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Research Article

Exact Solutions to the Sharma-Tasso-Olver Equation by Using Improved G'/G-Expansion Method

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This paper is concerned with a double nonlinear dispersive equation: the Sharma-Tasso-Olver equation. We propose an improved G'/G-expansion method which is employed to investigate the solitary and periodic traveling waves of this equation. As a result, some new traveling wave solutions involving hyperbolic functions, the trigonometric functions, are obtained. When the parameters are taken as special values, the solitary wave solutions are derived from the hyperbolic function solutions, and the periodic wave solutions are derived from the trigonometric function solutions. The improved G'/G-expansion method is straightforward, concise and effective and can be applied to other nonlinear evolution equations in mathematical physics.

1. Introduction

In this paper, we consider the following double nonlinear dispersive, integrable equation:

$$u_t + \alpha(u^3)_x + \frac{3}{2}\alpha\left(u^2\right)_{xx} + \alpha u_{xxx} = 0, \quad (1)$$

where α is a real parameter and $u(x,t)$ is the unknown function depending on the temporal variable t and the spatial variable x. This equation contains both linear dispersive term αu_{xxx} and the double nonlinear terms $\alpha(u^2)_{xx}$ and $\alpha(u^3)_x$. Equation (1) be called Sharma-Tasso-Olver equation in the literature. Many physicists and mathematicians have paied their attentions to the Sharma-Tasso-Olver equation in recent years due to its appearance in scientific applications. In [1], the tanh method, the extended tanh method, and other ansatz involving hyperbolic and exponential functions are efficiently used for the analytic study of this equation. The multiple solitons and kinks solutions are obtained. In [2], Yan investigated the Sharma-Tasso-Olver equation (1) by using the Cole-Hopf transformation method. The simple symmetry reduction procedure is repeatedly used in [3] to obtain exact solutions where soliton fission and fusion were examined. Wang et al. examined the soliton fission and fusion thoroughly by means of the Hirotas bilinear method and the Bäcklund transformation method in [4]. The generalized Kaup-Newell-type hierarchy of nonlinear evolution equations is explicitly related to Sharma-Tasso-Olver equation from [5]. Using the improved tanh function method in [6], the Sharma-Tasso-Olver equation with its fission and fusion has some exact solutions. In [7], some exact solution of the Sharma-Tasso-Olver equation is given by implying a generalized tanh function method for approximating some solutions which have been known.

In recent years, with the development of symbolic computation packages like Maple and Mathematica, which enable us to perform the tedious and complex computation on computer, much work has been focused on the direct methods to construct exact solutions of nonlinear evolution equations. The G'/G-expansion method proposed by Wang et al. [8] is one of the most effective direct methods to obtain travelling wave solutions of a large number of nonlinear evolution equations, such as the KdV equation, the mKdV equation, the variant Boussinesq equations, and the Hirota-Satsuma equations. Later, the further developed methods, named, the generalized G'/G-expansion method, the modified G'/G-expansion method, and the extended G'/G-expansion method have been proposed in [9–11], respectively. The aim of this paper is to derive more exact solitary wave solutions and periodic wave solutions of the

Sharma-Tasso-Olver equation. We will employ the improved G'/G method to solve Sharma-Tasso-Olver equation. Some entirely new exact solitary wave solutions and periodic wave solutions of the Sharma-Tasso-Olver equation are obtained.

The rest of the paper is organized as follows. In Section 2, we describe the method in brief. In Sections 3, we study the Sharma-Tasso-Olver equation by the improved G'/G-expansion method. Finally, we give the conclusion.

2. The Improved Method

The main steps of improved G'/G-expansion method [12, 13] are introduced as follows.

Step 1. Consider a general nonlinear PDE in the form

$$F\left(u, u_x, u_t, u_{xx}, u_{xt}, \ldots\right) = 0. \tag{2}$$

Using a wave variable

$$u\left(x, t\right) = U\left(\xi\right), \quad \xi = x - ct, \tag{3}$$

we can rewrite (2) as the following nonlinear ODE:

$$F\left(U, U', U'', \ldots\right) = 0, \tag{4}$$

where the prime denotes differentiation with respect to ξ.

