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Research Article

The Failure of Orthogonality under Nonstationarity: Should We Care About It?

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We consider two well-known facts in econometrics: (i) the failure of the orthogonality assumption (i.e., no independence between the regressors and the error term), which implies biased and inconsistent Least Squares (LS) estimates and (ii) the consequences of using nonstationary variables, acknowledged since the seventies; LS might yield spurious estimates when the variables do have a trend component, whether stochastic or deterministic. In this work, an optimistic corollary is provided: it is proven that the LS regression, employed in nonstationary and cointegrated variables where the orthogonality assumption is not satisfied, provides estimates that converge to their true values. Monte Carlo evidence suggests that this property is maintained in samples of a practical size.

1. Introduction

Two well-known facts lie behind this work: (i) the behavior of LS estimates whenever variables are nonstationary and (ii) the failure of the orthogonality assumption between independent variables and the error term, also in an LS regression.

(1) The reappraisal of the impact of unit roots in time-series observations, initiated in the late seventies, had profound consequences for modern econometrics. It became clear that (i) insufficient attention was being paid to trending mechanisms and (ii) most macroeconomic variables are probably nonstationary; such an appraisal gave

rise to an extraordinary development that substantially modified the way empirical studies in time-series econometrics are carried out. Research into nonstationarity has advanced significantly since it was reassessed in several important papers, such as those of [1–5].

(2) The orthogonality problem constitutes another significant research program in econometrics; its formal seed can be traced back to [6], where a proposal to solve the identification problem is made in the estimation of demand and supply curves (see [7]). Typically, in textbooks, the method of Instrumental Variables (IVs) is proposed as a solution to the problem of simultaneous equations and, broadly speaking, whenever there is no independence between the error term and the regressors, that is, when the orthogonality assumption is not satisfied.

This paper aims to study the consequences of using nonstationary variables in an LS regression when the regressor is related to the error term; this is done in a simple regression framework. The specification under particular scrutiny is

$$y_t = \alpha + \beta x_t + u_t. \tag{1.1}$$

To the best of our knowledge, the asymptotics—and the finite-sample properties—of the combination of nonstationarity, nonorthogonality between x_t and $u_{y,t}$, and LS estimates, have been scarcely studied (but see [8]). That said, we acknowledge that there are several comprehensive studies concerning the use of IV in the presence of nonstationarity [9, 10], for example, studied the asymptotics as well as the finite-sample properties of the IV estimator in the context of a cointegrated relationship, and proved that even spurious instruments (i.e., $I(1)$ instruments not structurally related to the regressors) provide consistent estimates. Phillips [11] proved that, when there is no structural relationship between the regressand and a single regressor, that is, when there is no cointegration between y and x, the use of spurious instruments does not prevent the phenomenon (this is a simple extension of [5]). We derive the asymptotic behavior of LS estimates, where the data generating processes (DGPs) consist of two cointegrated variables in which the regressor bears a relationship with the error term. In this case, LS provide consistent estimates. Additionally, some Monte Carlo evidence is presented to account for the adequacy of asymptotic results in finite samples. In other words, LS estimates of the true DGP parameters, μ_y and β_y (see (2.4) in the next section), do not require the information on the parameters of x, that is, x_t is weakly exogenous for the estimation of μ_y and β_y as defined by [12].

2. Relevant DGPs

This work aims to study the asymptotic properties of LS estimates when neither the orthogonality nor the stationarity assumptions are satisfied. Our approach is twofold: we assume (i) the variable x is statistically related to the innovations of y, as in the problem of independent variables measured with error and (ii) the DGPs of both variables are interdependent, as in the problem of simultaneity. All the cases studied consider nonstationary and cointegrated variables (DGP (2.1) is included because it eases the comprehension of the paper)

$$x_t = \mu_x + u_{x,t}, \tag{2.1}$$

$$x_t = \mu_x + x_{t-1} + u_{x,t}, \tag{2.2}$$

$$x_t = X_0 + (\mu_x + \rho_{y1})t + \xi_{x,t-1} + \rho_{y2}\xi_{y,t-1}, \tag{2.3}$$

$$y_t = \mu_y + \beta_y x_t + \underbrace{(u_{y,t} + \rho_x u_{x,t})}_{\text{innovations}}, \tag{2.4}$$

where $u_{z,t}$, for $z = x, y$, are independent white noises with zero-mean and constant variance σ_z^2, $\xi_{zt} = \sum_{i=0}^{t} u_{zi}$ and Z_0 is an initial condition. We may relax the assumptions made for the innovations; for example, we could force them to obey the general level conditions in [5, Assumption 1]. Nevertheless, although the asymptotic results would still hold in this case, our primary target concerns the problem of orthogonality between the regressor and the error term, not those of autocorrelation or heteroskedasticity. These DGPs allow for an interesting variety of cases (note that the asymptotics of the LS estimates when x and y have been independently generated by any of first three DGPs can be found, e.g., in [13]; notwithstanding, the authors can provide these cases as *mathematica* code upon request).

(1) Bookcase no. 1: DGP of x is (2.1) and DGP of y is (2.4) with $\rho_x = 0$. When the variables are generated in this manner, we fulfill the classical assumptions made in most basic econometrics textbooks. The variables are stationary, the innovations are homoskedastic and independent, and so forth. It is straightforward to show that: $\widehat{\alpha} \xrightarrow{p} \mu_y$, $\widehat{\beta} \xrightarrow{p} \beta_y$ and $\widehat{\sigma}^2 \xrightarrow{p} \sigma_y^2$.

(2) Bookcase no. 2: DGP of x is (2.1) and DGP of y is (2.4) with $\rho_x \neq 0$. These DGPs also represent a typical example of a problem of orthogonality in most basic econometrics textbooks. Although the variables are stationary and the innovations are homoskedastic and independent, the explanatory variable is related to the innovations of y. It is well known that the estimates do not converge to their true value. In particular, it is straightforward to show that: $\widehat{\alpha} \xrightarrow{p} \mu_y - \mu_x\rho_x$, $\widehat{\beta} \xrightarrow{p} \beta_y + \rho_x$ and $\widehat{\sigma}^2 \xrightarrow{p} \sigma_y^2$.

(3) Bookcase no. 3: DGP of x is (2.2) and DGP of y is (2.4) with $\rho_x = 0$. These DGPs allow the relationship between x and y to be cointegrated *à la* [14]. Once again, asymptotic results have been known for a long time, obtaining these does not entail any particular difficulty: $\widehat{\alpha} \xrightarrow{p} \mu_y$, $\widehat{\beta} \xrightarrow{p} \beta_y$, $\widehat{\sigma}^2 \xrightarrow{p} \sigma_y^2$ and $1 - R^2 = O_p(T^{-2})$.

