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Research Article

Adaptive Backstepping Fuzzy Control Based on Type-2 Fuzzy System

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A novel indirect adaptive backstepping control approach based on type-2 fuzzy system is developed for a class of nonlinear systems. This approach adopts type-2 fuzzy system instead of type-1 fuzzy system to approximate the unknown functions. With type-reduction, the type-2 fuzzy system is replaced by the average of two type-1 fuzzy systems. Ultimately, the adaptive laws, by means of backstepping design technique, will be developed to adjust the parameters to attenuate the approximation error and external disturbance. According to stability theorem, it is proved that the proposed Type-2 Adaptive Backstepping Fuzzy Control (T2ABFC) approach can guarantee global stability of closed-loop system and ensure all the signals bounded. Compared with existing Type-1 Adaptive Backstepping Fuzzy Control (T1ABFC), as the advantages of handling numerical and linguistic uncertainties, T2ABFC has the potential to produce better performances in many respects, such as stability and resistance to disturbances. Finally, a biological simulation example is provided to illustrate the feasibility of control scheme proposed in this paper.

1. Introduction

Early results on adaptive control for nonlinear systems are usually obtained based on the assumption that nonlinearities in systems satisfied matching conditions [1, 2]. To control the nonlinear systems with mismatched conditions, backstepping design technique has been developed in [3], and the papers [4, 5] addressed some robust adaptive control results by backstepping design. So far, backstepping approach has become one of the most popular design methods for a series of nonlinear systems. However, in many real plants, not only the nonlinearities in the system are unknown but also prior knowledge of the bounds of these nonlinearities is unavailable. In order to stabilize those nonlinear systems, many Approximator-Based Adaptive Backstepping Control (ABABC) methods have been developed by combining the concepts of adaptive backstepping and several universal approximators, like the Adaptive Backstepping Fuzzy Control (ABFC) [6–9], Adaptive Backstepping Neural Network Control (ABNNC) [10–13], and Adaptive Backstepping Wavelet Control (ABWC) [14, 15].

So far, lots of important results on ABFC have been reported. In [16], direct ABFC scheme has been proposed by combining the modified integral Lyapunov functions and the backstepping technique. In [17], the author introduced the further extended ABFC scheme to the time-delay setting. Recently, in [18], the fuzzy systems are used as feedforward compensators to model some system functions depending on the reference signal. Based on backstepping technique, in [19], adaptive fuzzy controller has been proposed for temperature control in a general class of continuous stirred tank reactors. In [20], the author developed a fuzzy adaptive backstepping design procedure for a class of nonlinear systems with nonlinear uncertainties, unmodeled dynamics, and dynamics disturbances. In [21], a fuzzy adaptive backstepping output feedback control approach is developed for a class of MIMO nonlinear systems with unmeasured states.

However, all the existing ABFC schemes have the common problem that they cannot fully handle or accommodate the uncertainties as they use precise type-1 fuzzy sets. In general, the uncertainty rules will be existed in the following three possible ways [22–28]: (i) the words that are used in antecedents and consequents of rules can mean different things to different people; (ii) consequents obtained by polling a group of experts will often be different for the same rule because the experts will not necessarily be in agreement; (iii) noisy training data. So when something is uncertain and the circumstances are fuzzy, we have trouble determining the membership grade even as a crisp number in [0, 1].

To overcome this drawback, we consider using fuzzy sets of type-2 in this paper. The concept of Type-2 Fuzzy Sets (T2FSs) was first introduced in [29] as an extension of the well-known ordinary fuzzy set, the Type-1 Fuzzy Sets (T1FSs). A T2FSs is characterized by a fuzzy membership function; that is, the membership grade for each element is also a fuzzy set in [0, 1]. The membership functions of T2FSs are three-dimensional and include a Footprint of Uncertainty (FOU), which is a new third dimension of T2FSs, and the FOU provides an additional degree of freedom. So compared to Type-1 Fuzzy Logic System (T1FLS), Type-2 Fuzzy Logic System (T2FLS) has many advantages as follows (i) As T2FSs are able to handle the numerical and linguistic uncertainties, T2FLC based on T2FSs will have the potential to produce a better performance than T1FLC. (ii) Using T2FSs to represent the FLC inputs and outputs will also result in the reduction of the FLC rule base compared to using T1FSs. (iii) In a T2FLC each input and output will be represented by a large number of T1FSs, which allows for greater accuracy in capturing the subtle behavior of the user in the environment. (iv) The T2FSs enable us to handle the uncertainty when trying to determine the exact membership functions for the fuzzy sets with the inputs and outputs of the FLC.

The papers [30] by Hagras and [31] by Melin and Castillo were the first two papers on T2FLC. Subsequently, Castillo, Hagras, and Sepulveda presented T2FLC designs, respectively; for details see [32–37]. Also, some results on T2 fuzzy sliding-mode controller have been presented in [38–40]. Moreover, in recent years, indirect adaptive interval T2 fuzzy control for SISO nonlinear system is proposed in [41] and direct adaptive interval T2 fuzzy control has been developed in [26] for a MIMO nonlinear system. Robust adaptive tracking control of multivariable nonlinear systems based on interval T2 fuzzy approach is developed in [42]. Adaptive control of two-axis motion control system using interval T2 fuzzy neural network is presented in [43]. The author introduced interval T2 fuzzy logic congestion control method for video streaming across IP networks in [44]. And in [45], optimization of interval T2 fuzzy logic controllers for a perturbed autonomous wheeled mobile robot has been developed.

Inspired by all of that, a novel ABFC approach based on T2FSs is drawn in this paper. Compared with traditional T1ABFC, T2ABFC can fully handle or accommodate the uncertainties and achieve higher performances. So, T2ABFC method proposed in this paper succeeds in solving the control problem of a series of nonlinear systems with not only mismatched conditions but also complicated uncertainties.

The rest of this paper is organized as follows. First is the problem formulation, with some preliminaries given in Section 2, and in Section 3, a brief introduction of the interval T2FLS. Indirect adaptive backstepping fuzzy controller design using interval T2FLS is presented in Section 4. In Section 5, a simulation example is provided to illustrate the feasibility of the proposed control scheme. In Section 6, we conclude the work of the paper.

2. Problem Formulation

Consider a class of SISO nonlinear systems described by the differential equations as

$$\begin{aligned} \dot{x}_i &= f_i(\overline{x}_i) + G_i(\overline{x}_i)x_{i+1} \quad 1 \le i \le n-1, \\ \dot{x}_n &= f_n(\overline{x}_n) + G_n(\overline{x}_n)u \quad n \ge 2, \\ y &= x_1, \end{aligned} \tag{2.1}$$

where $\overline{x}_i = [x_1, x_2 \dots x_i]^T \in R^i$ is the stable vector and $u \in R$ and $y \in R$ are the input and output of the system, respectively. $f_i(\overline{x}_i)$, $G_i(\overline{x}_i)$ $(i = 1, 2 \dots n)$ are unknown smooth nonlinear functions. The control objective of this paper is formulated as follows: for a given bounded reference signal $y_r(t) \in D$ with continuous and bounded derivatives up to order ρ, where $D \in R$ is a known compact set. Utilize fuzzy logic system and parameters adaptive laws such that

(1) all the signals involved in the closed-loop system are ultimately and uniformly bounded,

(2) the tracking errors converge to a small neighborhood around zero,

(3) the closed-loop system is global stable.

3. Interval Type-2 Fuzzy Logic Systems

In this section, the interval type-2 fuzzy set and the inference of the type-2 fuzzy logic system are presented.

Formally a type-2 fuzzy set $\widetilde{A}$ is characterized by a type-2 membership function $\mu_{\widetilde{A}}(x, u)$ [22], where $x \in X$ is the primary variable and $u \in J_x \subseteq [0, 1]$ is the secondary variable:

$$\widetilde{A} = \left\{ \left((x, u), \mu_{\widetilde{A}}(x, u) \right) \mid \forall x \in X \ \ \forall u \in J_x \subseteq [0, 1] \right\}, \tag{3.1}$$

in which $0 \le \mu_{\tilde{A}}(x,u) \le 1$. $\tilde{A}$ can also be expressed as follows [22]:

$$\tilde{A} = \int_{x\in X}\int_{u\in J_x} \frac{\mu_{\tilde{A}}(x,u)}{(x,u)} \qquad J_x \subseteq [0,1]. \tag{3.2}$$

Due to the facts that an Interval Type-2 Fuzzy Logic Control (IT2FLC) is computationally far less intensive than a general T2FLC and thus better suited for real-time computation in embedded computational artifacts, our learning and adaptation technique use an IT2FLC (using interval T2FSs to represent the inputs and outputs). The IT2FSs, currently the most widely used kind of T2FSs, are characterized by IT2 membership functions in which the secondary membership grades are equal to 1. The theoretic background of IT2 FLS can be seen in [23, 46–50]. It is described as

$$\tilde{A} = \int_{x\in X}\int_{u\in J_x} \frac{1}{(x,u)} = \int_{x\in X} \frac{\left[\int_{u\in J_x} 1/u\right]}{x}. \tag{3.3}$$

Also, a Gaussian primary membership function with uncertain mean and fixed standard deviation having an interval type-2 secondary membership function can be called an interval type-2 Gaussian membership function. Consider the case of a Gaussian primary membership function having an uncertain mean in $[m_1, m_2]$ and a fixed standard deviation σ. It can be expressed as

$$u_{\tilde{A}}(x) = \exp\left[-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2\right], \qquad m \in [m_1, m_2]. \tag{3.4}$$

Uncertainty about $\tilde{A}$ can be expressed by the union of all the primary memberships, and is bounded by an upper membership function and a lower membership function [22–24], which is called the FOU of $\tilde{A}$:

$$\mathrm{FOU}\left(\tilde{A}\right) = \bigcup_{\forall x\in X} J_x = \left\{(x,u) : u \in \left[\underline{\mu}_{\tilde{A}}(x), \overline{\mu}_{\tilde{A}}(x)\right]\right\}, \tag{3.5}$$

where

$$\begin{aligned} \underline{\mu}_{\tilde{A}}(x) &= \underline{\mathrm{FOU}\left(\tilde{A}\right)} \quad \forall x \in X, \\ \overline{\mu}_{\tilde{A}}(x) &= \overline{\mathrm{FOU}\left(\tilde{A}\right)} \quad \forall x \in X. \end{aligned} \tag{3.6}$$

The concept of FOU, associated with the concepts of lower and upper membership functions, models the uncertainties in the shape and position of the T1FSs.