Step 2. Suppose that the solution of ODE (4) can be written as follows:

$$U\left(\xi\right) = \sum_{i=-n}^{n} a_i \left(\frac{G'}{G + \sigma G'}\right)^i, \tag{5}$$

where σ, a_i $(i = -n, -n+1, \ldots)$ are constants to be determined later, n is a positive integer, and $G = G(\xi)$ satisfies the following second-order linear ordinary differential equation:

$$G'' + \lambda G' + \mu G = 0, \tag{6}$$

where λ, μ is a real constant. The general solutions of (6) can be listed as follows. When $\triangle = \lambda^2 - 4\mu > 0$, we obtain the hyperbolic function solution of (6)

$$G\left(\xi\right) = e^{-(\lambda/2)\xi}\left(A_1 \cosh\left(\frac{\sqrt{\triangle}}{2}\xi\right) + A_2 \sinh\left(\frac{\sqrt{\triangle}}{2}\xi\right)\right), \tag{7}$$

where A_1 and A_2 are arbitrary constants. When $\triangle = \lambda^2 - 4\mu < 0$, we obtain the following trigonometric function solution of (6):

$$G\left(\xi\right) = e^{-(\lambda/2)\xi}\left(A_1 \cos\left(\frac{\sqrt{-\triangle}}{2}\xi\right) + A_2 \sin\left(\frac{\sqrt{-\triangle}}{2}\xi\right)\right), \tag{8}$$

where A_1 and A_2 are arbitrary constants. When $\triangle = \lambda^2 - 4\mu = 0$, we obtain the solution of (6) as

$$G\left(\xi\right) = e^{-(\lambda/2)\xi}\left(A_1 + A_2\xi\right), \tag{9}$$

where A_1 and A_2 are arbitrary constants.

Step 3. Determine the positive integer n by balancing the highest order derivatives and nonlinear terms in (4).

Step 4. Substituting (5) along with (6) into (4) and then setting all the coefficients of $(G'/G)^k$ $(k = 1, 2, \ldots)$ of the resulting system's numerator to zero yield a set of overdetermined nonlinear algebraic equations for c, σ, and a_i $(i = -n, -n+1, \ldots)$.

Step 5. Assuming that the constants c, σ, and a_i $(i = -n, -n+1, \ldots)$ can be obtained by solving the algebraic equations in Step 4, then substituting these constants and the known general solutions of (6) into (5), we can obtain the explicit solutions of (2) immediately.

3. The Exact Solutions of the Sharma-Tasso-Olver Equation

In this section, we will construct travelling wave solutions of the Sharma-Tasso-Olver equation (1) by using the method described in Section 2.

Let $u(x, t) = \varphi(x - ct) = \varphi(\xi)$, where c is the wave speed. Substituting the above travelling wave variable $\xi = x - ct$ into Sharma-Tasso-Olver equation (1) yields

$$-c\varphi' + \alpha\left(\varphi^3\right)' + \frac{3}{2}\alpha\left(\varphi^2\right)'' + \alpha\varphi''' = 0. \tag{10}$$

By integrating (10) with respect to the variable ξ and assuming a zero constant of integration, we obtain the following nonlinear ordinary differential equation for the function φ:

$$-c\varphi + \alpha\varphi^3 + 3\alpha\varphi\varphi' + \alpha\varphi'' = 0. \tag{11}$$

Balancing φ'' with φ^3 in (11), we find $n + 2 = 3n$ so that $n = 1$, and we suppose that (11) owns the solutions in the form

$$\varphi\left(\xi\right) = a_0 + a1\frac{G'}{G + \sigma G'} + b1\left(\frac{G'}{G + \sigma G'}\right)^{-1}. \tag{12}$$

Substituting (12) along with (6) into (11) and then setting all the coefficients of $(G'/G)^k$ $(k = 0, 1, \ldots)$ of the resulting system's numerator to zero yield a set of overdetermined nonlinear algebraic equations about $a0, a1, b1, c$. Solving the over-determined algebraic equations, we can obtain the following results.

