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Research Article

A New Proof to the Necessity of a Second Moment Stability Condition of Discrete-Time Markov Jump Linear Systems with Real States

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This paper studies the second moment stability of a discrete-time jump linear system with real states and the system matrix switching in a Markovian fashion. A sufficient stability condition was proposed by Fang and Loparo (2002), which only needs to check the eigenvalues of a deterministic matrix and is much more computationally efficient than other equivalent conditions. The proof to the necessity of that condition, however, is a challenging problem. In the paper by Costa and Fragoso (2004), a proof was given by extending the state domain to the complex space. This paper proposes an alternative necessity proof, which does not need to extend the state domain. The proof in this paper demonstrates well the essential properties of the Markov jump systems and achieves the desired result in the real state space.

1. Introduction

1.1. Background of the Discrete-Time Markov Jump Linear Systems

This paper studies the stability condition of discrete-time jump linear systems in the real state domain. In a jump linear system, the system parameters are subject to abrupt jumps. We are concerned with the stability condition when these jumps are governed by a finite Markov chain. A general model is shown as follows:

$$\begin{gathered} x[k+1] = A\left[q[k]\right]x[k], \\ x[0] = x_0, \qquad q[k] = q_0, \end{gathered} \tag{1.1}$$

where $x[k] \in R^n$ is the state and $\{q[k]\}$ is a discrete-time Markov chain with a finite state space $\{q_1, q_2, \ldots, q_N\}$ and a transition matrix $Q = (q_{ij})_{N\times N}$, where $q_{ij} = P(q[k+1] = q_j \mid q[k] = q_i)$. $x_0 \in R^n$ is the initial state. q_0 is the initial Markov state, whose distribution is denoted as $p = [p_1 \;\; p_2 \;\; \cdots \;\; p_N]$ with $p_i = P(q_0 = q_i)$. $\{q[k]\}$ is assumed to be a *time-homogeneous aperiodic* Markov chain. When $q[k] = q_i$, $A[q[k]] = A_i (i = 1, \ldots, N)$, that is, $A[q[k]]$ switches among $\{A_i\}_{i=1}^N$. A compound matrix is constructed from A_i as

$$A_{[2]} = \left(Q^T \otimes I_{n^2}\right) \operatorname{diag}\left(A_i \otimes A_i\right)_{i=1}^N, \tag{1.2}$$

where I_{n^2} denotes an identity matrix with the order of n^2 and $\otimes$ denotes the Kronecker product [1]. A brief introduction on the Kronecker product will be given in Section 2.1.

For the jump linear system in (1.1), the first question to be asked is *"is the system stable?"* There has been plenty of work on this topic, especially in 90s, [2–6]. Recently this topic has caught academic interest again because of the emergence of networked control systems [7]. Networked control systems often suffer from the network delay and dropouts, which may be modelled as Markov chains, so that networked control systems can be classified into discrete-time jump linear systems [8–11]. Therefore, the stability of the networked control systems can be determined through studying the stability of the corresponding jump linear systems. Before proceeding further, we review the related work.

1.2. Related Work

At the beginning, the definitions of stability of jump linear systems are considered. In [6], three types of second moment stability are defined.

Definition 1.1. For the jump linear system in (1.1), the equilibrium point 0 is

(1) *stochastically stable*, if, for every initial condition $(x[0] = x_0, q[0] = q_0)$,

$$\mathbf{E}\left[\sum_{k=0}^{\infty} \|x[k]\|^2 \mid x_0, q_0\right] < \infty, \tag{1.3}$$

where $\|\cdot\|$ denotes the 2-norm of a vector;

(2) *mean square stable* (MSS), if, for every initial condition (x_0, q_0),

$$\lim_{k\to\infty} \mathbf{E}\left[\|x[k]\|^2 \mid x_0, q_0\right] = 0; \tag{1.4}$$

(3) *exponentially mean square stable*, if, for every initial condition (x_0, q_0), there exist constants $0 < \alpha < 1$ and $\beta > 0$ such that for all $k \geq 0$,

$$\mathbf{E}\left[\|x[k]\|^2 \mid x_0, q_0\right] < \beta\alpha^k \|x_0\|^2, \tag{1.5}$$

where α and β are independent of x_0 and q_0.

