P-convex function, by using Lemma 2.1 we get

$$
\begin{aligned}
&\left|\frac{f(a)+f(b)}{2} - \frac{1}{b-a}\int_a^b f(x)dx\right| \\
&\quad = \left|\frac{(b-a)^2}{16}\int_0^1 \left(1-t^2\right)\left(f''\left(\frac{1+t}{2}a + \frac{1-t}{2}b\right) + f''\left(\frac{1-t}{2}a + \frac{1+t}{2}b\right)\right)dt\right| \\
&\quad \le \frac{(b-a)^2}{16}\int_0^1 \left|1-t^2\right|\left(\left|f''\left(\frac{1+t}{2}a + \frac{1-t}{2}b\right)\right| + \left|f''\left(\frac{1-t}{2}a + \frac{1+t}{2}b\right)\right|\right)dt \\
&\quad = \frac{(b-a)^2}{24}\left[\left|f''(a)\right| + 2\left|f''\left(\frac{a+b}{2}\right)\right| + \left|f''(b)\right|\right].
\end{aligned}
\tag{2.3}
$$

□

An immediate consequence of Theorem 2.2 is as follows.

Corollary 2.3. *Let f be as in Theorem 2.2, if in addition*

(i) $f''((a+b)/2) = 0$, *then one has*

$$
\left|\frac{f(a)+f(b)}{2} - \frac{1}{b-a}\int_a^b f(x)dx\right| \le \frac{(b-a)^2}{24}\left[\left|f''(b)\right| + \left|f''(b)\right|\right], \tag{2.4}
$$

(ii) $f''(a) = f''(b) = 0$, *then one has*

$$\left|\frac{f(a)+f(b)}{2} - \frac{1}{b-a}\int_a^b f(x)dx\right| \le \frac{(b-a)^2}{12}\left|f''\left(\frac{a+b}{2}\right)\right|. \tag{2.5}$$

The corresponding version for powers of the absolute value of the second derivative is incorporated in the following theorem.

Theorem 2.4. *Let $f : I \to \mathbb{R}$ be a differentiable function on I°. Assume that $p \in \mathbb{R}$, $p > 1$ such that $|f''|^{p/(p-1)}$ is a P-convex function on I. Suppose that $a, b \in I^\circ$ with $a < b$ and $f'' \in L_1[a,b]$. Then, the following inequality holds:*

$$\begin{aligned}
&\left|\frac{f(a)+f(b)}{2} - \frac{1}{b-a}\int_a^b f(x)dx\right| \\
&\quad\le \frac{(b-a)^2}{24}\left(\frac{\sqrt{\pi}}{2}\right)^{1/p}\left(\frac{\Gamma(1+p)}{\Gamma(3/2+p)}\right)^{1/p}\left[\left(|f''(a)|^q + \left|f''\left(\frac{a+b}{2}\right)\right|^q\right)^{1/q}\right. \\
&\qquad\qquad \left.+\left(|f''(b)|^q + \left|f''\left(\frac{a+b}{2}\right)\right|^q\right)^{1/q}\right],
\end{aligned} \tag{2.6}$$

where $1/p + 1/q = 1$.

Proof. By assumption, Lemma 2.1 and Hölder's inequality, we have

$$\begin{aligned}
&\left|\frac{f(a)+f(b)}{2} - \frac{1}{b-a}\int_a^b f(x)dx\right| \\
&\quad\le \left|\frac{(b-a)^2}{16}\int_0^1 \left(1-t^2\right)\left(f''\left(\frac{1+t}{2}a + \frac{1-t}{2}b\right) + f''\left(\frac{1-t}{2}a + \frac{1+t}{2}b\right)\right)dt\right| \\
&\quad\le \frac{(b-a)^2}{16}\left(\int_0^1 \left(1-t^2\right)^p dt\right)^{1/p} \\
&\qquad\times\left[\left(\int_0^1 \left|f''\left(\frac{1+t}{2}a + \frac{1-t}{2}b\right)\right|^q dt\right)^{1/q} + \left(\int_0^1 \left|f''\left(\frac{1-t}{2}a + \frac{1+t}{2}b\right)\right|^q dt\right)^{1/q} dt\right] \\
&\quad\le \frac{(b-a)^2}{24}\left(\frac{\sqrt{\pi}}{2}\right)^{1/p}\left(\frac{\Gamma(1+p)}{\Gamma((3/2)+p)}\right)^{1/p} \\
&\qquad\times\left[\left(|f''(a)|^q + \left|f''\left(\frac{a+b}{2}\right)\right|^q\right)^{1/q} + \left(|f''(b)|^q + \left|f''\left(\frac{a+b}{2}\right)\right|^q\right)^{1/q}\right],
\end{aligned} \tag{2.7}$$