The distinction between T1 and T2 rules is associated with the nature of the membership functions; the structure of the rules remains exactly the same in the T1 case, but all the sets involved are T2 now. We can consider a T2FLS having n inputs $x_1 \in X_1, \ldots, x_n \in X_n$

and one output $y \in Y$, and assuming there are M rules, the ith rule of the IT2 SMC can be described as

$$R^l : \text{IF } x_1 \text{ is } \tilde{F}_1^l \ldots \text{ and } x_n \text{ is } \tilde{F}_n^l, \text{ THEN } y \text{ is } \tilde{G}^l \quad l = 1, \ldots, M. \tag{3.7}$$

Since the output of the inference-engine is a T2FS, it must be type-reduced and the defuzzifier is used to generate a crisp output. The type-reducer is an extension of T1 defuzzifier obtained by applying the extension principle [29]. There are many kinds of type-reduction methods [23, 24, 51, 52], such as the centroid, center of sets, center of sums, and height type-reduction, and these are elaborated upon in [25]. As in [23], the most commonly used type-reduction method is the center of sets type-reducer, as it has reasonable computational complexity that lies between the computationally expensive centroid type-reduction and the simple height and modified height type-reduction which have a problem when only one rule fires [25]. The type-reduced set using the center of sets type-reduction can be expressed as follows:

$$Y_{\cos}\left(Y^1, \ldots, Y^M, F^1, \ldots, F^M\right) = [y_l, y_r] = \int_{y^1} \cdots \int_{y^M} \int_{f^1} \cdots \int_{f^M} \frac{1}{\sum_{i=1}^M f^i y^i / \sum_{i=1}^M f^i}, \tag{3.8}$$

where $Y_{\cos}(x)$ is an interval output set determined by its left-most point y_l and its right-most point y_r, and $f^i \in F^i = [\underline{f}^i, \overline{f}^i]$. In the meantime, an IT2FLS with singleton fuzzification and meet under minimum or product t-norm $\underline{f}^i$ and $\overline{f}^i$ can be obtained as

$$\underline{f}^i = \underline{\mu}_{\tilde{F}_1^i}(x_1) * \cdots * \underline{\mu}_{\tilde{F}_n^i}(x_n), \tag{3.9}$$

and

$$\overline{f}^i = \overline{\mu}_{\tilde{F}_1^i}(x_1) * \cdots * \overline{\mu}_{\tilde{F}_n^i}(x_n). \tag{3.10}$$

Also, $y^i \in Y^i$ and $Y^i = [y_l^i, y_r^i]$ is the centroid of the IT2 consequent set $\tilde{G}^i$, the centroid of a T2FS, and for any value $y \in Y_{\cos}$, y can be expressed as

$$y = \frac{\sum_{i=1}^M f^i y^i}{\sum_{i=1}^M f^i}, \tag{3.11}$$

where y is a monotonic increasing function with respect to y^i. Also, y_l is the minimum associated only with y_l^i, and y_r is the maximum associated only with y_r^i. Note that y_l and y_r depend only on the mixture of $\underline{f}^i$ or $\overline{f}^i$ values. Therefore, the left-most point y_l and the right-most point y_r can be expressed as a Fuzzy Basis Function (FBF) expansion, that is,

$$y_l = \frac{\sum_{i=1}^M f_l^i y_l^i}{\sum_{i=1}^M f_l^i} = \sum_{i=1}^M y_l^i \xi_l^i = y_l^T \xi_l, \tag{3.12}$$

and

$$y_r = \frac{\sum_{i=1}^{M} f_r^i y_r^i}{\sum_{i=1}^{M} f_r^i} = \sum_{i=1}^{M} y_r^i \xi_r^i = y_r^T \xi_r, \tag{3.13}$$

respectively, where $\xi_l^i = f_l^i / \sum_{i=1}^{M} f^i$, $\xi_l = [\xi_l^1, \xi_l^2, \ldots, \xi_l^M]$, $y_l^T = [y_l^1, y_l^2, \ldots, y_l^M]$, and $\xi_r^i = f_r^i / \sum_{i=1}^{M} f^i$, $\xi_r = [\xi_r^1, \xi_r^2, \ldots, \xi_r^M]$, $y_r^T = [y_r^1, y_r^2, \ldots, y_r^M]$.

In order to compute y_l and y_r, the Karnik-Mendel iterative procedure is needed [25, 51]. It has been shown in [25, 26, 49, 51]. For illustrative purposes, we briefly provide the computation procedure for y_r. Without loosing of generality, assume y_r^i is arranged in ascending order, that is, $y_r^1 \leq y_r^2 \leq \cdots \leq y_r^M$.

Step 1. Compute y_r in (3.13) by initially setting $f_r^l = (\overline{f}^l + \underline{f}^l)/2$ for $l = 1, \ldots, M$, where $\overline{f}^l$ and $\underline{f}^l$ have been precomputed by (3.10) and let $y_r' = y_r$.

Step 2. Find R $(1 \leq R \leq M-1)$ such that $y_r^R \leq y_r' \leq y_r^{R+1}$.

Step 3. Compute y_r in (3.13) with $f_r^i = \underline{f}^i$ for $i \leq R$ and $f_r^i = \overline{f}^i$ for $i > R$ and let $y_r'' = y_r$.

Step 4. If $y_r'' \neq y_r$, then go to Step 5; if $y_r'' = y_r$, then stop and set $y_r = y_r''$.

Step 5. Set y_r' equal to y_r'' and return to Step 2.

The point to separate two sides by number R can be decided from the above algorithm, one side using lower firing strengths $\underline{f}^i$'s and another side using upper firing strengths $\overline{f}^i$'s. Therefore, y_r can be expressed as

$$y_r = \frac{\sum_{i=1}^{R} \underline{f}^i y_r^i + \sum_{i=R+1}^{M} \overline{f}^i y_r^i}{\sum_{i=1}^{R} \underline{f}^i + \sum_{i=R+1}^{M} \overline{f}^i} = \sum_{i=1}^{R} \underline{q}_r^i y_r^i + \sum_{i=R+1}^{M} \overline{q}_r^i y_r^i = \begin{bmatrix} \underline{Q}_r & \overline{Q}_r \end{bmatrix} \begin{bmatrix} \underline{y}_r \\ \overline{y}_r \end{bmatrix} = \xi_r^T \Theta_r, \tag{3.14}$$

where $\underline{q}_r^i = \underline{f}^i / D_r$, $\overline{q}_r^i = \overline{f}^i / D_r$ and $D_r = (\sum_{i=1}^{R} \underline{f}^i + \sum_{i=R+1}^{M} \overline{f}^i)$. In the meantime, we have $\underline{Q}_r = [\underline{q}_r^1, \underline{q}_r^2, \ldots, \underline{q}_r^R]$, $\overline{Q}_r = [\overline{q}_r^1, \overline{q}_r^2, \ldots, \overline{q}_r^R]$, $\xi_r^T = \lfloor \underline{Q}_r, \overline{Q}_r \rfloor$, and $\Theta_r^T = \lfloor \underline{y}_r \overline{y}_r \rfloor$.

The procedure to compute y_l is similar to compute y_r. In Step 2, it only determines L $(1 \leq L \leq M-1)$, such that $y_l^L \leq y_l' \leq y_l^{L+1}$. In Step 3, let $f_l^i = \overline{f}^i$ for $i \leq L$ and $f_l^i = \underline{f}^i$ for $i > L$. Therefore, y_l can be expressed as

$$y_l = \frac{\sum_{i=1}^{L} \underline{f}^i y_l^i + \sum_{i=L+1}^{M} \overline{f}^i y_l^i}{\sum_{i=1}^{L} \underline{f}^i + \sum_{i=L+1}^{M} \overline{f}^i} = \sum_{i=1}^{L} \underline{q}_l^i y_l^i + \sum_{i=L+1}^{M} \overline{q}_l^i y_l^i = \begin{bmatrix} \underline{Q}_l & \overline{Q}_l \end{bmatrix} \begin{bmatrix} \underline{y}_l \\ \overline{y}_l \end{bmatrix} = \xi_l^T \Theta_l, \tag{3.15}$$

where $\underline{q}_l^i = \underline{f}^i / D_l$, $\overline{q}_l^i = \overline{f}^i / D_l$ and $D_l = (\sum_{i=1}^{L} \underline{f}^i + \sum_{i=L+1}^{M} \overline{f}^i)$. In the meantime, we have $\underline{Q}_l = [\underline{q}_l^1, \underline{q}_l^2, \ldots, \underline{q}_l^R]$, $\overline{Q}_l = [\overline{q}_l^1, \overline{q}_l^2, \ldots, \overline{q}_l^R]$, $\xi_l^T = \lfloor \underline{Q}_l\ \overline{Q}_l \rfloor$, and $\Theta_l^T = \lfloor \underline{y}_l \overline{y}_l \rfloor$.

We defuzzify the interval set by using the average of y_l and y_r, hence, the defuzzified crisp output becomes

$$Y_{\text{fuzz2}}(x) = \frac{y_l + y_r}{2} = \frac{1}{2}\left(\xi_l^T \Theta_l + \xi_r^T \Theta_r\right) = \frac{1}{2}\left[\xi_l^T \xi_r^T\right]\begin{bmatrix}\Theta_l \\ \Theta_r\end{bmatrix} = \xi^T \Theta, \tag{3.16}$$

where $(1/2)[\xi_l^T \xi_r^T] = \xi^T$ and $[\Theta_l^T \Theta_r^T] = \Theta^T$.

Lemma 3.1. *(Wang [53]). Let $f(x)$ be a continuous function defined on a compact set Ω. Then for any constant $\varepsilon > 0$, there exists a fuzzy logic system* (3.16) *such as*

$$\sup_{x\in\Omega}\left|f(x) - \xi^T(x)\Theta\right| \le \varepsilon. \tag{3.17}$$

4. Adaptive Backstepping Fuzzy Controller Design Using IT2FLS

In this section, our objective is to use IT2FLS to approximate the nonlinear functions. With type-reduction, the IT2FLS is replaced by the average of two T1FLSs. Ultimately, the adaptive laws, by means of backstepping design technique, will be developed to adjust the parameters to attenuate the approximation error and external disturbance.

To begin with, some assumptions are given as follows.

Assumption 4.1. There exist positive constants $\overline{G}_i$ and $\underline{G}_i$ $(1 \le i \le n)$, such that $\underline{G}_i \le G_i(\overline{x}_i) \le \overline{G}_i$.

Assumption 4.2. There exist positive constant G_i^d $(1 \le i \le n)$, such that $|\dot{G}_i(\overline{x}_i)| \le G_i^d$.