In [6], the above 3 types of stabilities are proven to be equivalent. So we can study mean square stability without loss of generality. In [6], a necessary and sufficient stability condition is proposed.

Theorem 1.2 (see [6]). *The jump linear system in* (1.1) *is mean square stable, if and only if, for any given set of positive definite matrices* $\{W_i : i = 1, \dots, N\}$, *the following coupled matrix equations have unique positive definite solutions* $\{M_i : i = 1, \dots, N\}$:

$$\sum_{j=1}^{N} q_{ij} A_i^T M_j A_i - M_i = -W_i. \tag{1.6}$$

Although the above condition is necessary and sufficient, it is difficult to verify because it claims validity for any group of positive definite matrices $\{W_i : i = 1, \dots, N\}$. *A more computationally efficient testing criterion was, therefore, pursued [3, 4, 12–15]. Theorem 1.3 gives a sufficient mean square stability condition.*

Theorem 1.3 (see [4, 12]). *The jump linear system in* (1.1) *is mean square stable, if all eigenvalues of the compound matrix* $A_{[2]}$ *in* (1.2) *lie within the unit circle.*

Remark 1.4. By Theorem 1.3, the mean square stability of a jump linear system can be reduced to the stability of a deterministic system in the form $y_{k+1} = A_{[2]} y_k$ [13]. Thus the complexity of the stability problem is greatly reduced. Theorem 1.3 only provides a sufficient condition for stability. The condition was conjectured to be necessary as well [2, 15]. In the following, we briefly review the research results related to Theorem 1.3.

In [14], Theorem 1.3 was proven to be necessary and sufficient for a scalar case, that is, $A_i (i = 1, \dots, N)$ are scalar. In [15], the necessity of Theorem 1.3 was proven for a special case with $N = 2$ and $n = 2$. In [4, 12], Theorem 1.3 was asserted to be necessary and sufficient for more general jump linear systems. Specifically, Bhaurucha [12] considered a random sampling system with the sampling intervals governed by a Markov chain while Mariton [4] studied a continuous-time jump linear system. Although their sufficiency proof is convincing, their necessity proof is incomplete.

The work in [3] may shed light on the proof of the necessity of Theorem 1.3. In [3], a jump linear system model being a little different from (1.1) is considered. The difference lies in

(i) $x[k] \in C^n$, where C stands for the set of complex numbers,

(ii) $x_0 \in S_c$, where S_c is the set of complex vectors with finite second-order moments in the complex state space.

The mean square stability in [3] is defined as

$$\operatorname*{Lim}_{k \to \infty} \mathbf{E}\left[x[k] x^*[k] \mid x_0, q_0\right] = 0, \quad \forall x_0 \in S_c, \ \forall q_0, \tag{1.7}$$

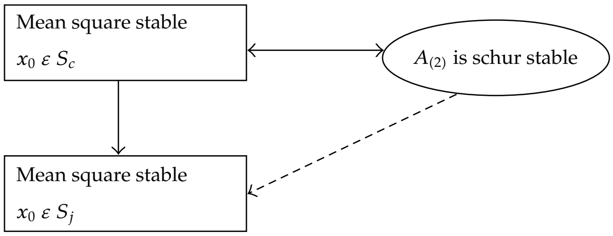

Figure 1: Relationship between different senses of mean square stability.

where * stands for the conjugate transpose. Corresponding to the definition in (1.7), the mean square stability in (1.4) can be eewritten into (because $x[k] \in R^n$ in (1.4), there is no difference between $x^T[k]$ and $x^*[k]$),

$$\operatorname*{Lim}_{k \to \infty} \mathbf{E}\left[x[k]x^*[k] \mid x_0, q_0\right] = 0, \quad \forall x_0 \in S_j, \forall q_0, \tag{1.8}$$

where S_j is the set of all vectors in R^n. For any vector $x \in R^n$, we can treat it as a random vector with a single element in R^n, and also a random vector in C^n. Of course, such random vectors have finite second-order moments. Therefore, we know

$$S_j \subset S_c, \qquad S_j \cap S_c \neq S_c. \tag{1.9}$$

It can be seen that the mean square stability in (1.7) requires *stronger* condition ($x_0 \in S_c$) than the one in (1.8) ($x_0 \in S_j$). When $A_i(i = 1, \ldots, N)$ are real matrices, a necessary and sufficient stability condition was given in the complex state domain.

