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Research Article

Exponential Passification of Markovian Jump Nonlinear Systems with Partially Known Transition Rates

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The problems of delay-dependent exponential passivity analysis and exponential passification of uncertain Markovian jump systems (MJSs) with partially known transition rates are investigated. In the deterministic model, the time-varying delay is in a given range and the uncertainties are assumed to be norm bounded. With constructing appropriate Lyapunov-Krasovskii functional (LKF) combining with Jensen's inequality and the free-weighting matrix method, delay-dependent exponential passification conditions are obtained in terms of linear matrix inequalities (LMI). Based on the condition, desired state-feedback controllers are designed, which guarantee that the closed-loop MJS is exponentially passive. Finally, a numerical example is given to illustrate the effectiveness of the proposed approach.

1. Introduction

In recent years, more and more attention has been devoted to the Markovian jump systems since they are introduced by Krasovskii and Lidskii [1]. It is known that systems with Markovian jump parameters are a set of systems with transition among the models governed by a Markov chain taking values in a finite set. They have the character of stochastic hybrid systems with two components in the state. The first one refers to the mode which is described by a continuous-time finite-state Markov process, and the second one refers to the state which is represented by a system of differential equations. Markovian jump systems have got the virtue of modeling the abrupt phenomena such as random failures and repairs of the components changes in the interconnections of subsystems, sudden environment changes, and so forth, which often takes place in many dynamical systems [2–4]. So due to extensive applications of such systems in manufacturing systems, power systems, communication systems, and network-based control systems, recently, many works have been reported about MJSs, which including filtering problems [5–7], stability analysis problems [8–12], and control problems [13–20], and so forth.

However, the aforementioned references almost considered that the transition probabilities are known exactly. In some practical applications, the mode information is transmitted through unreliable networks, it may be lost or observed simultaneously. That means the systems mode is neither totally accessible or inaccessible. So the ideal assumption on the transition probabilities inevitably limits the application of the traditional Markovian jump systems theory. Therefore, whether in theory or in practice, it is necessary to further consider more general systems with partially mode information [21–27].

Recently, the passivity problems for a variety of practical systems have been attracting renewing attention [28–31]. The passivity theory was first proposed in the circuit analysis [32] so it has played an efficient role in both electrical network and nonlinear control systems. The main point of passivity theory is that the passive properties of system can keep the system internal stability. Thus, the passivity theory provides a nice tool for analyzing the stability of a nonlinear system, and the passivity analysis has received a lot of attention and has found applications in diverse areas such as signal processing, complexity, chaos control and synchronization, and fuzzy control [33–38]. In [33] authors dealt with global robust passivity analysis for stochastic interval neural networks with interval time varying delays and Markovian jumping parameter; in [34] both delay-independent and delay-dependent stochastic passivity conditions are presented for uncertain neural networks; in [35–37] authors discussed the robust passivity and passification of Markovian jump systems and fuzzy time-delay systems; in [38], the exponential passivity of neural networks with time-varying are studied and the results are extended to two types of uncertainties.

In practice, input delays are often encountered in control systems because of the transmission of measurement information. Especially, in networked control systems, sensors controllers, and plants are often connected by a net medium hence it is quite meaningful to study the effect of the input delay in the design of controllers. However, to the best of the authors' knowledge providing less conservative delay-dependent exponential passification criteria for uncertain MJS with input delays and partially known transition rates to desired performance are still open problems.

Motivated by this observation, in this paper, we study the exponential passification problem of nonlinear Markovian jump systems with partially known transition rates, including state and input delays, the aim of this problem is to design a controller such that the resulting closed-loop systems satisfy a certain passivity performance index. Comparing with the large amount of the literature on the analysis of stability of Markovian jump systems, passivity analysis and passification for these systems have many obvious advantages. Thus, research in this area should be of both theoretical and practical importance, which motivates us to carry out the present work. Based on the LKF theory and the free-weighting matrix method, some desired exponentially passification controllers are designed, which guarantee that the closed-loop MJS is exponential passive. Finally, a numerical example is used to illustrate the designed method.

Notations. The notations are quite standard. Throughout this letter $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, resp., the n-dimensioned Euclidean space and the set of all $n \times m$ real matrices. The notation $X \geq Y$ (resp., $X > Y$) means that X and Y are symmetric matrices, and that $X - Y$ is positive semidefinitive (resp., positive definite). $\|\cdot\|$ is the Euclidean norm in $\mathbb{R}^n$. I is the identity matrix with compatible dimension. If A is a matrix, $\lambda_{\max}(A)$ (respective $\lambda_{\min}(A)$) means the largest (respective smallest) eigervalue of A. Moreover, let $(\Omega, \mathbb{F}, (\mathbb{F}_t)_{t\geq 0}, \mathbb{P})$ be a complete probability space with a filteration. $(\mathbb{F}_t)_{t\geq 0}$ satisfies the usual conditions (i.e, the filtration contains all P-null sets and is right continuous). $E\{\cdot\}$ stands for the mathematical expectation operator with respect to the given probability measure. Denote by $L^2_{\mathbb{F}_0}([-\overline{\tau}_2, 0] : \mathbb{R}^n)$ the family of all $\mathbb{F}_0$ measurable $C([-\overline{\tau}_2, 0] : \mathbb{R}^n)$-valued random variables $\varphi = \{\varphi(s) : -\overline{\tau}_2 \leq s \leq 0\}$ such that $\sup_{-\overline{\tau}_2 \leq s \leq 0} E\|\varphi(s)\|^2 < \infty$. The asterisk $*$ in a matrix is used to denote term that is induced by symmetry. Matrices, if not explicitly specified, are assumed to have appropriate dimensions. Sometimes, the arguments of function will be omitted in the analysis when no confusion can be arised.

2. Problem Formulation and Preliminaries

Consider the following uncertain MJS with time-varying delays

$$\begin{aligned} \dot{x}(t) = {} & A(t, r_t)x(t) + A_d(t, r_t)x(t - \tau(t, r_t)) + B_1(t, r_t)u(t) + E_1(t, r_t)u(t - \tau(t, r_t)) \\ & + D_0(r_t)f(x(t), r_t) + D_1(r_t)\omega(t). \end{aligned} \tag{2.1}$$

$$\begin{aligned} z(t) = {} & C(t, r_t)x(t) + C_d(t, r_t)x(t - \tau(t, r_t)) + B_2(t, r_t)u(t) + E_2(t, r_t)u(t - \tau(t, r_t)) \\ & + D_2(r_t)\omega(t), \end{aligned} \tag{2.2}$$

Here $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^p$ is the control input, $z(t) \in \mathbb{R}^q$ is the control output, and $\omega(t) \in \mathbb{R}^l$ is the exogenous disturbance input which belongs to $\mathbb{L}_2\ [0, \infty]$, $\{r_t, t \geq 0\}$ is a homogenous finite-state Markov process with right continuous trajectories, which takes value in a finite-state space $S = \{1, 2, \ldots, N\}$ with generator $\Pi = \{\pi_{ij}\}, i, j \in S$ and has the mode transition probabilities

$$\Pr\{r_{t+\Delta t} = j \mid r_t = i\} = \begin{cases} \pi_{ij}\Delta t + o(\Delta t) & i \neq j, \\ 1 + \pi_{ii}\Delta t + o(\Delta t) & i = j, \end{cases} \tag{2.3}$$

where $\Delta t > 0, \lim_{\Delta t \to 0}(o(\Delta t)/\Delta t) = 0$, π_{ij} is the transition rete from i to j, and

$$\pi_{ii} = -\sum_{j \neq i} \pi_{ij}, \qquad \pi_{ij} \geq 0,\ j \neq i. \tag{2.4}$$

For notational simplicity, which $r_t = i$, $i \in S$, the matrices $A(t, r_t)$, $A_d(t, r_t)$, $B_1(t, r_t), E_1(t, r_t), C(t, r_t)$, $C_d(t, r_t)$, $B_2(t, r_t)$, $E_2(t, r_t)$, $D_0(r_t)$, $D_1(r_t)$, and $D_2(r_t)$ will be described by $A_i(t)$, $A_{di}(t), B_{1i}(t), E_{1i}(t)$, $C_i(t), C_{di}(t), B_{2i}(t)$, $E_{2i}(t)$, D_{0i}, D_{1i}, and D_{2i}. We denote that

$$\begin{gathered} A_i(t) = A_i + \Delta A_i(t), \qquad A_{di}(t) = A_{di} + \Delta A_{di}(t), \qquad B_{1i}(t) = B_{1i} + \Delta B_{1i}(t), \\ E_{1i}(t) = E_{1i} + \Delta E_{1i}(t), \qquad C_i(t) = C_i + \Delta C_i(t), \qquad C_{di}(t) = C_{di} + \Delta C_{di}(t), \\ B_{2i}(t) = B_{2i} + \Delta B_{2i}(t), \qquad E_{2i}(t) = E_{2i} + \Delta E_{2i}(t), \end{gathered} \tag{2.5}$$

where $A_i, A_{di}, B_{1i}, E_{1i}, C_i, C_{di}, B_{2i}, E_{2i}$, and D_{0i}, D_{1i}, D_{2i} are known constant matrices with appropriate dimensions. In this paper, the transition rates of Markov chain are partially known, that is, some elements in matrix Π are unknown. We denote that

$$I_{\mathrm{kn}}^i = \{j : \text{if } \pi_{ij} \text{ is know}\} \qquad I_{\mathrm{uk}}^i = \{j : \text{if } \pi_{ij} \text{ is unknow}\} \tag{2.6}$$

moreover, if $I_{\mathrm{kn}}^i \neq \emptyset$, it is further described as $I_{\mathrm{kn}}^i = \{k_1^i, k_2^i, \ldots, k_m^i\}$, $1 \leq m \leq N-2$.

Remark 2.1. $k_l^i \in N^+$, $l \in \{1, 2, \ldots, m\}$ represents the index of the lth known element in the ith row of transition rate matrix. The case $m = N-1$ is excluded, which means if we have only one unknown element, one can naturally calculate it from the known elements in each row and the transition rate matrix property.

Now the mode-dependent state-feedback controller is taken to be as follows:

$$u(t) = K_i x(t), \tag{2.7}$$

then, the closed-loop MJS can be represented as

$$\begin{aligned}\dot{x}(t) &= (A_i(t) + B_{1i}(t)K_i)x(t) + (A_{di}(t) + E_{1i}(t)K_i)x(t-\tau_i(t)) + D_{0i}f(x(t), i) + D_{1i}\omega(t),\\ z(t) &= (C_i(t) + B_{2i}(t)K_i)x(t) + (C_{di}(t) + E_{2i}(t)K_i)x(t-\tau_i(t)) + D_{2i}\omega(t).\end{aligned} \tag{2.8}$$

Before proceeding further, we will introduce the following assumptions, definition and some lemmas which will be used in the next section.

