Hindawi Publishing Corporation
Journal of Inequalities and Applications
Volume 2010, Article ID 643768, 6 pages
doi:10.1155/2010/643768

Research Article

General Convexity of Some Functionals in Seminormed Spaces and Seminormed Algebras

Todor Stoyanov

Department of Mathematics, Varna University of Economics, Boulevard Knyaz Boris I 77, Varna 9002, Bulgaria

Correspondence should be addressed to Todor Stoyanov, todstoyanov@yahoo.com

Received 30 July 2010; Accepted 27 October 2010

Academic Editor: S. S. Dragomir

We prove some results for convex combination of nonnegative functionals, and some corollaries are established.

1. Introduction

Inequalities have been used in almost all the branches of mathematics. It is an important tool in the study of convex functions in seminormed space and seminormed algebras. Recently some works have been done by Altin et al. [1, 2], Tripathy et al. [1–6], Tripathy and Sarma [3, 4], Chandra and Tripathy [5], Tripathy and Mahanta [6], and many others involving inequalities in seminormed spaces and convex functions like the Orlicz function.

In this paper, inequalities for convex combinations of functionals satisfying conditions (a) and (b) are formulated in the theorems, and some corollaries are proved, using the theorems. Condition (a) relates to nonnegative functionals over which the inequalities in Theorems 1.1 and 1.4 on seminorm are proved. In Theorem 1.1, we consider seminormed spaces, and in Theorem 1.4 seminormed algebras. Condition (b) relates generally to the representations between seminormed spaces and seminormed algebras. The inequalities formulated in this way are proved in Corollaries 1.2 and 1.5. In this paper we consider the following generalization of the convexity in seminormed algebras. $A : \gamma f(\sum_{i=1}^{m} p_i x_i) \leq (\sum_{i=1}^{m} \|p_i\| f(x_i))$, where $\sum_{i=1}^{m} \|p_i\| = 1$, $p_i, x_i \in$ 1 for $i = 1, 2, \ldots, m$, $\| \cdot \|$ is the norm in A, and γ is a real number.

In order to justify our study, we have provided an example related to real functions of one variable, similar examples can be constructed. This has been used in the geometry of Banach spaces as found in [7, 8]. Similar statements related to functionals in finite-dimensional spaces and countable dimensional spaces have been provided in [9]. These results can be applied in the mentioned areas.

Theorem 1.1. *Let $\mathbb{X}$ be a seminormed space over $\mathbb{R}$ and the nonnegative functional f satisfy the following condition:*

(a) *$g(t) \cdot f(y) \leq f(x) \leq r(t) \cdot f(y)$, for all x,y with $\|x\|/\|y\| = t \in [0,1]$, where $g, r : [0,1] \to [0,1]$ are nondecreasing functions such that $g(t) \leq r(t)$. Then,*

(1) *there exists $\inf_{\alpha,t\in[0,1]}\delta(\alpha,t) = \gamma$, where*

$$\delta(\alpha,t) = \alpha \cdot g\left(\frac{t}{(\alpha \cdot t+\beta)}\right) + \frac{\beta}{r(\alpha \cdot t+\beta)}, \quad \text{for } \alpha \in [0,1] \text{ with } \alpha+\beta = 1, \tag{1.1}$$

(2) *the functions $g(t) : [0,1] \to [0,1]$ and $r^{-1}(r^{-1}) : [0,\sim] \to [0,\sim]$ are convex. Then, if $\alpha_i \geq 0$, $\sum_{i=1}^{m} \alpha_i = 1$, $x_i \in \mathbb{X}$, $(i = \overline{1,n})$ for $i = 1,2,\ldots,n$, the inequality $\gamma \cdot f(\sum_{i=1}^{m} \alpha_i \cdot x_i) \leq \sum_{i=1}^{m} \alpha_i \cdot f(x_i)$ is satisfied.*

Proof. Let $x, y \in \mathbb{X}$, as $\|x\| \leq \|y\|$. We put $\Delta = (\alpha \cdot f(x) + \beta \cdot f(y))/f(z)$, where $z = \alpha \cdot x + \beta \cdot y, \alpha \in [0,1], \alpha+\beta = 1$.