Theorem 1.5 (see [3]). *The jump linear system in* (1.1) *(with complex states) is mean square stable in the sense of* (1.7) *if and only if $A_{[2]}$ is Schur stable.*

Due to the relationship of $S_j \subset S_c$ and Theorem 1.5, we can establish the relationship diagram in Figure 1. As it shows, the Schur stability of $A_{[2]}$ is a sufficient condition for mean square stability with $x_0 \in S_j$ at the first look.

We are still wondering *"whether the condition in Theorem 1.3 is necessary too?"* the answer is definitely *"yes."* That necessity was conjectured in [2]. A proof to the necessity of that condition was first given in [16], which extends the state domain to the complex space and establishes the desired necessity in the stability sense of (1.7). As mentioned before, our concerned stability (in the sense of (1.8)) is weaker than that in (1.7). This paper proves that the weaker condition in (1.8) still yields the schur stability of $A_{[2]}$, that is, the necessity of theorem 1.3 is confirmed. This paper confines the state to the real space domain and makes the best use of the essential properties of the markov jump linear systems to reach the desired necessity goal. In Section 2, a necessary and sufficient version of Theorem 1.3 is stated and its necessity is strictly proven. In Section 3, final remarks are placed.

2. A Necessary and Sufficient Condition for Mean Square Stability

This section will give a necessary and sufficient version of Theorem 1.3. Throughout this section, we will define mean square stability in the sense of (1.4) ($x_0 \in S_j$). At the beginning, we will give a brief introduction to the Kronecker product and list some of its properties. After then, the main result, a necessary and sufficient condition for the mean square stability, is presented in Theorem 2.1 and its necessity is proven by direct matrix computations.

2.1. Mathematical Preliminaries

Some of the technical proofs in this paper make use of the *Kronecker product*, $\otimes$ [1]. The Kronecker product of two matrices $A = (a_{ij})_{M\times N}$, $B = (b_{pq})_{P\times Q}$ is defined as

$$A \otimes B = \begin{bmatrix} a_{11}B & a_{12}B & \cdots & a_{1N}B \\ a_{21}B & a_{22}B & \cdots & a_{2N}B \\ \vdots & \vdots & \ddots & \vdots \\ a_{M1}B & a_{M2}B & \cdots & a_{MN}B \end{bmatrix}_{MP\times NQ}. \tag{2.1}$$

For simplicity, $A \otimes A$ is denoted as $A^{[2]}$ and $A \otimes A^{[n]}$ is denoted as $A^{[n+1]}(n \geq 2)$.

For two vectors x and y, $x \otimes y$ simply rearranges the columns of xy^T into a vector. So for two stochastic processes $\{x[n]\}$ and $\{y[n]\}$, $\lim_{n\to\infty}\mathbf{E}[x[n] \otimes y[n]] = 0$ if and only if $\lim_{n\to\infty}\mathbf{E}[x[n]y^T[n]] = 0$. Furthermore, if $\lim_{n\to\infty}\mathbf{E}[x^{[2]}[n]] = 0$ and $\lim_{n\to\infty}\mathbf{E}[y^{[2]}[n]] = 0$, then

$$\lim_{n\to\infty}\mathbf{E}\big[x[n] \otimes y[n]\big] = 0. \tag{2.2}$$

The following property of the Kronecker product will be frequently used in the technical proofs

$$(A_1A_2\cdots A_n) \otimes (B_1B_2\cdots B_n) = (A_1 \otimes B_1)(A_2 \otimes B_2)\cdots(A_n \otimes B_n), \tag{2.3}$$

where A_i, $B_i(i = 1, 2, \ldots, n)$ are all matrices with appropriate dimensions.

Our computations need two linear operators, **vec** and **devec**. The **vec** operator transforms a matrix $A = (a_{ij})_{M\times N}$ into a vector as

$$\text{vec}(A) = [a_{11}\cdots a_{M1}a_{12}\cdots a_{M2}\cdots a_{1N}\cdots a_{MN}]^T. \tag{2.4}$$

The **devec** operator inverts the **vec** operator for a square matrix, that is,

$$\text{devec}(\text{vec}(A)) = A, \tag{2.5}$$

where A is a square matrix.