Case 1.

$$\begin{aligned} a0 &= \mu\sigma - \frac{\lambda}{2}, \qquad a1 = \sigma\lambda - \sigma^2\mu - 1, \\ b1 &= 0, \qquad c = -\frac{1}{4}\left(4\mu - \lambda^2\right)\alpha, \end{aligned} \tag{13}$$

where σ are arbitrary constants.

Case 2.

$$\begin{aligned} a0 &= -\lambda + 2\mu\sigma, \qquad a1 = 2\sigma\lambda - 2\sigma^2\mu - 2, \\ b1 &= 0, \qquad c = -\left(4\mu - \lambda^2\right)\alpha, \end{aligned} \tag{14}$$

where σ are arbitrary constants.

Case 3.

$$a0 = \mu\sigma - \frac{\lambda}{2} \pm \frac{\lambda^2 - 4\mu}{2}, \qquad a1 = \sigma\lambda - \sigma^2\mu - 1,$$
$$b1 = 0, \qquad c = -\left(4\mu - \lambda^2\right)\alpha, \tag{15}$$

where σ are arbitrary constants.

Case 4.

$$a0 = -2\mu\sigma + \lambda, \qquad a1 = 0,$$
$$b1 = 2\mu, \qquad c = \alpha\lambda^2 - 4\alpha\mu, \tag{16}$$

where σ are arbitrary constants.

Case 5.

$$a0 = 0, \qquad a1 = \sigma\lambda - \sigma^2\mu - 1,$$
$$b1 = \mu, \qquad c = \alpha\lambda^2 - 4\alpha\mu, \tag{17}$$

where σ are arbitrary constants.

Case 6.

$$a0 = -\mu\sigma + \frac{\lambda}{2}, \qquad a1 = 0,$$
$$b1 = \mu, \qquad c = \frac{1}{4}\alpha\lambda^2 - \alpha\mu, \tag{18}$$

where σ are arbitrary constants.

Case 7.

$$a0 = \mu\sigma - \frac{\lambda}{2} \pm \frac{\lambda^2 - 4\mu}{2}, \qquad a1 = 0,$$
$$b1 = \mu, \qquad c = -\left(4\mu - \lambda^2\right)\alpha, \tag{19}$$

where σ are arbitrary constants.

Using Case 5, (12), and the general solutions of (6), we can find the following travelling wave solutions of Sharma-Tasso-Olver equation (1).

When $\triangle = \lambda^2 - 4\mu > 0$, we obtain the hyperbolic function solutions of (1) as follows:

$$\begin{aligned}
u(x,t) &= \varphi(\xi) \\
&= a1 \frac{G'}{G + \sigma G'} + b1 \left(\frac{G'}{G + \sigma G'}\right)^{-1} \\
&= \left(\sigma\lambda - \sigma^2\mu - 1\right) \\
&\quad \times \left(\left(\left(-\lambda A_1 + A_2\sqrt{\triangle}\right) \cosh\left(\frac{\sqrt{\triangle}}{2}\xi\right) \right.\right. \\
&\quad \left. + \left(-\lambda A_2 + A_1\sqrt{\triangle}\right) \sinh\left(\frac{\sqrt{\triangle}}{2}\xi\right)\right) \\
&\quad \times \left(\left(2A_1 - \sigma\lambda A_1 + \sigma A_2\sqrt{\triangle}\right) \cosh\left(\frac{\sqrt{\triangle}}{2}\xi\right) \right. \\
&\quad + \left(2A_2 - \sigma\lambda A_2 + \sigma A_1\sqrt{\triangle}\right) \\
&\quad \left.\left. \times \sinh\left(\frac{\sqrt{\triangle}}{2}\xi\right)\right)^{-1} \right) \\
&\quad + \mu \left(\left(\left(2A_1 - \sigma\lambda A_1 + \sigma A_2\sqrt{\triangle}\right) \cosh\left(\frac{\sqrt{\triangle}}{2}\xi\right) \right.\right. \\
&\quad + \left(2A_2 - \sigma\lambda A_2 + \sigma A_1\sqrt{\triangle}\right) \\
&\quad \left. \times \sinh\left(\frac{\sqrt{\triangle}}{2}\xi\right)\right) \\
&\quad \times \left(\left(-\lambda A_1 + A_2\sqrt{\triangle}\right) \cosh\left(\frac{\sqrt{\triangle}}{2}\xi\right) \right. \\
&\quad + \left(-\lambda A_2 + A_1\sqrt{\triangle}\right) \\
&\quad \left.\left. \times \sinh\left(\frac{\sqrt{\triangle}}{2}\xi\right)\right)^{-1} \right),
\end{aligned} \tag{20}$$