(4) Nonstationarity and non-orthogonality case no. 1: DGP of x is (2.2) and DGP of y is (2.4). Notwithstanding, the obvious problem of orthogonality between x and the error term, the variables remain cointegrated. The artifact employed to induce the orthogonality problem can be considered as, for example, measurement errors in the explanatory variable. One should expect that, in the presence of this problem, estimates would not converge to their true value. We prove below that, contrary to expectations, this is not the case.

(5) Nonstationarity and non-orthogonality case no. 2: DGP of x is (2.3) and DGP of y is (2.4). As in the previous case, we have a cointegrated relationship between x

and y, only in this case, the problem of orthogonality between the regressor and the error term is even more explicit; the artifact employed to induce the orthogonality problem can be related to the typical simultaneous equations case. We also prove below that Least Squares (LS) provide consistent estimates.

The common belief as regards the last two cases is that the failure of the orthogonality assumption induces LS to generate inconsistent estimates, even in a cointegrated relationship. In fact, when the variables are generated as in (2.2)–(2.4), the estimates of the parameter converge to their true value (note that we did not consider the case where the orthogonality assumption is not satisfied because of the omission of a relevant variable; [15] studied the later case and proved that the LS estimates do not converge to their true values). This is proven in Theorem 2.1:

Theorem 2.1. *Let y_t be generated by* (2.4).

(i) *Let x_t be generated by* (2.2). *The innovations of both DGPs, $u_{z,t}$, for $z = y, x$, are independent white noises with zero-mean and constant variance σ_z^2; use y_t and x_t to estimate regression* (1.1) *by LS. Hence, as $T \to \infty$,*

(a) $\widehat{\alpha} \xrightarrow{p} \mu_y$,

(b) $\widehat{\beta} \xrightarrow{p} \beta_y$,

(c) $T^{-3/2} t_\beta = O_p(1)$,

(d) $\widehat{\sigma}^2 \xrightarrow{p} \sigma_y^2 + \rho_x^2 \sigma_x^2$,

(e) $T^2(1 - R^2) \xrightarrow{p} 12(\sigma_y^2 + \rho_x^2 \sigma_x^2)/(\beta_y \mu_x)^2$.

(ii) *Let x_t be generated by* (2.3). *The innovations of both DGPs, $u_{z,t}$, for $z = y, x$, are independent white noises with zero-mean and constant variance σ_z^2; use y_t and x_t to estimate regression* (1.1) *by LS. Hence, as $T \to \infty$,*

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(d) $\widehat{\sigma}^2 \xrightarrow{p} \sigma_y^2 + \rho_x^2 \sigma_x^2$,

(e) $T^2(1 - R^2) \xrightarrow{p} 12(\sigma_y^2 + \rho_x^2 \sigma_x^2)/[\beta_y(\mu_x + \rho_{y1})]^2$.

Proof. See Appendix A. □

These asymptotic results show that a relationship between the innovations of y_t and x_t—as stated by DGPs (2.2), (2.3), and (2.4)—does not obstruct the consistency of LS estimates when the variables are nonstationary and cointegrated (our results are in line with those of [8]). In other words, the failure of the orthogonality assumption does not preclude adequate asymptotic properties of LS. Furthermore, it can be said that x_t is weakly exogenous for the estimation of μ_y and β_y but not for the estimation of σ^2. The formula of the variance is noteworthy and the asymptotic expression of t_β depends on the values of σ_x^2, σ_y^2, and ρ_x.

In order to emphasize the relevance of this result, we modified the DGPs of the variables in an effort to strengthen the link between the DGPs and the literature on simultaneous equations. The modifications are twofold and appear in the following propositions. As in Theorem 2.1, the results in proposition 1 are made under the assumption that innovations are *i.i.d* processes.

Proposition 2.2. *Let y_t and x_t be generated by*

$$\begin{gathered} y_t - \beta_y x_t - \mu_y = u_{y,t}, \\ -\beta_x y_t + x_t - \mu_x - \gamma_x t = u_{x,t}, \end{gathered} \tag{2.5}$$

where $u_{z,t}$, for $z = x, y$, are independent white noises with zero mean and variance σ_z^2. Let these variables be used to estimate regression (1.1) *by LS. Hence, as $T \to \infty$,*

(1) $\widehat{\alpha} \xrightarrow{p} \mu_y$,

(2) $\widehat{\beta} \xrightarrow{p} \beta_y$,

(3) $T^{-3/2} t_\beta = O_p(1)$,

(4) $\widehat{\sigma}^2 \xrightarrow{p} \sigma_y^2$,

(5) $T^2(1 - R^2) \xrightarrow{p} 12\sigma_y^2(1 - \beta_x\beta_y)^2 / (\beta_y\gamma_x)^2$.

Proof. See Appendix A. □

Proposition 2.3. *Let y_t and x_t be generated by*

$$\begin{gathered} y_t - \beta_y x_t - \mu_y = u_{y,t}, \\ -\beta_x y_t + x_t - X_0 - \mu_x t = \xi_{x,t}, \end{gathered} \tag{2.6}$$

where $u_{z,t}$, for $z = x, y$, are independent white noises with zero mean and variance σ_z^2, and $\xi_{x,t} = \sum_{i=0}^t u_{x,i}$. Let these variables be used to estimate regression (1.1) *by LS. Hence, as $T \to \infty$,*

(1) $\widehat{\alpha} \xrightarrow{p} \mu_y$,

(2) $\widehat{\beta} \xrightarrow{p} \beta_y$,

(3) $T^{-3/2} t_\beta = O_p(1)$,

(4) $\widehat{\sigma}^2 \xrightarrow{p} \sigma_y^2$,

(5) $T^2(1 - R^2) \xrightarrow{p} 12\sigma_y^2(1 - \beta_x\beta_y)^2 / (\mu_x\beta_y)^2$.

Proof. See Appendix A. □

The two systems, represented in (2.5) and (2.6), bear a striking resemblance to classical examples of simultaneous equations in econometrics. The fundamental variations are, (i) a deterministic trend in the variable x_t in system (2.5) and (ii) a stochastic as well as a deterministic trend in system (2.6). The asymptotics of LS estimates do not show significant differences from those in Theorem 2.1. Note, however, that x_t is weakly exogenous for the estimation of μ_y, β_y, and σ_y^2. The main result is in fact identical, that is, the failure of orthogonality between x_t and the error term does not preclude the estimates from converging to their true values.