2.2. Main Results

Theorem 2.1. *The jump linear system in* (1.1) *is mean square stable if and only if* $A_{[2]}$ *is Schur stable, that is, all eigenvalues of* $A_{[2]}$ *lie within the unit circle.*

There are already some complete proofs for sufficiency of Theorem 2.1, [3, 12, 13]. So we will focus on the necessity proof. Throughout this section, the following notational conventions will be followed.

The initial condition of the jump linear system in (1.1) is denoted as $x[0] = x_0, q[0] = q_0$ and the distribution of q_0 is denoted as $p = [p_1 \;\; p_2 \;\; \cdots \;\; p_N]$ $(P(q[0] = q_i \mid q_0) = p_i)$.

The system transition matrix in (1.1) is defined as

$$\Phi(k;m) = \begin{cases} \prod_{l=m}^{k-1} A[q[l]], & \text{if } m < k, \\ I_n, & \text{if } m \geq k, \end{cases} \tag{2.6}$$

where I_n is an identity matrix with the order of n. With this matrix, the system's state at time instant k can be expressed as

$$x[k] = \Phi(k;0)x_0. \tag{2.7}$$

A conditional expectation is defined as

$$\Phi_i[k] = P\big(q[k] = q_i \mid q_0\big)\mathbf{E}\Big[(\Phi(k;0))^{[2]} \mid q[k] = q_i, q_0\Big], \tag{2.8}$$

where $i = 1, 2, \ldots, N$. Specially $\Phi_i[0] = p_i I_{n^2}$ $(i = 1, \ldots, N)$. Based on the definition of $\Phi_i[k]$, we obtain

$$\mathbf{E}\Big[(\Phi(k;0))^{[2]} \mid q_0\Big] = \sum_{i=1}^{N}\Phi_i[k]. \tag{2.9}$$

By combining all $\Phi_i[k](i = 1, 2, \ldots, N)$ into a bigger matrix, we define

$$V_\Phi[k] = \Big[\Phi_1^T[k] \;\; \Phi_2^T[k] \;\; \cdots \;\; \Phi_N^T[k]\Big]^T. \tag{2.10}$$

Thus, $V_\Phi[0] = p^T \otimes I_{n^2}$.

The necessity proof of Theorem 2.1 needs the following three preliminary Lemmas.

Lemma 2.2. *If the jump linear system in* (1.1) *is mean square stable, then*

$$\lim_{k\to\infty}\mathbf{E}\Big[(\Phi[k;0])^{[2]} \mid q_0\Big] = 0, \quad \forall q_0. \tag{2.11}$$

Proof of Lemma 2.2. Because the system is mean square stable, we get

$$\lim_{k\to\infty} \mathbf{E}\left[x^{[2]}[k] \mid x_0, q_0\right] = 0, \quad \forall x_0,\ q_0. \tag{2.12}$$

The expression of $x[k] = \Phi(k;0)x_0$ yields

$$\lim_{k\to\infty} \mathbf{E}\left[(\Phi(k;0)x_0)^{[2]} \mid x_0, q_0\right] = 0. \tag{2.13}$$

$\Phi(k;0)$ is an $n \times n$ matrix. So we can denote it as $\Phi(k;0) = [a_1(k), a_2(k), \ldots, a_n(k)]$, where $a_i(k)$ is a column vector. By choosing $x_0 = e_i$ (e_i is an $R^{n\times 1}$ vector with the ith element as 1 and the others as 0), (2.13) yields

$$\lim_{k\to\infty} \mathbf{E}\left[a_i^{[2]}[k] \mid q_0\right] = 0, \quad i = 1, 2, \ldots, n. \tag{2.14}$$

By the definition of the Kronecker product, we know

$$(\Phi(k;0))^{[2]} = [a_1[k] \otimes a_1[k], \ldots, a_1[k] \otimes a_n[k], \ldots, a_n[k] \otimes a_1[k], \ldots, a_n[k] \otimes a_n[k]]. \tag{2.15}$$