Assumption 1. The uncertain parameters are assumed to be of the form:

$$\begin{pmatrix} \Delta A_i(t) & \Delta A_{di}(t) & \Delta B_{1i}(t) & \Delta E_{1i}(t) \\ \Delta C_i(t) & \Delta C_{di}(t) & \Delta B_{2i}(t) & \Delta E_{2i}(t) \end{pmatrix} = \begin{pmatrix} T_{1i} \\ T_{2i} \end{pmatrix} F_i(t) \begin{pmatrix} N_{1i} & N_{2i} & N_{3i} & N_{4i} \end{pmatrix}, \tag{2.9}$$

where T_{1i}, T_{2i}, and N_{ki}, $k = 1, 2, 3, 4$, $i \in S$ are known real constant matrices with appropriate dimensions and $F_i(t)$, for all $i \in S$, are unknown time-varying matrix functions satisfying

$$F_i^T(t) F_i(t) \leq I. \tag{2.10}$$

Remark 2.2. It is assumed that all the elements $F_i(t)$, for all $i \in S$, are Lebesgue measurable. The matrices $\Delta A_i(t), \Delta A_{di}(t), \Delta B_{1i}(t), \Delta E_{1i}(t), \Delta C_i(t), \Delta C_{di}(t), \Delta B_{2i}(t)$, and $\Delta E_{2i}(t)$ are said to be admissible if and only if both (2.9) and (2.10) hold. The parameter uncertainty structure as in Assumption 1 is an extension of the so-called matching condition, which has been widely used in the problems of control and robust filtering of uncertain linear systems.

Assumption 2. The time-varying delay $\tau_i(t)$ satisfies $0 \leq \tau_{1i} \leq \tau_i(t) \leq \tau_{2i}$, $\dot{\tau}_i(t) \leq \mu_i$, with τ_{1i}, τ_{2i}, and μ_i being real constant scalars for each for all $i \in S$.

Assumption 3. For a fixed system mode $r_t = i \in S$, there exists a know real constant mode-dependent matrix $\Gamma_i = \mathrm{diag}(k_{1i}, k_{2i}, \dots, k_{ni}) > 0$ such that the nonlinear vector function $f(\cdot,\cdot)$ satisfy the following conditions:

$$f^T(x(t), i)\left(f(x(t), i) - \Gamma_i x(t)\right) \le 0. \tag{2.11}$$

Definition 2.3 (see [39]). The MJS (2.8) is said to be passive if there exists a constant δ such that

$$2E\left\{\int_0^T z^T(t)\omega(t)dt\right\} \ge \delta \tag{2.12}$$

holds for all $T \ge 0$.

Definition 2.4. The MJS (2.8) is said to be exponentially passive from input $\omega(t)$ to output $z(t)$, if there exists an exponential Lyapunov function (or called the exponential storage funtion) V defined on $\mathbb{R}^n$, and positive scalars ρ, γ such that for all $\omega(t)$, all initial conditions $x(0)$, all $t \ge 0$, the following inequality holds:

$$LV(x_t, r_t) + \rho V(x_t, r_t) - \gamma\omega^T(t)\omega(t) \le 2z^T(t)\omega(t). \tag{2.13}$$

Remark 2.5. From Definition 2.4, if $\rho = 0$, then the MJS in the form (2.8) is passive, in other words, exponential passivity implies passivity. It follows from (2.13) that

$$2E\left\{\int_0^T z^T(t)\omega(t)dt\right\} \ge -E\{V(x_0)\} - \gamma E\left\{\int_0^T \omega^T(t)\omega(t)dt\right\} = \delta. \tag{2.14}$$

Then from Definition 2.3, we can see that MJS (2.8) is passive. But the converse does not necessarily hold, that is, we can not obtain the exponential passive if systems are passive.

Lemma 2.6 (see [36]). *Let $Q(x) = Q^T(x), R(x) = R^T(x)$, and $S(x)$ depend affinely on x. Then the following linear matrix inequality:*

$$\begin{bmatrix} Q(x) & S(x) \\ S^T(x) & R(x) \end{bmatrix} > 0 \tag{2.15}$$

holds if and only if one of the following conditions holds:

(1) $R(x) > 0$, $Q(x) - S(x)R^{-1}(x)S^T(x) > 0$;

(2) $Q(x) > 0$, $R(x) - S^T(x)Q^{-1}(x)S(x) > 0$.

Lemma 2.7 (see [40]). *Let A, D, S, F, and P be real matrices of appropriate dimensions with $P > 0$ and F satisfy $F^T(t)F(t) \le I$. Then the following statement holds.*

(1) *For any scalar* $\varepsilon > 0$

$$DFS + (DFS)^T \leq \varepsilon^{-1} DD^T + \varepsilon S^T S. \tag{2.16}$$

(2) *For any vectors* x *and* y *with appropriate dimensions*

$$2x^T ADy \leq x^T APA^T x + y^T D^T P^{-1} Dy. \tag{2.17}$$

Lemma 2.8 (see [41]). *Let* A, X *be real matrices with appropriate dimensions. Then there exist a matrix* $P = P^T > 0$ *such that* $PA^T + AP + X < 0$, *if and only if, there exists a scalar* $\varepsilon > 0$ *and* Z *such that*

$$\begin{bmatrix} -Z - Z^T & Z^T A^T + P & Z^T \\ * & -\varepsilon^{-1} P + X & 0 \\ * & * & -\varepsilon P \end{bmatrix} < 0. \tag{2.18}$$

3. Main Results

3.1. Exponential Passivity Analysis

In this section, we assumed the transition rates are partially known and given the state-feedback controller gain matrix $K_i, i \in S$, at first, we will present a sufficient condition, which guarantees the MLS (2.8) is exponential passive.

Theorem 3.1. *Given the state-feedback controller gain matrix* K_i, *the uncertain MJS* (2.8) *is exponentially passive in the sense of expectation if there exists positive definite matrices* $P_i, Q_i, \overline{Q}_1, \overline{Q}_2, \overline{Q}_3, Q^*, Z_1, Z_2$, *positive scalars* $\gamma, \varepsilon_{1i}, \varepsilon_{2i}$, *and for any matrices* $G_i, M_i, R_i, U_i, V_i, H_i$ *with appropriate dimensions such that the following matrices inequalities hold for all* $i = 1, 2, \ldots, N$:

$$\begin{pmatrix} \Omega^1_{i,9\times 9} & \Lambda_1 & \Lambda_2 & \Lambda_3 & \Lambda_{4k} & \Lambda_{5k} & \Lambda_6 & \Lambda_7 \\ * & -Z_2 & 0 & 0 & 0 & 0 & \sqrt{2\overline{\tau}_2} Z_2 T_{1i} \varepsilon_{2i} & 0 \\ * & * & -Z_2 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -Z_2 & 0 & 0 & 0 & 0 \\ * & * & * & * & -Z_2 & 0 & 0 & 0 \\ * & * & * & * & * & -Z_2 & 0 & 0 \\ * & * & * & * & * & * & -\varepsilon_{2i} & 0 \\ * & * & * & * & * & * & * & -\varepsilon_{2i} \end{pmatrix} < 0 \quad k = 1, 2. \tag{3.1}$$

Case 1. If $\pi_{ii} \in I^i_{\mathrm{kn}}$

$$\begin{pmatrix} \pi_{ii} Q_i - Q^* & Q_j \\ * & -Q_j \end{pmatrix}_{\forall j \in I^i_{\mathrm{uk}}} < 0, \tag{3.2}$$

$$\begin{pmatrix} \left(1+\sum_{j\in I^i_{\mathrm{kn}}}\pi_{ij}\right)(\pi_{ii}Q_i - Q^*) & \sqrt{\pi_{ik^i_1}}Q_{k^i_1} & \cdots & \sqrt{\pi_{ik^i_m}}Q_{k^i_m} \\ * & -Q_{k^i_1} & 0 & 0 \\ * & * & \ddots & 0 \\ * & * & * & -Q_{k^i_m} \end{pmatrix} < 0, \tag{3.3}$$

$$P_iA_i + A_i^TP_i + P_j < 0 \quad \forall j \in I^i_{\mathrm{uk}}. \tag{3.4}$$

Case 2. If $\pi_{ii} \in I^i_{\mathrm{uk}}$

$$Q_j - Q^* > 0 \quad \forall j \in I^i_{\mathrm{uk}},\ j = i, \tag{3.5}$$

$$Q_j - Q^* < 0 \quad \forall j \in I^i_{\mathrm{uk}},\ j \neq i, \tag{3.6}$$

$$\begin{pmatrix} \left(-1-\sum_{j\in I^i_{\mathrm{kn}}}\pi_{ij}\right)Q^* & \sqrt{\pi_{ik^i_1}}Q_{k^i_1} & \cdots & \sqrt{\pi_{ik^i_m}}Q_{k^i_m} \\ * & -Q_{k^i_1} & 0 & 0 \\ * & * & \ddots & 0 \\ * & * & * & -Q_{k^i_m} \end{pmatrix} < 0, \tag{3.7}$$

$$P_iA_i + A_i^TP_i + P_j > 0 \quad \forall j \in I^i_{\mathrm{uk}},\ j = i, \tag{3.8}$$

$$P_iA_i + A_i^TP_i + P_j < 0 \quad \forall j \in I^i_{\mathrm{uk}},\ j \neq i, \tag{3.9}$$

where

$$\begin{aligned}
\Omega^1_{i,11} &= P_i\left(\left(1+\sum_{j\in I^i_{kn}}\pi_{ij}\right)A_i + B_{1i}K_i\right) + \left(\left(1+\sum_{j\in I^i_{kn}}\pi_{ij}\right)A_i + B_{1i}K_i\right)^T P_i \\
&\quad + \sum_{j\in I^i_{kn}}\pi_{ij}P_j + Q_i + \tau_{2i}Q^* + \overline{Q}_1 + \overline{Q}_2 + \overline{Q}_3 + (\tau_{2i}-\tau_{1i})Z_1 + G^T_{1i} + G_{1i}, \\
\Omega^1_{i,12} &= -G_{1i} + G^T_{2i} + M_{1i}, \qquad \Omega^1_{i,13} = R_{1i} + G^T_{3i} - M_{1i}, \\
\Omega^1_{i,14} &= -R_{1i} + G^T_{4i} + U_{1i} + P_i(A_{di} + E_{1i}K_i), \qquad \Omega^1_{i,15} = V_{1i} + G^T_{5i} - U_{1i}, \\
\Omega^1_{i,16} &= -V_{1i} + G^T_{6i} + H_{1i}, \qquad \Omega^1_{i,17} = G^T_{7i} - H_{1i} \qquad \Omega^1_{i,18} = G^T_{8i} + \varepsilon_{1i}\Gamma_i + P_iD_{0i}, \\
\Omega^1_{i,19} &= P_iD_{1i} - (C_i + B_{2i}K_i)^T, \qquad \Omega^1_{i,22} = -G^T_{2i} - G_{2i} + M^T_{2i} + M_{2i} - \overline{Q}_1, \\
\Omega^1_{i,23} &= -G^T_{3i} + R_{2i} + M^T_{3i} - M_{2i}, \qquad \Omega^1_{i,24} = -G^T_{4i} - R_{2i} + M^T_{4i} + U_{2i}, \\
pt\Omega^1_{i,25} &= -G^T_{5i} + V_{2i} + M^T_{5i} - U_{2i}, \qquad \Omega^1_{i,26} = -G^T_{6i} - V_{2i} + M^T_{6i} + H_{2i},
\end{aligned}$$

$$\Omega^1_{i,27} = -G^T_{7i} + M^T_{7i} - H_{2i}, \qquad \Omega^1_{i,28} = -G^T_{8i} + M^T_{8i}, \qquad \Omega^1_{i,29} = 0,$$

$$\Omega^1_{i,33} = R^T_{3i} + R_{3i} - M^T_{3i} - M_{3i}, \qquad \Omega^1_{i,34} = R^T_{4i} - R_{3i} - M^T_{4i} + U_{3i} \;,$$