Asymptotic properties of LS estimators clearly provide an encouraging perspective in time-series econometrics. Notwithstanding, we should bear in mind that asymptotic properties may be a poor finite-sample approximation. In order to observe the behavior of LS estimates in finite samples, we present two Monte Carlo experiments. Firstly, we represent graphically the convergence process of $\widehat{\beta}$ towards its true value, β. In accordance with asymptotic results, $\widehat{\beta} - \beta \xrightarrow{p} 0$ as $T \to \infty$. We reproduce the behavior of the later difference in figure 1. The variables x and y are generated according to (2.3) and (2.4), respectively. The sample size varies from 50 to 700 whilst β_y goes from −5 to 5. The remaining parameters appear below the figure.

A brief glance at Figure 1 reveals that the asymptotic results stated in Theorem 2.1 approximate conveniently the finite-sample results for $T > 150$. For smaller sample sizes, it can be seen that the difference between the parameter and its estimates corresponds usually to approximately 1.5% or less of the value of the former (we tried different variables in the y axis ($\rho_x, \rho_{y1}, \rho_{y2}, \sigma_y^2, \sigma_y^2, \ldots$); all of these trials produced similar figures).

The second Monte Carlo is built upon the same basis. In Table 1, each cell indicates the sample mean of $\widehat{\beta} - \beta_y$ and, below, its estimated standard deviation (in parentheses). The number of replications is 10,000. The parameter values used in the simulation are explicit within the table. The variables, x and y, are generated according to (2.3) and (2.4), respectively. Sample size ranges from $T = 50$–700; $\rho_{y1} = -0.15$; $\rho_x = 4$; $\sigma_y^2 = \sigma_x^2 = 1$; $\mu_y = 4.20$; the error term is a white noise with variance $\sigma_\epsilon^2 = 1$.

Table 1 shows that LS estimates of a nonstationary relationship with a nonorthogonality problem quickly converge to their true value; with a sample size as small as 50 observations, the difference between β_y and its estimate averages, at most, 0.015, and represents a deviation from the true value of 1.5%; in many other cases, the deviation is even smaller, of order 10^{-3}–10^{-4}. These differences tend to diminish further as the sample size grows. In fact, when there are 700 observations, the order of magnitude of such differences oscillates between 10^{-5}–10^{-8}. We performed the same experiment with autocorrelated disturbances AR(1) with $\phi = 0.7$ (data available upon request); using such disturbances severely deteriorates the efficiency of the LS estimates although $\widehat{\beta} - \beta_y$ still converges to zero; we do not focus on this issue because, as mentioned earlier, neither autocorrelation nor heteroskedasticity are under scrutiny in this work.

3. Concluding Remarks

Using cointegrated variables in an LS regression where the regressor is not independent of the error term does not preclude the method from yielding consistent estimates. In other words, it is proven that, under these circumstances, the regressor remains weakly exogenous for the estimation of μ_y and β_y (and for σ_y^2 in systems (2.5) and (2.6)) as defined by [12]. Furthermore, the finite-sample evidence indicates that LS provide good estimates even in samples of a practical size.

Notwithstanding, one should note the striking resemblance between the properties of the DGPs used in the propositions and those of variables belonging to a classical simultaneous-equation model. It may be possible that the estimation of such models, even if the macroeconomic variables they are nourished with are not stationary, would yield correct estimates. Of course, such a possibility rules out the existence of structural shifts, parameter instability, omission of a relevant variable, or any other major assumption failure.

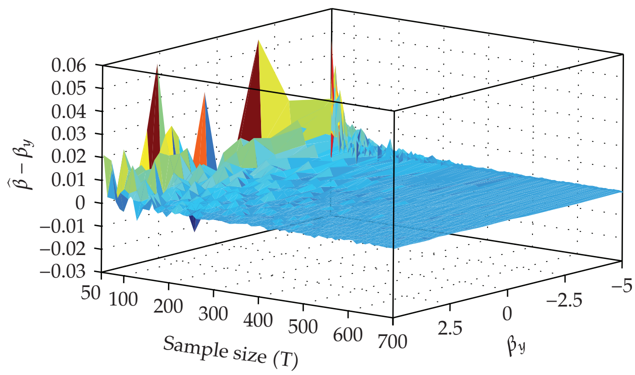

Figure 1: Finite-sample behavior of $\hat{\beta} - \beta_y$: sample size and β_y range from $T = 50$–700 and −5 to 5, respectively; $\rho_{y1} = -0.15$; $\rho_{y2} = 1.5$; $\rho_x = 4$; $\sigma_y^2 = \sigma_x^2 = 1$; $\mu_y = 4.20$; $\mu_x = 0.70$. Number of replications (for each coordinate): 20; grid density: 70 × 30.

Table 1: Finite-sample behavior of $\hat{\beta} - \beta_y$: mean and standard deviation.