Assumption 4.3. Define the optimal parameter vectors Θ_i^* as

$$\Theta_i^* = \arg\min_{\Theta_i\in\Omega_i}\left[\sup_{\overline{x}_i\in U_i}\left|f_i(\overline{x}_i \mid \Theta_i) - f_i(\overline{x}_i)\right|\right], \tag{4.1}$$

where Ω_i, U_i are compact regions for Θ_i, $\overline{x}_i$, respectively. The fuzzy logic system minimum approximation errors ω_i are defined as

$$\left|\xi_i^T \Theta_i^* - f_i(\overline{x}_i)\right| \le \omega_i. \tag{4.2}$$

T2ABFC method proposed in this paper is summarized in Theorem 4.4.

Theorem 4.4. *Suppose assumptions are 4.1–4.3 tenable, then the fuzzy adaptive output tracking design described by* (2.1)*, control law $u = -e_{n-1} - k_n e_n - \xi_n^T(\overline{x}_n)\widehat{\Theta}_n$, and parameter adaptive laws $\dot{\widehat{\Theta}}_{nl} = \dot{\widetilde{\Theta}}_{nl} = \gamma_{nl}(e_n \xi_{nl}^T(\overline{x}_n) - c_{nl}\widehat{\Theta}_{nl})$ $\dot{\widehat{\Theta}}_{nr} = \dot{\widetilde{\Theta}}_{nr} = \gamma_{nr}(e_n \xi_{nr}^T(\overline{x}_n) - c_{nr}\widehat{\Theta}_{nr})$ based on T2FLS guarantees that closed-loop system is globally uniform ultimately bounded, output tracking error converges to a small neighborhood of the origin and resistant to extern disturbance. (k_n, γ_{nl}, γ_{nr}, c_{nl}, c_{nr} are all design parameters).*

The detailed design and certification procedures are described in the following steps.

Step 1. Define the tracking error for the system as

$$e_1 = x_1 - y_r. \tag{4.3}$$

The time derivative of e_1 is

$$\dot{e}_1 = \dot{x}_1 - \dot{y}_r = f_1(\overline{x}_1) + G_1(\overline{x}_1)x_2 - \dot{y}_r. \tag{4.4}$$

Take x_2 as a virtual control, and define

$$\alpha_1^* = k_1 e_1 - \frac{1}{G_1(\overline{x}_1)}\left(f_1(\overline{x}_1) - \dot{y}_r\right), \tag{4.5}$$

where k_1 is a positive constant.

Since $f_1(\overline{x}_1)$ and $f_1(\overline{x}_1)$ are unknown, ideal controller is not available in practice. By Lemma 3.1, fuzzy logic systems are universal approximators, so we can assume that the unknown function $-(1/G_1\ (\overline{x}_1))(f_1(\overline{x}_1) - \dot{y}_r)$ can be approximated by the following type-2 fuzzy logic system $\xi_1^T(\overline{x}_1)\Theta_1$, and we obtain

$$\alpha_1^* = -k_1 e_1 - \xi_1^T(\overline{x}_1)\Theta_1^* - \omega_1. \tag{4.6}$$

Express x_2 as $x_2 = e_2 + \alpha_1$ and define

$$\alpha_1 = -k_1 e_1 - \xi_1^T(\overline{x}_1)\widehat{\Theta}_1 = -k_1 e_1 - \left(\xi_{1l}^T(\overline{x}_1)\widehat{\Theta}_{1l} + \xi_{1r}^T(\overline{x}_1)\widehat{\Theta}_{1r}\right), \tag{4.7}$$

then the time derivative of e_1 is

$$\begin{aligned}
\dot{e}_1 &= f_1(\overline{x}_1) + G_1(\overline{x}_1)(e_2 + \alpha_1) - \dot{y}_r \\
&= G_1(\overline{x}_1)\left(e_2 - k_1 e_1 - \xi_1^T(\overline{x}_1)\widetilde{\Theta}_1 + \omega_1\right) \\
&= G_1(\overline{x}_1)\left(e_2 - k_1 e_1 - \xi_{1l}^T(\overline{x}_1)\widetilde{\Theta}_{1l} - \xi_{1r}^T(\overline{x}_1)\widetilde{\Theta}_{1r} + \omega_1\right),
\end{aligned} \tag{4.8}$$

where $\widetilde{\Theta}_1 = \widehat{\Theta}_1 - \Theta_1^*$.

Consider the following Lyapunov function:

$$\begin{aligned}
V_1 &= \frac{1}{2G_1(\overline{x}_1)}e_1^2 + \frac{1}{2\gamma_1}\widetilde{\Theta}_1^T\widetilde{\Theta}_1 \\
&= \frac{1}{2G_1(\overline{x}_1)}e_1^2 + \frac{1}{2\gamma_{1l}}\widetilde{\Theta}_{1l}^T\widetilde{\Theta}_{1l} + \frac{1}{2\gamma_{1r}}\widetilde{\Theta}_{1r}^T\widetilde{\Theta}_{1r},
\end{aligned} \tag{4.9}$$

then the time derivative of V_1 is

$$\begin{aligned}
\dot{V}_1 &= \frac{e_1\dot{e}_1}{G_1(\overline{x}_1)} - \frac{\dot{G}_1(\overline{x}_1)}{2G_1^2(\overline{x}_1)}e_1^2 + \frac{1}{\gamma_1}\widetilde{\Theta}_1^T\dot{\widehat{\Theta}}_1 \\
&= \frac{e_1\dot{e}_1}{G_1(\overline{x}_1)} - \frac{\dot{G}_1(\overline{x}_1)}{2G_1^2(\overline{x}_1)}e_1^2 + \frac{1}{\gamma_{1l}}\widetilde{\Theta}_{1l}^T\dot{\widehat{\Theta}}_{1l} + \frac{1}{\gamma_{1r}}\widetilde{\Theta}_{1r}^T\dot{\widehat{\Theta}}_{1r} \\
&= e_1e_2 - k_1e_1^2 - e_1\left(\widetilde{\Theta}_{1l}^T\xi_{1l} + \widetilde{\Theta}_{1r}^T\xi_{1r}\right) + e_1\omega - \frac{\dot{G}_1(\overline{x}_1)}{2G_1^2(\overline{x}_1)}e_1^2 + \frac{1}{\gamma_{1l}}\widetilde{\Theta}_{1l}^T\dot{\widehat{\Theta}}_{1l} + \frac{1}{\gamma_{1r}}\widetilde{\Theta}_{1r}^T\dot{\widehat{\Theta}}_{1r} \\
&= e_1e_2 - k_1e_1^2 - \frac{\dot{G}_1(\overline{x}_1)}{2G_1^2(\overline{x}_1)}e_1^2 + e_1\omega_1 \\
&\quad + \widetilde{\Theta}_{1l}^T\left[-e_1\xi_{1l}^T(\overline{x}_1) + \frac{1}{\gamma_{1l}}\dot{\widehat{\Theta}}_{1l}\right] + \widetilde{\Theta}_{1r}^T\left[-e_1\xi_{1r}^T(\overline{x}_1) + \frac{1}{\gamma_{1r}}\dot{\widehat{\Theta}}_{1r}\right].
\end{aligned} \tag{4.10}$$

Choose the intermediate adaptive laws as

$$\begin{aligned}
\dot{\widehat{\Theta}}_{1l} &= \dot{\widetilde{\Theta}}_{1l} = \gamma_{1l}\left(e_1\xi_{1l}^T(\overline{x}_1) - c_{1l}\widehat{\Theta}_{1l}\right), \\
\dot{\widehat{\Theta}}_{1r} &= \dot{\widetilde{\Theta}}_{1r} = \gamma_{1r}\left(e_1\xi_{1r}^T(\overline{x}_1) - c_{1r}\widehat{\Theta}_{1r}\right),
\end{aligned} \tag{4.11}$$

where c_{1l} and c_{1r} are given positive constants.Substituting (4.11) into (4.10) yields

$$\dot{V}_1 = e_1e_2 + e_1\omega_1 - \left(k_1 + \frac{\dot{G}_1(\overline{x}_1)}{2G_1^2(\overline{x}_1)}\right)e_1^2 - \left(c_{1l}\widetilde{\Theta}_{1l}^T\widehat{\Theta}_{1l} + c_{1r}\widetilde{\Theta}_{1r}^T\widehat{\Theta}_{1r}\right), \tag{4.12}$$

where $k_1 = k_{1,0} + k_{1,1}$, $k_{1,0} > 0$, $k_{1,1} > 0$, $e_1\omega_1 - k_1e_1^2 \le e_1\omega_1 - k_{1,1}e_1^2 \le \varepsilon_1^2/4k_{1,1}$, $|\omega_1| \le |\varepsilon_1|$

We can obtain that

$$\begin{aligned}
-c_{1l}\widetilde{\Theta}_{1l}^T\widehat{\Theta}_{1l} &= -c_{1l}\widetilde{\Theta}_{1l}^T\left(\widetilde{\Theta}_{1l} + \Theta_{1l}^*\right) \le -c_{1l}\left\|\widetilde{\Theta}_{1l}\right\|^2 + c_{1l}\left\|\widetilde{\Theta}_{1l}\right\| \ \|\Theta_{1l}^*\| \le -\frac{c_{1l}\left\|\widetilde{\Theta}_{1l}\right\|^2}{2} + \frac{c_{1l}\|\Theta_{1l}^*\|^2}{2}, \\
-c_{1r}\widetilde{\Theta}_{1r}^T\widehat{\Theta}_{1r} &= -c_{1r}\widetilde{\Theta}_{1r}^T\left(\widetilde{\Theta}_{1r} + \Theta_{1r}^*\right) \le -c_{1r}\left\|\widetilde{\Theta}_{1r}\right\|^2 + c_{1r}\left\|\widetilde{\Theta}_{1r}\right\| \ \|\Theta_{1r}^*\| \le -\frac{c_{1r}\left\|\widetilde{\Theta}_{1r}\right\|^2}{2} + \frac{c_{1r}\|\Theta_{1r}^*\|^2}{2} \\
&-\left(k_{1,0} + \frac{\dot{G}_1}{2G_1^2}\right)e_1^2 \le -\left(k_{1,0} - \frac{G_1^d}{2G_1^2}\right)e_1^2, \quad \left(k_{1,0}^* = k_{1,0} - \frac{G_1^d}{2G_1^2} > 0\right).
\end{aligned} \tag{4.13}$$

From (4.12), (4.13), it follows that

$$\dot{V}_1 \le e_1e_2 - k_{1,0}^*e_1^2 + \frac{\varepsilon_1^2}{4k_{1,1}} - \frac{c_{1l}\left\|\widetilde{\Theta}_{1l}\right\|^2}{2} - \frac{c_{1r}\left\|\widetilde{\Theta}_{1r}\right\|^2}{2} + \frac{c_{1l}\|\Theta_{1l}^*\|^2}{2} + \frac{c_{1r}\|\Theta_{1r}^*\|^2}{2}. \tag{4.14}$$

Step 2. Differentiating e_2 yields

$$\dot{e}_2 = \dot{x}_2 - \dot{\alpha}_1 = f_2(\overline{x}_2) + G_2(\overline{x}_2)x_3 - \dot{\alpha}_1. \tag{4.15}$$

Take x_3 as a virtual control, and define

$$\alpha_2^* = -e_1 - k_2 e_2 - \frac{1}{G_2(\overline{x}_2)}\left(f_2(\overline{x}_2) - \dot{\alpha}_1\right), \tag{4.16}$$

where k_2 is a positive constant.