where $1/p + 1/q = 1$. We note that the Beta and Gamma functions are defined, respectively, as follows

$$\Gamma(x) = \int_0^1 e^{-x} t^{x-1} dt, \quad x > 0,$$
$$\beta(x, y) = \int_0^1 t^{x-1}(1-t)^{y-1} dt, \quad x > 0, \ y > 0 \tag{2.8}$$

and are used to evaluate the integral $\int_0^1 (1-t^2)^p dt$. Indeed, by setting $t^2 = u$, we get

$$dt = \frac{1}{2} u^{-1/2} du, \tag{2.9}$$

and using property

$$\beta(x, y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)} \tag{2.10}$$

of Beta function, we obtain

$$\begin{aligned}\int_0^1 \left(1-t^2\right)^p dt &= \frac{1}{2}\int_0^1 u^{-1/2}(1-u)^p du = \frac{1}{2}\beta\left(\frac{1}{2}, p+1\right) \\ &= 2^{-1}\frac{\Gamma(1/2)\Gamma(1+p)}{\Gamma(3/2+p)} = 2^{-1}\frac{\sqrt{\pi}\Gamma(1+p)}{\Gamma(3/2+p)} \\ &= \left(\frac{\sqrt{\pi}}{2}\right)\frac{\Gamma(1+p)}{\Gamma(3/2+p)},\end{aligned} \tag{2.11}$$

where $\Gamma(1/2) = \sqrt{\pi}$ and the proof is completed. □

The following corollary is an immediate consequence of Theorem 2.4.

Corollary 2.5. *Let f be as in Theorem 2.4, if in addition*

(i) $f''((a+b)/2) = 0$, *then one has*

$$\left|\frac{f(a)+f(b)}{2} - \frac{1}{b-a}\int_a^b f(x)dx\right| \leq \frac{(b-a)^2}{24}\left(\frac{\sqrt{\pi}}{2}\right)^{1/p}\left(\frac{\Gamma(1+p)}{\Gamma(3/2+p)}\right)^{1/p}\left(\left|f''(b)\right| + \left|f''(b)\right|\right), \tag{2.12}$$

(ii) $f''(a) = f''(b) = 0$, *then one has*

$$\left|\frac{f(a)+f(b)}{2}-\frac{1}{b-a}\int_a^b f(x)dx\right| \le \frac{(b-a)^2}{12}\left(\frac{\sqrt{\pi}}{2}\right)^{1/p}\left(\frac{\Gamma(1+p)}{\Gamma(3/2+p)}\right)^{1/p}\left|f''\left(\frac{a+b}{2}\right)\right|. \tag{2.13}$$

Another similar result may be extended in the following theorem.

Theorem 2.6. *Let $f : I \to \mathbb{R}$ be a differentiable function on I°. Assume that $q \ge 1$ such that $|f''|^q$ is a P-convex function on I. Suppose that $a, b \in I^\circ$ with $a < b$ and $f'' \in L_1[a,b]$. Then, the following inequality holds:*

$$\left|\frac{f(a)+f(b)}{2}-\frac{1}{b-a}\int_a^b f(x)dx\right|$$
$$\le \frac{(b-a)^2}{24}\left[\left(|f''(a)|^q+\left|f''\left(\frac{a+b}{2}\right)\right|^q\right)^{1/q}+\left(|f''(b)|^q+\left|f''\left(\frac{a+b}{2}\right)\right|^q\right)^{1/q}\right]. \tag{2.14}$$

Proof. Suppose that $a, b \in I^\circ$. From Lemma 2.1 and using well-known power mean inequality, we get