DGP parameters			Sample size				
β_y	μ_x	Statistic	50	100	200	500	700
−0.9	−1.5	Mean	$\mathbf{1.5 \cdot 10^{-2}}$	$\mathbf{3.1 \cdot 10^{-3}}$	$\mathbf{7.8 \cdot 10^{-4}}$	$\mathbf{8.6 \cdot 10^{-5}}$	$\mathbf{5.6 \cdot 10^{-5}}$
		Stand.dev.	$(1.8 \cdot 10^{-3})$	$(1.7 \cdot 10^{-4})$	$(1.8 \cdot 10^{-5})$	$(1.1 \cdot 10^{-6})$	$(3.9 \cdot 10^{-7})$
	−0.75	Mean	$\mathbf{1.5 \cdot 10^{-2}}$	$\mathbf{3.3 \cdot 10^{-3}}$	$\mathbf{7.6 \cdot 10^{-4}}$	$\mathbf{1.0 \cdot 10^{-4}}$	$\mathbf{5.3 \cdot 10^{-5}}$
		Stand.dev.	$(1.9 \cdot 10^{-3})$	$(1.6 \cdot 10^{-4})$	$(1.8 \cdot 10^{-5})$	$(1.1 \cdot 10^{-6})$	$(3.9 \cdot 10^{-7})$
	0.75	Mean	$\mathbf{1.4 \cdot 10^{-2}}$	$\mathbf{3.2 \cdot 10^{-3}}$	$\mathbf{8.0 \cdot 10^{-4}}$	$\mathbf{1.0 \cdot 10^{-4}}$	$\mathbf{5.8 \cdot 10^{-5}}$
		Stand.dev.	$(1.7 \cdot 10^{-3})$	$(1.6 \cdot 10^{-4})$	$(1.8 \cdot 10^{-5})$	$(1.0 \cdot 10^{-6})$	$(3.9 \cdot 10^{-7})$
	1.5	Mean	$\mathbf{1.5 \cdot 10^{-2}}$	$\mathbf{3.0 \cdot 10^{-3}}$	$\mathbf{7.2 \cdot 10^{-4}}$	$\mathbf{1.0 \cdot 10^{-4}}$	$\mathbf{5.8 \cdot 10^{-5}}$
		Stand.dev.	$(1.8 \cdot 10^{-3})$	$(1.6 \cdot 10^{-4})$	$(1.8 \cdot 10^{-5})$	$(1.1 \cdot 10^{-6})$	$(3.9 \cdot 10^{-7})$
0.9	−1.5	Mean	$\mathbf{2.2 \cdot 10^{-3}}$	$\mathbf{6.2 \cdot 10^{-4}}$	$\mathbf{1.4 \cdot 10^{-4}}$	$\mathbf{2.1 \cdot 10^{-5}}$	$\mathbf{1.1 \cdot 10^{-5}}$
		Stand.dev.	$(2.4 \cdot 10^{-4})$	$(2.8 \cdot 10^{-5})$	$(3.4 \cdot 10^{-6})$	$(2.1 \cdot 10^{-7})$	$(8.0 \cdot 10^{-8})$
	−0.75	Mean	$\mathbf{2.4 \cdot 10^{-3}}$	$\mathbf{6.0 \cdot 10^{-4}}$	$\mathbf{1.5 \cdot 10^{-4}}$	$\mathbf{1.9 \cdot 10^{-5}}$	$\mathbf{1.7 \cdot 10^{-5}}$
		Stand.dev.	$(2.3 \cdot 10^{-4})$	$(2.8 \cdot 10^{-5})$	$(3.4 \cdot 10^{-6})$	$(2.2 \cdot 10^{-7})$	$(7.9 \cdot 10^{-8})$
	0.75	Mean	$\mathbf{2.4 \cdot 10^{-3}}$	$\mathbf{5.8 \cdot 10^{-4}}$	$\mathbf{1.3 \cdot 10^{-4}}$	$\mathbf{2.2 \cdot 10^{-5}}$	$\mathbf{5.4 \cdot 10^{-6}}$
		Stand.dev.	$(2.3 \cdot 10^{-4})$	$(2.9 \cdot 10^{-5})$	$(3.5 \cdot 10^{-6})$	$(2.2 \cdot 10^{-7})$	$(8.0 \cdot 10^{-8})$
	1.5	Mean	$\mathbf{2.2 \cdot 10^{-3}}$	$\mathbf{5.6 \cdot 10^{-4}}$	$\mathbf{1.2 \cdot 10^{-4}}$	$\mathbf{2.1 \cdot 10^{-5}}$	$\mathbf{1.1 \cdot 10^{-5}}$
		Stand.dev.	$(2.4 \cdot 10^{-4})$	$(2.9 \cdot 10^{-5})$	$(3.4 \cdot 10^{-6})$	$(2.2 \cdot 10^{-7})$	$(7.9 \cdot 10^{-8})$

Appendix

A. Proof of Theorem 2.1 and Propositions 2.2 and 2.3

The estimated specification in Theorem 2.1 and Propositions 2.2 and 2.3 is $y_t = \alpha + \beta x_t + u_t$. In all three cases, we employ the following classical LS formulae (all sums run from $t = 1$ to T unless otherwise specified):

(i) $B = (X'X)^{-1}X'Y$,

(ii) $\hat{\sigma}^2 = T^{-1}\sum \hat{u}_t^2 = T^{-1}[\sum y_t^2 + \hat{\alpha}^2 T + \hat{\beta}^2 \sum x_t^2 - 2\hat{\alpha}\sum y_t - 2\hat{\beta}\sum y_t x_t + 2\hat{\alpha}\hat{\beta}\sum x_t]$,

(iii) $t_\beta = \hat{\beta}((X'X)_{22}^{-1}\hat{\sigma}^2)^{-1/2}$,

(iv) $R^2 = 1 - \mathrm{RSS}/\mathrm{TSS}$, where

$$X'X = \begin{pmatrix} T & \sum x_t \\ \sum x_t & \sum x_t^2 \end{pmatrix},$$
$$X'Y = \begin{pmatrix} \sum y_t \\ \sum y_t x_t \end{pmatrix}, \tag{A.1}$$
$$B = \begin{pmatrix} \widehat{\alpha} \\ \widehat{\beta} \end{pmatrix},$$

$\text{SSR} = \sum \widehat{u}_t^2$, $\text{TSS} = \sum (y_t - \overline{y})^2 = \sum y_t^2 - T^{-1}(\sum y_t)^2$, and $(X'X)^{-1}_{22}$ is the element in row 2, column 2, of the $(X'X)^{-1}$ matrix.

To obtain the asymptotics of $\widehat{\alpha}$, $\widehat{\beta}$, $\widehat{\sigma}^2$, t_β, and R^2 we need to ascertain the behaviour of the following expressions when $T \to \infty$: $\sum x_t$, $\sum y_t$, $\sum x_t^2$, $\sum y_t^2$, and $\sum x_t y_t$. The behavior of these expressions varies depending on the DGP of the variables x_t and y_t. We present such behavior for the DGPs underlying Theorem 2.1 and Propositions 2.2 and 2.3. All of the orders in probability stated in the underbraced sums can be found in [5, 13, 16–18]. It is important to clarify that the computation of the asymptotics follows [5] and was assisted by *Mathematica*; we thus rewrote below the expressions written as Mathematica code.

A.1. Theorem 2.1: First Result

The expressions needed to compute the asymptotic values of $\widehat{\alpha}$, $\widehat{\beta}$, $\widehat{\sigma}^2$, and R^2 are

$$\begin{aligned}
\sum x_t &= X_0 T + \mu_x \sum t + \underbrace{\sum \xi_{x,t-1}}_{O_p(T^{3/2})},\\
\sum x_t^2 &= X_0^2 T + \mu_x^2 \sum t^2 + \underbrace{\sum \xi_{x,t-1}^2}_{O_p(T^2)} + 2X_0\mu_x \sum t + 2X_0 \sum \xi_{x,t-1} + 2\mu_x \underbrace{\sum \xi_{x,t-1} t}_{O_p(T^{5/2})},\\
\sum x_t u_{x,t} &= X_0 \underbrace{\sum u_{x,t}}_{O_p(T^{1/2})} + \mu_x \underbrace{\sum u_{x,t} t}_{O_p(T^{3/2})} + \underbrace{\sum \xi_{x,t-1} u_{x,t}}_{O_p(T)},\\
\sum x_t u_{y,t} &= X_0 \sum u_{y,t} + \mu_x \sum u_{y,t} t + \sum \xi_{x,t-1} u_{y,t},\\
\sum y_t &= \mu_y T + \beta_y \sum x_t + \sum u_{y,t} + \rho_x \sum u_{x,t},\\
\sum y_t^2 &= \mu_y^2 T + \beta_y^2 \sum x_t^2 + \rho_x^2 \sum u_{x,t}^2 + \sum u_{y,t}^2 + 2\mu_y\beta_y \sum x_t\\
&\quad + 2\mu_y\left(\rho_x \sum u_{x,t} + \sum u_{y,t}\right) + 2\beta_y\left(\rho_x \sum x_t u_{x,t} + \sum x_t u_{y,t}\right) + 2\rho_x \underbrace{\sum u_{x,t} u_{y,t}}_{O_p(T^{1/2})},\\
\sum x_t y_t &= \mu_y \sum x_t + \beta_y \sum x_t^2 + \sum x_t u_{y,t} + \rho_x \sum x_t u_{x,t},
\end{aligned} \tag{A.1}$$