where $\xi = x - ct$, $c = \alpha\lambda^2 - 4\alpha\mu$, A_1, A_2, σ are arbitrary constants.

It is easy to see that the hyperbolic function solution can be rewritten at $A_1^2 < A_2^2$ and $A_1^2 > A_2^2$ as follows:

$$\begin{aligned}
u(x,t) &= \varphi(\xi) \\
&= \left(\sigma\lambda - \sigma^2\mu - 1\right) \\
&\quad \times \frac{-\lambda + \sqrt{\lambda^2 - 4\mu} \tanh\left(\left(\sqrt{\lambda^2 - 4\mu}/2\right)\xi + \xi_0\right)}{2 - \sigma\lambda + \sigma\sqrt{\lambda^2 - 4\mu} \tanh\left(\left(\sqrt{\lambda^2 - 4\mu}/2\right)\xi + \xi_0\right)} \\
&\quad + \mu \frac{2 - \sigma\lambda + \sigma\sqrt{\lambda^2 - 4\mu} \tanh\left(\left(\sqrt{\lambda^2 - 4\mu}/2\right)\xi + \xi_0\right)}{-\lambda + \sqrt{\lambda^2 - 4\mu} \tanh\left(\left(\sqrt{\lambda^2 - 4\mu}/2\right)\xi + \xi_0\right)},
\end{aligned} \tag{21a}$$

$$\begin{aligned}
u(x,t) &= \varphi(\xi) \\
&= \left(\sigma\lambda - \sigma^2\mu - 1\right)
\end{aligned}$$

$$\times \frac{-\lambda + \sqrt{\lambda^2 - 4\mu} \coth\left(\left(\sqrt{\lambda^2 - 4\mu}/2\right)\xi + \xi_0\right)}{2-\sigma\lambda+\sigma\sqrt{\lambda^2-4\mu}\coth\left(\left(\sqrt{\lambda^2-4\mu}/2\right)\xi+\xi_0\right)} + \mu\frac{2-\sigma\lambda+\sigma\sqrt{\lambda^2-4\mu}\coth\left(\left(\sqrt{\lambda^2-4\mu}/2\right)\xi+\xi_0\right)}{-\lambda + \sqrt{\lambda^2 - 4\mu}\coth\left(\left(\sqrt{\lambda^2 - 4\mu}/2\right)\xi + \xi_0\right)}, \tag{21b}$$

where $\xi = x - ct$, $c = -4\alpha\mu$, and $\xi_0 = \tanh^{-1}(A_2/A_1)$.

Specially, if $\sigma = 0$, (21a) and (21b) become

$$u(x,t) = \varphi(\xi) = \frac{\lambda}{2} - \frac{\sqrt{\lambda^2 - 4\mu}}{2}\tanh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}\xi + \xi_0\right) + \frac{2\mu}{-\lambda + \sqrt{\lambda^2 - 4\mu}\tanh\left(\left(\sqrt{\lambda^2 - 4\mu}/2\right)\xi + \xi_0\right)}, \tag{22a}$$

$$u(x,t) = \varphi(\xi) = \frac{\lambda}{2} - \frac{\sqrt{\lambda^2 - 4\mu}}{2}\coth\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}\xi + \xi_0\right) + \frac{2\mu}{-\lambda + \sqrt{\lambda^2 - 4\mu}\coth\left(\left(\sqrt{\lambda^2 - 4\mu}/2\right)\xi + \xi_0\right)}, \tag{22b}$$

where $\xi = x - ct, c = \alpha\lambda^2 - 4\alpha\mu$, and $\xi_0 = \tanh^{-1}(A_2/A_1)$.