(a) Let $\|x\| \leq \|z\| \leq \|y\|$. According to condition (a), we obtain

$$\Delta \geq \alpha \cdot g\left(\frac{\|x\|}{\|z\|}\right) + \frac{\beta}{r(\|z\|/\|y\|)}. \tag{1.2}$$

Knowing that g and r are nondecreasing, we obtain

$$\begin{aligned} \Delta &\geq \alpha \cdot g\left(\frac{\|x\|}{(\alpha \cdot \|x\| + \beta \cdot \|y\|)}\right) + \beta \cdot r^{-1} \cdot \left(\frac{(\alpha \cdot \|x\| + \beta \cdot \|y\|)}{\|y\|}\right) \\ &= \alpha \cdot g\left(\frac{t}{(\alpha \cdot t + \beta)}\right) + \beta \cdot r^{-1} \cdot (\alpha \cdot t + \beta) = \delta(\alpha, \mathrm{t}), \end{aligned} \tag{1.3}$$

where $t = \|x\|/\|y\|$.

There exists $\inf_{\alpha,t\in[0,1]}\delta(\alpha,t) = \gamma$ in compliance with (1). Therefore $\Delta \geq \gamma$.

If we put $x = y$ the result is $1 = \Delta \geq \gamma$, that is, $1 \geq \gamma$.

(b) Let $\|z\| \leq \|x\|$. Then, in view of (a), we have

$$\Delta \geq \alpha \cdot r^{-1}\left(\frac{\|z\|}{\|x\|}\right) + \beta \cdot r^{-1}\left(\frac{\|z\|}{\|x\|}\right) \geq \alpha+\beta = 1 \geq \gamma. \tag{1.4}$$

Let us consider n elements $x_i \in \mathbb{X}$, $(i = \overline{1,n})$, and we suppose $\|x_1\| \leq \|x_2\| \leq \cdots \leq \|x_n\|$.

Let $\Delta = (\sum_{i=1}^{m} \alpha_i \cdot f(x_i))/f(z)$, where $z = \sum_{i=1}^{m} \alpha_i \cdot x_i$, and $t_i = \|x_i\|/\|z\|$, $\|x_{k-1}\| \leq \|z\| < \|x_k\|$, as $1 \leq k \leq n$.

According to condition (a), we get

$$\Delta \geq \sum_{i=1}^{m} \alpha_i \cdot g(t_i) + \sum_{i=1}^{m} \alpha_i \cdot r^{-1}\left(t_i^{-1}\right) = \rho_n(\overline{\alpha}, \overline{x}), \tag{1.5}$$

where $\overline{\alpha} = (\alpha_1, \alpha_2, \ldots, \alpha_n)$, $\overline{x} = (x_1, x_2, \ldots, x_n)$.

Using the principle of induction over n, we will probe that $\inf_{n,\overline{\alpha,x}} \rho_n(\overline{\alpha,x}) \geq \gamma$.

We know that $\rho_2(\overline{\alpha,x}) = \delta(\alpha, t)$, and therefore about $n = 2$ the statement is proved. We assume the assertion about $(n-1)$ is correct.

(1) Let $k \leq 2$. Then, $\rho_n(\overline{\alpha}, \overline{x}) = S + (\alpha_{n-1} + \alpha_n) \cdot (\alpha \cdot r^{-1}(t_{n-1}^{-1}) + \beta \cdot r^{-1}(t_n^{-1}))$, where S is the rest of the sum, and $\alpha = \alpha_{n-1}/(\alpha_{n-1} + \alpha_n)$, $\beta = \alpha_n/(\alpha_{n-1} + \alpha_n)$. With condition (2) we have $\rho_n(\overline{\alpha}, \overline{x}) \geq S + (\alpha_{n-1} + \alpha_n) \cdot r^{-1}((\alpha t_{n-1}) + \beta t_n)^{-1}$, but $\|\alpha \cdot x_{n-1} + \beta \cdot x_n\| \leq \alpha \cdot \|x_{n-1}\| + \beta \cdot \|x_n\|$. Setting $x_{n-1} = \alpha \cdot x_{n-1} + \beta \cdot x_n$, $t'_{n-1} = \|x_{n-1}\|/\|z\|$ and knowing r is nondecreasing function, we obtain