Table 2: Glossary of the Mathematica code.

Term	Represents	Term	Represents	Term	Represents	Term	Represents
S_t	$\sum t$	S_{t2}	$\sum t^2$	$S_{\xi x,t}$	$\sum \xi_{x,t-1}t$	$S_{\xi y,t}$	$\sum \xi_{y,t-1}t$
S_x	$\sum x_t$	S_{x2}	$\sum x_t^2$	S_{ux}	$\sum u_{x,t}$	S_{uy}	$\sum u_{y,t}$
S_y	$\sum y_t$	S_{y2}	$\sum y_t^2$	S_{ux2}	$\sum u_{x,t}^2$	S_{uy2}	$\sum u_{y,t}^2$
S_{xux}	$\sum x_t u_{x,t}$	S_{xuy}	$\sum x_t u_{y,t}$	$S_{ux,t}$	$\sum u_{x,t}t$	$S_{uy,t}$	$\sum u_{y,t}t$
S_{xy}	$\sum x_t y_t$	Mx	$(X'X)^{-1}$	$S_{\xi x\xi y}$	$\sum \xi_{x,t-1}\xi_{y,t-1}$	S_{uxuy}	$\sum u_{x,t}u_{y,t}$
$S_{\xi x}$	$\sum \xi_{x,t-1}$	$S_{\xi y}$	$\sum \xi_{y,t-1}$	$S_{\xi xux}$	$\sum \xi_{x,t-1}u_{x,t}$	$S_{\xi yux}$	$\sum \xi_{y,t-1}u_{x,t}$
$S_{\xi x2}$	$\sum \xi_{x,t-1}^2$	$S_{\xi y2}$	$\sum \xi_{y,t-1}^2$	$S_{\xi xuy}$	$\sum \xi_{x,t-1}u_{y,t}$	$S_{\xi yuy}$	$\sum \xi_{y,t-1}u_{y,t}$

where $\xi_{y,t} = \sum_{i=1}^{t} u_{y,i}$ and Y_0 is an initial condition. The sums including solely the deterministic trend component are

$$\sum t = \frac{1}{2}\left(T^2 + T\right), \qquad \sum t^2 = \frac{1}{6}\left(2T^3 + 3T^2 + T\right). \tag{A.2}$$

The code in this case is represented below. To understand it, a brief glossary is required and appears in Table 2.

These expressions were written as *Mathematica 7.0* code.

$$\begin{aligned}
&\text{Clear All, } S_t = \frac{1}{2}\left(T^2+T\right), \qquad S_{t2} = \frac{1}{6}\left(2T^3+3T^2+T\right),\\
&S_x = X_0 T + \mu_x S_t + S_{\xi x} T^{3/2},\\
&S_{x2} = X_0^2 T + \mu_x^2 S_{t2} + S_{\xi x2} T^2 + 2X_0\mu_x S_t + 2X_0 S_{\xi x} T^{3/2} + 2\mu_x S_{\xi xt} T^{5/2},\\
&S_y = \mu_y T + \beta_y S_x + \rho_x S_{ux} T^{1/2} + S_{uy} T^{1/2},\\
&S_{xux} = X_0 S_{ux} T^{1/2} + \mu_x S_{uxt} T^{3/2} + S_{\xi xux} T,\\
&S_{xuy} = X_0 S_{uy} T^{1/2} + \mu_x S_{uyt} T^{3/2} + S_{\xi xuy} T,\\
&S_{y2} = \mu_y^2 T + \beta_y^2 S_{x2} + \rho_x^2 S_{ux2} T + S_{uy2} T + 2\mu_y\beta_y S_x\\
&\qquad + 2\mu_y\left(\rho_x S_{\text{ux}} T^{1/2} + S_{uy} T^{1/2}\right)\\
&\qquad + 2\beta_y\left(\rho_x S_{xux} + S_{xuy}\right) + 2\rho_x S_{uxuy} T^{1/2},\\
&S_{xy} = \mu_y S_x + \beta_y S_{x2} + \left(\rho_x S_{xux} + S_{xuy}\right).
\end{aligned} \tag{A.3}$$

A.2. Theorem 2.1: Second Result

The expressions needed to compute the asymptotic values of $\widehat{\alpha}$, $\widehat{\beta}$, $\widehat{\sigma}^2$, and R^2 appear below. Note that $\sum y_t$, $\sum y_t^2$, and $\sum y_t x_t$ are identical to the ones presented in the previous appendix and have been therefore omitted

$$\begin{aligned}
\sum x_t &= X_0T + (\mu_x + \rho_{y1}) \sum t + \sum \xi_{x,t-1} + \rho_{y2} \sum \xi_{y,t-1}, \\
\sum x_t^2 &= X_0^2T + (\mu_x + \rho_{y1})^2 \sum t^2 + \sum \xi_{x,t-1}^2 + \rho_{y2}^2 \sum \xi_{y,t-1}^2 \\
&\quad + 2X_0(\mu_x + \rho_{y1}) \sum t + 2X_0 \sum \xi_{x,t-1} + 2\rho_{y2}X_0 \sum \xi_{y,t-1} \\
&\quad + 2(\mu_x + \rho_{y1}) \sum \xi_{x,t-1}t + 2\rho_{y2}(\mu_x + \rho_{y1}) \sum \xi_{y,t-1}t + 2\rho_{y2}\underbrace{\sum \xi_{x,t-1}\xi_{y,t-1}}_{O_p(T^2)}, \qquad \text{(A.1)} \\
\sum x_tu_{x,t} &= X_0 \sum u_{x,t} + (\mu_x + \rho_{y1}) \sum u_{x,t}t + \sum \xi_{x,t-1}u_{x,t} + \rho_{y2} \sum \xi_{y,t-1}u_{x,t}, \\
\sum x_tu_{y,t} &= X_0 \sum u_{y,t} + (\mu_x + \rho_{y1}) \sum u_{y,t}t + \sum \xi_{x,t-1}u_{y,t} + \rho_{y2} \sum \xi_{y,t-1}u_{y,t}.
\end{aligned}$$