Taking $\lambda = 0$ in (22a) and (22b),we have

$$u(x,t) = \varphi(\xi) = \pm\left(\mu \coth\left(\sqrt{\mu}\xi + \xi_0\right) - \mu\tanh\left(\sqrt{\mu}\xi + \xi_0\right)\right), \tag{23}$$

where $\xi = x-ct$, $c = -4\alpha\mu$, and $\xi_0 = \tanh^{-1}(A_2/A_1)$, $\mu > 0$.

It is easy to see that if $A_1, A_2, \sigma, \lambda, \mu$ are taken as other special values in a proper way, more solitary wave solutions of (1) can be obtained. Here we omit them for simplicity.

When $\triangle = \lambda^2 - 4\mu < 0$, we get the trigonometric function solutions of (1) as follows:

$$\begin{aligned} u(x,t) &= \varphi(\xi) \\ &= a1\frac{G'}{G + \sigma G'} + b1\left(\frac{G'}{G + \sigma G'}\right)^{-1} \\ &= \left(\sigma\lambda - \sigma^2\mu - 1\right) \\ &\quad \times \left(\left(\left(-\lambda A_1 + A_2\sqrt{-\triangle}\right)\cos\left(\frac{\sqrt{-\triangle}}{2}\xi\right) + \left(-\lambda A_2 - A_1\sqrt{-\triangle}\right)\sin\left(\frac{\sqrt{-\triangle}}{2}\xi\right)\right)\right. \\ &\quad \times \left(\left(2A_1 - \sigma\lambda A_1 - \sigma A_2\sqrt{-\triangle}\right)\cos\left(\frac{\sqrt{-\triangle}}{2}\xi\right) + \left(2A_2 - \sigma\ \lambda A_2 + \sigma A_1\sqrt{-\triangle}\right)\sin\left(\frac{\sqrt{-\triangle}}{2}\xi\right)\right)^{-1} \\ &\quad + \mu\left(\left(\left(2A_1 - \sigma\lambda A_1 - \sigma A_2\sqrt{-\triangle}\right)\cos\left(\frac{\sqrt{-\triangle}}{2}\xi\right) + \left(2A_2 - \sigma\lambda A_2 + \sigma A_1\sqrt{-\triangle}\right)\sin\left(\frac{\sqrt{-\triangle}}{2}\xi\right)\right)\right. \\ &\quad \left.\times\left(\left(-\lambda A_1 + A_2\sqrt{-\triangle}\right)\cos\left(\frac{\sqrt{-\triangle}}{2}\xi\right) + \left(-\lambda A_2 - A_1\sqrt{-\triangle}\right)\sin\left(\frac{\sqrt{-\triangle}}{2}\xi\right)\right)^{-1}\right), \end{aligned} \tag{24}$$

where $\xi = x - ct$, $c = \alpha\lambda^2 - 4\alpha\mu$, A_1, A_2, σ are arbitrary constants.

It is easy to see that the trigonometric solution can be rewritten at $A_1^2 < A_2^2$ and $A_1^2 > A_2^2$ as follows:

$$\begin{aligned} u(x,t) &= \varphi(\xi) \\ &= \left(\sigma\lambda - \sigma^2\mu - 1\right) \\ &\quad \times \frac{-\lambda+\sqrt{-\lambda^2+4\mu}\tan\left(\left(\sqrt{-\lambda^2+4\mu}/2\right)\xi+\xi_0\right)}{2-\sigma\lambda+\sigma\sqrt{-\lambda^2+4\mu}\tan\left(\left(\sqrt{-\lambda^2+4\mu}/2\right)\xi+\xi_0\right)} \\ &\quad + \mu\frac{2-\sigma\lambda+\sigma\sqrt{-\lambda^2+4\mu}\tan\left(\left(\sqrt{-\lambda^2+4\mu}/2\right)\xi+\xi_0\right)}{-\lambda+\sqrt{-\lambda^2+4\mu}\tan\left(\left(\sqrt{-\lambda^2+4\mu}/2\right)\xi+\xi_0\right)}, \end{aligned} \tag{25a}$$