$$\Omega^1_{i,35} = R^T_{5i} + V_{3i} - M^T_{5i} - U_{3i}, \qquad \Omega^1_{i,36} = R^T_{6i} - V_{3i} - M^T_{6i} + H_{3i},$$

$$\Omega^1_{i,37} = R^T_{7i} - M^T_{7i} - H_{3i}, \qquad \Omega^1_{i,38} = R^T_{8i} - M^T_{8i} \qquad \Omega^1_{i,39} = 0 \;,$$

$$\Omega^1_{i,44} = -R^T_{4i} - R_{4i} + U^T_{4i} + U_{4i} - (1-\mu_i)Q_i, \qquad \Omega^1_{i,45} = -R^T_{5i} + V_{4i} + U^T_{5i} - U_{4i},$$

$$\Omega^1_{i,46} = -R^T_{6i} - V_{4i} + U^T_{6i} + H_{4i}, \qquad \Omega^1_{i,47} = -R^T_{7i} + U^T_{7i} - H_{4i} \;,$$

$$\Omega^1_{i,48} = -R^T_{8i} + U^T_{8i}, \qquad \Omega^1_{i,49} = -(C_{di} + E_{2i}K_i)^T,$$

$$\Omega^1_{i,55} = V^T_{5i} + V_{5i} - U^T_{5i} - U_{5i}, \qquad \Omega^1_{i,56} = -U^T_{6i} - V_{5i} + V^T_{6i} + H_{5i},$$

$$\Omega^1_{i,57} = V^T_{7i} - U^T_{7i} - H_{5i}, \qquad \Omega^1_{i,58} = V^T_{8i} - U^T_{8i} \qquad \Omega^1_{i,59} = 0,$$

$$\Omega^1_{i,66} = -V^T_{6i} - V_{6i} + H^T_{6i} + H_{6i} - \overline{Q}_2, \qquad \Omega^1_{i,67} = -V^T_{7i} + H^T_{7i} - H_{6i},$$

$$\Omega^1_{i,68} = -V^T_{8i} + H^T_{8i}, \qquad \Omega^1_{i,69} = 0, \qquad \Omega^1_{i,77} = -H^T_{7i} - H_{7i} - \overline{Q}_3,$$

$$\Omega^1_{i,78} = -H^T_{8i}, \qquad \Omega^1_{i,79} = 0, \qquad \Omega^1_{i,88} = -2\varepsilon_{1i}I, \qquad \Omega^1_{i,89} = 0,$$

$$\Omega^1_{i,99} = -D_{2i} - D^T_{2i} - \gamma I, \qquad \overline{\tau}_2 = \max_{i\in S}\{\tau_{2i}\}, \qquad \underline{\tau}_1 = \min_{i\in S}\{\tau_{1i}\},$$

$$\Lambda_1 = \left(\sqrt{2\overline{\tau}_2}Z_2(A_i + B_{1i}K_i), 0, 0, \sqrt{2\overline{\tau}_2}Z_2(A_{di} + E_{1i}K_i), 0, 0, 0, \sqrt{2\overline{\tau}_2}Z_2D_{0i}, \sqrt{2\overline{\tau}_2}Z_2D_{1i}\right)^T,$$

$$\Lambda_1(t) = \Big(\sqrt{2\overline{\tau}_2}Z_2(A_i(t) + B_{1i}(t)K_i), 0, 0, \sqrt{2\overline{\tau}_2}Z_2(A_{di}(t) + E_{1i}(t)K_i), 0, 0, 0, \sqrt{2\overline{\tau}_2}Z_2D_{0i},$$
$$\sqrt{2\overline{\tau}_2}Z_2D_{1i}\Big)^T,$$

$$\Lambda_2 = \sqrt{\overline{\tau}_2 - \tau_{2i}}H_i, \qquad \Lambda_3 = \sqrt{\tau_{1i}}G_i, \qquad \Lambda_{41} = \sqrt{\frac{\tau_{2i} - \tau_{1i}}{2}}M_i,$$

$$\Lambda_{42} = \sqrt{\frac{\tau_{2i} - \tau_{1i}}{2}}U_i, \qquad \Lambda_{51} = \sqrt{\frac{\tau_{2i} - \tau_{1i}}{2}}R_i, \qquad \Lambda_{52} = \sqrt{\frac{\tau_{2i} - \tau_{1i}}{2}}V_i,$$

$$\Lambda_6 = \left(\varepsilon_{2i}T^T_{1i}P_i, 0, 0, 0, 0, 0, 0, 0, -\varepsilon_{2i}T^T_{2i}\right)^T,$$

$$\Lambda_7 = (N_{1i} + N_{3i}K_i, 0, 0, N_{2i} + N_{4i}K_i, 0, 0, 0, 0, 0)^T,$$

$$G_i = \left(G^T_{1i} \;\; G^T_{2i} \;\; G^T_{3i} \;\; G^T_{4i} \;\; G^T_{5i} \;\; G^T_{6i} \;\; G^T_{7i} \;\; G^T_{8i} \;\; 0\right)^T$$

$$M_i = \left(M^T_{1i} \;\; M^T_{2i} \;\; M^T_{3i} \;\; M^T_{4i} \;\; M^T_{5i} \;\; M^T_{6i} \;\; M^T_{7i} \;\; M^T_{8i} \;\; 0\right)^T,$$

$$R_i = \left(R^T_{1i} \;\; R^T_{2i} \;\; R^T_{3i} \;\; R^T_{4i} \;\; R^T_{5i} \;\; R^T_{6i} \;\; R^T_{7i} \;\; R^T_{8i} \;\; 0\right)^T,$$

$$U_i = \left(U^T_{1i} \;\; U^T_{2i} \;\; U^T_{3i} \;\; U^T_{4i} \;\; U^T_{5i} \;\; U^T_{6i} \;\; U^T_{7i} \;\; U^T_{8i} \;\; 0\right)^T,$$

$$V_i = \left(V_{1i}^T \ V_{2i}^T \ V_{3i}^T \ V_{4i}^T \ V_{5i}^T \ V_{6i}^T \ V_{7i}^T \ V_{8i}^T \ 0\right)^T,$$
$$H_i = \left(H_{1i}^T \ H_{2i}^T \ H_{3i}^T \ H_{4i}^T \ H_{5i}^T \ H_{6i}^T \ H_{7i}^T \ H_{8i}^T \ 0\right)^T, \tag{3.10}$$

Proof. First, in order to cast our model involved in the framework of the Markov process, we define a new process $x_t(s) = x(t+s), \ s \in [-\overline{\tau}_2, 0]$, and let L be the weak infinitesimal generator of the random process $x_t(s), \ t \geq 0$ and

$$Lv(x_t, r_t) = \lim_{\Delta \to 0^+} \frac{1}{\Delta} \{E[v(x_{t+\Delta}, r_{t+\Delta}) \mid x_t, r_t = i] - v(x_t, r_t)\}. \tag{3.11}$$

Now consider the Lyapunov-Krasovskii functional as follows for $r_t = i, \ i \in 1, 2, \ldots, S$:

$$v(x_t, i) = v_1(x_t, i) + v_2(x_t, i) + v_3(x_t, i) + v_4(x_t, i) + v_5(x_t, i), \tag{3.12}$$

where

$$\begin{aligned}
v_1(x_t, i) &= x^T(t)P(i)x(t), \qquad v_2(x_t, i) = \int_{t-\tau_i(t)}^{t} x^T(s)Q(i)x(s)ds, \\
v_3(x_t, i) &= \int_{-\tau_{2i}}^{0} \int_{t+\theta}^{t} x^T(s)Q^* x(s)ds\, d\theta, \\
v_4(x_t, i) &= \int_{t-\tau_{1i}}^{t} x^T(s)\overline{Q}_1 x(s)ds + \int_{t-\tau_{2i}}^{t} x^T(s)\overline{Q}_2 x(s)ds + \int_{t-\overline{\tau}_2}^{t} x^T(s)\overline{Q}_3 x(s)d, \\
v_5(x_t, i) &= \int_{-\tau_{2i}}^{-\tau_{1i}} \int_{t+\theta}^{t} x^T(s)Z_1 x(s)ds\, d\theta + 2\int_{-\overline{\tau}_2}^{0} \int_{t+\theta}^{t} \dot{x}^T(s)Z_2 \dot{x}(s)ds\, d\theta,
\end{aligned} \tag{3.13}$$

where

$$\sum_{j=1}^{N} \pi_{ij} Q_j \leq Q^*. \tag{3.14}$$

In order to show the exponential passivity of the MJS (2.8) under the given controller gain matrix K_i, we set

$$J^* = Lv(x_t, i) - \gamma \omega^T(t)\omega(t) - 2z^T(t)\omega(t). \tag{3.15}$$

Notice that

$$\begin{aligned}
Lv_1(x_t,i) &= x^T(t)\Big(P_i(A_i(t)+B_{1i}(t)K_i)+(A_i(t)+B_{1i}(t)K_i)^T P_i\Big)x(t)+x^T(t)\sum_{j=1}^{N}\pi_{ij}P_j x(t)\\
&\quad+2x^T(t)P_i(A_{di}(t)+E_{1i}(t)K_i)x(t-\tau_i(t))+2x^T(t)P_iD_{0i}f(x(t),i)+2x^T(t)P_iD_{1i}\omega(t),\\
Lv_2(x_t,i) &\le x^T(t)Q_ix(t)-(1-\mu_i)x^T(t-\tau_i(t))Q_ix(t-\tau_i(t))+\int_{t-\tau_i(t)}^{t}x^T(s)\sum_{j=1}^{N}\pi_{ij}Q_jx(s)ds,\\
Lv_3(x_t,i) &\le \tau_{2i}x^T(t)Q^*x(t)-\textstyle\int_{t-\tau_i(t)}^{t}x^T(s)Q^*x(s)ds\ ,\\
Lv_4(x_t,i) &= x^T(t)\Big(\overline{Q}_1+\overline{Q}_2+\overline{Q}_3\Big)x(t)-x^T(t-\tau_{1i})\overline{Q}_1x(t-\tau_{1i})-x^T(t-\tau_{2i})\overline{Q}_2x(t-\tau_{2i})\\
&\quad-x^T(t-\overline{\tau}_2)\overline{Q}_3x(t-\overline{\tau}_2),\\
Lv_5(x_t,i) &= (\tau_{2i}-\tau_{1i})x^T(t)Z_1x(t)+2\overline{\tau}_2\dot{x}^T(t)Z_2\dot{x}(t)-\int_{t-\tau_{2i}}^{t-\tau_{1i}}x^T(s)Z_1x(s)ds\\
&\quad-2\int_{t-\overline{\tau}_2}^{t}\dot{x}^T(s)Z_2\dot{x}(s)ds\\
&= (\tau_{2i}-\tau_{1i})x^T(t)Z_1x(t)+2\overline{\tau}_2\dot{x}^T(t)Z_2\dot{x}(t)-\int_{t-\overline{\tau}_2}^{t-\tau_{2i}}\dot{x}^T(s)Z_2\dot{x}(s)ds\\
&\quad-\int_{t-\tau_{2i}}^{t-(\tau_i(t)+\tau_{2i})/2}\dot{x}^T(s)Z_2\dot{x}(s)ds-\int_{t-(\tau_i(t)+\tau_{2i})/2}^{t-\tau_i(t)}\dot{x}^T(s)Z_2\dot{x}(s)ds\\
&\quad-\int_{t-\tau_i(t)}^{t-(\tau_i(t)+\tau_{1i})/2}\dot{x}^T(s)Z_2\dot{x}(s)ds-\int_{t-(\tau_i(t)+\tau_{1i})/2}^{t-\tau_{1i}}\dot{x}^T(s)Z_2\dot{x}(s)ds\\
&\quad-\int_{t-\tau_{1i}}^{t}\dot{x}^T(s)Z_2\dot{x}(s)ds-\int_{t-\tau_{2i}}^{t-\tau_{1i}}x^T(s)Z_1x(s)ds-\int_{t-\overline{\tau}_2}^{t}\dot{x}^T(s)Z_2\dot{x}(s)ds.
\end{aligned}\tag{3.16}$$