The code in this case is represented below.

$$\begin{aligned}
&\text{Clear All}, S_t = \frac{1}{2}\left(T^2 + T\right), \qquad S_{t2} = \frac{1}{6}\left(2T^3 + 3T^2 + T\right), \\
&S_x = X_0T + (\mu_x + \rho_{y1})S_t + S_{\xi x}T^{3/2} + \rho_{y2}S_{\xi y}T^{3/2}, \\
&S_{x2} = X_0^2T + (\mu_x + \rho_{y1})^2S_{t2} + S_{\xi x}2T^2 + \rho_{y2}^2S_{\xi y}2T^2 \\
&\qquad + 2X_0(\mu_x + \rho_{y1})S_t + 2X_0S_{\xi x}T^{3/2} + 2X_0\rho_{y2}S_{\xi y}T^{3/2} \\
&\qquad + 2(\mu_x + \rho_{y1})S_{\xi xt}T^{5/2} + 2(\mu_x + \rho_{y1})\rho_{y2}S_{\xi yt}T^{5/2} + 2\rho_{y2}S_{\xi y\xi x}T^2, \\
&S_y = \mu_yT + \beta_yS_x + \rho_xS_{ux}T^{1/2} + S_{uy}T^{1/2}, \qquad \text{(A.2)} \\
&S_{xux} = X_0S_{ux}T^{1/2} + (\mu_x + \rho_{y1})S_{uxt}T^{3/2} + S_{\xi xux}T + \rho_{y2}S_{\xi yux}T, \\
&S_{xuy} = X_0S_{uy}T^{1/2} + (\mu_x + \rho_{y1})S_{uyt}T^{3/2} + S_{\xi xuy}T + \rho_{y2}S_{\xi yuy}T, \\
&S_{y2} = \mu_y^2T + \beta_y^2S_{x2} + \rho_x^2S_{ux2}T + S_{uy2}T + 2\mu_y\beta_yS_x \\
&\qquad + 2\mu_y\left(\rho_xS_{ux}T^{1/2} + S_{uy}T^{1/2}\right) + 2\beta_y(\rho_xS_{xux} + S_{xuy}) + 2\rho_xS_{uxuy}T^{1/2}, \\
&S_{xy} = \mu_yS_x + \beta_yS_{x2} + (\rho_xS_{xux} + S_{xuy}).
\end{aligned}$$

A.3. Proposition 2.2

First note that DGP (2.5) can be written as

$$\begin{aligned}
y_t &= \mu_y + \beta_yx_t + u_{y,t}, \\
x_t &= C_1 + C_2t + \frac{\beta_x}{C_0}u_{y,t} + \frac{u_{x,t}}{C_0},
\end{aligned} \qquad \text{(A.1)}$$

where, $C_0 = 1 - \beta_x\beta_y$, $C_1 = (\mu_x + \mu_y\beta_x)/C_0$, and $C_2 = \gamma_x/C_0$. The expressions needed to compute the asymptotic values of $\widehat{\alpha}$, $\widehat{\beta}$, $\widehat{\sigma}^2$, and R^2 are

$$\begin{aligned}
\sum x_t &= C_1T + C_2\sum t + \frac{\beta_x}{C_0}\sum u_{y,t} + \frac{1}{C_0}\sum u_{x,t},\\
\sum x_t^2 &= C_1^2T + C_2^2\sum t^2 + \left(\frac{\beta_x}{C_0}\right)^2\sum u_{y,t}^2 + \left(\frac{1}{C_0}\right)^2\sum u_{x,t}^2\\
&\quad + 2C_1C_2\sum t + 2\frac{C_1\beta_x}{C_0}\sum u_{y,t} + 2\frac{C_1}{C_0}\sum u_{x,t}\\
&\quad + 2\frac{C_2\beta_x}{C_0}\sum u_{y,t}t + 2\frac{C_2}{C_0}\sum u_{x,t}t + 2\frac{\beta_x}{C_0^2}\sum u_{x,t}u_{y,t},\\
\sum x_tu_{y,t} &= C_1\sum u_{y,t} + C_2\sum u_{y,t}t + \frac{\beta_x}{C_0}\sum u_{y,t}^2 + \frac{1}{C_0}\sum u_{x,t}u_{y,t},\\
\sum y_t &= \mu_yT + \beta_y\sum x_t + \sum u_{y,t},\\
\sum y_t^2 &= \mu_y^2T + \beta_y^2\sum x_t^2\sum u_{y,t}^2 + 2\mu_y\beta_y\sum x_t + 2\mu_y\sum u_{y,t} + 2\beta_y\sum x_tu_{y,t},\\
\sum x_ty_t &= \mu_y\sum x_t + \beta_y\sum x_t^2 + \sum x_tu_{y,t}.
\end{aligned} \tag{A.2}$$

The code in this case is represented below.

$$\begin{aligned}
&\text{Clear All}, S_t = \frac{1}{2}\left(T^2 + T\right), \qquad S_{t2} = \frac{1}{6}\left(2T^3 + 3T^2 + T\right),\\
&C_0 = 1 - \beta_x\beta_y, \qquad C_1 = \frac{\mu_x + \mu_y\beta_x}{1 - \beta_x\beta_y}, \qquad C_2 = \frac{\gamma_x}{1 - \beta_x\beta_y},\\
&S_x = C_1T + C_2S_t + \frac{\beta_x}{C_0}S_{uy}T^{1/2} + \frac{1}{C_0}S_{ux}T^{1/2},\\
&S_{x2} = {C_1}^2T + {C_2}^2S_{t2} + \left(\frac{\beta_x}{C_0}\right)^2S_{uy2}T + \left(\frac{1}{C_0}\right)^2S_{ux2}T + 2C_1C_2S_t + 2\frac{C_1\beta_x}{C_0}S_{uy}T^{1/2}\\
&\qquad + 2\frac{C_1}{C_0}S_{ux}T^{1/2} + 2\frac{C_2\beta_x}{C_0}S_{uyt}T^{3/2} + 2\frac{C_2}{C_0}S_{uxt}T^{3/2} + 2\frac{\beta_x}{{C_0}^2}S_{uxuy}T^{1/2},\\
&S_y = \mu_yT + \beta_yS_x + S_{uy}T^{1/2},\\
&S_{xuy} = C_1S_{uy}T^{1/2} + C_2S_{uyt}T^{3/2} + \frac{\beta_x}{C_0}S_{uy2}T + \frac{1}{C_0}S_{uxuy}T^{1/2},\\
&S_{y2} = {\mu_y}^2T + {\beta_y}^2S_{x2} + S_{uy2}T + 2\mu_y\beta_yS_x + 2\mu_yS_{uy}T^{1/2} + 2\beta_yS_{xuy},\\
&S_{xy} = \mu_yS_x + \beta_yS_{x2} + S_{xuy}.
\end{aligned} \tag{A.3}$$