So (2.14) yields

$$\lim_{k\to\infty} \mathbf{E}\left[(\Phi[k;0])^{[2]} \mid q_0\right] = 0, \quad \forall q_0. \tag{2.16}$$

□

Lemma 2.3. *If the jump linear system in* (1.1) *is mean square stable, then*

$$\lim_{k\to\infty} \Phi_i[k] = 0, \quad i = 1, \ldots, N,\ \forall q_0. \tag{2.17}$$

Proof of Lemma 2.3. Choose any z_0, $w_0 \in R^n$. Lemma 2.2 guarantees

$$\lim_{k\to\infty} \mathbf{E}\left[\left(z_0^{[2]}\right)^T (\Phi(k;0))^{[2]} w_0^{[2]} \mid q_0\right] = 0. \tag{2.18}$$

By the definition of the Kronecker product, we know

$$\mathbf{E}\left[\left(z_0^{[2]}\right)^T (\Phi(k;0))^{[2]} w_0^{[2]} \mid q_0\right] = \mathbf{E}\left[\left(z_0^T \Phi(k;0) w_0\right)^2 \mid q_0\right]. \tag{2.19}$$

By (2.8), (2.9), and (2.19), we get

$$\mathbf{E}\left[\left(z_0^T \Phi[k;0] w_0\right)^2 \mid q_0\right] = \sum_{i=1}^{N} P(q[k] = q_i \mid q_0) \mathbf{E}\left[\left(z_0^T \Phi[k,0] w_0\right)^2 \mid q[k] = q_i, q_0\right]. \tag{2.20}$$

Because $P(q[k]=q_i \mid q_0) \geq 0$ and $\mathbf{E}[(z_0^T\Phi[k;0]w_0)^2 \mid q[k]=q_i,q_0] \geq 0$, the combination of (2.18) and (2.20) yields

$$\lim_{k\to\infty} P(q[k]=q_i \mid q_0)\mathbf{E}\left[\left(z_0^T\Phi(k;0)w_0\right)^2 \mid q[k]=q_i,q_0\right]=0. \tag{2.21}$$

$\Phi(k;0)$ is an $n\times n$ matrix. So it can be denoted as $\Phi(k;0)=(a_{mj}(k))_{m=1,\dots,n;j=1,\dots,n}$. In (2.21), we choose $z_0=e_m$ and $w_0=e_j$ and get

$$\lim_{k\to\infty} P(q[k]=q_i \mid q_0)\mathbf{E}\left[(a_{mj}(n))^2 \mid q[k]=q_i,q_0\right]=0, \tag{2.22}$$

where $i=1,2,\dots,N$, $m=1,\dots,n$ and $j=1,\dots,n$. By the definition of $\Phi_i[k]$, we know the elements of $\Phi_i[k]$ take the form of

$$P(q[k]=q_i \mid q_0)\mathbf{E}\left[a_{m_1j_1}(k)a_{m_2j_2}(k) \mid q[k]=q_i,q_0\right], \tag{2.23}$$

where m_1, m_2, j_1, $j_2=1,\dots,n$. So (2.22) guarantees

$$\lim_{k\to\infty}\Phi_i[k]=0,\quad \forall q_0. \tag{2.24}$$

□

Lemma 2.4. *$V_\Phi[k]$ is governed by the following dynamic equation*

$$V_\Phi[k]=A_{[2]}\qquad V_\Phi[k-1], \tag{2.25}$$

with $V_\Phi[0]=p^T\otimes I_{n^2}$.

Proof of Lemma 2.4. By the definition in (2.8), we can recursively compute $\Phi_i[k]$ as follows:

$$\begin{aligned}
\Phi_i[k] &= P(q[k]=q_i \mid q_0)\mathbf{E}\left[(A[q[k-1]]\Phi(k-1;0))^{[2]} \mid q[k]=q_i,q_0\right]\\
&= P(q[k]=q_i \mid q_0)\mathbf{E}\left[(A[q[k-1]])^{[2]}(\Phi(k-1;0))^{[2]} \mid q[k]=q_i,q_0\right]\\
&= P(q[k]=q_i \mid q_0)\sum_{j=1}^{N}P(q[k-1]=q_j \mid q[k]=q_i,q_0)\\
&\qquad\times\mathbf{E}\left[(A[q[k-1]])^{[2]}(\Phi(k-1;0))^{[2]} \mid q[k]=q_i,q[k-1]=q_j,q_0\right]\\
&= \sum_{j=1}^{N}A_j^{[2]}P(q[k]=q_i \mid q_0)P(q[k-1]=q_j \mid q[k]=q_i,q_0)\\
&\qquad\times\mathbf{E}\left[(\Phi(k-1;0))^{[2]} \mid q[k]=q_i,q[k-1]=q_j,q_0\right].
\end{aligned} \tag{2.26}$$