From (4.7), we obtain

$$\dot{\alpha}_1 = \frac{\partial \alpha_1}{\partial x_1}\left(f_1 + G_1 x_2\right) + \frac{\partial \alpha_1}{\partial y_r}\dot{y}_r + \frac{\partial \alpha_1}{\partial \widehat{\Theta}_{1l}}\left[\gamma_{1l}\left(e_{1l}\xi_{1l} - c_{1l}\widehat{\Theta}_{1l}\right)\right] + \frac{\partial \alpha_1}{\partial \widehat{\Theta}_{1r}}\left[\gamma_{1r}\left(e_1\xi_{1r} - c_{1r}\widehat{\Theta}_{1r}\right)\right]. \tag{4.17}$$

Since $f_2(\overline{x}_2)$ and $G_2(\overline{x}_2)$ are unknown, ideal controller is not available in practice, so we can assume that the unknown function $-(1/G_2(\overline{x}_2))(f_2(\overline{x}_2) - \dot{\alpha}_1)$ can be approximated by the following type-2 fuzzy logic system $\xi_2^T(\overline{x}_2)\Theta_2$, and obtain

$$\alpha_2 = -e_1 - k_2 e_2 - \xi_2^T(\overline{x}_2)\widehat{\Theta}_2. \tag{4.18}$$

Express x_3 as $x_3 = e_3 + \alpha_2$ and the time derivative of e_2 is

$$\begin{aligned}\dot{e}_2 &= f_2(\overline{x}_2) + G_2(\overline{x}_2)(e_3 + \alpha_2) - \dot{\alpha}_1\\ &= G_2(\overline{x}_2)\left(e_3 - e_1 - k_2 e_2 - \xi_2^T(\overline{x}_2)\widetilde{\Theta}_2 + \omega_2\right),\end{aligned} \tag{4.19}$$

where $\widetilde{\Theta}_2 = \widehat{\Theta}_2 - \Theta_2^*$.

Consider the following Lyapunov function

$$\begin{aligned}V_2 &= V_1 + \frac{1}{2G_2(\overline{x}_2)}e_2^2 + \frac{1}{2\gamma_2}\widetilde{\Theta}_2^T\widetilde{\Theta}_2\\ &= V_1 + \frac{1}{2G_2(\overline{x}_2)}e_2^2 + \frac{1}{2\gamma_{2l}}\widetilde{\Theta}_{2l}^T\widetilde{\Theta}_{2l} + \frac{1}{2\gamma_{2r}}\widetilde{\Theta}_{2r}^T\widetilde{\Theta}_{2r},\end{aligned} \tag{4.20}$$

then the time derivative of V_2 is

$$\begin{aligned}\dot{V}_2 = \dot{V}_1 &- e_1 e_2 + e_2 e_3 - k_2 e_2^2 - \frac{\dot{G}_2(\overline{x}_2)}{2G_2^2(\overline{x}_2)}e_2^2 + e_2\omega_2\\ &- \widetilde{\Theta}_{2l}^T\left[e_2\xi_{2l}^T(\overline{x}_2) - \frac{1}{\gamma_{2l}}\dot{\widehat{\Theta}}_{2l}\right] - \widetilde{\Theta}_{2r}^T\left[e_2\xi_{2r}^T(\overline{x}_2) - \frac{1}{\gamma_{2r}}\dot{\widehat{\Theta}}_{2r}\right].\end{aligned} \tag{4.21}$$

Choose the intermediate adaptive laws as

$$\dot{\hat{\Theta}}_{2l} = \dot{\tilde{\Theta}}_{2l} = \gamma_{2l}\left(e_2\xi_{2l}^T(\overline{x}_2) - c_{2l}\hat{\Theta}_{2l}\right),$$
$$\dot{\hat{\Theta}}_{2r} = \dot{\tilde{\Theta}}_{2r} = \gamma_{2r}\left(e_2\xi_{2r}^T(\overline{x}_2) - c_{2r}\hat{\Theta}_{2r}\right). \tag{4.22}$$

Substituting (4.22) into (4.21) yields

$$\dot{V}_2 \le e_2e - k_{1,0}^*e_1^2 - k_{2,0}^*e_2^2 + \frac{\varepsilon_1^2}{4k_{1,1}} + \frac{\varepsilon_2^2}{4k_{2,1}} - \frac{c_{1l}\left\|\tilde{\Theta}_{1l}\right\|^2}{2} - \frac{c_{1r}\left\|\tilde{\Theta}_{1r}\right\|^2}{2} - \frac{c_{2l}\left\|\tilde{\Theta}_{2l}\right\|^2}{2}$$
$$- \frac{c_{2r}\left\|\tilde{\Theta}_{2r}\right\|^2}{2} + \frac{c_{1l}\left\|\Theta_{1l}^*\right\|^2}{2} + \frac{c_{1r}\left\|\Theta_{1r}^*\right\|^2}{2} + \frac{c_{2l}\left\|\Theta_{2l}^*\right\|^2}{2} + \frac{c_{2r}\left\|\Theta_{2r}^*\right\|^2}{2}. \tag{4.23}$$

Step i $(3 \le i \le n-1)$

A similar procedure is employed recursively at each step. By defining

$$e_i = x_i - \alpha_{i-1} \tag{4.24}$$

the time derivative of e_i is

$$\dot{e}_i = \dot{x}_i - \dot{\alpha}_{i-1} = f_i(\overline{x}_i) + G_i(\overline{x}_i)x_{i+1} - \dot{\alpha}_{i-1}. \tag{4.25}$$

Take x_{i+1} as a virtual control, and define

$$\alpha_i^* = -e_{i-1} - k_ie_i - \frac{1}{G_i(\overline{x}_i)}\left(f_i(\overline{x}_i) - \dot{\alpha}_{i-1}\right), \tag{4.26}$$

where k_i is a positive constant:

$$\dot{\alpha}_{i-1} = \sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_k}\left(G_kx_{k+1} + f_k\right) + \frac{\partial\alpha_{i-1}}{\partial y_r}\dot{y}_r + \sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{\Theta}_{kl}}\left[\gamma_{kl}\left(e_k\xi_{kl}^T - c_{kl}\hat{\Theta}_{kl}\right)\right]$$
$$+ \sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{\Theta}_{kr}}\left[\gamma_{kr}\left(e_k\xi_{kr}^T - c_{kr}\hat{\Theta}_{kr}\right)\right]. \tag{4.27}$$

The unknown function $-(1/G_i(\overline{x}_i))(f_i(\overline{x}_i)-\dot{\alpha}_{i-1})$ can be approximated by the following type-2 fuzzy logic system $\xi_i^T(\overline{x}_i)\Theta_i$, and we can obtain

$$\alpha_i = -e_{i-1} - k_ie_i - \xi_i^T(\overline{x}_i)\hat{\Theta}_i. \tag{4.28}$$

Express x_{i+1} as $x_{i+1} = e_{i+1} + \alpha_i$ and the time derivative of e_i is

$$\begin{aligned}\dot{e}_i &= f_i(\overline{x}_i) + G_i(\overline{x}_i)(e_{i+1} + \alpha_i) - \dot{\alpha}_{i-1} \\ &= G_i(\overline{x}_i)\Big(e_{i+1} - e_{i-1} - k_i e_i - \xi_i^T(\overline{x}_i)\widetilde{\Theta}_i + \omega_i\Big),\end{aligned} \tag{4.29}$$

where $\widetilde{\Theta}_i = \widehat{\Theta}_i - \Theta_i^*$.

Consider the following Lyapunov function:

$$\begin{aligned}V_i &= V_{i-1} + \frac{1}{2G_i(\overline{x}_i)}e_i^2 + \frac{1}{2\gamma_i}\widetilde{\Theta}_i^T\widetilde{\Theta}_i \\ &= V_{i-1} + \frac{1}{2G_i(\overline{x}_i)}e_i^2 + \frac{1}{2\gamma_{il}}\widetilde{\Theta}_{il}^T\widetilde{\Theta}_{il} + \frac{1}{2\gamma_{ir}}\widetilde{\Theta}_{ir}^T\widetilde{\Theta}_{ir},\end{aligned} \tag{4.30}$$

then the time derivative of V_i is

$$\begin{aligned}\dot{V}_i &= \dot{V}_{i-1} - e_{i-1}e_i + e_i e_{i+1} - k_i e_i^2 - \frac{\dot{G}_i(\overline{x}_i)}{2G_i^2(\overline{x}_i)}e_i^2 + e_i\omega_i \\ &\quad - \widetilde{\Theta}_{il}^T\left[e_i\xi_{il}^T(\overline{x}_i) - \frac{1}{\gamma_{il}}\dot{\widehat{\Theta}}_{il}\right] - \widetilde{\Theta}_{ir}^T\left[e_i\xi_{ir}^T(\overline{x}_i) - \frac{1}{\gamma_{ir}}\dot{\widehat{\Theta}}_{ir}\right].\end{aligned} \tag{4.31}$$

Choose the intermediate adaptive laws as

$$\begin{aligned}\dot{\widehat{\Theta}}_{il} &= \dot{\widetilde{\Theta}}_{il} = \gamma_{il}\Big(e_i\xi_{il}^T(\overline{x}_i) - c_{il}\widehat{\Theta}_{il}\Big), \\ \dot{\widehat{\Theta}}_{ir} &= \dot{\widetilde{\Theta}}_{ir} = \gamma_{ir}\Big(e_i\xi_{ir}^T(\overline{x}_i) - c_{ir}\widehat{\Theta}_{ir}\Big).\end{aligned} \tag{4.32}$$