$$\rho_n(\overline{\alpha}, \overline{x}) \geq S + (\alpha_{n-1} + \alpha_n) \cdot r^{-1}\left(\left(t'_n\right)^{-1}\right) = \rho_{n-1}(\overline{\alpha}', \overline{x}'), \tag{1.6}$$

where $\overline{\alpha}' = (\alpha_1, \alpha_2, \ldots, \alpha_{n-2}, \alpha_{n-1} + \alpha_n)$ and $\overline{x}' = (x_1, x_2, \ldots, x_{n-2}, x_{n-1})$. With the inductive assumption, $\rho_{n-1}(\overline{\alpha}', \overline{x}') \geq \gamma$, that is, $\rho_n(\overline{\alpha}, \overline{x}) \geq \gamma$, that is, $\Delta \geq \gamma$.

(2) Let $k \geq 3$. Then $\rho_n(\overline{\alpha}, \overline{x}) = (\alpha_1 + \alpha_2) \cdot (\alpha \cdot g(t_1) + \beta \cdot g(t_2)) + S$, where S is the rest of the sum, and $\alpha = \alpha_1/(\alpha_1 + \alpha_2)$, $\beta = \alpha_2/(\alpha_1 + \alpha_2)$. According to condition (2), we obtain $\rho_n(\overline{\alpha}, \overline{x}) \geq (\alpha_1 + \alpha_2) \cdot (\alpha \cdot g(t_1) + \beta \cdot g(t_2)) + S$. Let us place $t'_1 = \|x'_1\|/\|z\|$, where $x'_1 = \alpha x_1 + \beta x_2$, but $\|x'_1\| = \|\alpha \cdot x_1 + \beta \cdot x_2\| \leq \alpha \cdot \|x_1\| + \beta \cdot \|x_2\|$ and g is a nondecreasing function. Then, $\rho_n(\overline{\alpha}, \overline{x}) \geq (\alpha_1 + \alpha_2) \cdot g(t'_1) + S = \rho_{n-1}(\overline{\alpha}', \overline{x}')$, where $\overline{\alpha}' = (\alpha_1 + \alpha_2, \alpha_3, \ldots, \alpha_{n-1}, \alpha_n)$, and $\overline{x}' = (x_1, x_2, \ldots, x_{n-1}, x_n)$.

Applying the induction, we get $\Delta \geq \gamma$. □

Corollary 1.2. *Let $\mathbb{X}$ and $\mathbb{Y}$ be seminormed spaces over $\mathbb{R}$ and $f : \mathbb{X} \to \mathbb{Y}$. Then in Theorem 1.1, one replaces condition (a) by condition (b): $g(t) \cdot \|f(y)\|_y \leq \|f(x)\|_y \leq r(t) \cdot \|f(y)\|_Y$, for all $x, y \in \mathbb{X}$ with $\|x\|_X/\|y\|_X = t \in [0,1]$, and all the rest of the conditions are satisfied. Then, with $\alpha_i \geq 0$, $\sum_{i=1}^{m} \alpha_i = 1$, $x_i = \mathbb{X}$, $(i = \overline{1,n})$, the inequality $\gamma \cdot \|f(\sum_{i=1}^{m} \alpha_i \cdot x_i)\|_y \leq \sum_{i=1}^{m} \alpha_i \cdot \|f(x_i)\|_y$ is satisfied.*