Because $\Phi(k-1;0)$ depends on only $\{q[k-2], q[k-3], \ldots, q[0]\}$ and the jump sequence $\{q[k]\}$ is Markovian, we know

$$\mathbf{E}\Big[(\Phi(k-1;0))^{[2]} \mid q[k] = q_i, q[k-1] = q_j, q_0\Big] = \mathbf{E}\Big[(\Phi(k-1;0))^{[2]} \mid q[k-1] = q_j, q_0\Big]. \quad (2.27)$$

$P(q[k] = q_i \mid q_0)P(q[k-1] = q_j \mid q[k] = q_i, q_0)$ can be computed as

$$\begin{aligned} &P\big(q[k] = q_i \mid q_0\big)P\big(q[k-1] = q_j \mid q[k] = q_i, q_0\big) \\ &\quad = P\big(q[k-1] = q_j, q[k] = q_i \mid q_0\big) \\ &\quad = P\big(q[k] = q_i \mid q[k-1] = q_j, q_0\big)P\big(q[k-1] = q_j \mid q_0\big) \\ &\quad = P\big(q[k] = q_i \mid q[k-1] = q_j\big)P\big(q[k-1] = q_j \mid q_0\big) \\ &\quad = q_{ji}P\big(q[k-1] = q_j \mid q_0\big). \end{aligned} \quad (2.28)$$

Substituting (2.27) and (2.28) into the expression of $\Phi_i[k]$, we get

$$\Phi_i[k] = \sum_{j=1}^{N} q_{ji} A_j^{[2]} \Phi_j[k-1]. \quad (2.29)$$

After combining $\Phi_i[k] (i = 1, 2, \ldots, N)$ into $V_\Phi[k]$ as (2.10), we get

$$V_\Phi[k] = A_{[2]} V_\Phi[k-1]. \quad (2.30)$$

We can trivially get $V_\Phi[0]$ from $\Phi_i[0]$ by (2.10). □

Proof of Necessity of Theorem 2.1. By Lemma 2.3, we get

$$\lim_{k \to \infty} V_\Phi[k] = 0. \quad (2.31)$$

By Lemma 2.4, we get $V_\Phi[k] = A_{[2]}^k V_\Phi[0]$ and $V_\Phi[0] = p^T \otimes I_{n^2}$. Therefore, (2.31) yields

$$\lim_{k \to \infty} A_{[2]}^k \Big(p^T \otimes I_{n^2}\Big) = 0, \quad (2.32)$$

for any p (the initial distribution of q_0).

$A_{[2]}^k$ is an $Nn^2 \times Nn^2$ matrix. We can write $A_{[2]}^k$ as $A_{[2]}^k = [A^1(k), A^2(k), \ldots, A^N(k)]$ where $A^i(n) (i = 1, \ldots, N)$ is an $Nn^2 \times n^2$ matrix. By taking $p_i = 1$ and $p_j = 0 (j = 1, \ldots, i-1, i+1, \ldots, N)$, (2.32) yields

$$\lim_{k \to \infty} A^i(k) = 0. \quad (2.33)$$

Thus we can get

$$\lim_{k \to \infty} A_{[2]}^k = 0. \quad (2.34)$$

So $A_{[2]}$ is Schur stable. The proof is completed. □

3. Conclusion

This paper presents a necessary and sufficient condition for the second moment stability of a discrete-time Markovian jump linear system. Specifically this paper provides proof for the necessity part. Different from the previous necessity proof, this paper confines the state domain to the real space. It investigates the structures of relevant matrices and make a good use of the essential properties of Markov jump linear systems, which may guide the future research on such systems.

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