$$\begin{aligned}
&\left|\frac{f(a)+f(b)}{2}-\frac{1}{b-a}\int_a^b f(x)dx\right| \\
&\quad\le \left|\frac{(b-a)^2}{16}\int_0^1\left(1-t^2\right)\left(f''\left(\frac{1+t}{2}a+\frac{1-t}{2}b\right)+f''\left(\frac{1-t}{2}a+\frac{1+t}{2}b\right)\right)dt\right| \\
&\quad\le \frac{(b-a)^2}{16}\left(\int_0^1\left(1-t^2\right)dt\right)^{1-1/q}\left[\left(\int_0^1\left(1-t^2\right)\left|f''\left(\frac{1+t}{2}a+\frac{1-t}{2}b\right)\right|^q dt\right)^{1/q}\right. \\
&\qquad\left.+\left(\int_0^1\left(1-t^2\right)\left|f''\left(\frac{1-t}{2}a+\frac{1+t}{2}b\right)\right|^q dt\right)^{1/q}\right]dt \\
&\quad\le \frac{(b-a)^2}{16}\left(\frac{2}{3}\right)^{1-1/q}\left[\left(\frac{2}{3}\left\{|f''(a)|^q+\left|f''\left(\frac{a+b}{2}\right)\right|^q\right\}\right)^{1/q}\right. \\
&\qquad\left.+\left(\frac{2}{3}\left\{|f''(b)|^q+\left|f''\left(\frac{a+b}{2}\right)\right|^q\right\}\right)^{1/q}\right] \\
&\quad= \frac{(b-a)^2}{24}\left[\left(|f''(a)|^q+\left|f''\left(\frac{a+b}{2}\right)\right|^q\right)^{1/q}+\left(|f''(b)|^q+\left|f''\left(\frac{a+b}{2}\right)\right|^q\right)^{1/q}\right],
\end{aligned} \tag{2.15}$$

which completes the proof. □

Corollary 2.7. *Let f be as in Theorem 2.6, if in addition*

(i) $f''((a+b)/2)=0$, *then* (2.4) *holds,*

(ii) $f''(a)=f''(b)=0$, (2.5) *holds.*

3. Applications to Special Means

Now, we consider the applications of our theorems to the special means. We consider the means for arbitrary real numbers α, β $(\alpha\neq\beta)$. We take the following

(1) Arithmetic mean:

$$A(\alpha,\beta)=\frac{\alpha+\beta}{2},\quad \alpha,\beta\in\mathbb{R}. \tag{3.1}$$

(2) Logarithmic mean:

$$L(\alpha,\beta)=\frac{\alpha-\beta}{\ln|\alpha|-\ln|\beta|},\quad |\alpha|\neq|\beta|,\ \alpha,\beta\neq 0,\ \alpha,\beta\in\mathbb{R}. \tag{3.2}$$

(3) Generalized log-mean:

$$L_n(\alpha,\beta)=\left[\frac{\beta^{n+1}-\alpha^{n+1}}{(n+1)(\beta-\alpha)}\right]^{1/n},\quad n\in\mathbb{N},\ \alpha,\beta\in\mathbb{R},\ \alpha\neq\beta. \tag{3.3}$$

Now, using the results of Section 2, we give some applications for special means of real numbers.

Proposition 3.1. *Let $a,b\in\mathbb{R}$, $a<b$, and $n\in\mathbb{N}$, $n\geq 2$. Then, one has*

$$|L_n^n(a,b)-A(a^n,b^n)|\leq\frac{n(n-1)}{24}(b-a)^2\left[|a|^{n-2}+2\left|\frac{a+b}{2}\right|^{n-2}+|b|^{n-2}\right]. \tag{3.4}$$

Proof. The assertion follows from Theorem 2.2 applied to the P-convex function $f(x)=x^n$, $x\in\mathbb{R}$. □

Proposition 3.2. *Let $a, b \in \mathbb{R}$, $a < b$, and $0 \notin [0,1]$. Then, for all $p > 1$ one has*

$$\begin{aligned}
&\left|L^{-1}(a,b) - A\left(a^{-1}, b^{-1}\right)\right| \\
&\quad \leq \frac{(b-a)^2}{8}\left(\frac{\sqrt{\pi}}{2}\right)^{1/p}\left(\frac{\Gamma(1+p)}{\Gamma((3/2)+p)}\right)^{1/p} \\
&\qquad \times \left[\left(|a|^{-3q} + \left|\frac{a+b}{2}\right|^{-3q}\right)^{1/q} + \left(|b|^{-3q} + \left|\frac{a+b}{2}\right|^{-3q}\right)^{1/q}\right],
\end{aligned} \tag{3.5}$$

where $1/p + 1/q = 1$.