Proof. We consider the functional $\phi = \|f\|_y : \mathbb{X} \xrightarrow{f} \mathbb{Y} \xrightarrow{\|\cdot\|_y} \mathbb{R}_+$. Then, knowing (b), we conclude that ϕ satisfies Theorem 1.1's conditions and hence the needed inequality. □

Example 1.3. If we put in the conditions of Theorem 1.1, $g(t) = t^p$, $p > 1$, $p \in R$, $r(t) = t$, and $f : R \to R$, $t^p f(y) \leq f(ty) \leq t f(y)$, $t \in [0,1]$, then about $\alpha_1 \geq 0$, $\sum_{i=1}^{m} \alpha_i = 1$, $x_i = \mathbb{X}$, $(i = \overline{1,n})$, we will obtain the inequality

$$\gamma \cdot f(\alpha_1 \cdot x_1 + \cdots + \alpha_n \cdot x_n) \leq \left[\alpha_1 \cdot f(x_1) + \cdots + \alpha_n \cdot f(x_n)\right], \tag{1.7}$$

where

$$\gamma = 1 - p^{-(p-1)^{-1}} + p^{-p(p-1)^{-1}}. \tag{1.8}$$

Proof. Let us consider $\delta(\alpha,t)=\alpha\cdot g(t/(\alpha t+\beta))+\beta/r(\alpha\cdot t+\beta)$, where

$$g(t)=t^p,\quad r(t)=t,\quad \alpha\in[0,1],\quad t\in[0,1],\quad \beta=1-\alpha. \tag{1.9}$$

Then, $\delta(\alpha,t)=\alpha\cdot(t/(\alpha\cdot t+\beta))^p+\beta/(\alpha\cdot t+\beta)=h(t)$,

$$\frac{\partial\delta(\alpha,t)}{\partial t}=h'(t)=\alpha p\left(\frac{t}{\alpha\cdot t+\beta}\right)^{p-1}\frac{\beta}{(\alpha\cdot t+\beta)^2}-\frac{\alpha\beta}{(\alpha\cdot t+\beta)^2}=0, \tag{1.10}$$

when $(t/(\alpha\cdot t+\beta))^{p-1}=p^{-1}$, that is, $(t/(\alpha\cdot t+\beta))=p^{-(p-1)^{-1}}$; hence, $(\alpha\cdot t+\beta)/t=p^{(p-1)^{-1}}$. Further, we obtain $t=\beta(p^{(p-1)^{-1}}-\alpha)^{-1}$. It is obvious that we have a minimum at this point in the interval $[0,1]$.

Then, we obtain $(1/(\alpha\cdot t+\beta))=\beta^{-1}p^{(p-1)^{-1}}(p^{(p-1)^{-1}}-\alpha)$, and hence at the same point t

$$\begin{aligned}\delta(\alpha,t)&=\alpha\cdot\left(\frac{t}{\alpha\cdot t+\beta}\right)^p+\frac{\beta}{(\alpha\cdot t+\beta)}=\alpha\cdot p^{-p(p-1)^{-1}}+\left(p^{(p-1)^{-1}}-\alpha\right)p^{-p(p-1)^{-1}}\\&=1+\alpha\cdot\left(p^{-p(p-1)^{-1}}-p^{-(p-1)^{-1}}\right)\geq 1+\left(p^{-p(p-1)^{-1}}-p^{-(p-1)^{-1}}\right)=\gamma,\end{aligned} \tag{1.11}$$

since

$$\left(p^{-p(p-1)^{-1}}-p^{-(p-1)^{-1}}\right)\leq 0. \tag{1.12}$$

This confirms the assertion. □

If we put $p=2$ in the condition of the example, we receive $\gamma=3/4$. Therefore, $3f(\alpha_1x_1+\cdots+\alpha_nx_n)\leq 4[\alpha_1f(x_1)+\cdots+\alpha_nf(x_n)]$, when $\alpha_i\geq 0$, $\sum_{i=1}^n\alpha_i=1$, $x_i\in\mathbb{X}$, $(i=\overline{1,n})$.