Then using Newton-Leibniz formula, for any matrices $H_i, G_i, M_i, R_i, U_i, V_i$ we have

$$\begin{aligned}
&2\xi^T(t)G_i\left(x(t)-x(t-\tau_{1i})-\int_{t-\tau_{1i}}^{t}\dot{x}(s)ds\right)=0,\\
&2\xi^T(t)M_i\left(x(t-\tau_{1i})-x\left(t-\frac{\tau_{1i}+\tau_i(t)}{2}\right)-\int_{t-(\tau_{1i}+\tau_i(t))/2}^{t-\tau_{1i}}\dot{x}(s)ds\right)=0,\\
&2\xi^T(t)R_i\left(x\left(t-\frac{\tau_{1i}+\tau_i(t)}{2}\right)-x(t-\tau_i(t))-\int_{t-\tau_i(t)}^{t-(\tau_{1i}+\tau_i(t))/2}\dot{x}(s)ds\right)=0,\\
&2\xi^T(t)U_i\left(x(t-\tau_i(t))-x\left(t-\frac{\tau_{2i}+\tau_i(t)}{2}\right)-\int_{t-(\tau_{2i}+\tau_i(t))/2}^{t-\tau_i(t)}\dot{x}(s)ds\right)=0,
\end{aligned}$$

$$2\xi^T(t)V_i\left(x\left(t-\frac{\tau_{2i}+\tau_i(t)}{2}\right)-x(t-\tau_{2i})-\int_{t-\tau_{2i}}^{t-(\tau_{2i}+\tau_i(t))/2}\dot{x}(s)ds\right)=0,$$
$$2\xi^T(t)H_i\left(x(t-\tau_{2i})-x(t-\overline{\tau}_2)-\int_{t-\overline{\tau}_2}^{t-\tau_{2i}}\dot{x}(s)dt\right)=0, \tag{3.17}$$

where

$$\begin{aligned}\xi^T(t)=\Bigg(&x^T(t),x^T(t-\tau_{1i}),x^T\left(t-\frac{\tau_i(t)+\tau_{1i}}{2}\right),x^T(t-\tau_i(t)),x^T\left(t-\frac{\tau_i(t)+\tau_{2i}}{2}\right),\\&x^T(t-\tau_{2i})\ x^T(t-\overline{\tau}_2),f^T(x(t),i),\omega^T(t)\Bigg).\end{aligned} \tag{3.18}$$

From the Lemma 2.7 (2.2), it is easy to see that

$$\begin{aligned}
-2\xi^T(t)G_i\int_{t-\tau_{1i}}^{t}\dot{x}(s)ds&\le\tau_{1i}\xi^T(t)G_iZ_2^{-1}G_i^T\xi(t)+\int_{t-\tau_{1i}}^{t}\dot{x}^T(s)Z_2\dot{x}(s)ds\\
-2\xi^T(t)M_i\int_{t-(\tau_{1i}+\tau_i(t))/2}^{t-\tau_{1i}}\dot{x}(s)ds&\le\frac{\tau_i(t)-\tau_{1i}}{2}\xi^T(t)M_iZ_2^{-1}M_i^T\xi(t)+\int_{t-(\tau_{1i}+\tau_i(t))/2}^{t-\tau_{1i}}\dot{x}^T(s)Z_2\dot{x}(s)ds\\
-2\xi^T(t)R_i\int_{t-\tau_i(t)}^{t-(\tau_{1i}+\tau_i(t))/2}\dot{x}(s)ds&\le\frac{\tau_i(t)-\tau_{1i}}{2}\xi^T(t)R_iZ_2^{-1}R_i^T\xi(t)+\int_{t-\tau_i(t)}^{t-(\tau_{1i}+\tau_i(t))/2}\dot{x}^T(s)Z_2\dot{x}(s)ds\\
-2\xi^T(t)U_i\int_{t-(\tau_{2i}+\tau_i(t))/2}^{t-\tau_i(t)}\dot{x}(s)ds&\le\frac{\tau_{2i}-\tau_i(t)}{2}\xi^T(t)U_iZ_2^{-1}U_i^T\xi(t)+\int_{t-(\tau_{2i}+\tau_i(t))/2}^{t-\tau_i(t)}\dot{x}^T(s)Z_2\dot{x}(s)ds\\
-2\xi^T(t)V_i\int_{t-\tau_{2i}}^{t-(\tau_{2i}+\tau_i(t))/2}\dot{x}(s)ds&\le\frac{\tau_{2i}-\tau_i(t)}{2}\xi^T(t)V_iZ_2^{-1}V_i^T\xi(t)+\int_{t-\tau_{2i}}^{t-(\tau_{2i}+\tau_i(t))/2}\dot{x}^T(s)Z_2\dot{x}(s)ds\\
-2\xi^T(t)H_i\int_{t-\overline{\tau}_2}^{t-\tau_{2i}}\dot{x}(s)ds&\le(\overline{\tau}_2-\tau_{2i})\xi^T(t)H_iZ_2^{-1}H_i^T\xi(t)+\int_{t-\overline{\tau}_2}^{t-\tau_{2i}}\dot{x}^T(s)Z_2\dot{x}(s)ds.
\end{aligned} \tag{3.19}$$

Now by Assumption 3, it can be deduced that for any positive scalar $\varepsilon_{1i},\ i\in 1,2,\dots,S$,

$$2\varepsilon_{1i}f^T(x(t),i)\big(\Gamma_ix(t)-f(x(t),i)\big)\ge 0. \tag{3.20}$$

Then from the above discussion, we can see that

$$\begin{aligned}J^*\le{}&\xi^T(t)\left(\frac{\tau_i(t)-\tau_{1i}}{\tau_{2i}-\tau_{1i}}\Phi_1(t)+\frac{\tau_{2i}-\tau_i(t)}{\tau_{2i}-\tau_{1i}}\Phi_2(t)\right)\xi(t)-\int_{t-\tau_{2i}}^{t-\tau_{1i}}x^T(s)Z_1x(s)ds\\&-\int_{t-\overline{\tau}_2}^{t}\dot{x}^T(s)Z_2\dot{x}(s)ds,\end{aligned}$$

$$\Phi_1(t) = \Omega_{i,9\times 9}(t) + \Lambda_1(t)Z_2^{-1}\Lambda_1(t)^T + (\overline{\tau}_2 - \tau_{2i})H_iZ_2^{-1}H_i^T + \tau_{1i}G_iZ_2^{-1}G_i^T$$
$$+ \frac{\tau_{2i} - \tau_{1i}}{2}\left(M_iZ_2^{-1}M_i^T + R_iZ_2^{-1}R_i^T\right),$$
$$\Phi_2(t) = \Omega_{i,9\times 9}(t) + \Lambda_1(t)Z_2^{-1}\Lambda_1(t)^T + (\overline{\tau}_2 - \tau_{2i})H_iZ_2^{-1}H_i^T + \tau_{1i}G_iZ_2^{-1}G_i^T$$
$$+ \frac{\tau_{2i} - \tau_{1i}}{2}\left(U_iZ_2^{-1}U_i^T + V_iZ_2^{-1}V_i^T\right), \tag{3.21}$$

where

$$\Omega_{i,11}(t) = P_i(A_i(t) + B_{1i}(t)K_i) + (A_i(t) + B_{1i}(t)K_i)^T P_i$$
$$+ \sum_{j=1}^{N}\pi_{ij}P_j + Q_i + \tau_{2i}Q^* + \overline{Q}_1 + \overline{Q}_2 + \overline{Q}_3 + (\tau_{2i} - \tau_{1i})Z_1 + G_{1i}^T + G_{1i},$$
$$\Omega_{i,14}(t) = P_i(A_{di}(t) + E_{1i}(t)K_i) - R_{1i} + G_{4i}^T + U_{1i} \qquad \Omega_{i,19}(t) = P_iD_{1i} - (C_i(t) + B_{2i}(t)K_i)^T,$$
$$\Omega_{i,49}(t) = -(C_{di}(t) + E_{2i}(t)K_i)^T \tag{3.22}$$

other terms of $\Omega_{i,i\times j}(t)$ are similar to $\Omega^1_{i,i\times j}$. In order to get our results, we will describe that the $\Phi_1(t) < 0$ and $\Phi_2(t) < 0$.

By the Schur complement, $\Phi_1(t) < 0$ and $\Phi_2(t) < 0$ under the restriction of (3.14) if and only if

$$\begin{pmatrix} \Omega_{i,9\times 9} & \Lambda_1 & \Lambda_2 & \Lambda_3 & \Lambda_{4k} & \Lambda_{5k} \\ * & -Z_2 & 0 & 0 & 0 & 0 \\ * & * & -Z_2 & 0 & 0 & 0 \\ * & * & * & -Z_2 & 0 & 0 \\ * & * & * & * & -Z_2 & 0 \\ * & * & * & * & * & -Z_2 \end{pmatrix}_{k=1,2}$$
$$+ \left(T_{1i}^TP_i, 0_{n\times 7n}, -T_{2i}^T, \sqrt{2\overline{\tau}_2}T_{1i}^TZ_2, 0_{n\times 4n}\right)^T F_i(k)(N_{1i}+N_{3i}K_i, 0_{n\times 2n}, N_{2i} + N_{4i}K_i, 0_{n\times 10n})$$
$$+ (N_{1i} + N_{2i}K_i, 0_{n\times 2n}, N_{2i}+N_{4i}K_i, 0_{n\times 10n})^T F_i^T(k)\left(T_{1i}^TP_i, 0_{n\times 7n}, -T_{2i}^T, \sqrt{2\overline{\tau}_2}T_{1i}^TZ_2, 0_{n\times 4n}\right) < 0, \tag{3.23}$$

where $\Omega_{i,9\times 9}$ is the nominal matrix of $\Omega_{i,9\times 9}(t)$. Then from the Lemma 2.7 (2.1), above matrix inequality holds, which is equivalent to

$$\begin{pmatrix} \Omega_{i,9\times 9} & \Lambda_1 & \Lambda_2 & \Lambda_3 & \Lambda_{4k} & \Lambda_{5k} & \Lambda_6 & \Lambda_7 \\ * & -Z_2 & 0 & 0 & 0 & 0 & \sqrt{2\overline{\tau}_2}Z_2T_{1i}\varepsilon_{2i} & 0 \\ * & * & -Z_2 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -Z_2 & 0 & 0 & 0 & 0 \\ * & * & * & * & -Z_2 & 0 & 0 & 0 \\ * & * & * & * & * & -Z_2 & 0 & 0 \\ * & * & * & * & * & * & -\varepsilon_{2i} & 0 \\ * & * & * & * & * & * & * & -\varepsilon_{2i} \end{pmatrix}_{k=1,2} < 0. \tag{3.24}$$