Proof. The assertion follows from Theorem 2.4 applied to the P-convex function $f(x) = 1/x$, $x \in [a,b]$. □

Proposition 3.3. *Let $a, b \in \mathbb{R}$, $a < b$, and $n \in \mathbb{N}$, $n \geq 2$. Then, for all $q \geq 1$ one has*

$$\begin{aligned}
&|L_n^n(a,b) - A(a^n, b^n)| \\
&\quad \leq \frac{n(n-1)}{24}(b-a)^2\left[\left(|a|^{(n-2)q} + \left|\frac{a+b}{2}\right|^{(n-2)q}\right)^{1/q} + \left(|b|^{(n-2)q} + \left|\frac{a+b}{2}\right|^{(n-2)q}\right)^{1/q}\right].
\end{aligned} \tag{3.6}$$

Proof. The assertion follows from Theorem 2.6 applied to the P-convex function $f(x) = x^n$, $x \in \mathbb{R}$. □

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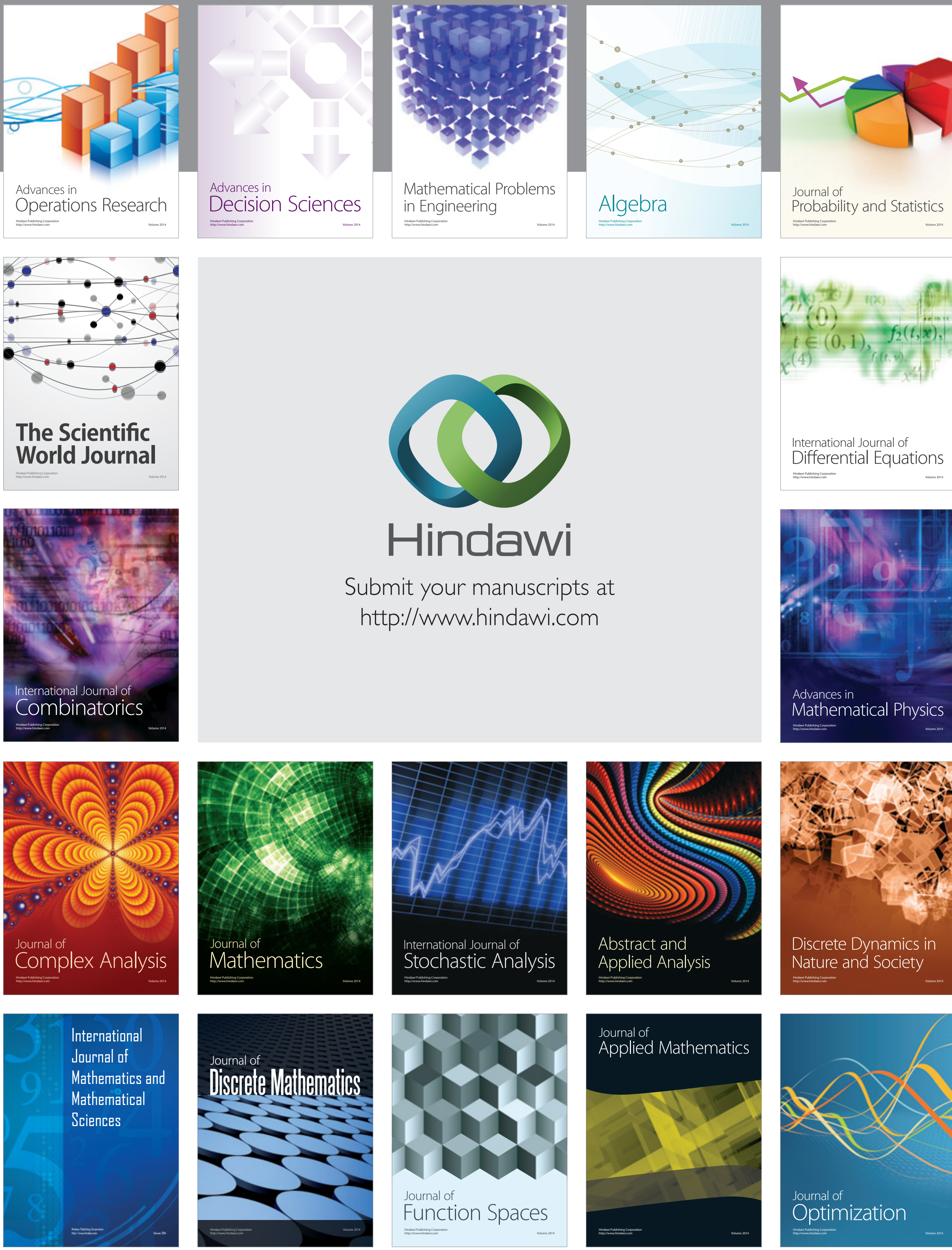
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