Substituting (4.32) into (4.31) yields

$$\begin{aligned}\dot{V}_i \le{}& e_i e_{i+1} - \sum_{k=1}^{i} k_{k,0}^* e_k^2 + \sum_{k=1}^{i}\frac{\varepsilon_k^2}{4k_{k,1}} - \sum_{k=1}^{i}\frac{c_{kl}\left\|\widetilde{\Theta}_{kl}\right\|^2}{2} - \sum_{k=1}^{i}\frac{c_{kr}\left\|\widetilde{\Theta}_{kr}\right\|^2}{2} \\ &+ \sum_{k=1}^{i}\frac{c_{kl}\left\|\Theta_{kl}^*\right\|^2}{2} + \sum_{k=1}^{i}\frac{c_{kr}\left\|\Theta_{kr}^*\right\|^2}{2}.\end{aligned} \tag{4.33}$$

Step n

In the final design step, the actual control input u will appears. Defining

$$e_n = x_n - \alpha_{n-1}, \tag{4.34}$$

the time derivative of e_n is

$$\dot{e}_n = \dot{x}_n - \dot{\alpha}_{n-1} = f_n(\overline{x}_n) + G_n(\overline{x}_n)u - \dot{\alpha}_{n-1}. \tag{4.35}$$

Define the actual control input u as

$$u = -e_{n-1} - k_n e_n - \xi_n^T(\overline{x}_n)\widehat{\Theta}_n, \tag{4.36}$$

where k_n is a positive constant, $\widetilde{\Theta}_i = \widehat{\Theta}_i - \Theta_i^*$.Choose the whole Lyapunov function as

$$\begin{aligned} V &= V_{n-1} + \frac{1}{2G_n(\overline{x}_n)}e_n^2 + \frac{1}{2\gamma_n}\widetilde{\Theta}_n^T\widetilde{\Theta}_n \\ &= V_{n-1} + \frac{1}{2G_n(\overline{x}_n)}e_n^2 + \frac{1}{2\gamma_{nl}}\widetilde{\Theta}_{nl}^T\widetilde{\Theta}_{nl} + \frac{1}{2\gamma_{nr}}\widetilde{\Theta}_{nr}^T\widetilde{\Theta}_{nr}, \end{aligned} \tag{4.37}$$

then the time derivative of V is

$$\begin{aligned} \dot{V} &= \dot{V}_{n-1} - e_{n-1}e_n - k_n e_n^2 - \frac{\dot{G}_n(\overline{x}_n)}{2G_n^2(\overline{x}_n)}e_n^2 + e_n\omega_n \\ &\quad - \widetilde{\Theta}_{nl}^T\left[e_n\xi_{nl}^T(\overline{x}_n) - \frac{1}{\gamma_{nl}}\dot{\widehat{\Theta}}_{nl}\right] - \widetilde{\Theta}_{nr}^T\left[e_n\xi_{nr}^T(\overline{x}_n) - \frac{1}{\gamma_{nr}}\dot{\widehat{\Theta}}_{nr}\right]. \end{aligned} \tag{4.38}$$

Choose the actual adaptive laws as

$$\begin{aligned} \dot{\widehat{\Theta}}_{nl} &= \dot{\widetilde{\Theta}}_{nl} = \gamma_{nl}\left(e_n\xi_{nl}^T(\overline{x}_n) - c_{nl}\widehat{\Theta}_{nl}\right), \\ \dot{\widehat{\Theta}}_{nr} &= \dot{\widetilde{\Theta}}_{nr} = \gamma_{nr}\left(e_n\xi_{nr}^T(\overline{x}_n) - c_{nr}\widehat{\Theta}_{nr}\right). \end{aligned} \tag{4.39}$$

Substituting (4.39) into (4.38) yields

$$\begin{aligned} \dot{V} &\leq -\sum_{k=1}^{n} k_{k,0}^* e_k^2 + \sum_{k=1}^{n}\frac{\varepsilon_k^2}{4k_{k,1}} - \sum_{k=1}^{n}\frac{c_{kl}\left\|\widetilde{\Theta}_{kl}\right\|^2}{2} - \sum_{k=1}^{n}\frac{c_{kr}\left\|\widetilde{\Theta}_{kr}\right\|^2}{2} \\ &\quad + \sum_{k=1}^{n}\frac{c_{kl}\left\|\Theta_{kl}^*\right\|^2}{2} + \sum_{k=1}^{n}\frac{c_{kr}\left\|\Theta_{kr}^*\right\|^2}{2}. \end{aligned} \tag{4.40}$$

$k_{k,0}^*$ is chosen such that $k_{k,0}^* > (\rho/2G_k)$, then

$$k_{k,0} > \left(\frac{\rho}{2G_k}\right) + \left(\frac{\dot{G}_k}{2G_k^2}\right) \quad (k = 1, \ldots, n), \tag{4.41}$$

where ρ is a positive constant and

$$\delta \leq \sum_{k=1}^{n} \frac{\varepsilon_k^2}{4k_{k,1}} + \sum_{k=1}^{n} \frac{c_{kl}\left\|\Theta_{kl}^*\right\|^2}{2} + \sum_{k=1}^{n} \frac{c_{kr}\left\|\Theta_{kr}^*\right\|^2}{2}. \tag{4.42}$$

Then (4.40) becomes

$$\begin{aligned}
\dot{V} &\leq -\sum_{k=1}^{n} k_{k,0}^* e_k^2 + \delta - \sum_{k=1}^{n} \frac{c_{kl}\left\|\tilde{\Theta}_{kl}\right\|^2}{2} - \sum_{k=1}^{n} \frac{c_{kr}\left\|\tilde{\Theta}_{kr}\right\|^2}{2} \\
&< -\sum_{k=1}^{n} \frac{\rho}{2G_k} e_k^2 + \delta - \sum_{k=1}^{n} \frac{\rho\tilde{\Theta}_{kl}^T\tilde{\Theta}_{kl}}{2} - \sum_{k=1}^{n} \frac{\rho\tilde{\Theta}_{kr}^T\tilde{\Theta}_{kr}}{2} \\
&< -\rho\left(\sum_{k=1}^{n} \frac{1}{2G_k} e_k^2 + \sum_{k=1}^{n} \frac{\tilde{\Theta}_{kl}^T\tilde{\Theta}_{kl}}{2} + \sum_{k=1}^{n} \frac{\tilde{\Theta}_{kr}^T\tilde{\Theta}_{kr}}{2}\right) + \delta \\
&< -\rho V + \delta.
\end{aligned} \tag{4.43}$$

From (4.43) we have

$$V(t) \leq V(t_0)e^{-\rho(t-t_0)} + \frac{\delta}{\rho}. \tag{4.44}$$

It can be shown that the signals $x(t), e(t), \Theta_l(t), \Theta_r(t)$, and $u(t)$ are globally uniformly ultimately bounded and that $|y_n(t) - y_r(t)| \leq \sqrt{2V(t)}e^{(\rho/2)(t-t_0)} + \sqrt{2\delta/\rho}$. In order to achieve the tracking error convergences to a small neighborhood around zero, the parameters ρ and δ should be chosen appropriately, then it is possible to make $\sqrt{2\delta/\rho}$ as small as desired. Denote $\phi > \sqrt{2\delta/\rho}$. Since as $t \to \infty$, $e^{-(\delta/2)(t-t_0)} \to 0$, therefore, it follows that there exists T, when $t \geq T$, $|y(t) - y_r(t)| \leq \phi$.

5. Simulation

In this section, we provide a biological simulation example to illustrate the feasibility of the control scheme proposed in this paper.

In recent years, interest in adaptive control systems has increased rapidly along with interest and progress in control topics. The adaptive control has a variety of specific meanings, but it often implies that the system is capable of accommodating unpredictable environmental changes, whether these changes arise within the system or external to it.

Adaptation is a fundamental characteristic of living organisms such as prey-predator systems and many other biological models since these systems attempt to maintain physiological equilibrium in the midst changing environmental conditions.

Background Knowledge

The Salton Sea, which is located in the southeast desert of California, came into the limelight due to deaths of fish and fish catching birds on a massive scale. Recently, Chattopadhyay and

Bairagi [54] proposed and analyzed an eco-epidemiological model on Salton Sea.We assume that there are two populations.

(1) The prey population, Tilapia fish, whose population density is denoted by N, which is the number of Tilapia fish per unit designated area.

(2) In the absence of bacterial infection, the fish population grows according to a logistic law with carrying capacity K ($K \in R_+$), with an intrinsic birth rate constant r ($r \in R_+$), such that

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right). \tag{5.1}$$

(3) In the presence of bacterial infection, the total fish population N is divided into two classes, namely, susceptible fish population, denoted by S, and infected fish population, denoted by I. Therefore, at any time t, the total density of prey (i.e., fish) population is

$$N(t) = S(t) + I(t). \tag{5.2}$$

(4) Only the susceptible fish population S is capable of reproducing with the logistic law, and the infected fish population I dies before having the capacity of reproduction. However, the infected fish I still contributes with S to population growth towards the carrying capacity.

(5) Liu et al. [55] concluded that the bilinear mass action incidence rate due to saturation or multiple exposures before infection could lead to a nonlinear incidence rate as $\lambda S^p I^q$ with p and q near 1 and without a periodic forcing term, which have much wider range of dynamical behaviors in comparison to bilinear incidence rate λSI. Here, $\lambda \in R_+$ is the force of infection or rate of transmission. Therefore, the evolution equation for the susceptible fish population S can be written as

$$\frac{dS}{dt} = rS\left(1 - \frac{S+I}{K}\right) - \lambda SI. \tag{5.3}$$

(6) The predator population, Pelican birds, whose population density is denoted by P, which is the number of birds per unit designated area.

(7) It is assumed that Pelicans cannot distinguish the infected and healthy fish. They consume the fish that are readily available. Since the prey population is infected by a disease, infected preys are weakened and become easier to predate, while susceptible (healthy) preys easily escape predation. Considering this fact, it is assumed that the Pelicans mostly consume the infected fish only. The natural death rate of infected prey (not due to predation) is denoted by μ ($\mu \in R_+$). d is the total death of predator population (including natural death and death due to predation of infected prey). m is the search rate, θ is the conversion factor, and a is the half saturation coefficient.

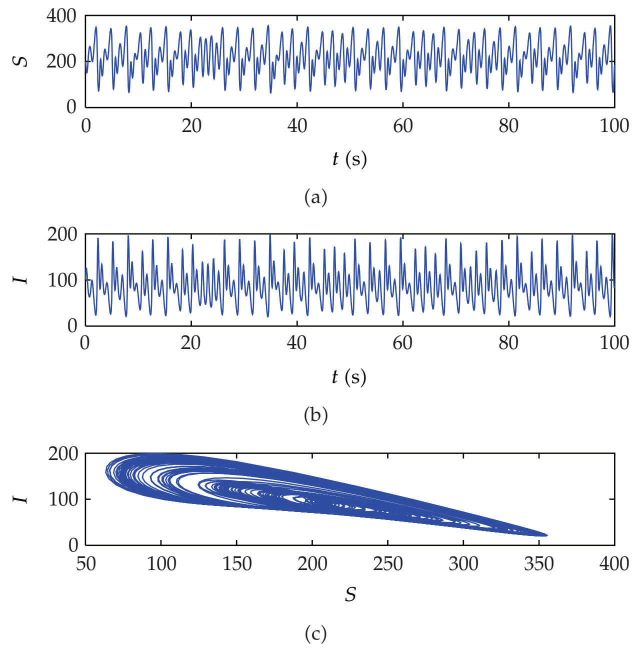

Figure 1: Typical chaotic attractor for the model system.