A.4. Proposition 2.3

As in the previous appendix, first rewrite DGP (2.5) as

$$y_t = \mu_y + \beta_y x_t + u_{y,t},$$
$$x_t = D_1 + D_2 t + \frac{\beta_x}{D_0} u_{y,t} + \frac{\xi_{x,t-1}}{D_0}, \tag{A.1}$$

where, $D_0 = 1 - \beta_x \beta_y$, $D_1 = (X_0 + \mu_y \beta_x)/D_0$, and $D_2 = \mu_x / D_0$. The expressions needed to compute the asymptotic values of $\widehat{\alpha}$, $\widehat{\beta}$, $\widehat{\sigma}^2$, and R^2 appear below. Note that $\sum y_t$, $\sum y_t^2$, and $\sum y_t x_t$ are identical to the ones presented in the previous appendix and have been therefore omitted

$$\sum x_t = D_1 T + D_2 \sum t + \frac{\beta_x}{D_0} \sum u_{y,t} + \frac{1}{D_0} \sum \xi_{x,t-1},$$
$$\sum x_t^2 = D_1^2 T + D_2^2 \sum t^2 + \left(\frac{\beta_x}{D_0}\right)^2 \sum u_{y,t}^2 + \left(\frac{1}{D_0}\right)^2 \sum \xi_{x,t-1}^2$$
$$+ 2D_1 D_2 \sum t + 2\frac{D_1 \beta_x}{D_0} \sum u_{y,t} + 2\frac{D_1}{D_0} \sum \xi_{x,t-1} \tag{A.2}$$
$$+ 2\frac{D_2 \beta_x}{D_0} \sum u_{y,t-1} t + 2\frac{D_2}{D_0} \sum \xi_{x,t-1} t + 2\frac{\beta_x}{D_0^2} \sum \xi_{x,t-1} u_{y,t},$$
$$\sum x_t u_{y,t} = D_1 \sum u_{y,t} + D_2 \sum u_{y,t} t + \frac{\beta_x}{D_0} \sum u_{y,t}^2 + \frac{1}{D_0} \sum \xi_{x,t-1} u_{y,t}.$$

The code in this case is represented below:

$$\text{Clear All}, S_t = \frac{1}{2}\left(T^2 + T\right), \qquad S_{t2} = \frac{1}{6}\left(2T^3 + 3T^2 + T\right),$$

$$D_0 = 1 - \beta_x \beta_y, \qquad D_1 = \frac{X_0 + \mu_y \beta_x}{1 - \beta_x \beta_y}, \qquad D_2 = \frac{\mu_x}{1 - \beta_x \beta_y},$$

$$S_x = D_1 T + D_2 S_t + \frac{\beta_x}{D_0} S_{uy} T^{1/2} + \frac{1}{D_0} S_{\xi x} T^{3/2},$$

$$S_{x2} = D_1{}^2 T + D_2{}^2 S_{t2} + \left(\frac{\beta_x}{D_0}\right)^2 S_{uy2} T + \left(\frac{1}{D_0}\right)^2 S_\xi x2\ T^2 + 2D_1 D_2 S_t$$
$$+ 2\frac{D_1 \beta_x}{D_0} S_{uy} T^{1/2} + 2\frac{D_1}{D_0} S_{\xi x} T^{3/2} + 2\frac{D_2 \beta_x}{D_0} S_{uyt} T^{3/2} + 2\frac{D_2}{D_0} S_{\xi xt} T^{5/2} + 2\frac{\beta_x}{D_0{}^2} S_{\xi xuy} T;$$

$$S_y = \mu_y T + \beta_y S_x + S_{uy} T^{1/2},$$

$$S_{xuy} = D_1 S_{uy} T^{1/2} + D_2 S_{uyt} T^{3/2} + \frac{\beta_x}{D_0} S_{uy2} T + \frac{1}{D_0} S_{\xi xuy} T,$$

$$S_{y2} = {\mu_y}^2 T + {\beta_y}^2 S_{x2} + S_{uy2} T + 2\mu_y \beta_y S_x + 2\mu_y S_{uy} T^{1/2} + 2\beta_y S_{xuy},$$

$$S_{xy} = \mu_y S_x + \beta_y S_{x2} + S_{xuy}. \tag{A.3}$$

A.5. Computation of the Asymptotics

The previous three appendices provide the Mathematica code of $\sum x_t$, $\sum y_t$, $\sum x_t^2$, $\sum y_t^2$, and $\sum x_t y_t$ for different DGP combinations. We now present the code that computes the asymptotics of (1.1) in any such combination. Note that the code computes the asymptotics in the following order: the matrix $(X'X)^{-1}$, $(X'X)^{-1}_{22}$, $\hat{\alpha}$, $\hat{\beta}$, $\hat{\sigma}^2$, t_β, and $1 - R^2$. Comments appear inside parentheses (*—*).

(*Matrix $(X'X)$*)

Mx = $\begin{pmatrix} T & S_x \\ S_x & S_{x2} \end{pmatrix}$;

(*Inverse of Matrix $(X'X)$*)iMx = Inverse[Mx];

(*Element 1, 1 of $(X'X)^{-1}$*) R1 = Extract[iMx, {1, 1}];
(*Element 1, 2 of $(X'X)^{-1}$*) R2 = Extract[iMx, {1, 2}];
(*Element 2, 1 of $(X'X)^{-1}$*) R3 = Extract[iMx, {2, 1}];
(*Element 2, 2 of $(X'X)^{-1}$*) R4 = Extract[iMx, {2, 2}];

(*Factorization*) R40 = Factor [R4];
(*Numerator*) R4num = Numerator [R40];
(*Denominator*) R4den = Denominator [R40];

(*Highest power of T in numerator*)

K1 = Exponent [R4num, T],

(*Highest power of T in denominator*)

K2 = Exponent [R4den, T];
(*Limit of the numerator divided by T^{K1}*)

R4num2 = Limit [Expand [R4num/T^{K1}], T $\to \infty$];

(*Limit of the denominator divided by T^{K2}*)