Case 1. If $\pi_{ii} \in I^i_{\mathrm{kn}}$ then (3.24) is equivalent to

$$\begin{pmatrix}
\Omega^1_{i,9\times 9} & \Lambda_1 & \Lambda_2 & \Lambda_3 & \Lambda_{4k} & \Lambda_{5k} & \Lambda_6 & \Lambda_7 \\
* & -Z_2 & 0 & 0 & 0 & 0 & \sqrt{2\overline{\tau}_2}Z_2T_{1i}\varepsilon_{2i} & 0 \\
* & * & -Z_2 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & -Z_2 & 0 & 0 & 0 & 0 \\
* & * & * & * & -Z_2 & 0 & 0 & 0 \\
* & * & * & * & * & -Z_2 & 0 & 0 \\
* & * & * & * & * & * & -\varepsilon_{2i} & 0 \\
* & * & * & * & * & * & * & -\varepsilon_{2i}
\end{pmatrix}_{k=1,2}$$
$$+ \ \mathrm{diag}\left(\sum_{\substack{j\in I^i_{\mathrm{uk}}\\ j\neq i}} \pi_{ij}P_j, \overbrace{0,\ldots,0}^{15}\right) + \mathrm{diag}\left(\sum_{\substack{j\in I^i_{\mathrm{uk}}\\ j\neq i}} \pi_{ij}\left(P_iA_i + A_i^TP_i\right), \overbrace{0,\ldots,0}^{15}\right) < 0. \tag{3.25}$$

Obviously, we can see that if (3.1) and (3.4) hold, then $\Phi_1(t) < 0$ and $\Phi_2(t) < 0$ under the restriction of (3.14). Next we will further consider the equivalent form of (3.14).
$\sum_{j=1}^N \pi_{ij}Q_j < Q^*$ is equivalent to

$$\sum_{\substack{j\in I^i_{\mathrm{kn}}\\ j\neq i}} \pi_{ij}Q_j + \sum_{\substack{j\in I^i_{\mathrm{uk}}\\ j\neq i}} \pi_{ij}Q_j + \pi_{ii}Q_i - Q^* + \sum_{j\in I^i_{\mathrm{kn}}} \pi_{ij}(\pi_{ii}Q_i - Q^*) + \sum_{j\in I^i_{\mathrm{uk}}} \pi_{ij}(\pi_{ii}Q_i - Q^*) < 0. \tag{3.26}$$

If we have the following matrix inequalities hold, we can have that (3.14) is satisfied

$$\begin{gathered}
\left(1 + \sum_{j\in I^i_{\mathrm{kn}}} \pi_{ij}\right)(\pi_{ii}Q_i - Q^*) + \sum_{\substack{j\in I^i_{\mathrm{kn}}\\ j\neq i}} \pi_{ij}Q_j < 0, \\
\pi_{ii}Q_i - Q^* + Q_j < 0 \qquad j \in I^i_{\mathrm{uk}}.
\end{gathered} \tag{3.27}$$

Obviously, (3.27) is equivalent to (3.2) and (3.3) by the Schur complement.

Case 2. If $\pi_{ii} \in I^i_{\mathrm{uk}}$ then (3.24) is equivalent to

$$\begin{pmatrix} \Omega^1_{i,9\times 9} & \Lambda_1 & \Lambda_2 & \Lambda_3 & \Lambda_{4k} & \Lambda_{5k} & \Lambda_6 & \Lambda_7 \\ * & -Z_2 & 0 & 0 & 0 & 0 & \sqrt{2\overline{\tau}_2} Z_2 T_{1i}\varepsilon_{2i} & 0 \\ * & * & -Z_2 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -Z_2 & 0 & 0 & 0 & 0 \\ * & * & * & * & -Z_2 & 0 & 0 & 0 \\ * & * & * & * & * & -Z_2 & 0 & 0 \\ * & * & * & * & * & * & -\varepsilon_{2i} & 0 \\ * & * & * & * & * & * & * & -\varepsilon_{2i} \end{pmatrix}_{k=1,2} + \operatorname{diag}\left(\sum_{\substack{j\in I^i_{\mathrm{uk}} \\ j\neq i}} \pi_{ij} P_j, \overbrace{0,\ldots,0}^{15} \right)$$
$$+ \operatorname{diag}\left(\sum_{\substack{j\in I^i_{\mathrm{uk}} \\ j\neq i}} \pi_{ij} \left(P_i A_i + A_i^T P_i \right), \overbrace{0,\ldots,0}^{15} \right) + \pi_{ii} \operatorname{diag}\left(P_i A_i + A_i^T P_i + P_i, \overbrace{0,\ldots,0}^{15} \right) < 0. \tag{3.28}$$

Then if (3.1), (3.8), and (3.9) hold, then $\Phi_1(t) < 0$ and $\Phi_2(t) < 0$ under the restriction of (3.14), furthermore, with the similar consideration, we can deduce that if (3.5)–(3.7) are established, then (3.14) is founded. So there exists a positive scalar ρ_1, then

$$J^* \leq -\rho_1 \|x(t)\|^2 - \lambda_{\min}(Z_1) \int_{t-\overline{\tau}_2}^{t} \|x(s)\|^2 ds - \lambda_{\min}(Z_2) \int_{t-\overline{\tau}_2}^{t} \|\dot{x}(s)\|^2 ds. \tag{3.29}$$

On the other hand, it is easy to obtain that

$$\begin{aligned} v(x(t), i) \leq{} & \|P\| \|x(t)\|^2 + \left(\|Q\| + \|Q^*\| + \left\|\overline{Q}_1\right\| + \left\|\overline{Q}_2\right\| + \left\|\overline{Q}_3\right\| + (\overline{\tau}_2 - \underline{\tau}_1)\|Z_1\| \right) \\ & \times \int_{t-\overline{\tau}_2}^{t} \|x(s)\|^2 ds + 2\overline{\tau}_2 \|Z_2\| \int_{t-\overline{\tau}_2}^{t} \|\dot{x}(s)\|^2 ds, \end{aligned} \tag{3.30}$$

where $\|P\| = \max_{i\in S}\{\|P_i\|\}$, $\|Q\| = \max_{i\in S}\{\|Q_i\|\}$.

Let $\rho > 0$ be sufficiently small such that

$$\begin{gathered} \rho\|P\| - \rho_1 < 0, \\ \rho\left(\|Q\| + \|Q^*\| + \left\|\overline{Q}_1\right\| + \left\|\overline{Q}_2\right\| + \left\|\overline{Q}_3\right\| + (\overline{\tau}_2 - \underline{\tau}_1)\|Z_1\| \right) - \lambda_{\min}(Z_1) < 0, \\ 2\overline{\tau}_2 \rho \|Z_2\| - \lambda_{\min}(Z_2) < 0. \end{gathered} \tag{3.31}$$

So, by Definition 2.4, the MJS (2.8) is exponentially passive. This completes the proof. □

Remark 3.2. It is easy to derive that the MJS (2.8) is exponential mean square stability with $\omega(t) = 0$ if the MJS (2.8) is exponentially passive. Moreover, the result of Theorem 3.1 makes use of the information of the subsystems upper bounds of the time varying delays, which may bring us less conservativeness, and from the free-weighting matrix and Newton-Leibnitz formula, the upper bounds of μ_i are not restricted to be less than 1 in this paper. Therefore, our result is more natural and reasonable to the Markovian jump systems.

Remark 3.3. In order to obtain the gain matrices K_i for convenience in the next section, (3.1) is not LMI, if we substitute ε_{2i} by ε_{2i}^{-1} and use the Lemma 2.7 (2.1), we can obtain the equivalent form of LMI.

3.2. Exponential Passification

In this section, we will determine the feedback controller gain matrices $K_i, i \in S$ in (2.7), which guarantee that the closed-loop MJS (2.8) is exponentially passive with partially known transition rates.

Theorem 3.4. *Given a positive constant ε, there exists a state-feedback controller in the form* (2.7) *such that the closed-loop MJS* (2.8) *is exponentially passive if there exist positive definite matrices $\overline{P}_i, \overline{Q}_i, \widetilde{Q}_1, \widetilde{Q}_2, \widetilde{Q}_3, \overline{Q}^*, \overline{Z}_1, \overline{Z}_2$, positive scalar $\varepsilon_{1i}, \varepsilon_{2i}$, and for any matrices $\overline{G}_i, \overline{M}_i, \overline{R}_i, \overline{U}_i, \overline{V}_i, \overline{H}_i, Z_{ii}$ with appropriate dimensions satisfying the following LMIs under the two cases for all $i = 1, 2, \ldots, N$.*

Case 1. If $\pi_{ii} \in I_{\mathrm{kn}}^i$

$$\begin{pmatrix} \pi_{ii}\overline{Q}_i - \overline{Q}^* & \overline{Q}_j \\ * & -\overline{Q}_j \end{pmatrix}_{\forall j \in I_{\mathrm{uk}}^i} < 0, \tag{3.32}$$

$$\begin{pmatrix} \left(1 + \sum_{j \in I_{\mathrm{kn}}^i} \pi_{ij}\right)\left(\pi_{ii}\overline{Q}_i - \overline{Q}^*\right) & \sqrt{\pi_{ik_1^i}}\overline{Q}_{k_1^i} & \cdots & \sqrt{\pi_{ik_m^i}}\overline{Q}_{k_m^i} \\ * & -\overline{Q}_{k_1^i} & 0 & 0 \\ * & * & \ddots & 0 \\ * & * & * & -\overline{Q}_{k_m^i} \end{pmatrix} < 0, \tag{3.33}$$

$$\begin{pmatrix} \overline{\Omega}_{i,9\times 9}^{1} & \overline{\Lambda}_1 & \overline{\Lambda}_2 & \overline{\Lambda}_3 & \overline{\Lambda}_{4k} & \overline{\Lambda}_{5k} & \overline{\Lambda}_6 & \overline{\Lambda}_7 & \overline{\Lambda}_8 \\ * & -\overline{Z}_2 & 0 & 0 & 0 & 0 & \sqrt{2\overline{\tau}_2}T_{1i}\varepsilon_{2i} & 0 & 0 \\ * & * & \Theta & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Theta & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \Theta & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \Theta & 0 & 0 & 0 \\ * & * & * & * & * & * & -\varepsilon_{2i} & 0 & 0 \\ * & * & * & * & * & * & * & -\varepsilon_{2i} & 0 \\ * & * & * & * & * & * & * & * & \overline{\Lambda}_9 \end{pmatrix} < 0 \quad k = 1, 2, \tag{3.34}$$

$$\begin{pmatrix} -Z_{ii}-Z_{ii}^T & Z_{ii}^T A_i^T+\overline{P}_i & Z_{ii}^T & 0 \\ * & -\varepsilon^{-1}\overline{P}_i & 0 & \overline{P}_i \\ * & * & -\varepsilon\overline{P}_i & 0 \\ * & * & * & -\overline{P}_j \end{pmatrix}_{\forall j\in I_{uk}^i} < 0, \tag{3.35}$$