(8) The system appears to exhibit a chaotic behavior for a range of parametric values. The range of the system parameters for which the subsystems converge to limit cycles is determined. In Figure 1, typical chaotic attractor for the model system is obtained for the parameter values as see [54]:

$$r = 22 \quad K = 400 \quad \lambda = 0.06 \quad \mu = 3.4 \quad m = 15.5 \quad a = 15 \quad d = 8.3 \quad \theta = 10.0. \tag{5.4}$$

Practically, the populations of prey and predator are supposed to be stable or constant, so controller will be used to achieve the intended target. In this model, we add a controller to the third differential equation, that is to say, we control the population of Tilapia fish to be constant by changing the population of Pelican birds in some ways.

From the above assumptions and substituting S, I, P by x_1, x_2, x_3, respectively, we can write down the following differential equations:

$$\begin{aligned}
\frac{dx_1}{dt} &= rx_1\left(1 - \frac{x_1}{K}\right) - \left(\frac{r}{K} + \lambda\right)x_1x_2, \\
\frac{dx_2}{dt} &= \lambda x_1x_2 - \mu x_2 - \frac{mx_2}{x_2 + a}x_3, \\
\frac{dx_3}{dt} &= \frac{\theta x_2x_3}{x_2 + a} - dx_3 + u.
\end{aligned} \tag{5.5}$$

The backstepping design algorithm for type-2 adaptive fuzzy control is proposed as follows.

Step 1. Define the type-2 membership functions as

$$\mu_{M_1}^1(x_1) = e^{-(x_1-50*(1\pm10\%)/10)^2}, \quad \mu_{M_1}^2(x_1) = e^{-(x_1-60*(1\pm10\%)/10)^2}, \quad \mu_{M_1}^3(x_1) = e^{-(x_1-70*(1\pm10\%)/10)^2},$$
$$\mu_{M_2}^1(x_2) = e^{-(x_2-50*(1\pm10\%)/10)^2}, \quad \mu_{M_2}^2(x_2) = e^{-(x_2-60*(1\pm10\%)/10)^2}, \quad \mu_{M_2}^3(x_2), = e^{-(x_2-70*(1\pm10\%)/10)^2},$$
$$\mu_{M_3}^1(x_3) = e^{-(x_3-20*(1\pm10\%)/10)^2}, \quad \mu_{M_3}^2(x_3) = e^{-(x_3-30*(1\pm10\%)/10)^2}, \quad \mu_{M_3}^3(x_3) = e^{-(x_3-40*(1\pm10\%)/10)^2}, \tag{5.6}$$

then the fuzzy basis functions are obtained as

$$\xi_{1l} = \left[\frac{\underline{\mu}_{M_1}^1}{A}, \frac{\underline{\mu}_{M_1}^2}{A}, \frac{\overline{\mu}_{M_1}^3}{A}\right], \qquad A = \underline{\mu}_{M_1}^1 + \underline{\mu}_{M_1}^2 + \overline{\mu}_{M_1}^3,$$

$$\xi_{1r} = \left[\frac{\overline{\mu}_{M_1}^1}{B}, \frac{\overline{\mu}_{M_1}^2}{B}, \frac{\underline{\mu}_{M_1}^3}{B}\right], \qquad B = \overline{\mu}_{M_1}^1 + \overline{\mu}_{M_1}^2 + \underline{\mu}_{M_1}^3,$$

$$\xi_{2l} = \left[\frac{\underline{\mu}_{M_1}^1 * \underline{\mu}_{M_2}^1}{C}, \frac{\underline{\mu}_{M_1}^1 * \underline{\mu}_{M_2}^2}{C}, \frac{\underline{\mu}_{M_1}^1 * \overline{\mu}_{M_2}^3}{C}, \frac{\underline{\mu}_{M_1}^2 * \underline{\mu}_{M_2}^1}{C}, \frac{\underline{\mu}_{M_1}^2 * \underline{\mu}_{M_2}^2}{C}, \frac{\underline{\mu}_{M_1}^2 * \overline{\mu}_{M_2}^3}{C},\right.$$
$$\left.\frac{\overline{\mu}_{M_1}^3 * \underline{\mu}_{M_2}^1}{C}, \frac{\overline{\mu}_{M_1}^3 * \underline{\mu}_{M_2}^2}{C}, \frac{\overline{\mu}_{M_1}^3 * \overline{\mu}_{M_2}^3}{C}\right],$$

$$C = \underline{\mu}_{M_1}^1 * \underline{\mu}_{M_2}^1 + \underline{\mu}_{M_1}^1 * \underline{\mu}_{M_2}^2 + \underline{\mu}_{M_1}^1 * \overline{\mu}_{M_2}^3 + \underline{\mu}_{M_1}^2 * \underline{\mu}_{M_2}^1 + \underline{\mu}_{M_1}^2 * \underline{\mu}_{M_2}^2 + \underline{\mu}_{M_1}^2 * \overline{\mu}_{M_2}^3$$
$$+ \overline{\mu}_{M_1}^3 * \underline{\mu}_{M_2}^1 + \overline{\mu}_{M_1}^3 * \underline{\mu}_{M_2}^2 + \overline{\mu}_{M_1}^3 * \overline{\mu}_{M_2}^3,$$

$$\xi_{1r} = \left[\frac{\overline{\mu}_{M_1}^1 * \overline{\mu}_{M_2}^1}{D}, \frac{\overline{\mu}_{M_1}^1 * \overline{\mu}_{M_2}^2}{D}, \frac{\overline{\mu}_{M_1}^1 * \underline{\mu}_{M_2}^3}{D}, \frac{\overline{\mu}_{M_1}^2 * \overline{\mu}_{M_2}^1}{D}, \frac{\overline{\mu}_{M_1}^2 * \overline{\mu}_{M_2}^2}{D}, \frac{\overline{\mu}_{M_1}^2 * \underline{\mu}_{M_2}^3}{D},\right.$$
$$\left.\frac{\underline{\mu}_{M_1}^3 * \overline{\mu}_{M_2}^1}{D}, \frac{\underline{\mu}_{M_1}^3 * \overline{\mu}_{M_2}^2}{D}, \frac{\underline{\mu}_{M_1}^3 * \underline{\mu}_{M_2}^3}{D}\right],$$

$$D = \overline{\mu}_{M_1}^1 * \overline{\mu}_{M_2}^1 + \overline{\mu}_{M_1}^1 * \overline{\mu}_{M_2}^2 + \overline{\mu}_{M_1}^1 * \underline{\mu}_{M_2}^3 + \overline{\mu}_{M_1}^2 * \overline{\mu}_{M_2}^1 + \overline{\mu}_{M_1}^2 * \overline{\mu}_{M_2}^2 + \overline{\mu}_{M_1}^2 * \underline{\mu}_{M_2}^3$$
$$+ \underline{\mu}_{M_1}^3 * \overline{\mu}_{M_2}^1 + \underline{\mu}_{M_1}^3 * \overline{\mu}_{M_2}^2 + \underline{\mu}_{M_1}^3 * \underline{\mu}_{M_2}^3,$$

$$
\begin{aligned}
\xi_{3l} &= \Bigg[\frac{\underline{\mu}^{1}_{M_2} * \underline{\mu}^{1}_{M_3}}{E}, \frac{\underline{\mu}^{1}_{M_2} * \underline{\mu}^{2}_{M_3}}{E}, \frac{\underline{\mu}^{1}_{M_2} * \overline{\mu}^{3}_{M_3}}{E}, \frac{\underline{\mu}^{2}_{M_2} * \underline{\mu}^{1}_{M_3}}{E}, \frac{\underline{\mu}^{2}_{M_2} * \underline{\mu}^{2}_{M_3}}{E}, \frac{\underline{\mu}^{2}_{M_2} * \overline{\mu}^{3}_{M_3}}{E}, \\
&\qquad \frac{\overline{\mu}^{3}_{M_2} * \underline{\mu}^{1}_{M_3}}{E}, \frac{\overline{\mu}^{3}_{M_2} * \underline{\mu}^{2}_{M_3}}{E}, \frac{\overline{\mu}^{3}_{M_2} * \overline{\mu}^{3}_{M_3}}{E}\Bigg], \\
E &= \underline{\mu}^{1}_{M_2} * \underline{\mu}^{1}_{M_3} + \underline{\mu}^{1}_{M_2} * \underline{\mu}^{2}_{M_3} + \underline{\mu}^{1}_{M_2} * \overline{\mu}^{3}_{M_3} + \underline{\mu}^{2}_{M_2} * \underline{\mu}^{1}_{M_3} + \underline{\mu}^{2}_{M_2} * \underline{\mu}^{2}_{M_3} + \underline{\mu}^{2}_{M_2} * \overline{\mu}^{3}_{M_3} \\
&\quad + \overline{\mu}^{3}_{M_2} * \underline{\mu}^{1}_{M_3} + \overline{\mu}^{3}_{M_2} * \underline{\mu}^{2}_{M_3} + \overline{\mu}^{3}_{M_2} * \overline{\mu}^{3}_{M_3}, \\
\xi_{3r} &= \Bigg[\frac{\overline{\mu}^{1}_{M_2} * \overline{\mu}^{1}_{M_3}}{F}, \frac{\overline{\mu}^{1}_{M_2} * \overline{\mu}^{2}_{M_3}}{F}, \frac{\overline{\mu}^{1}_{M_2} * \underline{\mu}^{3}_{M_3}}{F}, \frac{\overline{\mu}^{2}_{M_2} * \overline{\mu}^{1}_{M_3}}{F}, \frac{\overline{\mu}^{2}_{M_2} * \overline{\mu}^{2}_{M_3}}{F}, \frac{\overline{\mu}^{2}_{M_2} * \underline{\mu}^{3}_{M_3}}{F}, \\
&\qquad \frac{\underline{\mu}^{3}_{M_2} * \overline{\mu}^{1}_{M_3}}{F}, \frac{\underline{\mu}^{3}_{M_2} * \overline{\mu}^{2}_{M_3}}{F}, \frac{\underline{\mu}^{3}_{M_2} * \underline{\mu}^{3}_{M_3}}{F}\Bigg], \\
F &= \overline{\mu}^{1}_{M_2} * \overline{\mu}^{1}_{M_3} + \overline{\mu}^{1}_{M_2} * \overline{\mu}^{2}_{M_3} + \overline{\mu}^{1}_{M_2} * \underline{\mu}^{3}_{M_3} + \overline{\mu}^{2}_{M_2} * \overline{\mu}^{1}_{M_3} + \overline{\mu}^{2}_{M_2} * \overline{\mu}^{2}_{M_3} + \overline{\mu}^{2}_{M_2} * \underline{\mu}^{3}_{M_3} \\
&\quad + \underline{\mu}^{3}_{M_2} * \overline{\mu}^{1}_{M_3} + \underline{\mu}^{3}_{M_2} * \overline{\mu}^{2}_{M_3} + \underline{\mu}^{3}_{M_2} * \underline{\mu}^{3}_{M_3}, \\
\xi_{1l} &= \left[\xi^{1}_{1l}, \xi^{2}_{1l}, \xi^{3}_{1l}\right]^{T}, \qquad \xi_{1r} = \left[\xi^{1}_{1r}, \xi^{2}_{1r}, \xi^{3}_{1r}\right]^{T}, \\
\xi_{2l} &= \left[\xi^{1}_{2l}, \xi^{2}_{2l} \dots \xi^{9}_{2l}\right]^{T}, \qquad \xi_{2r} = \left[\xi^{1}_{2r}, \xi^{2}_{2r} \dots \xi^{9}_{2r}\right]^{T}, \\
\xi_{3l} &= \left[\xi^{1}_{3l}, \xi^{2}_{3l} \dots \xi^{9}_{3l}\right]^{T}, \qquad \xi_{3r} = \left[\xi^{1}_{3r}, \xi^{2}_{3r} \dots \xi^{9}_{3r}\right]^{T}.
\end{aligned} \tag{5.7}
$$