Theorem 1.4. *Let $\mathbb{A}$ be a seminormed algebra over R with a unit. The functional $f:\mathbb{A}\to\mathbb{R}_+$ satisfies condition (a): $g(t)\cdot f(y)\leq f(x)\leq r(t)\cdot f(y)$, for x,y as $\|x\|/\|y\|=t\in[0,1]$, where $g,r:[0,1]\to[0,1]$ are nondecreasing functions such that $g(t)\leq r(t)$.*

Besides, the following requirements are fulfilled

(1) *There exists $\inf_{\alpha,t\in[0,1]}\delta(\alpha,t)=\lambda$, where*

$$\delta(\alpha,t)=\alpha\cdot g\left(\frac{t}{(\alpha\cdot t+\beta)}\right)+\frac{\beta}{r(\alpha\cdot t+\beta)},\quad \text{for } \alpha\in[0,1],\ \text{with } \alpha+\beta=1. \tag{1.13}$$

(2) *The function, $g(t):[0,1]\to[0,1]$ and $r^{-1}(t^{-1}):[1,\sim]\to[1,\sim]$ are convex. Then, if $p_i,x_i\in\mathbb{A}$, $(i=\overline{1\cdot n})$, $\sum_{i=1}^m\|p_i\|=1$, one receives the inequality*

$$\gamma\cdot f\left(\sum_{i=1}^m p_ix_i\right)\leq\sum_{i=1}^m\|p_i\|\cdot f(x_i). \tag{1.14}$$

Proof. Let $p, q, x, y \in \mathbb{A}$, as $\|x\| \leq \|y\|$, $\|p\| + \|q\| = 1$.

We put $\Delta = (\|p\| \cdot f(x) + \|q\| \cdot f(y))/f(z)$, where $z = p \cdot x + q \cdot y$.

(a) Let $\|x\| \leq \|z\| \leq \|y\|$. According to condition (a), we have $\Delta \geq \|p\| \cdot g(\|x\|/\|z\|) + \|q\|/r(\|z\|/\|y\| \geq \|p\| \cdot g(\|x\|/\|p\| \cdot \|x\|))$.

Here, we have $\|p \cdot x + q \cdot y\| \leq \|p\| \cdot \|x\| + \|q\| \cdot \|y\|$, and g, r are nondecreasing.

If $\alpha = \|p\|$, $\beta = \|q\|$, then $\Delta \geq \alpha \cdot g(t/(\alpha \cdot t + \beta)) + \beta \cdot r^{-1} \cdot (\alpha \cdot t + \beta) = \delta(\alpha, t)$, where $t = \|x\|/\|y\|$.

Then, $\inf_{\alpha, t \in [0,1]} \delta(\alpha, t) = \gamma$ exist in compliance with (1). Therefore $\Delta \geq \gamma$.

If we put $x = y$, the result is $1 = \Delta \geq \gamma$, that is, $1 \geq \gamma$.

(b) Let $\|z\| \leq \|x\|$. Then, in view of the fact that (a), we get

$$\Delta \geq \|p\| \cdot r^{-1} \cdot \left(\frac{\|z\|}{\|x\|}\right) + \frac{\|q\|}{r(\|z\|/\|y\|)} \geq \|p\| + \|q\| = 1 \geq \gamma. \tag{1.15}$$

Let $p_i, x_i \in \mathbb{A}$, $(i = \overline{1 \cdot n})$, as $\sum_{i=1}^{m} \|p_i\| = 1$. Let us put $\Delta = (\sum_{i=1}^{m} \|p_i\| \cdot f(x_i))/f(z)$, where $z = \sum_{i=1}^{m} p_i \cdot x_i$.

We can accept $\|x_1\| \leq \|x_2\| \leq \cdots \leq \|x_n\|$. Let $1 \leq k \leq n$ and $\|x_{k-1}\| \leq \|z\| \leq \|x_k\|$.