Case 2. If $\pi_{ii} \in I_{\text{uk}}^i$

$$\overline{Q}_j - \overline{Q}^* > 0 \quad \forall j \in I_{\text{uk}}^i,\ j = i, \tag{3.36}$$

$$\overline{Q}_j - \overline{Q}^* < 0 \quad \forall j \in I_{\text{uk}}^i,\ j \neq i, \tag{3.37}$$

$$\begin{pmatrix} \left(-1-\sum_{j\in I_{kn}^i}\pi_{ij}\right)\overline{Q}^* & \sqrt{\pi_{ik_1^i}}\overline{Q}_{k_1^i} & \cdots & \sqrt{\pi_{ik_m^i}}\overline{Q}_{k_m^i} \\ * & -\overline{Q}_{k_1^i} & 0 & 0 \\ * & * & \ddots & 0 \\ * & * & * & -\overline{Q}_{k_m^i} \end{pmatrix} < 0, \tag{3.38}$$

$$\begin{pmatrix} \overline{\Omega}_{i,9\times 9}^2 & \overline{\Lambda}_1 & \overline{\Lambda}_2 & \overline{\Lambda}_3 & \overline{\Lambda}_{4k} & \overline{\Lambda}_{5k} & \overline{\Lambda}_6 & \overline{\Lambda}_7 & \overline{\Lambda}_8 \\ * & -\overline{Z}_2 & 0 & 0 & 0 & 0 & \sqrt{2\overline{\tau}_2}T_{1i}\varepsilon_{2i} & 0 & 0 \\ * & * & \Theta & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Theta & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \Theta & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \Theta & 0 & 0 & 0 \\ * & * & * & * & * & * & -\varepsilon_{2i} & 0 & 0 \\ * & * & * & * & * & * & * & -\varepsilon_{2i} & 0 \\ * & * & * & * & * & * & * & * & \overline{\Lambda}_9 \end{pmatrix} < 0 \quad k = 1, 2, \tag{3.39}$$

$$\begin{pmatrix} -Z_{ii}-Z_{ii}^T & -Z_{ii}^T A_i^T+\overline{P}_i & Z_{ii}^T \\ * & -\varepsilon^{-1}\overline{P}_i-\overline{P}_j & 0 \\ * & * & -\varepsilon\overline{P}_i \end{pmatrix}_{\substack{\forall j\in I_{uk}^i \\ j=i}} < 0, \tag{3.40}$$

$$\begin{pmatrix} -Z_{ii}-Z_{ii}^T & Z_{ii}^T A_i^T+\overline{P}_i & Z_{ii}^T & 0 \\ * & -\varepsilon^{-1}\overline{P}_i & 0 & \overline{P}_i \\ * & * & -\varepsilon\overline{P}_i & 0 \\ * & * & * & -\overline{P}_j \end{pmatrix}_{\substack{\forall j\in I_{uk}^i \\ j\neq i}} < 0, \tag{3.41}$$

where

$$
\begin{aligned}
\overline{\Omega}_{i,11}^{1} &= \left(\left(1+\sum_{j\in I_{kn}^{i}}\pi_{ij}\right)A_i\overline{P}_i + B_{1i}Y_i\right) + \left(\left(1+\sum_{j\in I_{kn}^{i}}\pi_{ij}\right)A_i\overline{P}_i + B_{1i}Y_i\right)^T + \pi_{ii}\overline{P}_i + \overline{Q}_i + \tau_{2i}\overline{Q}^* \\
&\quad + \widetilde{Q}_1 + \widetilde{Q}_2 + \widetilde{Q}_3 + (\tau_{2i}-\tau_{1i})\overline{Z}_1 + \overline{G}_{1i}^T + \overline{G}_{1i}, \\
\overline{\Omega}_{i,11}^{2} &= \left(\left(1+\sum_{j\in I_{kn}^{i}}\pi_{ij}\right)A_i\overline{P}_i + B_{1i}Y_i\right) + \left(\left(1+\sum_{j\in I_{kn}^{i}}\pi_{ij}\right)A_i\overline{P}_i + B_{1i}Y_i\right)^T + \overline{Q}_i + \tau_{2i}\overline{Q}^* + \widetilde{Q}_1 \\
&\quad + \widetilde{Q}_2 + \widetilde{Q}_3 + (\tau_{2i}-\tau_{1i})\overline{Z}_1 + \overline{G}_{1i}^T + \overline{G}_{1i}, \\
\overline{\Omega}_{i,12}^{1} &= \overline{\Omega}_{i,12}^{2} = -\overline{G}_{1i} + \overline{G}_{2i}^T + \overline{M}_{1i}, \qquad \overline{\Omega}_{i,13}^{1} = \overline{\Omega}_{i,13}^{2} = \overline{R}_{1i} + \overline{G}_{3i}^T - \overline{M}_{1i}, \\
\overline{\Omega}_{i,14}^{1} &= \overline{\Omega}_{i,14}^{2} = -\overline{R}_{1i} + \overline{G}_{4i}^T + \overline{U}_{1i} + \left(A_{di}\overline{P}_i + E_{1i}Y_i\right) \qquad \overline{\Omega}_{i,15}^{1} = \overline{\Omega}_{i,15}^{2} = \overline{V}_{1i} + \overline{G}_{5i}^T - \overline{U}_{1i}, \\
\overline{\Omega}_{i,16}^{1} &= \overline{\Omega}_{i,16}^{2} = -\overline{V}_{1i} + \overline{G}_{6i}^T + \overline{H}_{1i}, \qquad \overline{\Omega}_{i,17}^{1} = \overline{\Omega}_{i,17}^{2} = \overline{G}_{7i}^T - \overline{H}_{1i}, \\
\overline{\Omega}_{i,18}^{1} &= \overline{\Omega}_{i,18}^{2} = \overline{G}_{8i}^T + \overline{P}_i\Gamma_i + D_{0i}\varepsilon_{1i}, \\
\overline{\Omega}_{i,19}^{1} &= \overline{\Omega}_{i,19}^{2} = D_{1i} - \left(C_i\overline{P}_i + B_{2i}Y_i\right)^T, \qquad \overline{\Omega}_{i,22}^{1} = \overline{\Omega}_{i,22}^{2} = -\overline{G}_{2i}^T - \overline{G}_{2i} + \overline{M}_{2i}^T + \overline{M}_{2i} - \widetilde{Q}_1, \\
\overline{\Omega}_{i,23}^{1} &= \overline{\Omega}_{i,23}^{2} = -\overline{G}_{3i}^T + \overline{R}_{2i} + \overline{M}_{3i}^T - \overline{M}_{2i}, \qquad \overline{\Omega}_{i,24}^{1} = \overline{\Omega}_{i,24}^{2} = -\overline{G}_{4i}^T - \overline{R}_{2i} + \overline{M}_{4i}^T + \overline{U}_{2i}, \\
\overline{\Omega}_{i,25}^{1} &= \overline{\Omega}_{i,25}^{2} = -\overline{G}_{5i}^T + \overline{V}_{2i} + \overline{M}_{5i}^T - \overline{U}_{2i}, \qquad \overline{\Omega}_{i,26}^{1} = \overline{\Omega}_{i,26}^{2} = -\overline{G}_{6i}^T - \overline{V}_{2i} + \overline{M}_{6i}^T + \overline{H}_{2i}, \\
\overline{\Omega}_{i,27}^{1} &= \overline{\Omega}_{i,27}^{2} = -\overline{G}_{7i}^T + \overline{M}_{7i}^T - \overline{H}_{2i}, \qquad \overline{\Omega}_{i,28}^{1} = \overline{\Omega}_{i,28}^{2} = -\overline{G}_{8i}^T + \overline{M}_{8i}^T, \qquad \overline{\Omega}_{i,29}^{1} = \overline{\Omega}_{i,29}^{2} = 0, \\
\overline{\Omega}_{i,33}^{1} &= \overline{\Omega}_{i,33}^{2} = \overline{R}_{3i}^T + \overline{R}_{3i} - \overline{M}_{3i}^T - \overline{M}_{3i}, \qquad \overline{\Omega}_{i,34}^{1} = \overline{\Omega}_{i,34}^{2} = \overline{R}_{4i}^T - \overline{R}_{3i} - \overline{M}_{4i}^T + \overline{U}_{3i}, \\
\overline{\Omega}_{i,35}^{1} &= \overline{\Omega}_{i,35}^{2} = \overline{R}_{5i}^T + \overline{V}_{3i} - \overline{M}_{5i}^T - \overline{U}_{3i}, \qquad \overline{\Omega}_{i,36}^{1} = \overline{\Omega}_{i,36}^{2} = \overline{R}_{6i}^T - \overline{V}_{3i} - \overline{M}_{6i}^T + \overline{H}_{3i}, \\
\overline{\Omega}_{i,37}^{1} &= \overline{\Omega}_{i,37}^{2} = \overline{R}_{7i}^T - \overline{M}_{7i}^T - \overline{H}_{3i}, \qquad \overline{\Omega}_{i,38}^{1} = \overline{\Omega}_{i,38}^{2} = \overline{R}_{8i}^T - \overline{M}_{8i}^T, \qquad \overline{\Omega}_{i,39}^{1} = \overline{\Omega}_{i,39}^{2} = 0, \\
\overline{\Omega}_{i,44}^{1} &= \overline{\Omega}_{i,44}^{2} = -\overline{R}_{4i}^T - \overline{R}_{4i} + \overline{U}_{4i}^T + \overline{U}_{4i} - (1-\mu_i)\overline{Q}_i, \\
\overline{\Omega}_{i,45}^{1} &= \overline{\Omega}_{i,45}^{2} = -\overline{R}_{5i}^T + \overline{V}_{4i} + \overline{U}_{5i}^T - \overline{U}_{4i}, \\
\overline{\Omega}_{i,46}^{1} &= \overline{\Omega}_{i,46}^{2} = -\overline{R}_{6i}^T - \overline{V}_{4i} + \overline{U}_{6i}^T + \overline{H}_{4i}, \qquad \overline{\Omega}_{i,47}^{1} = \overline{\Omega}_{i,47}^{2} = -\overline{R}_{7i}^T + \overline{U}_{7i}^T - \overline{H}_{4i}, \\
\overline{\Omega}_{i,48}^{1} &= \overline{\Omega}_{i,48}^{2} = -\overline{R}_{8i}^T + \overline{U}_{8i}^T, \qquad \overline{\Omega}_{i,49}^{1} = \overline{\Omega}_{i,49}^{2} = -\left(C_{di}\overline{P}_i + E_{2i}Y_i\right)^T, \\
\overline{\Omega}_{i,55}^{1} &= \overline{\Omega}_{i,55}^{2} = \overline{V}_{5i}^T + \overline{V}_{5i} - \overline{U}_{5i}^T - \overline{U}_{5i}, \qquad \overline{\Omega}_{i,56}^{1} = \overline{\Omega}_{i,56}^{2} = -\overline{U}_{6i}^T - \overline{V}_{5i} + \overline{V}_{6i}^T + \overline{H}_{5i}, \\
\overline{\Omega}_{i,57}^{1} &= \overline{\Omega}_{i,57}^{2} = \overline{V}_{7i}^T - \overline{U}_{7i}^T - \overline{H}_{5i}, \qquad \overline{\Omega}_{i,58}^{1} = \overline{\Omega}_{i,58}^{2} = \overline{V}_{8i}^T - \overline{U}_{8i}^T \qquad \overline{\Omega}_{i,59}^{1} = \overline{\Omega}_{i,59}^{2} = 0,
\end{aligned}
$$