$$\begin{aligned}u(x,t) &= \varphi(\xi) \\ &= \left(\sigma\lambda - \sigma^2\mu - 1\right) \\ &\quad\times \frac{-\lambda+\sqrt{-\lambda^2+4\mu}\cot\left(\left(\sqrt{-\lambda^2+4\mu}/2\right)\xi+\xi_0\right)}{2-\sigma\lambda+\sigma\sqrt{-\lambda^2+4\mu}\cot\left(\left(\sqrt{-\lambda^2+4\mu}/2\right)\xi+\xi_0\right)} \\ &\quad+\mu\frac{2-\sigma\lambda+\sigma\sqrt{-\lambda^2+4\mu}\cot\left(\left(\sqrt{-\lambda^2+4\mu}/2\right)\xi+\xi_0\right)}{-\lambda+\sqrt{-\lambda^2+4\mu}\cot\left(\left(\sqrt{-\lambda^2+4\mu}/2\right)\xi+\xi_0\right)},\end{aligned} \tag{25b}$$

where $\xi = x - ct$, $c = -4\alpha\mu$, and $\xi_0 = \tan^{-1}(A_2/A_1)$.

Specially, if $\sigma = 0$, (25a) and (25b) become

$$\begin{aligned}u(x,t) &= \varphi(\xi) \\ &= \frac{\lambda}{2} - \frac{\sqrt{-\lambda^2+4\mu}}{2}\tan\left(\frac{\sqrt{-\lambda^2+4\mu}}{2}\xi+\xi_0\right) \\ &\quad+\frac{2\mu}{-\lambda+\sqrt{-\lambda^2+4\mu}\tan\left(\left(\sqrt{-\lambda^2+4\mu}/2\right)\xi+\xi_0\right)},\end{aligned} \tag{26a}$$

$$\begin{aligned}u(x,t) &= \varphi(\xi) \\ &= \frac{\lambda}{2} - \frac{\sqrt{-\lambda^2+4\mu}}{2}\cot\left(\frac{\sqrt{-\lambda^2+4\mu}}{2}\xi+\xi_0\right) \\ &\quad+\frac{2\mu}{-\lambda+\sqrt{-\lambda^2+4\mu}\cot\left(\left(\sqrt{-\lambda^2+4\mu}/2\right)\xi+\xi_0\right)},\end{aligned} \tag{26b}$$

where $\xi = x - ct$, $c = \alpha\lambda^2 - 4\alpha\mu$, and $\xi_0 = \tan^{-1}(A_2/A_1)$.

Taking $\lambda = 0$ in (26a) and (26b), we have

$$\begin{aligned}u(x,t) &= \varphi(\xi) \\ &= \pm\left(\mu\cot\left(\sqrt{\mu}\xi+\xi_0\right) - \mu\tan\left(\sqrt{\mu}\xi+\xi_0\right)\right),\end{aligned} \tag{27}$$

where $\xi = x - ct$, $c = -4\alpha\mu$, and $\xi_0 = \tan^{-1}(A_2/A_1)$, $\mu > 0$.

Using other 6 cases, (12) and the general solutions of (6), we could obtain abundant exact solutions of (1), and here we do not list all of them.

4. Conclusions

In this paper, the Sharma-Tasso-Olver equation is studied by using improved G'/G-expansion method. And we got many general solutions expressed by hyperbolic functions, the trigonometric functions, the validity of which is verified. The ansatz (2) is more general than the ansatz in G'/G-expansion method [8] and modified G'/G-expansion method [10]. If we set the parameters in (5) and (6) to special values, the above two methods can be recovered by this method. Therefore, the improved method is more powerful than the G'/G-expansion method and modified G'/G-expansion method, and some new types of travelling wave solutions and solitary wave solutions would be expected for some nonlinear evolution equations.

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