R4den2 = Limit [Expand [R4den/T^{K2}], $T \to \infty$];
(*Limit of the Element 2, 2 of $(X'X)^{-1}$ multiplied by T^{k1}/T^{K2}*)
R42 = Factor [Expand [(R4num2/R4den2) $*\, T^{K1}/T^{K2}$]];

(*Parameter $\hat{\alpha}$*)

P10 = Factor [Expand [R1S_y + R2S_{xy}]];
P11num = Numerator [P10];
K5 = Exponent [P11num, T];
Anum = Limit [Expand [P11num/T^{K5}], $T \to \infty$];
P12den = Denominator [P10];
K6 = Exponent [P12den, T];
Aden = Limit [Expand [P12den/T^{K6}], $T \to \infty$];
Apar = Factor [Expand [(Anum/Aden) $*\, T^{K5}/T^{K6}$]]

(*Parameter $\hat{\beta}$*)

P20 = Factor [Expand [R3S_y + R4S_{xy}]];
P21num = Numerator [P20];
K7 = Exponent [P21num, T];
Bnum = Limit [Expand [P21num/T^{K7}], $T \to \infty$];
P22den = Denominator [P20];
K8 = Exponent [P22den, T];
Bden = Limit [Expand [P22den/T^{K8}], $T \to \infty$];
Bpar = Factor [Expand [(Bnum/Bden) $*\, T^{K7}/T^{K8}$]]

(*Parameter $\hat{\sigma}^2$*)

P40 = Factor [Expand [S_{y2} + P10^2T + P20$^2S_{x2}$ − 2P10S_y − 2P20S_{xy} + 2P10P20S_x]];
P41num = Numerator [P40];
K11 = Exponent [P41num, T];
Vnum = Factor [Limit [Expand [P41num/T^{K11}], $T \to \infty$]];
P42den = Denominator [P40];
K12 = Exponent [P42den, T];
Vden = Factor [Limit [Expand [P42den/T^{K12}], $T \to \infty$]];
Vpar = Factor [Expand [T^{-1} * (Vnum/Vden) $*\, T^{K11}/T^{K12}$]]

(*SSR/TSS*)

P50 = Factor [Expand [P40/($S_{y2} - T(S_y/T)^2$)]];
P51num = Numerator [P50];
K13 = Exponent [P51num, T];
Rcnum = Factor [Limit [Expand [P51num/T^{K13}], $T \to \infty$]];
P52den = Denominator [P50];
K14 = Exponent [P52den, T];

Rcden = Factor [Limit [Expand [P52den/T^{K14}], $T \to \infty$]];
Rc = Factor [Expand [(Rcnum/Rcden) $* T^{K13}/T^{K14}$]];

(*t_β*)

t_β = Full Simplify [Bpar/(Vpar * R42)$^{-1/2}$]
(*$1 - R^2$*)

P70 = Full Simplify [Rc].

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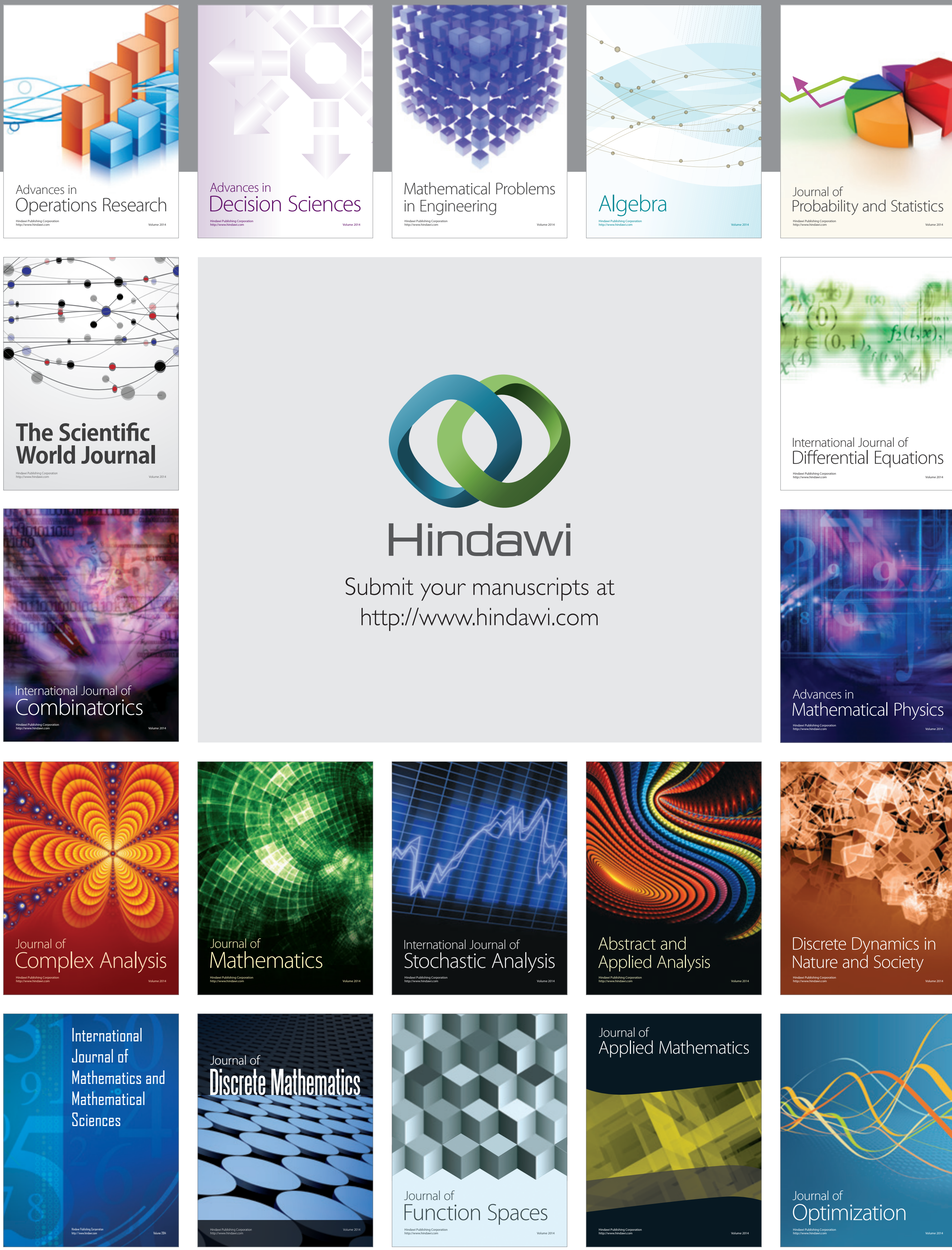
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