$$\overline{\Omega}^1_{i,66} = \overline{\Omega}^2_{i,66} = -\overline{V}^T_{6i} - \overline{V}_{6i} + \overline{H}^T_{6i} + \overline{H}_{6i} - \widetilde{Q}_2, \qquad \overline{\Omega}^1_{i,67} = \overline{\Omega}^2_{i,67} = -\overline{V}^T_{7i} + \overline{H}^T_{7i} - \overline{H}_{6i},$$

$$\overline{\Omega}^1_{i,68} = \overline{\Omega}^2_{i,68} = -\overline{V}^T_{8i} + \overline{H}^T_{8i}, \qquad \overline{\Omega}^1_{i,69} = \overline{\Omega}^2_{i,69} = 0, \qquad \overline{\Omega}^1_{i,77} = \overline{\Omega}^2_{i,77} = -\overline{H}^T_{7i} - \overline{H}_{7i} - \widetilde{Q}_3,$$

$$\overline{\Omega}^1_{i,78} = \overline{\Omega}^2_{i,78} = -\overline{H}^T_{8i}, \qquad \overline{\Omega}^1_{i,79} = \overline{\Omega}^2_{i,79} = 0, \qquad \overline{\Omega}^1_{i,88} = \overline{\Omega}^2_{i,88} = -2\overline{\varepsilon}_{1i}I, \qquad \overline{\Omega}^1_{i,89} = \overline{\Omega}^2_{i,89} = 0,$$

$$\overline{\Omega}^1_{i,99} = \overline{\Omega}^2_{i,99} = -D_{2i} - D^T_{2i} - \gamma, \qquad \Theta = J^T\overline{Z}_2 J - J^T\overline{P}_i - \overline{P}_i,$$

$$\overline{\Lambda}_1 = \left(\sqrt{2\overline{\tau}_2}\left(A_i\overline{P}_i + B_{1i}Y_i\right), 0, 0, \sqrt{2\overline{\tau}_2}\left(A_{di}\overline{P}_i + E_{1i}Y_i\right), 0, 0, 0, \sqrt{2\overline{\tau}_2}D_{0i}\overline{\varepsilon}_{1i}, \sqrt{2\overline{\tau}_2}D_{1i}\right)^T,$$

$$\overline{\Lambda}_2 = \sqrt{\overline{\tau}_2 - \tau_{2i}}\overline{H}_i, \qquad \overline{\Lambda}_3 = \sqrt{\tau_{1i}}\overline{G}_i, \qquad \overline{\Lambda}_{41} = \sqrt{\frac{\tau_{2i} - \tau_{1i}}{2}}\overline{M}_i,$$

$$\overline{\Lambda}_{42} = \sqrt{\frac{\tau_{2i} - \tau_{1i}}{2}}\overline{U}_i, \qquad \overline{\Lambda}_{51} = \sqrt{\frac{\tau_{2i} - \tau_{1i}}{2}}\overline{R}_i, \qquad \overline{\Lambda}_{52} = \sqrt{\frac{\tau_{2i} - \tau_{1i}}{2}}\overline{V}_i,$$

$$\overline{\Lambda}_6 = \left(\varepsilon_{2i}T^T_{1i}, 0, 0, 0, 0, 0, 0, 0, -\varepsilon_{2i}T^T_{2i}\right)^T,$$

$$\overline{\Lambda}_7 = \left(N_{1i}\overline{P}_i + N_{3i}Y_i, 0, 0, N_{2i}\overline{P}_i + N_{4i}Y_i, 0, 0, 0, 0, 0\right)^T,$$

$$\overline{\Lambda}_8 = \begin{pmatrix} \sqrt{\pi_{ik^i_1}}\overline{P}_i & \sqrt{\pi_{ik^i_2}}\overline{P}_i & \cdots & \sqrt{\pi_{ik^i_m}}\overline{P}_i \\ 0_{8n\times n} & 0_{8n\times n} & \cdots & 0_{8n\times n} \end{pmatrix},$$

$$\overline{\Lambda}_9 = \operatorname{diag}\left(-\overline{P}_{k^i_1}, -\overline{P}_{k^i_2}, \ldots, -\overline{P}_{k^i_m}\right), \qquad \overline{Z}_2 = Z_2^{-1}, \qquad Y_i = K_i\overline{P}_i, \qquad \overline{\varepsilon}_{1i} = \varepsilon_{1i}^{-1},$$

$$\overline{P}_i = P_i^{-1}, \qquad \overline{Q}_i = P_i^{-1}Q_iP_i^{-1}, \qquad \widetilde{Q}_1 = P_i^{-1}\overline{Q}_1P_i^{-1}, \qquad \widetilde{Q}_2 = P_i^{-1}\overline{Q}_2P_i^{-1},$$

$$\widetilde{Q}_3 = P_i^{-1}\overline{Q}_3P_i^{-1},$$

$$\Sigma = \operatorname{diag}\left(\overbrace{\overline{P}_i, \ldots, \overline{P}_i}^{7}\right), \qquad \overline{G}_i = \left(\overline{G}^T_{1i}, \ldots, \overline{G}^T_{8i}, 0\right)^T = \left(\overline{P}_i\left(G^T_{1i}, \ldots, G^T_{7i}\right)\Sigma, \overline{\varepsilon}_{1i}\overline{P}_iG^T_{8i}, 0\right)^T,$$

$$\overline{H}_i = \left(\overline{H}^T_{1i}, \ldots, \overline{H}^T_{8i}, 0\right)^T = \left(\overline{P}_i\left(H^T_{1i}, \ldots, H^T_{7i}\right)\Sigma, \overline{\varepsilon}_{1i}\overline{P}_iH^T_{8i}, 0\right)^T,$$

$$\overline{M}_i = \left(\overline{M}^T_{1i}, \ldots, \overline{M}^T_{8i}, 0\right)^T = \left(\overline{P}_i\left(M^T_{1i}, \ldots, M^T_{7i}\right)\Sigma, \overline{\varepsilon}_{1i}\overline{P}_iM^T_{8i}, 0\right)^T,$$

$$\overline{R}_i = \left(\overline{R}^T_{1i}, \ldots, \overline{R}^T_{8i}, 0\right)^T = \left(\overline{P}_i\left(R^T_{1i}, \ldots, R^T_{7i}\right)\Sigma, \overline{\varepsilon}_{1i}\overline{P}_iR^T_{8i}, 0\right)^T,$$

$$\overline{U}_i = \left(\overline{U}^T_{1i}, \ldots, \overline{U}^T_{8i}, 0\right)^T = \left(\overline{P}_i\left(\overline{U}^T_{1i}, \ldots, \overline{U}^T_{7i}\right)\Sigma, \overline{\varepsilon}_{1i}\overline{P}_i\overline{U}^T_{8i}, 0\right)^T,$$

$$\overline{V}_i = \left(\overline{V}^T_{1i}, \ldots, \overline{V}^T_{8i}, 0\right)^T = \left(\overline{P}_i\left(V^T_{1i}, \ldots, V^T_{7i}\right)\Sigma, \overline{\varepsilon}_{1i}\overline{P}_iV^T_{8i}, 0\right)^T, \tag{3.42}$$

when the LMIs are feasible, a desired state-feedback controller can be obtained in the form of (2.7) *with the controller gains given by* $K_i = Y_i P_i$ *for all* $i \in S$.

Proof. At first, we list the following fact:

$$J^T \overline{Z}_2 J - J^T \overline{P}_i - \overline{P}_i J + \overline{P}_i Z_2 \overline{P}_i = \left(\overline{Z}_2 J - \overline{P}_i\right)^T Z_2 \left(\overline{Z}_2 J - \overline{P}_i\right) \geq 0 \tag{3.43}$$

which implies that

$$-\overline{P}_i Z_2 \overline{P} \leq \Theta = J^T \overline{Z}_2 J - J^T \overline{P}_i - \overline{P}_i J. \tag{3.44}$$

Now perform a congruence transformation to (3.1) by

$$\operatorname{diag}\left(\overbrace{P_i^{-1}, \ldots, P_i^{-1}}^{7}, \varepsilon_{1i}^{-1}, I, Z_2^{-1}, \overbrace{P_i^{-1}, \ldots, P_i^{-1}}^{4}, I, I\right). \tag{3.45}$$

If $\pi_{ii} \in I_{\mathrm{kn}}^i$, then by the Schur complement and (3.44), we can infer that (3.34) is established. In the same way, if $\pi_{ii} \in I_{\mathrm{uk}}^i$, (3.39) is established.

From Lemma 2.8, we can see that (3.35) is equivalent to $A_i \overline{P}_i + \overline{P}_i A_i^T + \overline{P}_i P_j \overline{P}_i < 0$ for all $j \in I_{\mathrm{uk}}^i$, so (3.4) can be established. Furthermore, using the same method that proposed above, we can deduced that (3.32), (3.33), (3.36)–(3.38), and (3.40) are equivalent to (3.2) (3.3), (3.5)–(3.7), and (3.8), respectively. In conclusion, the gain matrix of desired controller in the form of (2.7) is given by $K_i = Y_i P_i$. This completes the proof. □

Remark 3.5. To reduce the conservatism, when estimating $Lv_5(x_t, i), -\int_{t-\overline{\tau}_2}^{t} \dot{x}^T(s) Z_2 \dot{x}(s) ds$ is not simply enlarged as $-\int_{t-\tau_i(t)}^{t} \dot{x}^T(s) Z_2 \dot{x}(s) ds$, but $-\int_{t-\tau_i(t)}^{t-(\tau_{1i}+\tau_i(t))/2} \dot{x}^T(s) Z_2 \dot{x}(s) ds$, $-\int_{t-(\tau_{2i}+\tau_i(t))/2}^{t-\tau_i(t)} \dot{x}^T(s) Z_2 \dot{x}(s) ds$ are considered as well, and different free-weighting matrices are introduced. This method above may lead to obtain improved feasible region for delay-dependent exponential passivity criteria.

Remark 3.6. In fact, Theorem 3.1 gives a exponential passivity criteria for MJS (2.8) with $\tau_{1i} \leq \tau_i(t) \leq \tau_{2i}$, $\dot{\tau}_i(t) \leq \mu_i$, where μ_i is a given constant. In many cases, μ_i is unknown. Considering this case, a rate-independent criteria for a delay satisfying $\tau_{1i} \leq \tau_i(t) \leq \tau_{2i}$ is derived as follows by setting $Q_i = Q^* = 0$, for all $i \in S$ in the proof of Theorem 3.1.