We have $\Delta \geq \sum_{i=1}^{m} \|p\|_i \cdot g(t_i) + \sum_{i=1}^{m} \|p\|_i \cdot r^{-1} \cdot t_i^{-1} = \rho_n(\overline{p}, \overline{x})$, where

$$\overline{p} = (p_1, p_2, \ldots, p_n), \qquad \overline{x} = (x_1, x_2, \ldots, x_n), \qquad {}_i = \frac{\|x_i\|}{\|z\|}. \tag{1.16}$$

Applying the principle of induction over n we will prove that $\rho_n(\overline{p}, \overline{x}) \geq \gamma$. In view of the fact that was mentioned at the beginning, we get $\rho_2(\overline{p}, \overline{x}) = \delta(\alpha, t) \geq \gamma$. Assuming the statement for $(n-1)$ holds, we will prove it for n.

(1) Let $k \leq 2$.

Putting $\alpha = \|p_{n-1}\|/(\|p_{n-1}\| + \|p_n\|)$, $\beta = \|p_n\|/(\|p_{n-1}\| + \|p_n\|)$, we have $\rho_n(\overline{p}, \overline{x}) = S + (\|p_{n-1}\| + \|p_n\|) \cdot (\alpha \cdot r^{-1}(t_{n-1}^{-1}) + \beta \cdot r^{-1}(t_n^{-1}))$ where S is the rest of the sum. Using condition (2), we get

$$\rho_n(\overline{p}, \overline{x}) \geq S + (\|p_{n-1}\| + \|p_n\|) r^{-1}\left((\alpha \cdot t_{n-1}) + \beta \cdot t_n)^{-1}\right). \tag{1.17}$$

Let $x'_{n-1} = (p_{n-1} \cdot x_{n-1} + p_n \cdot x_n)/(\|p_{n-1}\| + \|p_n\|)$, $t'_{n-1} = \|x'_{n-1}\|/\|z\|)$.

Since r does not decrease, and $\|x'_{n-1}\| \leq \alpha \cdot \|x_{n-1}\| + \beta \cdot \|x_n\|$, then $\rho_n(\overline{p}, \overline{x}) \geq S + (\|p_{n-1}\| + \|p_n\|) \cdot r^{-1}((t'_{n-1})^{-1}) = \rho_{n-1}(\overline{p}', \overline{x}')$, where $\overline{p}' = (p_1, p_2, \ldots, p_{n-2}, p'_{n-1})$, $p'_{n-1} = (\|p_{n-1}\| + \|p_n\|) \cdot e$, and $\overline{x}' = (x_1, x_2, \ldots, x_{n-2}, x'_{n-1})$.

By e we denote the unit of the algebra $\mathbb{A}$. According to the inductive suggestion, we obtain $\rho_n(\overline{p}, \overline{x}) \geq \rho_{n-1}(\overline{p}', \overline{x}') \geq \gamma$.

(2) Let $k \geq 3$.

We set $\alpha = \|p_1\|/(\|p_1\| + \|p_2\|)$, $\beta = \|p_2\|/(\|p_1\| + \|p_2\|)$. As (2), we have $\rho_n(\overline{p}, \overline{x}) \geq (\|p_1\| + \|p_2\|) \cdot g(\alpha \cdot t_1 + \beta \cdot t_2) + S$, where S is the rest of the sum.

Let $x'_1 = (p_1 \cdot x_1 + p_2 \cdot x_2)/(\|p_1\| + \|p_2\|)$, $t'_1 = \|x'_1\|/\|z\|$.