Step 2. We assume that there exist some language rules of unknown functions 1, 2, and 3, respectively.

The unknown function 1:

$$\frac{1}{(r/K)+\lambda}\left[rx_1\left(1-\frac{x_1}{K}\right)-\dot{y}_r\right]. \tag{5.8}$$

The unknown function 2:

$$\frac{x_2+a}{mx_2}\left(\lambda x_1 x_2 - \mu x_2 - \dot{\alpha}_1\right). \tag{5.9}$$

The unknown function 3:

$$\frac{\theta x_2 x_3}{x_2+a} - \dot{\alpha}_2 \frac{\theta x_2 x_3}{x_2+a} - \dot{\alpha}_2. \tag{5.10}$$

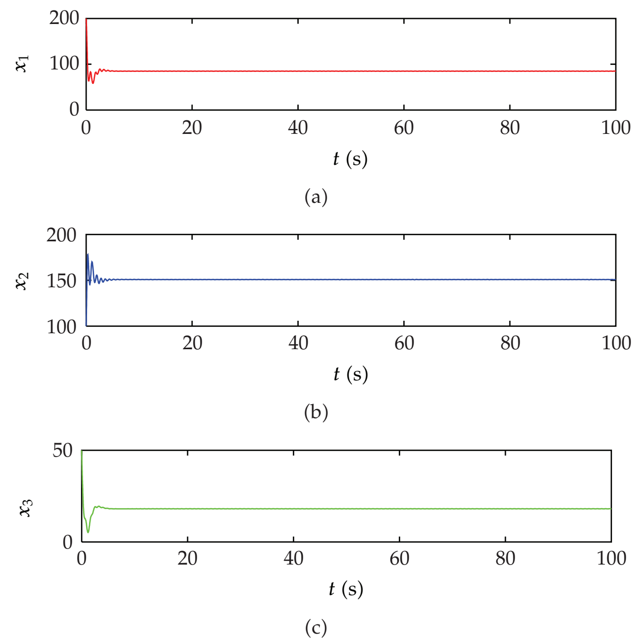

Figure 2: State variable outputs with T2ABFC.

The fuzzy rules of the Unknown Function 1 (UF1) are as follows.

*R*1: if x_1 is Small (S), then UF1 is Powerful (P).

*R*2: if x_1 is Medium (M), then UF1 is Weak (W).

*R*3: if x_1 is Large (L), then UF1 is Modest (M).

The fuzzy rules of the Unknown Function 2 (UF2) are as follows.

*R*1: if x_1 is S, x_2 is S, then UF2 is W.

*R*2: if x_1 is S, x_2 is M, then UF2 is M.

*R*3: if x_1 is S, x_2 is L, then UF2 is P.

*R*4: if x_1 is M, x_2 is S, then UF2 is W.

*R*5: if x_1 is M, x_2 is M, then UF2 is M.

*R*6: if x_1 is M, x_2 is L, then UF2 is P.

*R*7: if x_1 is L, x_2 is S, then UF2 is P.

*R*8: if x_1 is L, x_2 is M, then UF2 is M.

*R*9: if x_1 is L, x_2 is L, then UF2 is W.

The fuzzy rules of the Unknown Function 3 (UF3) are as follows.

*R*1: if x_2 is S, x_3 is S, then UF3 is M.

*R*2: if x_2 is S, x_3 is M, then UF3 is W.

*R*3: if x_2 is S, x_3 is L, then UF3 is P.

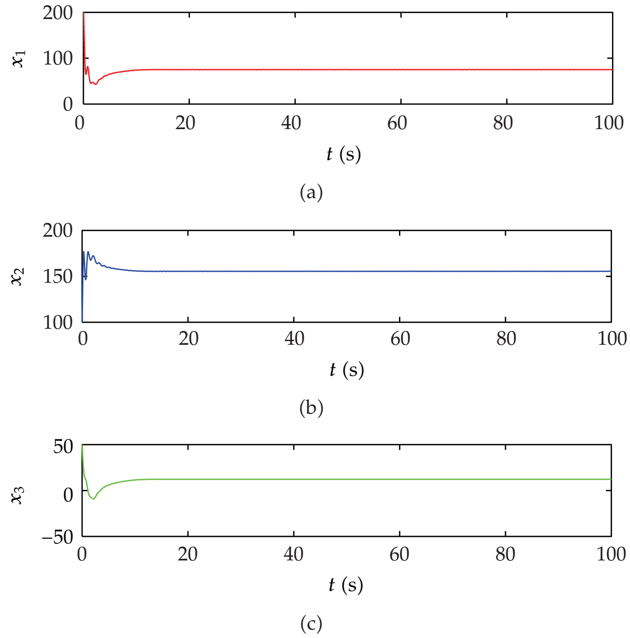

Figure 3: State variable outputs with T1ABFC.

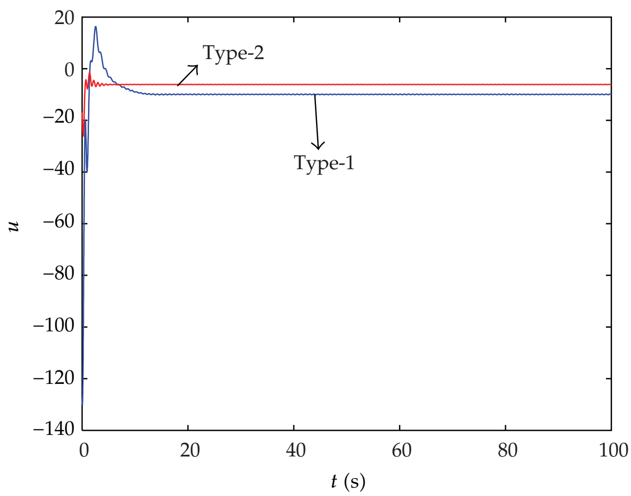

Figure 4: Trajectories of controller.

$R4$: if x_2 is M, x_3 is S, then UF3 is W.

$R5$: if x_2 is M, x_3 is M, then UF3 is M.

$R6$: if x_2 is M, x_3 is L, then UF3 is P.

$R7$: if x_2 is L, x_3 is S, then UF3 is M.

$R8$: if x_2 is L, x_3 is M, then UF3 is W.

$R9$: if x_2 is L, x_3 is L, then UF3 is P.

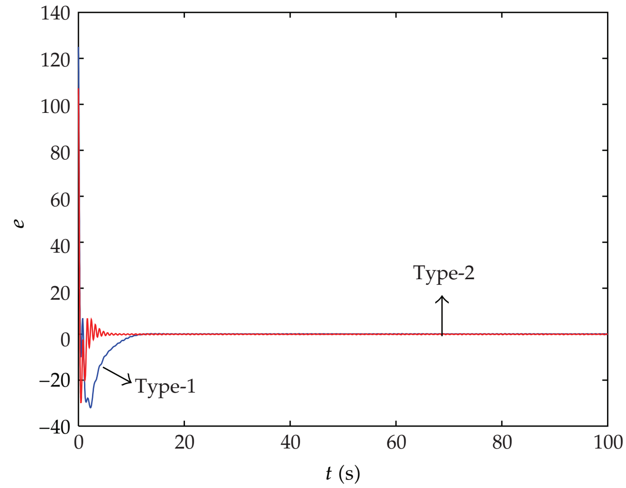

Figure 5: Trajectories of tracking error.

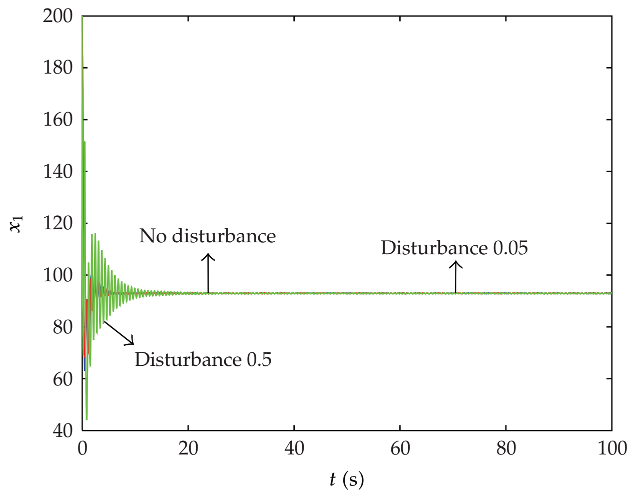

Figure 6: Trajectories of the state variable x_1 by T2ABFC with external disturbances.