4. Examples

In this section, we will consider a interval time-varying delay MJS in the form of (2.8) with three modes, and the parameters of the system are given as follows:

$$
\begin{gathered}
A_1 = \begin{pmatrix} -0.05 & -0.05 \\ 0.5 & -0.5 \end{pmatrix}, \qquad A_2 = \begin{pmatrix} -0.05 & -0.09 \\ 1.5 & -0.1 \end{pmatrix}, \qquad A_3 = \begin{pmatrix} -0.03 & -0.015 \\ 0.05 & -0.01 \end{pmatrix}, \\
A_{d1} = \begin{pmatrix} 0.11 & 0.24 \\ -0.53 & -0.37 \end{pmatrix}, \qquad A_{d2} = \begin{pmatrix} -0.59 & 0.01 \\ -0.07 & -0.61 \end{pmatrix}, \qquad A_{d3} = \begin{pmatrix} 0.52 & 0.24 \\ 0.02 & -0.45 \end{pmatrix}, \\
D_{01} = \begin{pmatrix} 0 & 0 \\ -0.02 & 0 \end{pmatrix}, \qquad D_{02} = \begin{pmatrix} -0.02 & 0 \\ 0 & -0.02 \end{pmatrix}, \qquad D_{03} = \begin{pmatrix} 0 & -1.2 \\ 0 & 0 \end{pmatrix}, \\
B_{11} = \begin{pmatrix} 2.0 \\ 1.0 \end{pmatrix}, \qquad B_{12} = \begin{pmatrix} 1.0 \\ 0.5 \end{pmatrix}, \qquad B_{13} = \begin{pmatrix} 1.0 \\ 2.0 \end{pmatrix}, \qquad E_{11} = \begin{pmatrix} 0.5 \\ 1.0 \end{pmatrix}, \\
E_{12} = \begin{pmatrix} 0.8 \\ 2.0 \end{pmatrix}, \qquad E_{13} = \begin{pmatrix} 1.0 \\ 0.5 \end{pmatrix}, \qquad D_{11} = \begin{pmatrix} 1.0 \\ 0.2 \end{pmatrix}, \qquad D_{12} = \begin{pmatrix} 1.0 \\ 1.0 \end{pmatrix}, \\
D_{13} = \begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix}, \qquad C_1 = \begin{pmatrix} 1.0 & 0.2 \end{pmatrix}, \qquad C_2 = \begin{pmatrix} 0.5 & 1.0 \end{pmatrix}, \\
C_3 = \begin{pmatrix} 0.5 & 0.5 \end{pmatrix}, \qquad C_{d1} = \begin{pmatrix} -1.0 & 0.2 \end{pmatrix}, \qquad C_{d2} = \begin{pmatrix} 0.1 & -0.1 \end{pmatrix}, \\
C_{d3} = \begin{pmatrix} 0.5 & -0.5 \end{pmatrix}, \qquad B_{21} = 1.0, \qquad B_{22} = -0.5, \qquad B_{23} = 0.5, \\
D_{21} = 1.0, \qquad D_{22} = 0.5, \qquad D_{23} = -0.5, \qquad T_{11} = T_{12} = T_{13} = \begin{pmatrix} 0.02 \\ 0.01 \end{pmatrix}, \\
N_{11} = N_{12} = N_{13} = \begin{pmatrix} 0.02 & 0.01 \end{pmatrix}, \qquad N_{21} = N_{22} = N_{23} = \begin{pmatrix} 0.01 & 0.02 \end{pmatrix}, \\
N_{31} = N_{32} = N_{33} = N_{41} = N_{42} = N_{43} = 0.01, \qquad T_{21} = T_{22} = T_{23} = 0.1, \\
\mu_1 = 0.2, \qquad \mu_2 = 0.3, \qquad \mu_3 = 0.1, \qquad \Gamma_1 = \Gamma_2 = \Gamma_3 = \begin{pmatrix} 0.06 & 0 \\ 0 & 0.06 \end{pmatrix}, \\
\tau_{11} = 0.12, \qquad \tau_{12} = 0.11, \qquad \tau_{13} = 0.13, \qquad \tau_{21} = 0.23, \\
\tau_{22} = 0.28, \qquad \tau_{23} = 0.25,
\end{gathered}
\tag{4.1}
$$

The two cases of the transition rates matrices are described as follows:

$$
\begin{gathered}
\text{Case 1}: \quad \Pi = \begin{pmatrix} -0.5 & 0.2 & 0.3 \\ 0.2 & -0.6 & 0.4 \\ 0.5 & 0.3 & -0.8 \end{pmatrix}, \\
\text{Case 2}: \quad \Pi = \begin{pmatrix} -0.5 & ? & ? \\ 0.2 & -0.6 & 0.4 \\ 0.5 & ? & ? \end{pmatrix},
\end{gathered}
\tag{4.2}
$$

Table 1: Calculated the controller gains matrix for different cases.

	Case 1	Case 2
K_1	(−0.2256, 0.0218)	(−2.4527, 0.2749)
K_2	(−0.4330, −0.1248)	(−0.4842, 0.1157)
K_3	(−0.2800, −0.1641)	(−1.0109, −0.1008)

where ? means the unknown element. With the choice of $\varepsilon = 0.2$ and $J = \begin{pmatrix} 0.3 & 0 \\ 0 & 0.3 \end{pmatrix}$, we can obtain the feasibility solution of case 1 and case 2 as follows.
Case 1:

$$\begin{gathered}
\overline{P}_1 = \begin{pmatrix} 3.2525 & -0.1821 \\ -0.1821 & 4.2081 \end{pmatrix}, \qquad \overline{P}_2 = \begin{pmatrix} 2.6050 & -0.2795 \\ -0.2795 & 3.6119 \end{pmatrix}, \qquad \overline{P}_3 = \begin{pmatrix} 3.2386 & -0.2026 \\ -0.2026 & 3.8813 \end{pmatrix}, \\
\overline{Q}_1 = \begin{pmatrix} 1.8478 & -0.0727 \\ -0.0727 & 2.0824 \end{pmatrix}, \qquad \overline{Q}_2 = \begin{pmatrix} 1.5779 & -0.5056 \\ -0.5056 & 1.7197 \end{pmatrix}, \qquad \overline{Q}_3 = \begin{pmatrix} 1.4462 & 0.2109 \\ 0.2109 & 1.8895 \end{pmatrix}, \\
\tilde{Q}_1 = \begin{pmatrix} 0.5074 & -0.1035 \\ -0.1035 & 0.7639 \end{pmatrix}, \qquad \tilde{Q}_2 = \begin{pmatrix} 0.3321 & -0.0760 \\ -0.0760 & 0.5493 \end{pmatrix}, \qquad \overline{Z}_1 = \begin{pmatrix} 1.4315 & -0.2159 \\ -0.2159 & 1.8214 \end{pmatrix}, \\
\overline{Z}_2 = \begin{pmatrix} 3.0280 & -0.2085 \\ -0.2085 & 4.1893 \end{pmatrix}, \qquad Y_1 = \begin{pmatrix} -0.7376 & 0.1326 \end{pmatrix}, \qquad Y_2 = \begin{pmatrix} -1.0932 & -0.3296 \end{pmatrix}, \\
Y_3 = \begin{pmatrix} -0.8734 & -0.5801 \end{pmatrix}.
\end{gathered} \tag{4.3}$$

case 2:

$$\begin{gathered}
\overline{P}_1 = 1.0e+004 * \begin{pmatrix} 0.1304 & 0.3586 \\ 0.3586 & 2.6532 \end{pmatrix}, \qquad \overline{P}_2 = 1.0e+004 * \begin{pmatrix} 0.5768 & 0.4664 \\ 0.4664 & 3.9812 \end{pmatrix}, \\
\overline{P}_3 = 1.0e+004 * \begin{pmatrix} 0.4962 & 0.7608 \\ 0.7608 & 4.3848 \end{pmatrix}, \qquad \overline{Q}_1 = 1.0e+004 * \begin{pmatrix} 0.0864 & 0.2330 \\ 0.2330 & 1.7220 \end{pmatrix}, \\
\overline{Q}_2 = 1.0e+003 * \begin{pmatrix} 0.1918 & 0.3925 \\ 0.3925 & 3.7184 \end{pmatrix}, \qquad \overline{Q}_3 = 1.0e+004 * \begin{pmatrix} 0.0537 & 0.2133 \\ 0.2133 & 1.5653 \end{pmatrix}, \\
\tilde{Q}_1 = 1.0e+003 * \begin{pmatrix} 0.0658 & 0.3281 \\ 0.3281 & 2.4649 \end{pmatrix}, \qquad \tilde{Q}_2 = 1.0e+003 * \begin{pmatrix} 0.0429 & 0.2206 \\ 0.2206 & 1.6811 \end{pmatrix}, \\
\overline{Z}_1 = 1.0e+003 * \begin{pmatrix} 0.2312 & 0.9273 \\ 0.9273 & 7.3269 \end{pmatrix}, \qquad \overline{Z}_2 = 1.0e+004 * \begin{pmatrix} 0.6754 & 1.4246 \\ 1.4246 & 5.4089 \end{pmatrix}, \\
Y_1 = 1.0e+003 * \begin{pmatrix} -2.2126 & -1.5022 \end{pmatrix}, \qquad Y_2 = 1.0e+003 * \begin{pmatrix} -2.2530 & 2.3481 \end{pmatrix}, \\
Y_3 = 1.0e+004 * \begin{pmatrix} -0.5783 & -1.2112 \end{pmatrix}.
\end{gathered} \tag{4.4}$$

Under the two cases above, Table 1 lists the state-feedback controller gains matrix K_i, which can be determined by the method of Theorem 3.4. If the ρ is sufficiently small, we can check that the MJS (2.8) is exponentially passive under the condition of Theorem 3.4. Given

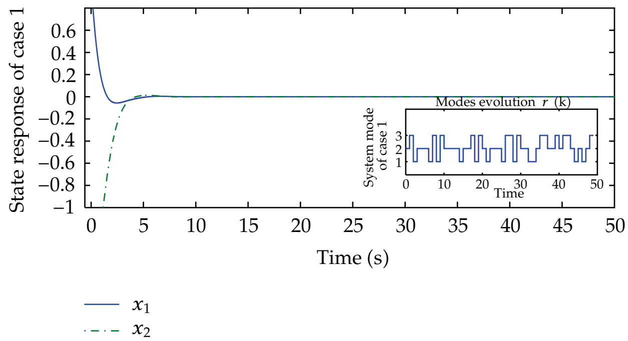

Figure 1: State response of case 1 and the switch signal.

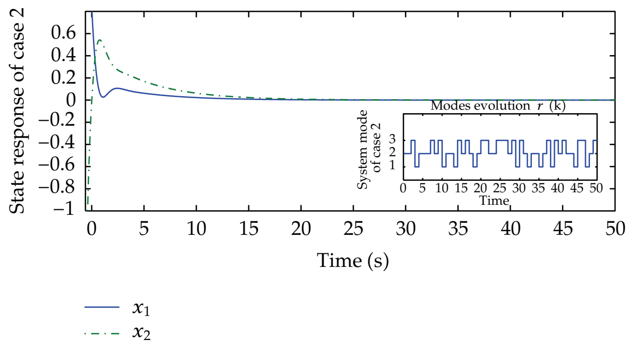

Figure 2: State response of case 2 and the switch signal.

the initial condition as $x(t) = (2.0 \quad -2.0)^T$ and $r(t) = 2$, from Figures 1 and 2, we can easily see that the closed-loop system in (2.8) is mean square exponential stable with $\omega(t) = 0$.

Remark 4.1. In order to illustrate the effectiveness of the proposed approach, a numerical example is given which included two cases, that is, case 1, the transition rate matrix is completely known; case 2, some elements in the transition rate matrix are inaccessible. By using Matlab Toolbox, we can obtain the gain matrix K_i, which guarantees that the Markovian jump systems (2.8) is robust exponential passivity. If we choose the switch signal as Figures 1 and 2, we can know that the closed-loop system (2.8) is exponentially stable in the mean square under the state-feedback controllers obtained above, which have been listed in Table 1.

5. Conclusions

In this paper, the problems of exponential passification of uncertain MJS have been investigated. To reflect more realistic dynamical behaviors of the system, both the partially known transition rates, state and input delays have been considered. With utilizing the Lyapunov functional method and free-weighting matrix method, delay-dependent exponential passivity conditions are established. Finally, an illustrative example has been given to demonstrate the effectiveness of the proposed approach.

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