Since g does not decrease, and $\|x'_1\| \leq \alpha \cdot \|x_1\| + \beta \cdot \|x_2\|$, then $\rho_n(\overline{p}, \overline{x}) \geq (\|p_1\| + \|p_2\|) \cdot g(t'_1) + S = \rho_{n-1}(\overline{p}', \overline{x}')$, where $\overline{p}' = (p'_1, p_3, \ldots, p_{n-1}, p_n), p'_1 = (\|p_1\| + \|p_2\|) \cdot e$, and $\overline{x}' = (x_1, x_2, \ldots, x_{n-2}, x'_{n-1})$. According to the induction principle, we obtain $\rho_n(\overline{p}, \overline{x}) \geq \rho_{n-1}(\overline{p}', \overline{x}') \geq \gamma$. □

Corollary 1.5. *Let $\mathbb{A}$ be a seminormed algebra above $\mathbb{R}$ with a unit, and let $\mathbb{X}$ be a seminormed space over $\mathbb{R}$, and $f : \mathbb{A} \to \mathbb{X}$.*

Then, if one replaces the condition (a) in Theorem 1.4 by condition (c): $g(t) \cdot \|f(y)\|_X \leq \|f(x)\|_X \leq r(t) \cdot \|f(y)\|_X$, for all $x, y \in A$ with $\|x\|_a / \|y\|_a = t \in [0,1]$, and all the rest of the conditions are satisfied. One denotes by $\|\cdot\|_x$ the norm in $\mathbb{X}$, and the norm in $\mathbb{A}$ with $\|\cdot\|_a$. Then if $p_i, x_i \in A$, $(i = \overline{1,n})$ $\sum_{i=1}^{m} \|p_i\|_a = 1$ one receives the inequality

$$\gamma \cdot \left\| f\left(\sum_{i=1}^{m} p_i \cdot x_i\right)\right\|_x \leq \sum_{i=1}^{m} \|p_i\|_\alpha \cdot \|f(x_i)\|_x. \tag{1.18}$$

Proof. We consider the functional $\phi = \|f\|_x : \mathbb{A} \xrightarrow{f} \mathbb{X} \xrightarrow{\|\cdot\|_x} \mathbb{R}_+$. Then, knowing (c), we get that ϕ satisfies Theorem 1.1's conditions and hence the needed inequality. □

References

[1] Y. Altin, M. Et, and B. C. Tripathy, "The sequence space $|Np|(M, r, q, s)$ on seminormed spaces," *Applied Mathematics and Computation*, vol. 154, no. 2, pp. 423–430, 2004.

[2] B. C. Tripathy, Y. Altin, and M. Et, "Generalized difference sequence spaces on seminormed space defined by Orlicz functions," *Mathematica Slovaca*, vol. 58, no. 3, pp. 315–324, 2008.

[3] B. C. Tripathy and B. Sarma, "Sequence spaces of fuzzy real numbers defined by Orlicz functions," *Mathematica Slovaca*, vol. 58, no. 5, pp. 621–628, 2008.

[4] B. C. Tripathy and B. Sarma, "Vector valued double sequence spaces defined by Orlicz function," *Mathematica Slovaca*, vol. 59, no. 6, pp. 767–776, 2009.

[5] P. Chandra and B. C. Tripathy, "On generalised Köthe-Toeplitz duals of some sequence spaces," *Indian Journal of Pure and Applied Mathematics*, vol. 33, no. 8, pp. 1301–1306, 2002.

[6] B. C. Tripathy and S. Mahanta, "On a class of generalized lacunary difference sequence spaces defined by Orlicz functions," *Acta Mathematicae Applicatae Sinica*, vol. 20, no. 2, pp. 231–238, 2004.

[7] R. P. Maleev and S. L. Troyanski, "On the moduli of convexity and smoothness in Orlicz spaces," *Studia Mathematica*, vol. 54, no. 2, pp. 131–141, 1975.

[8] R. P. Maleev and S. L. Troyanski, "On cotypes of Banach lattices," in *Constructive Function Theory*, pp. 429–441, Publishing House of the Slovak Academy of Sciences, Sofia, Bulgaria, 1983.

[9] T. S. Stoyanov, "Inequalities for convex combinations of functions," in *Proceedings of the 18th Spring-Conference of the Union of Bulgarian Mathematicians*, Albena, Bulgaria, April 1989.