Step 3. Specify positive design parameters as follows:

$$k_1 = k_2 = k_3 = 0.5, \qquad \gamma_{1l} = \gamma_{2l} = \gamma_{3l} = \gamma_{1r} = \gamma_{2r} = \gamma_{3r} = 1,$$
$$c_{1l} = c_{2l} = c_{3l} = 0.5, \qquad c_{1r} = c_{2r} = c_{3r} = 0.6. \tag{5.11}$$

The first intermediate controller is chosen as

$$\alpha_1 = -k_1 e_1 - \left(\xi_{1l}^T(\overline{x}_1)\widehat{\Theta}_{1l} + \xi_{1r}^T(\overline{x}_1)\widehat{\Theta}_{1r}\right). \tag{5.12}$$

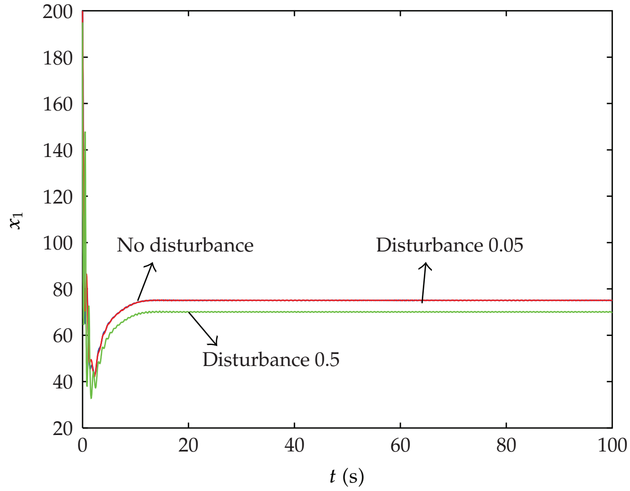

Figure 7: Trajectories of the state variable x_1 by T1ABFC with external disturbances.

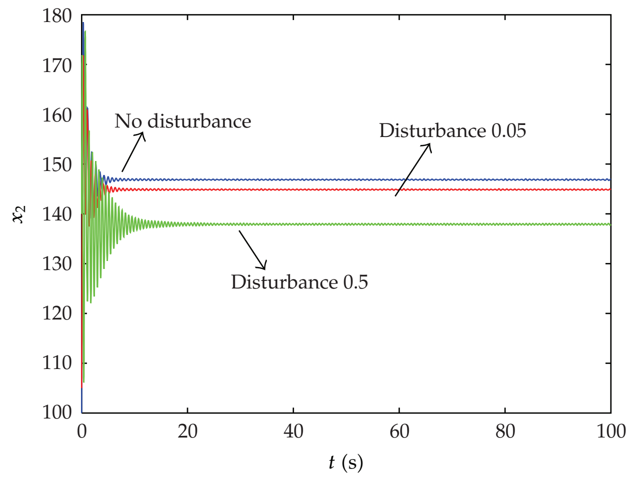

Figure 8: Trajectories of the state variable x_2 by T2ABFC with external disturbances.

The first adaptive laws are designed as

$$\begin{aligned}\dot{\widehat{\Theta}}_{1l} &= \dot{\tilde{\Theta}}_{1l} = \gamma_{1l}\left(e_1\xi_{1l}^T(\overline{x}_1) - c_{1l}\widehat{\Theta}_{1l}\right),\\ \dot{\widehat{\Theta}}_{1r} &= \dot{\tilde{\Theta}}_{1r} = \gamma_{1r}\left(e_1\xi_{1r}^T(\overline{x}_1) - c_{1r}\widehat{\Theta}_{1r}\right).\end{aligned} \tag{5.13}$$

The second intermediate controller is chosen as

$$\alpha_2 = -e_1 - k_2e_2 - \left(\xi_{2l}^T(\overline{x}_2)\widehat{\Theta}_{2l} + \xi_{2r}^T(\overline{x}_2)\widehat{\Theta}_{2r}\right). \tag{5.14}$$

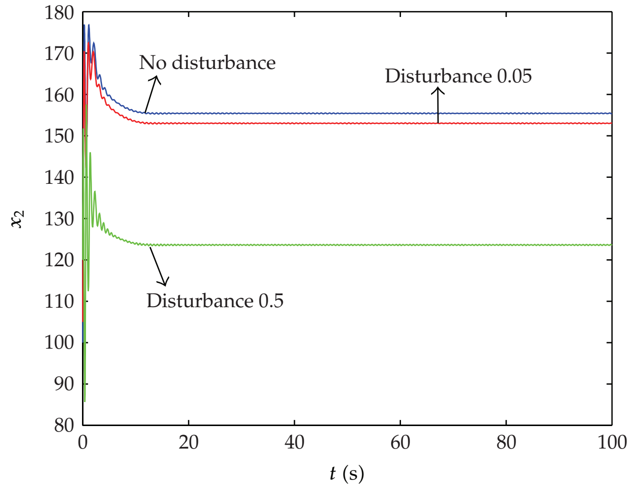

Figure 9: Trajectories of the state variable x_2 by T1ABFC with external disturbances.

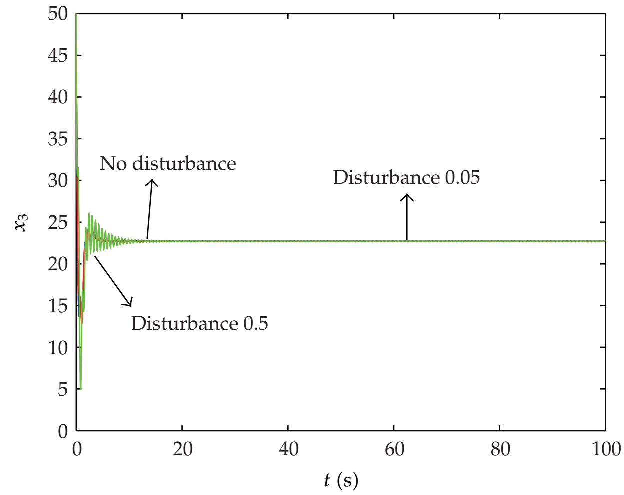

Figure 10: Trajectories of the state variable x_3 by T2ABFC with external disturbances.

The second adaptive laws are designed as

$$\begin{aligned} \dot{\widehat{\Theta}}_{2l} &= \dot{\widetilde{\Theta}}_{2l} = \gamma_{2l}\left(e_2\xi_{2l}^T(\overline{x}_2) - c_{2l}\widehat{\Theta}_{2l}\right), \\ \dot{\widehat{\Theta}}_{2r} &= \dot{\widetilde{\Theta}}_{2r} = \gamma_{2r}\left(e_2\xi_{2r}^T(\overline{x}_2) - c_{2r}\widehat{\Theta}_{2r}\right). \end{aligned} \tag{5.15}$$

The actual controller is chosen as

$$u = -e_2 - k_3e_3 - \left(\xi_{3l}^T(\overline{x}_3)\widehat{\Theta}_{3l} + \xi_{3r}^T(\overline{x}_3)\widehat{\Theta}_{3r}\right). \tag{5.16}$$

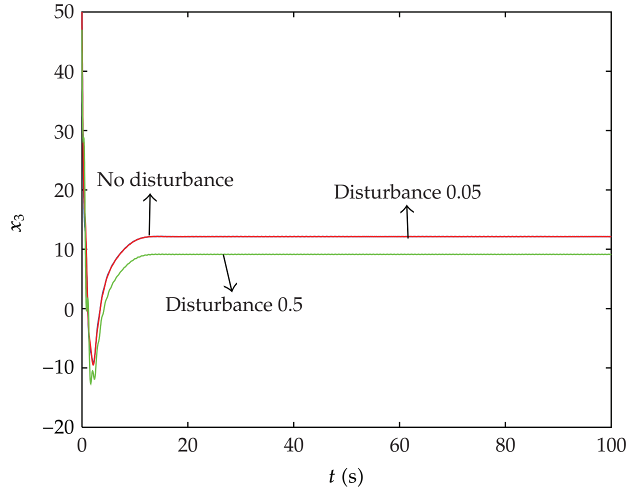

Figure 11: Trajectories of the state variable x_3 by T1ABFC with external disturbances.

The actual adaptive laws are designed as

$$\begin{aligned}\dot{\hat{\Theta}}_{3l} &= \dot{\tilde{\Theta}}_{3l} = \gamma_{3l}\left(e_3\xi_{3l}^T(\overline{x}_3) - c_{3l}\widehat{\Theta}_{3l}\right),\\ \dot{\hat{\Theta}}_{3r} &= \dot{\tilde{\Theta}}_{3r} = \gamma_{3r}\left(e_3\xi_{3r}^T(\overline{x}_3) - c_{3r}\widehat{\Theta}_{3r}\right).\end{aligned} \tag{5.17}$$

The reference output is specified as $y_r(t) = 93$.

The performance of T2ABFC is compared by traditional T1ABFC in state variable outputs, controller trajectory, tracking error, and resistance to disturbances. The simulation results are shown in Figures 1–11.

(1) Figures 2 and 3 show the state variable output with T2ABFC and T1ABFC, respectively. We can obtain that the T2ABFC has a higher performance in terms of response speed than T1ABFC.

(2) Figure 4 shows the trajectories of controller based on T2ABFC and T1ABFC, respectively.

(3) Figure 5 shows the trajectories of tracking error based on T2ABFC and T1ABFC, respectively. By comparing, the superiority of T2ABFC in tracking performance is obvious.

In order to show the property of resistance to disturbances, training data is corrupted by a random noise $\pm 0.05\ x$ and $\pm 0.5\ x$, that is, x is replaced by $(1\pm \text{random}(0.05))\ x$ and $(1 \pm \text{random}\ (0.5))\ x$.

(4) Figures 6 and 7 show the responses of state variables x_1 with disturbances based on T2ABFC and T1ABFC, respectively.

(5) Figures 8 and 9 show the responses of state variables x_2 with disturbances based on T2ABFC and T1ABFC, respectively.

(6) Figures 10 amd 11 show the responses of state variables x_3 with disturbances based on T2ABFC and T1ABFC respectively. It is obvious that the system based on T2ABFC has the better property of resistance to disturbances.

From all the outputs of simulation, we can obtain that T2ABFC method proposed in this paper guaranteeS the higher tracking performance and resistance to external disturbances than traditional T1ABFC method.

6. Conclusion

In this paper, we solve the globally stable adaptive backstepping fuzzy control problem for a class of nonlinear systems and T2ABFC is recommended in this approach. From the simulation results, the main conclusions can be drawn.

(1) T2ABFC guarantees the outputs of the closed-loop system follow the reference signal, and all the signals in the closed-loop system are uniform ultimately bounded.

(2) Compared by traditional T1ABFC, T2ABFC has higher performances, in terms of stability, response speed, and resistance to external disturbances.

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